Exploring the role of lesson study on educators' pedagogical content knowledge in teaching mathematics.

Submitted in fulfilment of the requirements of the degree of Doctor of Education (Mathematics Education)

in the School of Education in the Faculty of Arts at the Durban University of Technology

By

Sikhumbuzo Sithembiso Dhlamini

Student No.: 20428545

April 2022

Supervisor: Dr J.P. Abraham ate: 30 March 2022

Co-Supervisor: Dr V.Z. Masuku Date: 30 March 2022

ABSTRACT

Findings from local, sub-Saharan and international assessment conducted with South African learners at both primary and secondary levels indicate that South African learners perform poorly in mathematics. Although there are numerous explanations for learners′ poor performance, one of the most important explanations is that educators lack pedagogical content knowledge of mathematics.

This study examines the role of lesson study in educators′ pedagogical content knowledge in teaching mathematics. Lesson study is a professional development method for educators that has been used in Japan for decades. In the current study, lesson study seeks to answer 3 research questions: i) What do educators need to enhance their pedagogical content knowledge of teaching numeric and geometric patterns? ii) How does educators′ participation in lesson study enhance their pedagogical content knowledge about teaching of numeric and geometric patterns? iii) What are the educators′ views about the lesson study when teaching numeric and geometric patterns?

The investigation of the role of lesson study was conducted through a thematic analysis and a qualitative method. The study focused on 2 of 5 accessible Professional Learning Communities in the Ilembe District of KwaZulu-Natal Province, South Africa, to conduct this research. The reason for selecting these 2 Professional Learning Communities had a total of 7 schools, with Professional Learning Community 1 having 3 schools and 10 educators and Professional Learning Community 2 with 4 schools and 13 educators while other Professional Learning Communities have less number of educators.

Participants in the 2 Professional Learning Communities participated in the lesson study process for 9 months. In each lesson study cycle, participants were required to plan, discuss, teach, observe and reflect on lessons together. Due to various constraints, both Professional Learning Communities completed 2 lesson study cycles at the end of this study. Multiple sources of data were collected through group discussions, observations, reflections, semi-structured interviews, and participants′ diary entries.

The findings of this study highlight the role of lesson study in improving educators′ pedagogical content knowledge in teaching mathematics. Analysis of the qualitative data revealed that lesson study as a professional development activity provided educators with an encouraging opportunity. Moreover, lesson study became a vehicle for participants to collaborate, discuss and share their teaching knowledge and experiences. Consequently, participants expressed the view that their pedagogical content knowledge was enhanced by participating in the lesson study. However, the extent of enhancement depends on each participant′s attitude and commitment to the teaching profession.

The findings underscore the need for lesson study as an educator-led and bottom-up approach to innovative professional development. It became clear that educators′ pedagogical content knowledge is critical to good mathematics teaching and learner understanding. The findings are consistent with the literature on lesson study that show educators need lesson study to enhance their pedagogical content knowledge for teaching numeric and geometric patterns: their mathematical pedagogical knowledge, mathematical content knowledge, and contextual knowledge.

Subsequently lesson study offers an alternative, innovative and effective approach to educator professional development that ultimately impacts instruction, therefore, lesson study needs to be promoted in the South African educational context. Although lesson study is a model of educator-led professional development, the involvement of policy makers to promote lesson study in the early stages is still important and relevant. This research recommends collaboration with curriculum specialists to gain insight and expert ideas. Educators need more support from School Management Teams, who are a great help in managing and planning lesson study. All in all, there is a need to foster a culture of lesson study that helps generate enthusiasm and commitment to learning in the lesson study among educators.

STUDENT DECLARATION

The study described in this thesis was carried out with 23 primary school mathematics educators, in Ilembe District of KwaZulu-Natal Province, South Africa. The study commenced on January 2019 to February 2022 under the supervision of Dr J. Abraham of the Durban University of Technology, Pietermaritzburg in Indumiso Campus (School of Education) and co-Supervisor Dr V.Z. Masuku of University of KwaZulu-Nata, Durban in Howard College Campus (Teaching and Learning Division).

This study represents the original work by the author and has not been submitted in any form for any diploma or degree to any other tertiary institution. Where the author has made use of work of other authors, it is duly acknowledged in the text.

…… Date: 30 March 2022

Sikhumbuzo Sithembiso Dhlamini

ACKNOWLEDGEMENTS

First and foremost, I would like to give thanks to God Almighty for keeping me alive and giving me strength to pursue my studies from undergraduate qualification (Bachelor of Education), postgraduate qualifications (Bachelor of Education Honours, Master of Education, Post Graduate Diploma in Project Management), until this one (Doctor of Education).

I wish deeply and sincerely to thank my supervisor, Dr Jose Abraham and co-supervisor, Dr Veronica Zanele Masuku, for their guidance, continuous encouragement throughout this journey of D. Ed., offering thoughtful critical suggestions, and most importantly the role they played and being continuously available as supervisors. Thank you sincerely to the entire School of Education and the Durban University of Technology community.

I am incredibly grateful to all the educators who participated in this research, gave so freely of their time, were engaged with the process, and allowed me to be part of their school lives during the academic school year of 2021. I would also like to extend my genuine appreciation to the principals of the seven schools and their School Management Teams for securing meeting venues and allowing me to observe educators teaching.

Thank you sincerely to everyone within the community of KwaZulu-Natal Department of Education, Zululand District (TLS-GET) – friends and colleagues whose academic and social support has allowed me to pursue and enjoy conducting this research. Thank you to Dr David Sekao for your continuous support and guidance throughout this journey. Thank you to Innocent Ncwane for always being my referral person. Thank you sincerely to all my colleagues and friends who were studying their PhD – particularly Mrs. Nozipho Khumalo, Ms. Maureen Thobela, Mr. Vusumuzi Ndlovu, Mr. Hendry Madide and Mr. Siboniso Zondi.

My genuine gratitude also goes to my entire family (Dhlamini and Ngema), spiritual family (Evangelical Lutheran Church especially Enqabeni Parish under the leadership

of Revd Z.K. Sibisi, Mrs. N.C. Sithole and Mrs. A. Sindane), my friends, who have always been beside me, encouraging me to continue with my studies.

Finally, special thanks go to my wife (Thandazile “Lesson Study”), my daughters: Asimbonge Gcinizwi and Ntokomalo Uyabongwa Singethwe, and Ms. Zanele Nkala (our helper), parents (MaNgema and Muzi-omuhle “Mubizana”) and siblings (Mndeni “Nkwe”) and Nokubongwa “Tano”) for their continuous support.

LIST OF ABBREVIATIONS USED

ANA	Annual National Assessment
ANC	African National Congress
ATPs	Annual Teaching Plans
CA	Content Area
CAs	Content Areas
CAPS	Curriculum and Assessment Policy Statement
CHAT	Cultural Historical Activity Theory
CK	Content Knowledge
DBE	Department of Basic Education
DHs	Departmental Heads
DHET	Department of Higher Education and Training
DoE	Department of Education
DRC	Departmental Research Committee
DTDCs	District Educators Development Centres
DUT	Durban University of Technology
EFAL	English First Additional Language
ELRC	Education Labour Relations Council
FRC	Faculty Research Committee
GET	General Education and Training
HL	Home Language
IREC	Institution Research Ethics Committee
JICA	Japan International Cooperation Agency
KZN	KwaZulu-Natal
LAC	Language Across the Curriculum
LoLT	Language of Learning and Teaching
LTSM	Learner and Teacher Support Material
MECs	Members of Executive Councils
MPCK	Mathematical Pedagogical Content Knowledge
NSC	National Senior Certificate
PED	Provincial Department of Education
PEDs	Provincial Department of Educations
PCK	Pedagogical Content Knowledge

PISA	Programme of International Student Assessment
PK	Pedagogical Knowledge
PLC	Professional Learning Community
PLCs	Professional Learning Communities
PoAs	Programme of Assessments
RATP	Revised Annual Teaching Plan
SACE	South African Council of Educators
SMT	School Management Team
SMTs	School Management Teams
SMK	Subject Matter Knowledge
TIMSS	Trend in International Mathematics and Science Study
WEF	World Economic Forum

LIST OF FIGURES

LIST OF TABLES

LIST OF APPENDICES

TABLE OF CONTENTS

CHAPTER 3 METHODOLOGY

CHAPTER 5 DISCUSSIONS, RECOMMENDATIONS AND CONCLUSION

CHAPTER 1 INTRODUCTION AND OVERVIEW

1.1. Introduction

This study looked at how participating in a collaborative professional development activity known as lesson study enhanced mathematics educators to improve their pedagogical content knowledge (PCK). Educators are central in educational reform and professional development, enhancing understanding of taught content. Education, according to Schofer, Ramirez, and Meyer (2021), is crucial to the total development of developing countries and the global economy. As a result, variety opportunities of professional development exist to benefit educators in improving PCK and skills (Chai, 2019; Lai & Lin, 2018).

One such opportunity is lesson study, which was preferred in this study to enhance the knowledge and skills of primary school mathematics educators. Essential disciplines like art, language, mathematics, and science must be taught in schools. Mathematics is the focus of this study since it is a fundamental subject in many sectors, including business, commerce, engineering, science, and technology (Maass et al., 2019; Pratama & Retnawati, 2018). Furthermore, mathematics is utilised as an assessment instrument to assess a country's educational progress. The Trend in International Mathematics and Science Study (TIMSS) and the Programme for International Student Assessment (PISA) are two instances of worldwide benchmarking that monitor and compare progress in education of mathematics in different nations on a regular basis (Pratama & Retnawati, 2018).

The researcher explores the role of lesson study as professional development for educators' PCK in teaching mathematics in primary schools through a case study. Two professional learning communities (PLCs) were chosen from a group of primary mathematics educators in the Ilembe District of KwaZulu-Natal, South Africa, at the start of the study. Their district mathematics subject advisors provided professional development seminars on lesson study to these PLCs.

What is pedagogical content knowledge?

Lee Shulman used the idiom pedagogical content knowledge in 1986 to illustrate the portion of pedagogical knowledge that an educator utilises to translate subject knowledge into pedagogical products and teaching strategies (Tuithof et al., 2021). PCK is about teaching in general, not about a certain subject. It is a vital component of educator education and a fundamental component of effective teaching (Baki & Arslan, 2022). PCK refers to the way educators connect pedagogical knowledge (what educators comprehend about teaching) with subject knowledge (what educators comprehend about what to teach) (Shulman, 1987).

What is lesson study?

The term ′lesson study′ is a literal decoded of the Japanese word ***jugyokenkyu***, where ***jugyo*** refers to a lesson and ***kenkyu*** to study or research (Ni Shúilleabháin, 2015). Lesson study dates back to the late 1890s, when educators at primary schools linked with regular schools began to observe and critically examine lessons (Makinae, 2019). Lesson study is a mode of enhancing mathematics teaching and learning by involving a group of educators in a specific type of activity. It offers educators valuable learning opportunities by allowing them to collaborate with their peers to study topic content, student thinking and learning, and modify classroom practices (Hervas & Median, 2022).

Lesson study, a Japanese method to school education, has piqued the curiosity of the worldwide education community. *The Teaching Gap* (Stigler & Hiebert, 1999) and the video from TIMSS research, which highlighted the usual format of mathematics teaching in Japanese classroom, brought lesson study to the concentration of educators abroad of Japan (Fujii, 2018; 2019). As more countries embrace this strategy for their needs, the focus of lesson study to improving mathematics teaching and learning has shifted (Sekao, 2019). Lesson study is a professional development programme in which educators collaborate in small groups to prepare lessons for learners with a similar learning goal (Vermunt et al., 2019).

Why mathematics?

The term mathematics was coined by the Pythagoreans, with whom the study of mathematics for its own sake began. The phrase mathematics is translated from the Greek word ***máthēma***, which means “that which is learnt,” “what one gets to know,” thus also “study” and “science.” Mathematics is technically a noun in the plural – geometry, algebra, calculus: all of these are mathematics, but are treated as singular (Hudson, 2020).

Mathematics is defined by the Department of Basic Education (DBE) as a language that uses notations and symbols to represent geometric, numerical and pictorial relationships (DBE, 2011). A language is a structured communication system used by humans, so mathematics as a language fits ‘best’ for this study. Language can be based not only on speech and gestures (spoken language), but also includes signs, or writing (symbols and notations). Moreover, the language of mathematics is an everyday language that is not even the same as the language used in school, let alone the language used by mathematics experts (Hudson, 2020).

Mathematics can be used to refer to a specific set of national, social, and domain-specific areas in which any individual or group of people has experience and expertise. One of the challenges of today′s globalised societies is that language diversity itself is becoming more diverse, which plays a role in mathematics education (Barwell, 2016). The teaching and learning of mathematics fundamentally depends on language. In addition, mathematics is one of the “gateway” subjects in South Africa, considered critical to the country′s economic growth, development, science and technology, so, mathematics is relevant to this case study.

Definition of professional learning community.

Shirley Hord coined the term professional learning community (PLC) in 1997 (Hord, 2009), but it was not used in South African education until 2011 (Ndlovu, 2021). A PLC is a cluster of educators who have decided to meet on a regular basis to share and learn from one another about needs they have recognized (Admiraal et al., 2021). They go on to say that a PLC is a professional body that comes together voluntarily. Learning that focuses on knowledge from practice and knowledge from study is referred to as professional learning. Unlike clusters, which must be within a circuit, a

PLC can be within a school, either local, or international. PLC addresses quality instruction and voluntary, ongoing professional learning.

Before ending, the remainder of this chapter addresses the study's history and context, the research problem, the research questions, the study's structure and relevance, the thesis outline, and lastly a summary of all the thesis chapters.

1.2. Background and Motivation of the Study

Mathematics is considered as one of the utmost important subjects in the school curriculum internationally. In South Africa, mathematics is a mandatory subject in the General and Education Training (GET) band (grades 1 to 9). Learning mathematics in the lower grades has a useful purpose in the life of a country and can equip learners with logical, analytical, systematic and creative thinking skills (Kenedi et al., 2019). It has been established as the subject that is directly related to other subjects, especially science and technology (Nelson, Voithofer & Cheng, 2019).

South Africa is ranked as one of the worst performing countries in mathematics in the World Economic Forum's (WEF) Global Competitiveness Report issued in 2019 (Barrichello et al., 2020; Jolejole-Caube, Dumlao & Abocejo, 2019). National Senior Certificate (NSC) scores in mathematics improved by 3.8% in 2021 compared to 2020 results (DBE, 2022). Although mathematics performance in NSC results has improved slightly, it has never been above 60% and has always ranked last among gateway subjects (11^{th} position) for 5 consecutive years. In addition, mathematics results for NSC or grade 12 have fluctuated for 5 years (2017 – 2021). As one of the measures to improve performance, the DBE Minister, Mrs. Angie Motshekga, sent a team of curriculum officials to Japan in 2016 and 2017 to observe lesson study as part of professional development.

Learner performance is more significantly affected by the merit of the educator in terms of PCK than by the learners′ preceding academic success or the previous grade they attended, according to a growing body of data (Khoza & Biyela, 2020; Lee, Capraro & Capraro, 2018; Odumosu & Fisayi, 2018;). In terms of PCK, research reveals that learners make the maximum progress when they are assigned to successful educators (Akturk & Ozturk, 2019). Indeed, these findings prompted the creation of this research

to look at the function of lesson study in enhancing educators' PCK in mathematics teaching. At all stages of education, the primary goal of teaching is to effect fundamental change in learners. A mathematics educator is expected to have the knowledge and a high level of PCK in teaching mathematical concepts to accomplish this.

The cornerstone for educators' teaching practice in their classrooms is knowledge, which comprises subject matter knowledge (SMK) and PCK. PCK varies from SMK in that it is used to characterise and identify educators' understanding of their learners′ challenges with subject matter and their ability to link mathematical ideas (Lee, Capraro & Capraro, 2018). Research (Lertdechapat & Faikhamta, 2021; Wood, 2021; Rochintaniawati et al., 2019; Murtafiah & Lukitasari, 2019) shows that lesson study can develop educators′ PCK and help educators improve their PCK. In addition, lesson study develops mathematics educators′ skills and knowledge; and affects the progress of learners′ understanding of the subject taught.

Although lesson study was introduced in South Africa more than two decade ago, it has not yet gained popularity. However, there have been some studies on lesson study, especially among primary school educators (Sekao & Engelbrecht, 2021; Helmbold, Venketsamy & Van Heerden, 2021; Kgothego, 2020; Mhakure, 2019). According to my knowledge, I could not find any study conducted in South Africa on numeric and geometric patterns employing lesson study as professional development. As a result, the aspiration of this study was to explore the role of lesson study in primary school educators' PCK in teaching patterns.

Across the world, research has found that there are barriers to teaching mathematics at the school level, and to address them, some references have recommended conducting lesson study with the support of PCK to solve problems (Shimizu, 2022; Willems & Van den Bossche, 2019; Seleznyov, 2018). Purwaningsih et al. (2020) conducted a study in Indonesia proving that lesson study enhances educators′ PCK. Ni Shúilleabháin and Bjuland (2019) studied two different educational cultures in Europe. Their study found that lesson study as professional development improves educators′ PCK and learners′ performance in mathematics. For these reasons, this

study is conducted to enhance educators′ PCK when teaching mathematics in a primary school to improve learners′ achievement in mathematics.

As a paradigm of professional development, lesson study will assist educators in enhancing their teaching quality. Lesson study could also assist educators improve their professionalism without having to take time away from their regular responsibilities or incur additional costs. If educators make the attempt and commit to enhancing the merit of mathematics teaching in South Africa, it could be a competent strategy for boosting educators' PCK. Lesson study allows educators to share their understanding and knowledge while also providing constructive feedback to their colleagues, resulting in more learner-centred teaching and better outcome.

1.3. Focus of the Study

There are five content areas (CAs) in the South African mathematics curriculum for GET band. The following is how the CAs are organised: i) Numbers, Operations, and Relationships; ii) Patterns, Functions, and Algebra; iii) Space and Shape (Geometry); iv) Measurement; and v) Data Handling (DBE, 2011). Each content area (CA) aids in the development of various skills. This study focuses on content area 2: *Patterns*, *Functions* and *Algebra*, which provides nearly seamless opportunities for research.

The study's focus and extent were constrained by time, distance, and the COVID-19 epidemic. This research focused on educators' PCK using lesson study as a model of a professional development paradigm. The purpose of this study was to document the PCK of mathematics educators in grades 4 to 6 in the domain of mathematics teaching, with a focus on numeric and geometric patterns. The understanding of numeric and geometric patterns is demanded of all mathematics educators from grades 1 to 12. Rather than at all primary educators, this study concentrated on the Intermediate Phase (grades 4 to 6). The fact that only one of twelve districts in KwaZulu-Natal (Ilembe District) was included in this study was also a limitation.

1.4. Brief Overview of the Literature

This study was informed by a several studies and builds on previous studies (Baki & Arslan, 2022; Sekao & Engelbrecht, 2021; Redmond & Lock, 2019; Ogegbo, Gaigher & Salagaram, 2019; Yurnetti, 2018; Stols & Ono, 2016; Ni Shúilleabháin, 2015; Van Putten, Stols & Howie, 2014; Zvobwo, 2013; Yoshida, 2012). While most educational research and innovation has occurred worldwide, the lesson study is undoubtedly a Japanese invention (Ni Shúilleabháin, 2015). In this section, only a few studies are mentioned.

The practice of lesson study is said to have been adopted in Japan in the 1960s (Fernandez & Yoshida, 2012), however it is also said to have started in the late 1890s (Makinae, 2019). In the United States as well as other countries, the book *The Teaching Gap* (Stigler & Hiebert, 1999) is thought to have inspired interest in lesson study. According to the 2019 TIMSS results, three aspects of Japanese classes were highly rated: i) arithmetic material, ii) content coherence and relationships, and iii) learner reasoning and reflection throughout class (Fishbein, Foy & Yin, 2021).

Educators' knowledge on what needs to be taught has been a matter of debate for decades. More recently, educators' knowledge has been divided into two categories: SMK and PCK, which are not fundamentally different (Kutluca, 2021; Chan & Hume, 2019; Lee, Capraro & Capraro, 2018). The way educators connect pedagogical knowledge (what educators comprehend about teaching) with their SMK determines their PCK (what educators comprehend about actual teaching). The fusion or unification of educators' pedagogical knowledge and their SMK is referred to as PCK (Rowland, Huckstep & Thwaites, 2005).

PCK in mathematics education is sometimes referred to as mathematics pedagogical content knowledge (MPCK) (Gasteiger et al., 2020). They further said, this type of knowledge, called MPCK, is characterised as a unique blend of information and pedagogy that mathematics educators possess. MPCK is made up of two main components, according to a thorough examination of the literature. The first is knowledge of mathematics teaching tactics, such as the use of analogies, demonstrations, explanations, examples, and illustrations (Ernest, 2018; Johnson,

2017; Depaepe, Verschael & Kelchtermans, 2013). It goes on to say that the second factor is understanding learners' perceptions of mathematics.

This study focuses on PCK since it is a significant part of the South African primary mathematics curriculum, not only what mathematics educators teach their learners, but also the procedure of teaching and learning mathematics (Leung et al., 2021). As a result, the focus of this research is on how educators might be assisted in developing and enhancing their PCK in the context of mathematics education through lesson study.

1.5. Aim of the Study

The aim of this research was to explore the role of lesson study in educators' pedagogical content knowledge regarding teaching mathematics. Lesson study has recently been recognised in research studies as a way to enhance educators' teaching. As a result, there is a lot of curiosity about whether these findings and outcomes can be repeated in the South African school setting. Lesson study has a direct impact on educators' teaching in four areas: PCK, SMK, content knowledge (CK), and reflective practice, according to the literature (Spiteri & Chang Rundgren, 2020; Vermunt et al., 2019; McCulloch et al., 2018).

According to Shimizu (2022), there is little research on teaching and learning mathematics in immersion groups using lesson study as a professional development approach. As a result, lesson study as a method of enhancing educators' professional development is still a relatively new concept and not yet widely used in the South African educational system (Shimizu, 2022; Kgothego, 2020). It is critical that participating educators have reliable experience with the lesson study model as conceptualised in the Japanese model of lesson study to explore the role of lesson study.

Participants completed two lesson study cycles per PLC to learn and envision the idea and concept. However, because of the disparities in education systems and teaching cultures between Japan and South Africa, the lesson study is expected to meet constraints and obstacles in its implementation (Adler & Alshwaikh, 2019). As a result,

the school context that effects the execution of the lesson study in the South African educational context will be examined in this study.

1.6. Research Questions

Lesson study emerged to be an adequate strategy model that supports educators in enhancing their PCK and mathematical practice, based on the existing literature. Some studies have explained the processes, features and settings of lesson study. However, in another study context, processes, features and settings may be added or differ during this study. Considering these objectives, this research intends to answer the following research questions:

1. What do educators need to enhance their pedagogical content knowledge of teaching numeric and geometric patterns?
2. How does educators′ participation in lesson study enhance their pedagogical content knowledge in the teaching of numeric and geometric patterns?
3. What are the educators′ view about the lesson study when teaching numeric and geometric patterns?

1.7. Research Methods

This study employs a qualitative research approach and is based on the interpretive paradigm. The researcher who uses an interpretive paradigm believes that there is no single correct or precise method for collecting knowledge (Lune & Berg, 2017; Merriam, 2015). The role of lesson study in enhancing educators' pedagogical content knowledge when teaching numeric and geometric patterns was investigated using a case study design.

The participants in this study are 31 educators from 7 primary schools in Ilembe District who teach mathematics in grades 4 to 6. The recruitment criteria were that a participant must be an Intermediate Phase mathematics educator in 2 selected PLCs, regardless of qualification, teaching experience or gender.

In the Ilembe District, non-probability sampling was employed to find 2 accessible PLCs out of 5. In this study, documents were analysed, observations were conducted, reflective journals were kept and semi-structured interviews were conducted. These

tools were appropriate for collecting qualitative data in line with the research paradigm, design and approach.

1.8. Analysis of the Data

The transcribed video, audio, and recordings were identified, coded, and organised into themes. The analysis of this study was done using the Thematic Analysis Approach. The researcher read the transcribed lesson study cycle, lesson observation notes, audio, interview, and video recordings repeatedly to understand what the participants were doing and saying. The trustworthiness of qualitative research was established through the use of four strategies: audibility, confirmability, credibility and transferability.

1.9. Ethical Clearance

The researcher presented the research proposal to the School of Education Departmental Research Committee and requested ethical clearance from the Durban University of Technology's Institutional Research Ethics Committee (IREC). The IREC granted full approval (see Appendix B: Ethical Clearance Letter). The Department of Education in KwaZulu-Natal (DoE) also granted approval and authorisation for the research to be conducted in designated schools in the Ilembe District (see Appendix A: Permission to Conduct the Research).

Informed consent letters, consent forms and information sheets were given to all participants (see Appendix C: Informed Consent Form). Participants were informed that participation was completely voluntary and that they might opt out at any moment. Participants were also guaranteed privacy and anonymity. These documents' annexures are included in the Appendices portion of this thesis.

1.10. Limitations of the Study

In the Ilembe District of KwaZulu-Natal Province, it would be appropriate to explore the role of lesson study in Intermediate Phase educators' PCK in teaching mathematics. Since the study was conducted by a single researcher, this study was limited to two Professional Learning Communities and recruited 31 primary mathematics educators in Ilembe District. The inclusion of such a tiny sample could cast doubt on the findings' external validity, and it is possible that educators who could

have made meaningful contributions were excluded from the study since they were not part of the PLCs chosen.

Therefore, this study does not aim to generalise the research findings, but contextualises its findings within the case study and the participants involved. This study involved 1 of the 12 education districts in the province of KwaZulu-Natal. There were two lesson study cycles per PLC, which was convenient for the researcher and participants because they were planned and conducted during official teaching and learning time. Only 8 research lessons were observed instead of 9, which was a limitation of the study. Nevertheless, the researcher was able to collect detailed qualitative data regardless of the limitations of the study.

1.11. Definition of Key Concepts

Lesson study: an approach of professional development in which educators collaborate in small groups to prepare lessons with a similar learning objective for learners (Vermunt et al., 2019).

Pedagogical Content Knowledge: is a form of knowledge that only educators have (Loughran, 2020).

Subject Matter Knowledge: this refers to an educator's knowledge of the discipline in which he or she teaches (Luft et al., 2020).

Cultural Historical Action Theory: is a theoretical framework for understanding and analysing the relationship between the human mind (what a human being thinks and feels) and activity (what a human being does) (Roth & Lee, 2007; Kuutti, 1995).

Department of Education: President Jacob G. Zuma reorganised the old National Department of Education into two departments, the Department of Basic Education (DBE) and the Department of Higher Education and Training (DHET), during the 3rd democratic administration.

Mathematics: is the study of numbers and how they relate to each other and to the real world (Russell, 2020).

Teaching: is the practice of engaging with learners to help them understand and apply concepts, knowledge, and procedures.

Educator: a person who gives intellectual, moral, and social instruction, previously an educator in South Africa was called a teacher.

1.12. Organisation and Structure of the Thesis

While the main reasons for the research have been presented in this chapter, further reasons are given and explained in detail in the following chapters. The thesis is segmented into five chapters, in which the results and research questions are interpreted and discussed.

Chapter 1

This chapter provides a basic understanding of the study by outlining the study: study background and rationale, study focus, major overview of study, study purpose, research questions, research methods, data analysis, ethics, study limitations, and thesis design and structure.

Chapter 2

This chapter reviews relevant and important studies that explore the role of lesson study on educators′ pedagogical content knowledge in teaching mathematics. It provides an overview of the literature concerned with enhancing mathematics educators′ teaching through lesson study, Cultural Historical Activity Theory (CHAT) as a theoretical framework, the rationale for using this theory, its limitations, and how it will ultimately shape the study.

Chapter 3

This chapter outlines the research methodology and process, such as: research paradigm, research design and methodology, research approach, research process, data generation methods, data analysis approach, trustworthiness issues and finally the limitations of the study.

Chapter 4

This chapter focuses on the analysis of the data collected in the field.

Chapter 5

This chapter addresses the research questions by providing a summary of the study, recommendations that emerge from the findings that would pave the way for future research, and a conclusion of the study.

1.13. Conclusion

Educators and governments face a difficult task in providing meaningful professional development. Teaching mathematics in a typical classroom to a diverse group of learners is a difficult task (Tan, Smith & O′Halloran, 2015). As a result, this study takes into account both educator and learner needs, namely, how critical it is for educators to improve their professionalism and equally critical for learners to receive mathematics instruction that helps them to attain their objectives based on their own preferences, preparedness, and capacity. Lesson Study was investigated as a possible way to meet both requirements (Pirozzo, 2014). This chapter introduced the study theme and outlined the research's goals and objectives, as well as its importance. With Ilembe District primary schools as a case study, the chapter drew the relevant intents and questions aimed at understanding the role of lesson study on PCK. This chapter went on to provide an overview of the predicted subjects of the following chapters, establishing the framework for further investigation to be conducted within the study. The second chapter will focus on the investigation's accumulated efforts and varied literature. It will shed additional light on the research topic and provide a more detailed examination of it against the backdrop of past research and literature on the issue.

CHAPTER 2 LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2.1. Introduction

The researcher introduced the readers to the research field in which this study is situated in the preceding chapter. The researcher builds on this in this chapter by reviewing the literature relevant to the current study. In doing so, a school-based conceptualisation of mathematics was initially presented. This was followed by a detailed description of how mathematics teaching can be enhanced in schools. The researcher begins with a brief discussion showing connections between the three concepts, which are: PCK, Lesson Study and PLC. After a detailed presentation of the literature related to the study, this chapter also explains a theoretical framework. In this theoretical framework section, Cultural Historical Activity Theory (CHAT) is explained in more detail. The seven components of CHAT are also explained in detail and applied to the study. The last two sections of this chapter discuss research on CHAT education and CHAT influence on lesson study.

2.2. Conceptualising Mathematics in the School Context

The word **mathematics** derives its origin from the Greek term ***máthēma***, which refers to any type of discipline, study or teaching. Mathematics is defined as "a language that uses symbols and notations to represent numerical, geometric, and pictorial relationships" in the South African curriculum (DBE, 2011, p. 8). It is a human activity that entails seeing, representing, and researching patterns and quantitative correlations in physical and social phenomena, as well as among mathematical objects. It contributes to decision-making by assisting in the development of mental processes that increase logical and critical thinking, accuracy, and problem-solving. In addition, Fatima (2012) describes mathematics as the abstract study of change, space, structure, and numbers (quantity).

Mathematics is considered 1 of the 11 “gateway” subjects in South Africa, which is considered critical to the technological, scientific and economic growth; and development of the country. The current study is therefore inspired by the statement that says, *one size does not fit all* (Furlong, 2020), meaning that every learner is distinctive. Therefore, educators need to be able to differentiate their teaching strategies during the teaching and learning process to accommodate diverse learners

within a class. Furthermore, Ewens (2014) and Forsten, Grant and Hollas (2002) explain that it is a requirement for educators to know both their learners and pedagogy to differentiate their instruction. This idea means that a nation that wants to achieve good performance in all subjects and especially in mathematics, needs highly qualified educators, which in turn requires meaningful teaching and learning for all learners. Moreover, it (mathematics) is directly related to other subjects, predominantly science and technology (Kaiser, 2020). Although this is so, the current situation regarding the academic achievement of mathematics learners is contrary to the above view.

According to WEF (Barrichello et al., 2020; Serrao, 2019), the Global Competitiveness Report published that South Africa was ranked as one of the lowest performing countries in mathematics. The researcher assumes that this poor performance led the South African government to consider mathematics education as one of the country's priority subjects, as it appears in almost all aspects of life. This has forced South Africa to import mathematics and science educators from surrounding countries such as Zimbabwe, Kenya and the Congo (DBE, 2014). Zvobwo (2013) presents instances of how mathematics is used in everyday life, such as reading time and maps, calculating areas and volumes, estimating, understanding patterns and home plans, and adhering to timetables. This explains the importance and various phenomena and applications of mathematics in our daily lives. Furthermore, Stols et al. (2015) defined mathematics as a subject that produces an ever-changing world because humans cannot avoid using numbers and instead must interact with them in some way. Despite emphasising the value and necessity of mathematics, learners continue to perform poorly or fail the subject.

A study conducted by Volmink (2020) shows that the Council for Quality Assurance in in Education and Training has identified a worrying trend of poor learner performance in mathematics at all grade levels, so the study raises major concerns about the teaching of this subject in schools. The study also shows that the number of candidates choosing mathematics as a subject decrease in grade 10 through to grade 12. Specifically, the 2020 NSC results dwindled by 0.8% compared to 2019, while the 2021 NSC results increased by 3.8% compared to 2020 (DBE, 2021; DBE, 2022). Mathematics has never been above 60% and has consistently ranked last among gateway subjects (11th position) for 5 consecutive years. In addition, mathematics

results for NSC or grade 12 have fluctuated for the past 5 years (2017 – 2021). This explains the extent to which learners are performing poorly in mathematics. Table 1 below shows the number of grade 12 candidates who wrote and passed mathematics in South Africa for 5 consecutive years (2017 – 2021) and the percentage who passed.

Table 1: Mathematics National Senior Certificate

	2017	2018	2019	2020	2021
Enrolled	245 506	233 858	222 985	233 579	259 401
Wrote	245 103	233 858	222 034	233 315	259 143
Passed	127 197	135 638	121 179	125 526	149 177
% Passed	51.9%	58.0%	54.6%	53.8%	57.6%

An indication of the importance of mathematics is that it has been made a compulsory subject to be considered and passed from grades 1 to 9, with each learner having to achieve at least Level 3 (40% - 49%). Since the Annual National Assessment (ANA) was the only common assessment required by law for grades 1 to 9, learners′ performance in grade 6 and 9 was below 50%. Table 2 below shows learners′ performance at the national level. The last administration of ANA was in 2014. In 2015, there was a stalemate between the DBE and educators′ unions regarding influence and writing ANA.

Table 2: ANA National Average Percentage Marks for Mathematics

GRADE	MATHEMATICS AVERAGE PERCENTAGE MARK			
	2011	**2012**	**2013**	**2014**
3	39%	42%	53%	56%
6	35%	27%	39%	43%
9	11%	13%	14%	11%

According to Circular D2 of 2017, there is an absence of a standardised national assessment (DBE, 2017). Primary schools are considered low-performing if less than 60% of learners scored at Level 4 (50 - 59%) and above in mathematics in grade 6. However, the Circular D2 of 2017 further states that a secondary school is considered low-performing if its pass percentage in the NSC is below 65%, and it has achieved less than 30% of bachelor and diploma passes, combined.

The above results suggest that educators lack PCK in mathematics. Poor learner performance, according to Kabutu-Njekwa (2019), Kandjinga (2018), and Ni Shúilleabháin (2015), indicates a serious problem in teaching and learning approaches. As a subject advisor for mathematics in grades 4 to 9 in the KZN DoE, Zululand District, the researcher has found that educators lack PCK. According to Van Putten, Stols, and Howie (2014), the educator's knowledge of what is to be learnt and how it is to be conveyed to the learner is the starting point of teaching. Khan (2015) also asserts that a thorough understanding of mathematical topics and how to teach them is required of all mathematics educators.

Clearly, the South African DBE is devoted to improving mathematics performance. As one of the measures to improve the PCK of mathematics educators and the performance learners, the DBE Minister, Mrs. Angie Motshekga, sent a team of curriculum officials to Japan in 2016 and 2017 to observe lesson study. Concerning this, the researcher feels prompted to conduct a study to investigate the role of lesson study on educators′ PCK in teaching mathematics.

Mathematics is a subject that has a poor pass rate in South Africa (Volmink, 2020). As a result, three concepts are considered effective in improving pass rates in the current study. These are PCK, lesson study and PLCs. A discussion of the three concepts is detailed below.

2.3. Pedagogical Content Knowledge

In a paper titled "*The Missing Paradigm in Research about Teaching*," Shulman introduced PCK at a conference at the University of Texas in 1983. (Gao, Damico & Gelfuso, 2021; Tuithof et al., 2021). The research article by Professor Lee S. Shulman was initially published in 1986. He described PCK as the unique combination of content and pedagogy that educators possess (Jacob & Gwany, 2020; Shulman, 1987). According to the literature, there is a major variation in PCK classification between Gess-Newsome (1999a), Grossman (1990), and Shulman (1986).

Shulman (1987; 1986) claims that SMK has a horizontal relationship with PCK and curriculum knowledge to produce the content knowledge (CK). Unlike Shulman, Gess-Newsome (1999a) and Grossman (1990) define PCK as a combination of SMK, pedagogical knowledge (PK), and CK. Thus, PCK is a dual activity that is very useful in implementing instructional activities that impact the intellectual being of all learners. This amalgamation of SMK and PK enables an educator to help learners develop an appropriate understanding of the concepts being taught (Blömeke & Delaney, 2012; Voss, Kunter& Baumert, 2011). In other words, PCK results from the union of content information with pedagogical procedures.

Schmidt, Cogan, and Houang (2011) and Shulman (1986) divide educators' general knowledge into seven categories: i) CK; ii) general PK; iii) PCK; iv) curriculum knowledge; v) knowledge about educational contexts; vi) knowledge about learners and their characteristics; and vii) knowledge about education history and philosophy. Shulman (1986) further argues that PCK can be delineated as a divergent form of knowledge that distinguishes educators from mere content experts. Furthermore, Chiva-Bartoll and Fernández-Rio (2021) view PCK as a “transformative prototype” in which educators must demonstrate a sense of flexibility in the delivery of instruction. This idea reflects that educators have adequate PCK when they are able to combine

new content each day with the appropriate strategies to try not to pass knowledge unilaterally to learners, but to transform it in ways that maximise understanding and learning.

Werner and Woessmann (2021); and Remillard (2005) explain the educator as one of the essential subdivisions in the framework of education. This brings into play the fact that regardless of the type of educational framework (that is, government formal, non-formal, or casual), educators are needed to make that framework a veracity. From ancient to modern times, educators have been required in the South African school system, as well as in the education systems of other countries. At all levels of education, it is fundamental that the educator is the source of the teaching and learning process. Contextual analysis has shown that educators are generally responsible for fulfilling the educational framework (Kioupi and Voulvoulis, 2019).

Therefore, this study considered CK and PK as two things that are essential in explaining educator quality. According to Roy and Roy (2015), the educator needs to know "what to teach" (that is CK) and "how to teach" (that is PK). Moreover, regarding the nature of the educator and his/her effectiveness, it is better to have an integrated knowledge of content and pedagogy that is PCK (Van Driel, 2021; Akyuz, 2018). As a result, the goal of this research is to demonstrate the importance of PCK in mathematics teaching. Figure 1 below shows how CK and PK lead to PCK.

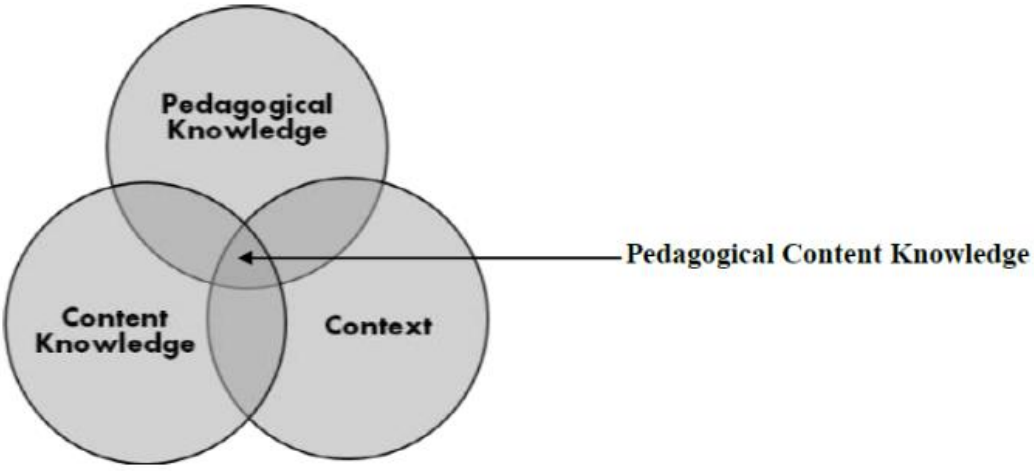


Figure 1: Pedagogical Content Knowledge

Numerous research studies Schmid, Brianza and Petko (2020); Murtafiah and Lukitasari (2019); Sibuyi (2012); Carlsen (1999); Shulman (1987; 1986) have been conducted on the topic of PCK and they all examine its importance to education in general. The studies do not apply PCK specifically to mathematics education and it has not been linked to lesson study. For this reason, the present study focuses on PCK in mathematics teaching, since educators who specialise in this subject must also have PCK. PCK must encompass knowledge of learners and their features, knowledge of pedagogical context, acquaintance of pedagogical degrees, goals and standards, and their theoretical and chronological grounds, according to Shulman (1987). Consequently, Shulman (1986, p.8) described PCK as "knowledge on how to supervise and teach a particular subject that is adaptable to learners with varying "interests and skills." He emphasises the idea that content and methodology merge into a single unit that can support a meaningful instructional process.

PCK, according to Baumert et al. (2010) and Magnusson, Krajcik, and Borko (1999), is the way an educator conceptualises how students might be helped to grasp a subject. This suggests that PCK can be very effective in making mathematics teaching and learning more meaningful, and according to this study, thus improving academic performance in the subject. They (Baumert et al., 2010; Magnusson, Krajcik & Borko, 1999) go on to say that this knowledge comprises knowing how to handle, teach, and change certain themes within a subject, as well as how to offer them to students of various interests and skills.

In addition, PCK has also been described as having the great potential to link SMK with knowledge of teaching and learning procedures (Scheiner et al., 2019; Lee, Capraro & Capraro, 2018). Van Driel (2021) further elaborated on this view, saying that an educator's understanding and presentation contribute to a group of learners being well-informed about a particular subject matter. They further say using various teaching strategies, representations and assessments, takes into account learners' learning conditions.

PCK, according to Shulman (1986), is made up of ways for presenting and expressing a topic in such a way that it is intelligible to everyone, as well as a grasp of what makes learning particular topics easy or difficult. More so, an educator′s PCK is important and has a great impact on learners′ academic progress (Khoza & Biyela; 2020; Malcolm, Mavhunga & Rollnick, 2019). The above views are taken as justification of PCK in mathematics teaching. In addition, Monteiro et al. (2021) state that researchers and educational legislators believe that PCKs make a superior contribution to the effectiveness of the teaching and learning process.

In this study, PCK is considered as a concept that encourages educators to have appropriate knowledge, understanding and is useful for empowering learners. Educators use PCK to encounter the diverse necessities of learners in heterogeneous sessions, taking into account the curricular requirements of the topic. In the context of PCK, educators need to obtain subject knowledge that relates to the methods and concepts of SMK to be taught. Furthermore, PK relates to how they teach the subject matter and context knowledge that relates to various elements that influence teaching and learning, such as policies and conditions.

Numerous research studies Murtafiah and Lukitasari (2019); Ni Shúilleabháin (2015); Meng and Sam (2013); and Lee (2010) have been conducted on PCK in relation to lesson study and mathematics. Although these studies applied PCK in mathematics teaching especially in relation to lesson study as a means of professional development, they were not related to numeric and geometric patterns. For this reason, the current study focuses on PCK in mathematics teaching, particularly numeric and geometric patterns.

2.4. Lesson Study as Professional Development

This section focuses on lesson study as a model for educator professional development in this study. Lesson study, according to Ogegbo, Gaigher, and Salagaram (2019), is school-based professional development that may be applied for any subject at any level of education. In different countries, however, different terminology is used, such as "Learning Communities" in Singapore (Huat, 2009), "Action Education" in China (Paine & Fang, 2006), and "Learning Study" in Hong Kong (Ling et al., 2005). Lesson study, according to Uştuk and Çomoğlu (2021); and Stigler

and Hiebert (1999), is a model that allows educators to question their teaching and learning practices.

2.4.1. The Origins and Understanding of Lesson Study

The Meiji administration introduced a different school system called the objective lesson slant in 1872 (Makinae, 2019). The objective lesson technique was offered as an updated teaching style for the new primary schools at the time. Lesson study dates back to the late 1890s, when educators at primary schools linked with regular schools began to observe and critically examine lessons (Makinae, 2019). Lesson study is increasingly being employed in countries such as Singapore, the United States, Ghana, Kenya, and other African countries, according to the literature (Adler & Alshwaikh, 2019; Cheng & Yee, 2011; Akyeampong et al., 2011; Coe, Carl & Frick, 2010). Lesson study is distinct from other types of educator professional development in that it takes place within the teaching and learning process.

Lesson study is an educator-focused and educator-led practise in which PLC members set a specific objective and work out how to examine their elective goal based on their evaluation of the practice. Basically, lesson study can potentially be used to improve educators′ PCK (Ni Shúilleabháin, 2015). Lesson study is a popular practice utilised by educators in the same subject, and it can be used as a practice for them in many schools (Vermunt et al., 2019; Likando, 2018; Cajkler et al., 2013; Lewis & Hurd, 2011; Coe, 2010). Based on the above opinions, the researcher thought it was worthwhile to conduct this study to investigate how the lesson study can be utilised to enhance mathematics teaching of patterns.

Leavy and Hourigan (2016) postulate that participation in lesson study has the potential to reduce educators′ sense of isolation as a group and familiarise them with the norms of working together as part of an educator community. Educators from the same or different schools in the same circuit (formerly called ward) work together to improve their teaching. Lesson study is a bottom-up approach to professional development in which educators collaborate to plan (research) a lesson, implement, observe, and reflect on it (Stols & Ono, 2016). This means that lesson study is a model that encourages educators from different schools within a circuit to work as a team to teach a particular subject, in this case mathematics.

Additional guests or outside professionals, such as Department Heads, subject advisors, and the Circuit Manager, may be invited to engage in lesson planning, observation, and reflection (Lewis, 2002). As a result, the goal of this research is to explore how lesson study might assist educators enhance their PCK in mathematics instruction.

Not only is the flawless lesson the main goal of lesson study, but also the lesson plan that is prepared inside this model (Bjuland & Mosvold, 2015; Friedman, 2005). Therefore, the lesson study in this study ought not be understood as a recommendation or advocacy for the use of strictly prescribed lessons, but rather as a developmental model that enhances the PCK for mathematics teaching. Moreover, as a prototype for professional growth, lesson study can be an approach to improve both educators′ mathematical knowledge and learners′ outcomes (Bjuland & Mosvold, 2015; Kuramoto & Shi, 2012; Murata et al., 2012; Fernandez, 2005). Through lesson study collaborative planning can identify ways to improve learners′ learning experiences. According to Ni Shúilleabháin (2015), the lesson study can also serve to enhance educators′ PCK as it provides a way for educators to find out how their learners participate in mathematics. Learner learning can be enhanced and provides educators with an opportunity to learn about new teaching strategies and to learn from their colleagues (peers).

The researcher understands the above view to mean that lesson study helps educators understand how learners can learn effectively, using appropriate methods. Participating in lesson study can airy up dialogues regarding the imperceptible aspects of teaching, such as how learners engage in activities and how best to foster classroom debates (Putri & Ilma, 2011). In addition, Lewis, Perry, and Hurd (2009) argue that the lesson study of teaching is essentially based on the fact that educators should face challenges in their classroom to implement new approaches to teaching. This means that educators learn from what other educators have accomplished and what has not worked to meet the learning objectives of a particular lesson. As a result, it is believed in this study that lesson study has a great deal of potential to enhance educators' PCK in mathematics teaching.

2.4.2. Lesson Study Mechanisms, Benefits and Challenges

Lesson study must be integrated into the school day and framework, according to Sekao and Engelbrecht (2021); McMillan and Jess (2021); Ni Shúilleabháin and Seery (2017); and Cheng and Yee (2011). Therefore, it is critical that educators who participate in lesson study, as well as those who support it within a school system, are aware of the model's mechanisms that affect school structures. PLCs emerge because of integrating lesson learning into the educational system (Younus, 2021; Lewis & Perry, 2017; Lieberman, 2009).

Chichibu and Kihara (2013) note that in South Africa, lesson study implementation is comparable to that in Japan and is common. In their year-long study conducted in all schools, they found that PLCs are given time as various individuals collaborate throughout the school calendar within the schools. In addition, a lesson study is a powerful method that can lead to 'systemic' improvement where educators, subject advisors, researchers and material developers learn within a PLC (Yarema 2010; Fang & Lee 2009). The above information clearly shows the importance of a lesson study for teaching in all subjects and especially for teaching mathematics in the current study.

From the previous literature, lesson study can have the following 5 benefits: i) improving teaching skills, ii) improving CK, iii) developing educators′ PCK through reflection and creating opportunities for educators to share "best" practises, iv) positively impacting learners′ learning, and v) building a community of educators (Lewis & Perry, 2017; Ni Shúilleabháin & Seery, 2017; Stols & Ono, 2016; Lieberman, 2009). In addition, hundreds of leading educators who worked with "coasting schools" and used lesson study as a coaching approach to improve learner progress in writing and mathematics at age 11 have shown significant year-to-year impact (Dudley, 2012). These views indicate that lesson study is beneficial to both educators and learners.

Cheng and Yee (2011) conducted a study involving a team of four educators, the Departmental Head and the grade level mathematics leader from Singapore. The study showed that even though the lesson study required a lot of energy and time from the educators, there were positive results. Firstly, educators understood that the use

of mathematical terms should be accurate and consistent so that learners could understand the topics more easily or smoothly. Secondly, the lesson study urged educators to reconstruct learners′ thinking and design lessons that address learners′ misconceptions. A study conducted by Lee (2008) in Hong Kong showed that educators became more independent, learned from their colleagues through discussions and perceived learners′ perspectives during the lesson study cycle.

Ozawa (2009) explains that lesson study was first introduced to mathematics and science educators in the Mpumalanga province of South African in 1999. Even though educators had to stay at work after school hours longer than required, there were quite a few positive results. Again, the KwaZulu-Natal Curriculum GET Sub-directorate encouraged only contemporary primary school educators to participate in lesson study as a method of improving teaching. Concerning this, I recognize that the current study is not the first to employ lesson study. However, this study is unique in that it uses lesson study to enhance educators' PCK in mathematics teaching.

Educators benefit from lesson study, according to Ozawa, Ono, and Chikamori (2010). To begin, educators discussed CK, teaching methods, and assessment strategies with one another. Secondly, they believed that reflection meetings were crucial because other educators gave various insights or alternative viewpoints on the teachings being delivered. Thirdly, they recommended presenting lesson study to other educators because they viewed lesson study as a type of beneficiary model for curriculum. This is especially important when all members of the PLC share common goals (Stewart & Brendefur, 2005). Fourthly, lesson study appears to be a method of responding to the demands of learners on a constant basis. Despite the DBE's stated assumption that officials responsible for curriculum visit and observe Japanese lesson study; a professional development strategy, in 2016 and 2017, lesson study has yet to gain much traction in South Africa (Kgothego, 2020; DBE, 2017). This suggests that not much has been done in South Africa to implement the lesson study model into mainstream teaching and into enhancing educators′ PCK in mathematics teaching.

Essentially, Hix (2008) asserts that a multi-year examination of a lesson study can result in a favourable shift in educators' attitude toward teaching and learning of mathematics. In this study, the previous statement shows that the process of lesson

study is a persuasive technique for educators to question their practise, talk about their everyday teaching, and be freely guided by it. Furthermore, depending on their expertise, each educator who participated in the lesson study procedure acquires something different (Hiles, 2019). Lesson study can have an impact on various aspects of educator development, including practice, knowledge, educators, and collaboration with their colleagues. It is critical to comprehend how each element of development interacts over time (Harsono, 2016). This idea is fulfilled in this study, which integrates lesson study and PLCs to enhance educators' PCK in mathematics teaching. Furthermore, as discussed below, this integration plays a key role in transforming educators' perspectives and learning through i) commute in norms, ii) revamp in participation possibilities, and iii) alterations in tools.

Changing norms also involves lesson study by helping to shape educators′ beliefs, expectations, values and perspectives. Educators no longer have to struggle with feeling accountable for their own actions, as they are quick to question and observe other educators and can feel accountable for each other′s practise in changing norms (Lewis, Perry & Hurd, 2009). Educators will have the opportunity to share their expertise and experiences about learner responses, collegial observation, and the negotiation of a common research lesson plan when participation opportunities change (Tepylo & Moss, 2011). As a result, these elements can influence community norms. Changes in tools are critical because educators can obtain the necessary tools through PLCs. For example, lesson plans serve as agendas for post-lesson discussions that can help educators expand their knowledge (Perry & Lewis, 2009). The researcher recognised the need to conduct this study to investigate the role of lesson study in educators' PCK in teaching mathematics, based on the above benefits.

Despite the obvious advantages of implementing lesson study, there are some challenges. One of the most frequently recognised challenges is the lack of time for collaboration. Lesson study is time-consuming, nonetheless, if lesson study is to be effective, educators must be given the time they need (Coenders & Verhoef 2019; Cerbin & Kopp, 2011). As indicated by Widjaja, Groves and Ersozlu (2021); and Stigler and Hiebert (1999), educators need two hours per week of uninterrupted learning for PLC to be quantifiable. Working on a common lesson study cycle, according to Lewis and Hurd (2011), typically requires 6 - 10 hours of PLC sessions spread out over 3 to

4 weeks (without re-teaching a lesson) and separated by only a few days. Therefore, in this study, a typical cycle spans a period of 3 to 4-weeks, with a minimum of 2 hours per PLC meeting.

In addition, many educators are reluctant to engage in baseline or critical peer analysis (Stewart & Brendefur, 2005). Another challenge to conducting the lesson study may arise from how the programme is to be implemented. Lesson study in South Africa is currently imposed from the outside by subject advisors, consequently it may face resistance (Lewis & Perry, 2017; Wallace, 1998). In this study, one of the biggest challenges to providing resources for lesson study implementation is the lack of outcomes that can be assessed through observation. Lesson study does not satisfy this criterion in a culture where immediate, positive outcomes are expected for nearly any type of educational reform (Coe, 2010).

Another challenge is that PLC members come from different schools, so they must travel and may require additional time to meet and discuss. In any event, according to Murata (2011), an in-school PLC group is beneficial since educators may share information about the learners and their respective communities. Be that as it may, South African schools have few educators (one educator teaches one or more grades) which is not possible due to enrolment numbers. In addition, a lot of educator energy is required for lesson study (Lee, 2008).

2.5. The Facilitator in the Lesson Study

Even though lesson study is becoming more popular around the world, (Aykan & Yildirim, 2021; Sekao & Engelbrecht, 2021; Isoda & Olfos, 2021; Ni Shúilleabháin, 2015) argue that high-quality implementation necessitates skilled facilitation. It is crucial to mention in the context of this study that this type of professional development is relatively new in South Africa, and few educators have engaged in lesson study. Most educators who conduct lesson study in PLCs lack the necessary competence to make the programme run well.

As a result, it is recognised that developers are required to establish lesson study in the South African environment, even though some educators are unaware of how new facilitators learn to accomplish this work. Most professional development in South

Africa is led by lecturers, department officials, or school principals. Little is known about how educators learn to promote professional development in situations when they oversee a significant portion of the process (Lewis, 2006). This means that the educators who are the key players in the implementation of this model (lesson study) are not more involved in the lesson study model, but those who sit in the offices are the ones who play an active role in it.

As one of the subject advisors for mathematics in KwaZulu-Natal, the researcher has found that professional development days often consist of workshops that are only one session. The workshops address either a new policy or approach that impacts educators (for example Mediation of CAPS Section 4 in 2020) or new teaching techniques (appropriate teaching during the COVID-19 pandemic in 2020). The presentation and practice of these workshops are separated, allowing educators to apply them to their work. Long-term, discipline-specific professional development activities, for example, do not imply practice. Coe (2010) and Fernandez and Chokshi (2002), emphasise that the educators' role in lesson study is to be both producers and consumers of teaching knowledge, and that diverse types of educational research are relevant and beneficial. This implies that educators should take an active role in promoting lesson study to enhance their PCK in general and mathematics teaching in particular. Since lesson study consolidates practice, facilitators are unable to arrange entire sessions in advance.

The facilitator plays a crucial role in lesson study by guiding PLC members through the cycle. The facilitator keeps the focus on the research lesson's main purpose and learning objectives while allowing all members to participate in the cycle (Meyer & Wilkerson, 2011). This is taken to suggest that the organiser oversees ensuring that all participants' ideas are captured throughout the process. Because he or she has strong subject CK and PK knowledge, a facilitator can be any internal member of the PLC or may be asked from outside the PLC to participate (Lewis & Hurd, 2011; Lewis, 2002). However, in South Africa, PLC members are encouraged to choose the facilitator from within their own ranks and to change roles when a new lesson study cycle is introduced. Shared leadership, according to Lin, Lee, and Riordan (2018), means that all members of the PLC can engage equally in observations and debates.

2.6. Lesson Study in South Africa

Before I illustrate the lesson study in South Africa, I will provide some historical background on education reform in South Africa as a developing country following the end of apartheid. South Africa was governed by a system of government known as 'apartheid' from 1948 to the early 1990s. It was a racial segregation and inequality-based system. According to Fiske and Ladd (2004), all parts of education, including administration, funding, professional training, and curriculum, were created and operated in a shockingly unequal manner along racial lines during the National Party's apartheid government. With a set hierarchy, the system was both supremacist and authoritarian. There was a ranking system in every school, regardless of racial mix, that placed the administrator at the top and the learners at the bottom (Christie, 1991).

Since the end of apartheid in 1994, education in South Africa has evolved considerably, and educators have been regularly challenged with new curricula (Jackson & Rothmann, 2005). The African National Congress (ANC)-led administration has made education equality one of its top priorities. Every citizen of South Africa has an unfettered right to a basic education, according to the 1996 Constitution (Assembly, 1996). The constitution guaranteed the right to education to all South Africans, regardless of race.

Many black township schools confront the same issues as they did during the apartheid era (Jackson & Rothmann, 2005; Nkabinde, 1997). The teaching profession was stressed by the political changes, yet there are still unfavourable factors that promote educator burnout in South Africa today (Spaull, 2013). Such as the duty load, a big number of learners in a class, tasks management, and a lack of material instruments to successfully handle tasks. In South Africa, the educational culture is still one of seclusion (isolation) (Le Grange, 2021; Bantwini, 2019). Even within schools, educators regard this culture as more collaborative, with fewer chances for knowledge and insight sharing. Lesson study could be viewed as a useful model for empowerment.

Prior to this study, a lesson study was conducted in South Africa, to the best of my knowledge and that of my co-promoter and promoter (Sekao & Engelbrecht, 2021; Helmbold, Venketsamy & Van Heerden, 2021; Alder & Alshwaikh, 2019; Coe, Carl & Frick, 2010; Ono & Ferreira, 2010). Lesson study was initially implemented in South Africa in two provinces, the Free State and Mpumalanga, in 2010. Lesson study, on the other hand, has not taken off across the country. According to the literature, lesson study has failed to resonate in South Africa (Ono & Ferreira, 2010; Coe, Carl & Frick, 2010). The DBE and the Japan International Cooperation Agency (JICA) co-hosted a workshop in 2016 for GET mathematics advisers from the Eastern Cape and the North West in Japan to observe lesson study implementation. In 2017, the programme was expanded to include GET mathematics advisors from the Free State and KwaZulu-Natal.

Despite the 2 opportunities to participate in lesson study, it is not yet a common practice in the 4 provinces indicated, let alone in South Africa. There were 15 articles and 5 dissertations on lesson study published in South African authorised journals before the end of 2021. Lesson study is a common practice in all levels of educator education, including initial and in-service. However, in this study, a group of educators from primary schools in KwaZulu-Natal Province DoE, Ilembe District, was introduced to the technique of lesson study to explore the role of lesson study in educators' PCK in mathematics teaching.

Lesson study could be viewed as a tool that educators appreciate to help close the gap in mathematics educators′ PCK. If South African educators believe that the preparation they acquired during their tertiary education was insufficient, they may respect the lesson study process. Lesson study is a method of working together with colleagues to improve classroom education and can assist educators in moving away from their traditional role as rule-followers (Fiske & Ladd, 2004). Educators in post-apartheid assume roles as learning facilitators, translators and designers of learning materials and programmes, administrators, leaders and managers, lifelong learners, researchers and scholars, citizens, community members and pastoral caregivers, assessors and specialists in various learning domains and phases (DoE, 2000a).

Lesson study is not yet ubiquitous in South African schools, nor is it a common subject of research in primary schools, according to the literature evaluated in the preceding sections of this study. There are significant distinctions between primary schools and secondary schools, therefore, lesson study practices in the two contexts may differ. Educators in South Africa do not teach only one grade; instead, depending on their duty load, they may teach all grades in a phase (grades 4 to 6) or even grade 7 in one day. As a result, in South Africa, both primary and secondary school educators are obliged or encouraged to participate in professional development. Even though subject-specific departments are promoted, there is frequently no formal structure in place where educators who teach the same subject meet or prepare together. In this way, the lesson study is based on research on collaborative planning in primary schools in South Africa.

2.7. Exploring the South African Lesson Study Cycle

Various people have different ideas about how many steps there are in the lesson study cycle. The British model, according to Dudley (2014), comprises a cycle of at least 3 research lessons that PLC members create, teach, observe, and analyse collectively. According to Coenders and Verhoef (2019), the Dutch model consists of a 6-steps process that begins with orientation to the theme and ends with sharing of findings or experiences. In Zambia, the lesson study was first introduced in 2005, with the lesson study cycle consisting of 8-steps (MoGE & JICA, 2016). However, since the current study is being conducted in the Republic of South Africa, the researcher adopted the South African lesson study cycle that was supported by JICA in 2016. This lesson study cycle consists of a 5-step process:

1. Identify problematic concepts
2. Plan the lesson
3. Present and observe lesson
4. Reflect
5. Improve the lesson

Figure 2 below illustrates the South African lesson study cycle.

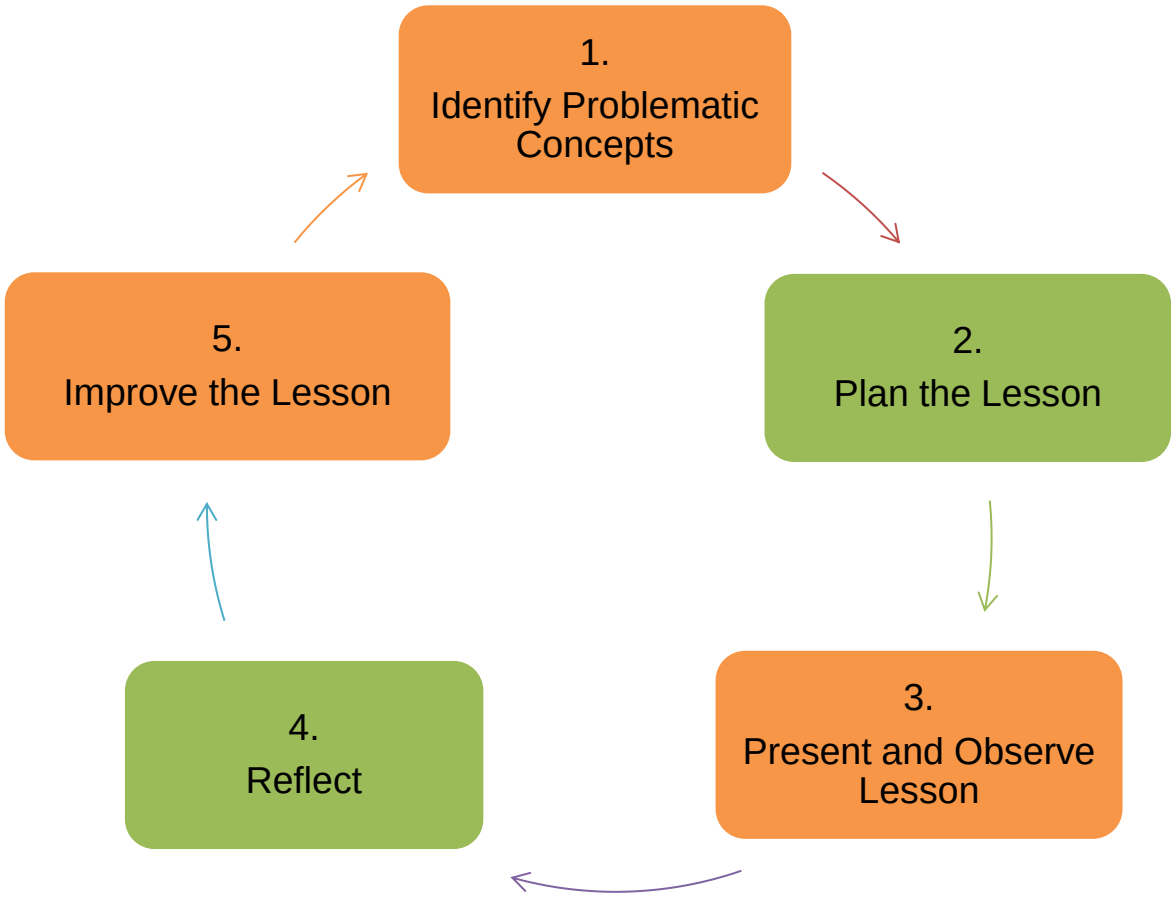


Figure 2: Lesson Study Cycle

Even though the purpose of lesson study may appear to be very straightforward, Doig, Groves, and Fujii (2011) argue that the lesson study process is an iceberg on which the visible component of the research lesson is an institution of that research lesson. The steps of the lesson study cycle are outlined below in greater detail:

2.7.1. Identify Problematic Concepts

In the introductory segment of the lesson study cycle, educators must agree on a common goal for the period in which they will be involved (Meyer & Wilkerson, 2011). The above idea reflects that in the first phase of the lesson study cycle, educators should agree on a goal that they want to achieve by participating in this cycle. It is not normal for every participating member of a PLC to hold exactly the same values (Kloser et al., 2021; Weick, 1969). This means that members may fail to establish a common goal, which may result in inadequate outcomes from the activity. Therefore,

it is important for educators to agree on a general aim that is specific and relevant to their practise and that they believe can be achieved.

In addition, Cerbin and Kopp (2011) require that the verge of the lesson study be characterised by needs of learners or the pedagogical challenge. Therefore, in the preparatory phase of the lesson study cycle, mathematics educators can directly scrutinise their annual teaching plans (ATPs) and programme of assessments (PoAs) in determining the diagnostic analysis so that they can identify the appropriate methods for specific topics. They should use the common assessment administered in all PLC schools to identify common misconceptions that arise from learner responses to identify the challenging topic that ought to be addressed in the lesson study session. Otherwise, educators participating in the lesson study may choose a topic that is challenging for (some of) them to effectively teach and use it as the basis for the lesson study. Once educators have agreed on which topic or concept they want to focus on, they can proceed to the following step of lesson planning.

2.7.2. Planning Research Lesson

Ni Shúilleabháin (2015) and Lynch (2011) explain that the second stage of the sequence is for educators to engage in collaborative lesson planning that provides a comprehensive explanation for designing a research lesson. PLC participants work with guests to design the various methods that can be used to teach the recognised mathematics topic. In addition, brainstorming can be conducted for individual data collection on the best approach to teaching the identified problematic topic (Coe, 2010; Fernandez & Chokshi, 2002). In any case, PLC members should regularly remind themselves that the lesson study implies the collaborative creation of a lesson plan by all educators participating in the PLC (Ni Shúilleabháin & Bjuland, 2019).

Planning of a research lesson is grounded on the metacognitive teaching-learning techniques for a particular CA that determine how to teach mathematics (Van der Walt & Maree, 2007). In organising a research lesson, according to Ni Shúilleabháin (2015), Chichibu and Kihara (2013), and Yoshida (2002), the following concepts should be considered: i) identify a topic that is confusing to learners or challenging to teach, ii) identify the objectives of the research lesson, and iii) design your research lesson. According to the findings of this study, these principles aimed to design an excellent

research lesson that makes learners′ thinking transparent and allows different educators to find out how their learners learn.

When working in collaboration to prepare lessons, educators consider the SMK, PK, knowledge of learners' thinking, and the lesson's desired objectives or outcomes (Yoshida, 2012; Schoenfeld & Klipatrick, 2008; Shulman, 1986). This means that it is easy to enhance PCK for teaching mathematics in schools if the 4 aspects are integrated. Moreover, Gurl (2011) adds that educators must not only use curriculum materials but also consult sources outside the classroom when planning research lessons during teaching.

Documents such as CAPS and ATPs should be consulted in the context of the South African education system to ensure that lessons are aligned with the DBE expectations. It is more helpful when educators collaborate to examine existing lessons (Hix, 2008). Similarly, it is significant for educators to repurpose old lessons that need to be modified (Hix, 2008). Regular topics or concepts can be utilised to design research lessons, and an excellent educator's purpose is to conduct and create useful lessons in this manner (Yoshida et al., 2003). This stage of the lesson study sequence is considered very important in the present study because it has a great part in enhancing educators′ PCK in teaching in general and in mathematics in particular.

2.7.3. Lesson Presentation and Observation

This stage consists of hands-on teaching of the planned lesson. One PLC member volunteers to teach the collaborative lesson to his or her learners on a mutually agreed-upon date, whereas the other members of the PLC look on and take detailed notes (Coenders & Verhoef, 2019). This implies that it is a stage of implementing all the ideas that would have been proposed in the second stage. In addition, Shimizu (2022) conducted research that shows that an educator who volunteers or is assigned to teach should have in-depth knowledge of the proposed teaching strategies.

To capture the structure, process, and conclusion of the lesson, observers should be trustworthy, punctual, and remain in the room throughout the presentation of the research lesson (Ni Shúilleabháin & PDST, 2018). Any other knowledgeable outside guest (such as, subject advisor) who was not involved in lesson preparation may be

invited to attend the research lesson presentation. As part of their preparations, educators choose what to observe during the research lesson (Sekao & Engelbrecht, 2021); and the observation instrument should be available prior to the lesson observation. According to Helmbold, Venketsamy and Van Heerden (2021), the most beneficial strategy for observing a research lesson is to observe a learner or cluster of learners during the lesson, document their observations, and not intervene in the lesson.

Ni Shúilleabháin (2015); Chichibu and Kihara (2013); and Gurl (2011) explain that observing lessons provides educators with a unique and brand new opportunity to glimpse inside a class they helped design. With only one educator leading the lesson, observing educators can observe how learners engage with and respond to mathematics (Breen et al., 2014). They go on to state that this is different from educators observing in their own classrooms because they must be able to identify specific moments in learners′ learning while simultaneously teaching.

This stage, according to the researcher, is equally crucial since it allows the observing educator to examine lessons step by step while they are in progress. Lesson study begins with observation of research lessons (Murata, 2011; Corcoran, 2009), and educators might reflect on how learners arrived at a particular solution. According to Coe (2010), videotaping a lesson is another alternative, albeit it is unlikely to succeed in replacing the educator observers due to its narrow emphasis. This is crucial because it captures all the steps of the lesson so that it can be easily referred to when reviewing the lesson.

2.7.4. Post-lesson Reflection

This is a stage where educators reflect on what they saw in the post-discussion based on the notes they took during the research lesson (Ni Shúilleabháin, 2015). This suggests that the main purpose of the meeting is for the educators to discuss what they observed in relation to the learners′ learning in the post-lesson discussion, rather than to criticise or object to the presenter. Furthermore, Coe (2010) defines the goal of this process as evaluating how effective the research lesson was in achieving the intended objectives. The meeting should also take place on the same day as the observation. The facilitator as an observer should lead the post-lesson reflection

meeting; however, the lesson presenter should be the first to reflect on the research lesson.

The presenter can give his or her opinion on what worked and what did not work in the lesson. The objectives of the lesson should be the focus of the reflection - what factors contributed to achieving them or not. Other educators can provide input based on the information gathered. Throughout the process, it is critical to keep feedback focused on the observed research lesson. Observers should begin by praising the educator who delivered the research session and commenting on some excellent aspects of the lesson, according to Fernandez and Chokshi (2002). All comments made by observing educators should be supported by solid evidence from their observations.

Educators are encouraged to share their thoughts, comments, and ideas about the research lessons in terms of pedagogical practices, knowledge transmitted, and learner engagement during the lesson study (Ni Shúilleabháin, 2015). Educators′ reflection will encourage them to enhance their understanding of pedagogical content (Lynch, 2011). The PLC should also use this time to celebrate their successes in completing a lesson study cycle.

2.7.5. Lesson Improvement

This is the final and most important stage of the lesson study cycle, as all ideas generated during the post-lesson reflection are consolidated and applied to the research lesson (Lewis & Hurd, 2011; Meyer & Wilkerson, 2011). This may require re-teaching the research lesson to assess the appropriateness and/or success of the proposed methodology. Different modifications could be tried, for example, the same educator could conduct the lesson again with different learners, or a different educator could conduct the lesson with different learners, as shown by to Ni Shúilleabháin (2015), Lynch (2011), and Yoshida et al. (2003). I gave the highlighted concepts a lot of thought, therefore I decided to undertake a study to explore the role of lesson study in enhancing educators' PCK in mathematics teaching.

2.8. Lesson Study Versus Educators′ PCK Understanding

Although, collaboration is gaining momentum as a laudable pedagogical strategy, the big question for the researcher is whether it is a very important instrument for changing the isolationist culture. Rapanta et al. (2020) agree that, despite substantial evidence that peer tutoring or sharing is a hopeful approach to educator development, educators are often resistant to it. The above thought suggests that lesson study is not a welcome model within the education system, although it is very useful for enhancing educators′ PCK.

In this regard, some research recommends that, while collaboration fosters a sense of community among participating educators, it has minimal impact on actual practice (Ogegbo, Gaigher & Salagaram, 2019; Huang & Shimizu, 2016; Stols & Ono, 2016; Olson, White & Sparrow, 2011; Puchner & Taylor, 2006). Furthermore, Van de Pol, Volman, and Beishuizen (2010) argue that the interactions that focus on actual classroom performance are the most valuable and challenging for educators. The use of lesson study in this study has the potential to play a central role in enhancing educators' PCK in mathematics teaching. The goal, according to Burney (2004), is to replace isolating tactics with more demanding and supportive ones by developing relationships between educators and students across the classroom.

Furthermore, Huang and Shimizu (2016); and Puchner and Taylor (2006) confirm that lesson study can assist educators enhance their PCK by developing their CK. As a result, the findings of this study show that lesson study can help mathematics educators in South Africa enhance their PCK. In this regard, the lesson study has a good impact on learners' understanding of the topics presented, allowing for the formation of school PLCs and the development of their professional expertise linked to PCK as they proceed through the stages involved (Cerbin, 2011; Coe, 2010; Yamping & Lee, 2010; Fernandez, 2002).

Olson, White and Sparrow (2011) examined the impact of lesson study on the PCK of mathematics educators in the United States and found that 3 out of 5 educators who completed the lesson study process changed their PCK for more than a year and a half. Stols and Ono (2016) point out that one of the benefits of lesson study is improving educators′ PCK through reflection. They go on to say that history shows that

each of these pedagogical solutions that attempt to bypass the educator fails. This means that the combination of educator PCK and lesson study in teaching mathematics in South Africa can lead to good results in mathematics. In the study conducted by Ogegbo, Gaigher and Salagaram (2019), Physical Sciences educators from South Africa showed how collaborating with another educator improved their PCK. Posthuma (2012) focused on the advantages of lesson study as reported by participants from the Free State province. They expressed satisfaction with the lesson study process, but it remains to be seen whether they will adjust their classroom behaviour in response to the findings.

2.9. Professional Learning Community in South Africa

A PLC is a group of professionals or experts who voluntarily come together to learn with and from each other when they themselves identified a need. PLCs can occur within a school or between schools, as opposed to clusters, which must occur between schools. A PLC is the work of a professional; this means that participants in a PLC adopt a professional attitude. Professional learning involves learning that focuses on knowledge from practice and knowledge from research. PLCs can be local (within a school, circuit, district, province or national) and international, as opposed to clusters, which must be within a circuit. School clusters focus primarily on assessment development and facilitation tasks, whereas PLCs are about quality teaching and ongoing professional learning that is voluntary.

The Integrated Strategic Planning Framework for Teacher Education and Development (ISPFTED, 2011 - 2025) was launched in 2011 by the Ministers of DBE, Mrs. Angie Motshekga, and Minister of DHET, Dr. Blade Nzimande, to strengthen the process and address challenges in improving educator quality (Ndlovu, 2021). The establishment of educator PLCs to promote collective participation in professional development activities is one of ISPFTED's mandates. The strategy seeks to encourage all educators, including principals, to take responsibility for their own professional development (DBE, 2018; Nkengbeza & Heystek, 2017). Professional development can take a variety of forms, including learning with and from peers in PLCs, according to the strategy.

The term PLC was coined by Shirley Hord in 1997 (Bissessar, 2021; Hord, 2009), but has been applied to a variety of communities. Nonetheless, the term “professional learning community” is frequently used in current educational research. Many educational advances have been accompanied by terms like "community," "learning communities," "discourse communities," and "communities of practice" (Mehrtash, 2021; Ni Shúilleabháin, 2015). A PLC is a cluster of professionals or experts who voluntarily come together to learn with and from each other about what they themselves have identified as a need. PLCs can occur both within a school and between schools. Some schools are small, such as schools with multi-grade consisting of 2 educators, so it is best to partner with neighbouring schools. Participants of a PLC assume responsibility for their very own personal development.

PLCs should not replace workshops and trainings inside or outside the school, but they can help put the knowledge gained in workshops into practice. PLC participation is not mandatory, yet, all educators are expected to participate in lifelong professional development. PLC at the school level is classified as a Type 2 activity, while PLC between schools is classified as a Type 1 activity, according to the South African Council for Educators (SACE) (Bernadine, 2019). The PLC is made up of a mix of new and seasoned educators, each with their own set of learning requirements and expertise. Furthermore, according to Brown, Horn, and King (2018), experienced educators can learn new enthusiasm, ideas, and technology, such as the 4th Industrial Revolution (4IR), that new educators can bring to the PLC.

The PLC can be customized for a specific subject or grade, or it can span multiple subjects and grades. For Foundation and intermediate Phases educators, it may be better to create grade- or phase-specific PLCs, while PLCs at secondary schools are often subject-specific groups. There is no set schedule, yet some regularity is essential for learning. According to Admiraal et al. (2021) the more intensive, frequent, and longer PLC sessions, the more individuals can benefit from them. Professional, Learning, and Community are the three words that make up PLC.

Professional

A PLC is the result of a professional's work. This indicates that PLCs′ members have a professional demeanour which means that the learners' needs are prioritised and their learning is supported (Wiseman, Arroyo & Richter, 2013). PLC work is supposedly motivated by knowledge and research, according to this study. According to Gabriel and Farmer (2016) and Hilliard (2012) educators discuss how to implement pedagogical innovations into their practice. Educators may also add to the research foundation by examining and reviewing their own practice.

Learning

Learner, educator, leader, and school learning are all addressed in PLCs (Gabriel & Farmer, 2016). Furthermore, Wiseman, Arroyo, and Richter (2013); and Hilliard (2012) assert that professional learning involves learning based on both practice and research knowledge. As a result, this study was conducted with educators as participants in their classrooms, allowing for a comparison of the impact of different instructional styles on learning.

Community

PLCs are founded on the belief that learning is more effective when it takes place in a community of professionals (Wiseman, Arroyo & Richter, 2013). Thus, in this study, schools may have created the framework and culture to support such collaborative learning, resulting in enhancing educators PCK for teaching mathematics.

2.9.1. The Characteristics of PLC

Snyders (2017), Nkengbeza and Heystek (2017); and Hord (2008) identified ten characteristics that institutions need for their capacity building to be successful.

Mutual Trust and Respect

PLCs will fail if members do not trust one another and do not respect one another's differences (Ostovar-Nameghi & Sheikhahmadi, 2016; Pirtle & Tobia 2014). This shows that for a PLC to be successful, it must be transparent, sincere and consistent in everything it does as a team to build trust between the members. Gabriel and Farmer (2016); and Wiseman, Arroyo and Richter (2013) confirm that mutual trust and respect are not automatic but must be earned. This means that creating them will take

a lot of effort and time, but once they are in place, real and profound learning may occur.

Support Challenge and Constructive Critique

Another characteristic of a successful PLC that Provini (2012) and Weber (2011) point out is that participants should challenge each other with useful thoughts and questions. This view indicates that members participating in the PLC need to be critical in their discussions to arrive at comprehensive ideas that can lead to the enhancement of educators′ PCK in mathematics teaching. Halla (2015) reinforces the above idea by stating that constructive criticism is desired and valued so that members can align and learn their thoughts on learning.

Shared Vision and Clear Focus on Ensuring Learning for all Learners

Another crucial aspect of PLCs is a shared vision of what institutes high-quality learning and teaching. According to Provini (2012) and Weber (2011), learning takes centre stage when educators embrace this commitment. That is, when there is a shared vision and clear focus, all stakeholders will work together. Furthermore, the vision must be transformed into a concrete, truthful and useful learning focus. The vision must challenge educators to interrogate their present practices, make modifications and identify learners′ learning desires (Halla, 2015; Hord, 2009). In this study, this feature is considered crucial as educators should reflect on their traditional practices so that they can move to the most appropriate approaches.

Collaborative and Reflective Inquiry

This fourth characteristic contrasts with traditional practices, where preparing, teaching and assessing learners was the responsibility of a single educator. Gabriel and Farmer (2016); and Hilliard (2012) affirm that effective PLCs challenge the individualisation of teaching. Colleague learning, collaborative teaching, observing, and mentorship, encourage the opening of one's classroom. This means that in a PLC, increasing learning is a communal responsibility for all members. Furthermore, educators who engage in reflective inquiry have in-depth dialogues regarding their learning and teaching. To put it in another way, it is critical for educators to examine their practices on a frequent basis through mutual observation, collaborative planning,

and the application of innovative ideas and information so that all participants of a PLC get a better grasp of educational processes.

Inclusive Membership and Openness

Halla (2015); and Pirtle and Tobia (2014) state that diversity within a PLC is very important as it helps create a stimulating teaching and learning atmosphere. Therefore, PLCs should be open to new members and work against any kind of gender discrimination to allow for free and meaningful participation. This is supported by Powell (2016) who says that effective PLCs do not consist of isolated communities. Gabriel and Farmer (2016) emphasise networking with other institutions such as schools and outside members or experts from other PLCs without mislaying focus. The researcher is aware that PLCs are very important for classroom research and therefore enhancing educators′ PCK in mathematics teaching.

Leadership

According to Ostovar-Nameghi and Sheikhahmadi (2016); and Hilliard (2012), a supportive School Management Team (SMT) is an important requirement for effective PLCs. This indicates that the PLC is not led by a single person, but rather by a group of people, regardless of gender. Setting an agenda, creating materials, coordinating group activities, and assisting colleagues in their learning are just a few examples. Therefore, it is important that all participants respect those who hold leadership positions during PLC instructional activities.

Collective Responsibility for Leaners' Learning

Members of a PLC collaborate to have a better knowledge of how all their learners learn and how to enhance learning (Halla, 2015; Hord, 2009). Educators learn to think outside the box of their own classrooms through lesson study, action research, and cooperative teaching. Ostovar-Nameghi and Sheikhahmadi (2016), in particular, argue that learners' learning is influenced by factors other than what happens in the classroom, what occurred in previous grades and what would happen in future grades. This indicates that it is not only the educators of the current grade who should be considered for participation in a lesson study.

Coherent, Responsive Change in Practice

PLC engaged educators' professional development can be more cohesive and sensitive to changes in practice than traditional educators' professional development (Wiseman, Arroyo & Richter, 2013). In the current study, educators may learn and discuss something in a PLC, relate it to their practice, then reflect on it in the following PLC session. PLC talks that result in changes in practice, according to Ostovar-Nameghi and Sheikhahmadi (2016) and Weber (2011), can be followed up by monitoring that practice. Error analysis and classroom observations are examples of practices that improve professional development coherence.

Regularity

Accordingly, an effective PLC entails that members meet frequently (Wiseman, Arroyo & Richter, 2013; Provini, 2012; Weber, 2011). Thus, the regularity with which participating members meet can result in deepening discussions, progressive knowledge growth and lasting effects on teaching and learning. Weber (2011) argues that a minimum regularity for PLC sessions is difficult to determine, but follow up sessions can be conducted using electronic communication tools or virtual meetings.

Systematic, Rigorous Enquiry into Practice

According to Halla (2015) and Provini (2012) educators must collect data regarding learning and teaching in their classrooms for PLCs to be successful. They go on to say that data can comprise results from internal, provincial, national, and international assessments. As artefacts or tools of practice, these assessments include tests, ANAs, and TIMSS, as well as learner activities, thinking, and instructional practices. This data enable error analysis and serve as a foundation for debates on the efficacy of different instructional methods.

2.9.2. The Roles in the PLC

Provincial Education Departments (PEDs) and educators bear the main responsibility for establishing and supporting PLCs. Many individuals and associations, on the other hand, have responsibilities to assist PLCs. Policymakers, School Management Teams (SMTs), educators, and knowledge must all work together to improve learning outcomes and alter professional practice, as shown in Figure 3. Effective PLCs necessitate a common vision for professional development in which educators, SMTs,

and policymakers collaborate to promote every South African learner's engagement, learning, and well-being.

Figure 3: Components of Professional Learning Communities (Weber, 2011, p. 63)

The educator's role is to be motivated to participate effectively in PLCs to enhance educator professionalism and learning outcomes. The educator should be willing to actively participate and learn as part of the instructional process. In addition, the educator should take leadership roles in the PLCs, for example facilitating, developing an agenda, and stating the objectives. The role of an SMT is to encourage educators to participate in the PLCs and to create the conditions for them to be successful. The principal's responsibility is to assist with resource allocation, logistics, and scheduling rather than leading each PLC session.

District Educators Development Centres (DTDCs) are responsible for providing resources and expertise in facilitation skills, as well as highlighting concerns for discussion at the provincial and national levels. Within the district, DTDCs serve as focal points for sharing PLC practices. DTDCs also work with district subject committees and PLCs to create synergies and provide annual progress reports to the provincial level. Provinces have the primary responsibility for creating an enabling environment for PLCs to operate effectively. Through subject advisors or qualified educators as mentors, provinces can also provide external guidance to PLCs.

2.9.3. Lesson Study as a Form of PLC

Lesson study should be considered a system of PLC and research in educator learning. PLCs have shown that lesson study is an appropriate alternative for comprehensive learning of educators and subsequently learners learning in a cohort of physics educators in the United States (Perry & Lewis, 2009).

As a result of the lesson study, an educator PLC was built and developed, which aided in the development of educator knowledge as well as the relationship between different types of knowledge (Murata, 2011). The lesson study helps educators break out of the isolation described by Remillard and Bryans (2004). Educators can contribute their ideas, goals, and teaching experiences by participating in a lesson study. Educators' participation in such a PLC might help them understand their pedagogical responsibilities and encourage them to enhance their teaching (Grossman, Wineburg & Woolworth, 2001). Therefore, these characteristics serve as a solid foundation for the present project's design.

Many of the guidelines and qualities of educator PLC are included in the highlights of the lesson study. Educators set a shared aim for their lesson study; there is no solo front-runner, and the educator facilitator role is cycled (Lewis & Hurd, 2011). Educators collaborate to generate curriculum materials and increase their knowledge as a team and individually during lesson study sessions (Murata, 2011; Lewis, Perry & Hurd, 2009). Shared trust and respect are required inside a lesson study PLC, and may emerge over time for a newly formed PLC (Grossman, Wineburg & Woolworth, 2001). As with any in-school professional development programme, a lesson study PLC must

be able to operate in stable conditions inside the school (Lewis & Hurd, 2011; Kruse, Louis & Bryk, 1994).

2.10. Theoretical Framework

The work of Lev Vygotsky, who developed the concept of intercession in the 1920s, is heavily influenced by Cultural Historical Activity Theory (CHAT) (Engeström, 2001). Hasan and Kazlauskas (2014, p.164) give a streamlined meaning of CHAT as a theory of "who does what, why and how." As the lesson study is characterised as professional development (Ni Shúilleabháin & PDST, 2018), CHAT affords a structure to examine how educators′ practices are inserted into a model of impact.

2.11. Genesis of CHAT

CHAT has advanced over 3 generations of study (Daniels, 2004; Engeström, 2001; 1987), being the most important. The original focuses on Vygotsky's notion of mediated action, or how people's interactions with their surroundings are mediated by artefacts rather than being direct (Fleer, 2010). This shows that the CHAT is a theory that emphasises the collaboration of individuals in a particular environment to enhance lesson study and the PLC's qualities. The combination of object, subject, and mediating artefact is presented as Vygotsky's concept of mediation cultural of activities (Daniels, 2006). Nevertheless, the first generation CHAT had the limitation that the examination focuses only on individuals.

The second generation is based on Leontiev's idea of activity, where the unit of analysis is broadened from the individual mediated action to a system of aggregated activity. This implies that there must be some reciprocal activities involved in the lesson study process. Leontiev (1978) contends that action is the least significant context for understanding individual engagement. Individual activity is thus influenced by his/her social position, the circumstances in which they find themselves, and a variety of distinctive personal traits. According to Leontiev (1978), the concept of activity is inextricably tied to motivational perceptions. According to Powell (2016) and Leontiev (1978) the fundamental factor that distinguishes one activity from another is the disparity between its objects that is the goal of the activity, which enhances it in a particular way. The utilisation of lesson study in the current study gives a fantastic chance for educators to enhance their PCK in mathematics teaching.

CHAT third generation extends the component of analysis to include the relationships amongst different systems activity (Wei, 2019; Provini, 2012; Weber, 2011). The current generation has a connection with the cycle of lesson study, in which there must be a good connection between the people involved in the lesson study process. Following Wei (2019) Engeström modifies mechanisms of improvement. Artefacts, and objects and subjects′ response, correspondingly, and extends this model of action mediated (Bourke & McGee, 2012). Thus, the goal describes the complex social systems that shape the actions (Yamagata-Lynch, 2010). It characterises the development of the component of analysis from a solo action classification to at least 2 interconnected action classifications. This generation of CHAT provides educators with a model to analyse activity structures and explore meaning within each system and where they converge. Researchers who embrace the 3rd CHAT generation analyse human acts through numerous activity classifications. The role of debate is different from points of view and power issues in managing collaborating activity systems as networks (Roth & Lee, 2007). In this regard, it is the way of lesson study to enhance educators′ PCK in mathematics teaching.

2.12. An Outline of the Seven Components of CHAT

The following Figure 4 shows a schematic representation of a non-exclusive activity classification as per the concept of Engeström (1987).

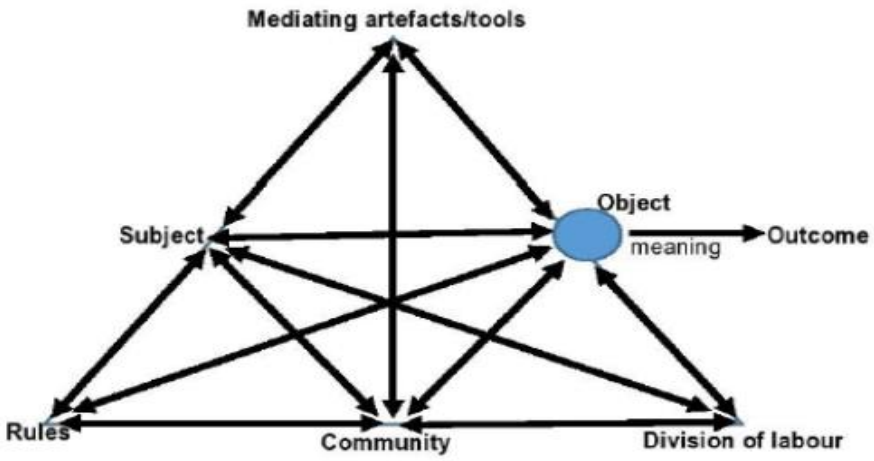


Figure 4: Cultural Historical Activity Theory

It demonstrates how subject-object relationships are not instantaneous, but are mediated by many elements such as artefacts, community, rules, and division of labour. The arrows between the subdivisions indicate that they are not separate elements that exist in isolation. Arrows, on the other hand, reliably interact and are dynamic with the other segments, defining the activity system (Wei, 2019; Gronn, 2002). The model provides the opportunity to analyse many relationships within the triangular structure of the activity (Kabutu-Njekwa, 2019; Wei, 2019; Bourke & McGee, 2012).

Individual and group acts in an activity system are represented by the upper section of the triangle. The lower part refers to community members who supervise the division of labour between the rules and the division of labour that supervises the activity itself. The object-oriented behaviours are described by interpretation and potential for change (Engeström, 2001; Leontiev, 1981) using oval representations of the object. I will reiterate each of these segments in detail below.

2.12.1. Subject

The subject refers to teams or individuals who share a similar goal or reason for performing an activity (Kabutu-Njekwa, 2019; Wei, 2019). In this study, the subjects are educators who teach grades 4 to 6 mathematics. Subjects are motivated to transform objects into outcomes/goals (a better professional development model that can lead to improved PCK) or desired outcomes. As subjects pursue the object, their identities and knowledge are shaped and changed by educators′ interactions with the other elements of the activity system (Roth & Lee, 2007).

Subjects′ characters, the object of their actions and the artefacts they use are generally reproduced over an indefinite period (Russel, 1997). This means that change takes a certain amount of time, therefore, lesson study process is appropriate for facilitating the acquisition of PCK by mathematics educators. This is because collective participants and relationships within the activity are not fixed and can change dynamically as social and cultural conditions change (Engeström & Sannino, 2010). Therefore, activity theory is also referred to as cultural-historical activity theory (CHAT) (Engeström, 2009).

2.12.2. Object

An object defines an activity in CHAT and is defined as any tangible or ethereal thing that drives a certain activity (Kabutu-Njekwa, 2019; Yamagata-Lynch, 2010). Therefore, in this study, it is crucial that actions are real in relation to all steps within the cycle of the lesson study. The participants in the activity can control an object to accomplish the desired result (Kuutti, 1995). The object in mathematics learning and teaching might vary depending on the subject, that is, whether it is the educator, the researcher educator, or the learner. If the educator is the subject of the activity, the object may represent long-term goals that emerge to fulfil the educator's requirements (Bourke & McGee, 2012).

In this study, educators gather throughout a lesson study cycle to improve PCK during collaborative planning with the purpose of achieving a long-term goal, such as enhancing mathematics learning and teaching methods. When the learner is the subject, the object is usually short-term goals like solving arithmetic problems (Jaworski, 2010; Venkat & Alder, 2008; Hardman, 2005). Kuutti (1995) backs up this claim by stating that things are not immutable and can change during an activity. As Kaptelinin (2005, p.10) states, “the object of activity has a dual status; it is both a projection of the human mind onto the objective world and a projection of the world onto the human mind.”

According to Kaptelinin (2005), the concept of the object of activity has the potential to be an analytical instrument that improves the assessment of what people do and why they do it. According to Bourke and McGee (2012), the purpose of action might be considered a plausible rationale for various behaviours of individuals, organizations, or associations. The object can be defined as “the sense-maker” that provides meaning to different substances and phenomena and regulates their standards. Therefore, the researcher believes that interviews can be useful artefacts to decipher the motives that lead to the enhancement of educators′ PCK in the teaching of mathematics (Jaworski, 2010). As the object predicts the intention for presence of the activity and acts as an inspirational factor that coordinates the subject′s actions.

2.12.3. Outcomes

The item passes through several transformations, including persons and their experiences, artefacts, and the action at hand, before stabilising as a finished product (Kabutu-Njekwa, 2019; Bourke & McGee, 2012). The age of new artefacts, the construction of new societies, new practices, and so forth are all desired results. To attain these goals, subject educators must work together to identify any systemic inconsistencies that may arise throughout the object's change. These results have the potential to encourage the individual to participate in future activities (Engeström, 1987). In the present study, the outcomes of the lesson study promoted the improvement of educators′ PCK in teaching mathematics.

2.12.4. Artefacts/Tools

Activity systems are typically historically redesigned by participants using artefacts. Artefacts serve to activate participants to enhance collaborative activities and instructional procedures (Halverson, 2006). Artefacts serve as mediators (Bedny, Seglin, & Meister, 2000) and are influenced by the historical and local context (Virkkunen & Kuutti, 2000). Artefacts are tangible objects that have significance that are carefully built into and through the actions of humans engaged in social practice (Blunden, 2007).

The construction mediated by artefacts suggests a collaborative procedure in which alternative viewpoints and voices encounter, impact, and amalgamate (Engeström, 1999). From a disseminated perception of mathematics educators, Spillane, Halverson and Diamond (2004, p.13) note that “educators practise is situated in an environment composed of artefacts that represent, in reified form, the accomplishments and problem-solving initiatives of previous human action.” Current research illustrates that mathematics educators use artefacts to enhance PLC members’ relationships (Halverson, 2004) to increase confidence among learners (Halverson, 2006).

Artefacts can be conceptual and/or physical tools of practise used by subjects (Postholm & Vennebo, 2019). In this study, material artefacts are lesson plan templates, reflective tools and teaching aids that mediate each educator′s actions and experience. Educators′ expertise and experience influence the selection of material

artefacts/tools to be used. Artefacts are used by people to change their external environment and the objects that surround them (Bedny, Seglin & Meister, 2000). With activity systems, artefacts set norms and adhere to boundaries (Rock, 1998).

2.12.5. Rules

Individually, rules provide instruction so that a subject can successfully participate in the PLC. Rules are a way for the organization to communicate with its members about what it desires (Stryker, 1994). Rules define the procedures and permissible interactions for members of the PLC to participate in and with each other (Engeström, 2007; 1993). According to Mills and Murgatroid (1991) the configuration of implicit and explicit rules is a phenomenon whose primary characteristic is that it controls, constrains, guides, and defines social action in general.

Rules are object-oriented, useful, and highly exemplified by material artefacts. From the perspective of CHAT, rules represent constraint and power. Rules involve agreements among community members about who does what, in what order and at what time. Mills and Murgatroid (1991) suggest that rules are assets that are personally interwoven with the communicative practises that subjects use to make sense of the social orders that have been constructed. There are 2 types of rules: explicit and implicit:

i. Explicit rules are generally accepted guidelines that are documented and developed through formal procedures (e.g. guidelines, educational policies).
ii. Implicit norms are unspoken principles that may influence the course of an action to varied degrees (e.g. rules that specify how the school division resolves problems and who is responsible for discussions). The basic purpose of both implicit and explicit rules is to guide subjects' activities to attain the activity's aim.

The rules can direct the temporal rhythm of work, resource consumption, and behaviour rules (Engeström, 1987). The arrangement and coordination of educators' actions to attain their purpose is referred to as temporal rhythms of work. Individual actions with some regularity and consistency can be condensed using temporal work rhythms, which emphasize the continuous flow of activities with occasional repeating events (Anderson, 1968; 1966).

The department's rules of conduct convey common suspicions about behaviour and establish the permissive and prescriptive range of consensus. They reflect the limits within which change can take place freely, intentionally, and without causing social harm. Violations of rules of behaviour are associated with a loss of regard, trust, and authority (Adler & Borys, 1996).

2.12.6. Community

Community alludes to the social group to which each subject correlates while engaging in an activity (Kabutu-Njekwa, 2019; Wei, 2019). Thus, a community is an activity system with different perceptions, advantages and backgrounds. Furthermore, Mwanza-Simwami, Engeström and Amon (2009) affirm that community also represents the physical surroundings as the context in which the activity is performed. In this study, mathematics educators are members of the community within their professional learning communities (PLCs).

2.12.7. Division of Labour

Subjects constantly negotiate their labour division within the community, implying that everyone′s actions can only make meaning in the context of the interdependent members' collective activity (Wei, 2019; Yamagata-Lynch, 2010; Hatcher, 2005). In this study, opinion is understood to mean that ideas originating from different individuals become meaningful to all participants. The way a community is built to change an object into an outcome is referred to as division of labour (Engeström, 1987).

The division of labour can be horizontal as tasks are distributed among individuals in the community with equal status, and vertical as undertakings are distributed here and there through power sharing (Daniels, Cole & Wertsch, 2007; Barab et al., 2002). Members of the PLC negotiate the division of labour and the rules (expectations, curriculum requirements) for participation in the PLC. Educators will have different roles within a PLC because a single subject does not have all the knowledge and skills necessary to perform all the actions within the lesson study as professional development. Therefore, the seven components of CHAT play a role in conducting this

study to explore the role of the lesson study in enhancing educators′ PCK in the teaching of mathematics.

2.13. Research on CHAT in Educational Setting

Bakhurst (2009) expressed that CHAT can be an appropriate theory for studying educational issues. He claims that if you have a well-defined goal, a good sense of desired results, a self-identifying set of subjects, a good sense of what counts as an instrument or tool, and so on, you will be alright. CHAT is a term that has been used to describe educational reform. According to Roth and Lee (2007), the spread of studies utilising CHAT in educational contexts is due to its effectiveness in both analysing data collected in real classrooms and structuring change. This section provides an overview of these studies.

In a primary school, Engeström, Engeström, and Suntio (2002a) conducted a study of change in troublesome learners and educators. The study employed CHAT's conceptual tools to examine and analyse educators' daily practices, highlighting learner disinterest as the root of the difficulties. Feldman and Weiss (2010) analyse educators who are participating in collaborative action research to adopt a new form of instructional technology in their classrooms using the third generation of CHAT. Venkat and Adler (2008) looked at two sub-sections of mathematics at the early phases of their interaction with innovation. As a result of interacting with the new policy, CHAT approaches were able to broaden their scope.

Saka, Southerland, and Brook (2009) investigated the negotiation processes between two aspiring science educators and their respective school communities. CHAT was employed by the researchers to examine how aspiring educators became experienced community members. The research of Beswick, Watson, and DeGeest (2010) demonstrates how activity systems can be used to analyse school departments. The researchers were able to uncover discrepancies in the interpretation of the object of departments and their impact on educator marginalization by using CHAT as a hypothetical framework.

Bourke and McGee (2012) employed the second generation of CHAT in a New Zealand study to investigate the intricacy of the three-year cultural innovation process. CHAT was utilised by the researchers to identify a single activity system that encompasses interactions between supervised and unsupervised educators. Kabutu-Njekwa (2019) investigated educators' and students' perceptions of the causes of poor mathematics performance among grade 9 students in a South African study. CHAT provided a lens through which mathematics instruction might be understood as a cohesive system within a specific context or setting.

2.14. CHAT Influences Lesson Study

This section argues the use of CHAT to theoretically impact or inspire lesson study based on the presentation of CHAT and its contributions to educational research. CHAT proposes that humans are not isolated individuals who interact with their surroundings on a purely natural basis; rather, our relationships with others and the cultural-historical framework in which we live are influenced by others and the context in which we live (Yamagata-Lynch, 2010).

As a result, the researcher views the lesson study as a collection of practices used in various sociocultural contexts. The study of educators' professional development through lesson analysis should widen the perspective from individual educators to the entire community. During the dialogic process of lesson study, each individual member of the lesson study group (subject of learning), regardless of whether they are experienced educators (active object), learns new practical knowledge (Wei, 2019).

Educators rather than educational researchers conduct lesson studies, which means that valid educator voices should be treated as a resource for understanding educators' professional learning. Diverse perspectives are more likely to cause tension, conflict, and even logical contradictions (Wei, 2019). CHAT seeks to help participants better grasp the processes and highlight any key steps needed to improve practice by exposing discrepancies that occur inside an educational system. CHAT, according to Wei (2019), provides educators and researchers participating in or analysing lesson studies with a theoretical framework for examining the vital behaviours and imperceptible reasons of varied individuals by inspiring new professional learning outcomes.

2.15. Chapter Conclusion

The literature review and theoretical frameworks that underpin this study were provided and addressed in this chapter. The first half of this chapter was devoted to a review of the literature that was relevant to this research. It looked at the international, sub-Sahara African, and local literature on the impact of lesson study on educators' PCK in mathematics teaching. The literature focused on the conceptualisation of mathematics in the school context. The review also focused on educators′ pedagogy for enhancing mathematics teaching in schools. Under this main heading, PCK, the lesson study as professional development, the origins and understanding of the lesson study; the mechanisms of the lesson study, the benefits and challenges, and the exploration of the South African lesson study sequence were detailed.

In addition, the facilitator of lesson study; lesson study in South Africa, and lesson study compared to the PCK understanding of educators′ were explained in detail. The professional learning community in South Africa was explained as was the characteristics of the PLC, the role in the PLC, and lesson study as a form of PLC. In the second section of the chapter, the genesis of the CHAT and the seven components that underlie the CHAT were explained in detail and shown how they are related to the objectives of the study. By applying the interpretivist paradigm, the CHAT was of great help in exploring the role of lesson study in educators′ PCK in teaching mathematics. The final subheadings to be addressed were the research on CHAT in the educational setting and the influence of CHAT on lesson study.

CHAPTER 3 METHODOLOGY

3.1. Introduction

The previous chapter above provided a review of the international, sub-Saharan, and local literature on the role of lesson study in educators' PCK in teaching mathematics. Chapter 3 discusses the research paradigm, research design and research methodology used in the study of the role of lesson study on educators' PCK in teaching mathematics. The interpretivist paradigm, case study research design, and qualitative research approach were used to present how the study could be conducted to produce high-quality results. The sampling methods and procedures used to select the research participants are explained in detail. The tools used to generate data are document analysis, observation and the reflective meeting or diary, as well as thematic analysis as an approach to analysis. The chapter finishes with a review of the issues of trustworthiness and ethical considerations that were considered.

3.2. Research Questions

3.2.1. Main Research Question

What are the roles of lesson study on educators' pedagogical content knowledge in teaching mathematics?

3.2.2. Research Sub-questions

The following sub-questions were developed to answer the main research question:

i) What do educators need to enhance their pedagogical content knowledge of teaching numeric and geometric patterns?

ii) How does educators' participation in lesson study enhance their pedagogical content knowledge about the teaching of numeric and geometric patterns?

iii) What are the educators' views about the lesson study when teaching numeric and geometric patterns?

3.3. Research Paradigm

A research paradigm is a scholarly orientation or research model that is accepted as a significant and relevant approach to guiding the development of a research methodology (Mertens, 2015; Punch & Oancea, 2014; Huitt, 2011). According to Shaw (2022) there are three major research paradigms: Positivist, Critical and Interpretivist paradigms.

3.3.1. Positivist Paradigm

Positivist paradigm was not used in this study since there is only one concrete reality that can be understood, identified and measured (Taylor & Medina, 2013). Causal inferences are based on:

- temporal precedence (in other words, for X to be the cause of Y, X must be temporally prior to Y),
- association (in other words, X and Y are related), and
- the absence of confounding factors.

Within a causal framework, this provides for natural explanation and prediction. As a result, no causes other than those mentioned have an impact on the outcome; X is the lone cause of Y within the range of possibilities.

According to Al Riyami (2015), knowledge may and must be developed objectively, free of the values of researchers or participants. When knowledge is correctly created, it is truth: it is certain, exact, and in tune with reality. Absolute isolation between the research participant and the researcher is required to develop truth efficiently. To achieve this distinction, Positivism employs dualism and objectivity. To put it in another way, positivist thought asserts that participant and researcher are distinct (dualism). Furthermore, the two entities are separated in the study by adherence of strict standards (objectivity).

3.3.2. Critical Paradigm

The critical paradigm was not used in this study as well, it revolves around power, inequality and social change at its core (Johnson & Onwuegbuzie, 2004). The critical paradigm often incorporates ideas developed by early social theorists such as Max

Horkheimer (Bourdieu et al., 2007) and later work by feminist scholars such as Nancy Fraser. The critical paradigm, in contrast to the positivist paradigm, asserts that social science can never be completely objective or value-free. Moreover, this paradigm is based on the idea that scientific research should be conducted with the explicit objective of bringing about social change. Moreover, its research programmes aim to promote positive change in the research participants and the systems under study and to generate useful data.

3.3.3. Interpretive paradigm

This study is located in the interpretive paradigm, which is very suitable for qualitative research. The interpretive research paradigm is used by those researchers who believe that reality is based on people's prejudiced experiences of the outside world. This means that they conform to the epistemological and ontological views that reality is a social construct.

By adopting an interpretive paradigm, the researcher holds that there is no single correct way or specific method for gaining knowledge (Lune & Berg, 2017; Merriam, 2015). Ramírez et al. (2013) argue that when conducting interpretive research, there are no theories that are considered accurate or inaccurate, but rather they are all judged by how well they appear to the researcher and the participants taking part in the research process. The constructs of the study are derived from the area in which the research is being conducted through in-depth assessment and evaluation of the phenomenon of interest.

The researcher believes that interpretive researchers assume that their knowledge and meaning result from the way the researcher interprets the participants′ findings and there is no unbiased knowledge that is independent of the thinking, reasoning individual. The dependent and independent variables are not predetermined in interpretive research. Rather, it concentrates on the complete range of human sense-making as it develops (Freshwater & Cahill, 2013). The interpretive method seeks to comprehend the subjective motivations and meanings that drive social behaviour.

Interpretivists are more interested in assessing, evaluating, and refining interpretive ideas than developing new theories. Merriam and Tisdell (2015) describe three different uses of theory in interpretive case studies: theory as a guide for data collection and design, theory as an iterative process of data collection and analysis, and theory as a case study outcome. In this work, theory was used as an iterative process between data collection and analysis. According to Mertens (2015), interpretivism is a large family of different paradigms rather than just one.

In conducting the study, I chose to use the interpretivist paradigm as it is a central effort to understand the world of social engagement and human experience (Guba & Lincoln, 2011). This tactic seeks to understand and interpret what the participants are thinking or the meaning they attach to of the context (Punch & Oancea, 2014). In this study, I engaged in interactive processes where participants asked questions and shared their knowledge collaboratively. By engaging participants in a real-life situation, I obtained clear information about the role of educators' PCK in teaching mathematics.

3.4. Research Design

The research design, according to Rozakis (2007), refers to the researcher's overall approach for integrating the many components of the study in a logical and coherent manner. It is the plan for data collecting, measurement, and analysis that ensures that the research problem is adequately handled. The type of design the researcher chooses is determined by the problem of the study. The purpose of a study design is to ensure that the knowledge gained will enable the researcher successfully and uniquely solve the research problem. Obtaining relevant information for a research problem usually requires defining the type of evidence needed to test a theory, evaluate a programme, or adequately describe and assess the significance of an observable phenomenon (Schwab & Borgdorff, 2014).

This study is designed to provide careful observation of primary school mathematics educators in the Ilembe District of KwaZulu Natal as they perform their duties particularly in this area. The design will allow for proper recording, collection and analysis of data. The design will in turn help in the interpretation and formulation of conclusions and recommendations by the researcher.

3.4.1. Methodology

According to Shields and Rangarjan (2013), the procedures or strategies used to find, select, process and analyse information about a topic are referred to as research methodology. The methodology section of a research article allows the reader to critically examine the overall validity and dependability of the study. The methodology section addresses two important questions: How was the information collected or generated? What method was used to study it?

Rocco, Hatcher and Creswell (2011) add that a coherent and logical structure based on perspectives, ideas and values that guide the researchers′ decisions, provides a contextual framework for the research. It involves a theoretical analysis of a field's corpus of procedures and principles, with techniques varying across disciplines based on their historical development. This results in a variety of possibilities that encompass different viewpoints on how information and reality should be considered. This places the procedures within a larger framework of concepts and methods (Trochim, 2006).

Methodology can be conceptualised as a spectrum that ranges from a purely quantitative to a purely qualitative approach. Although a methodology may traditionally fit into one of these categories, researchers may mix techniques to address their research questions, resulting in multimethod and/or interdisciplinary methodologies. In general, a methodology differs from a technique in that it does not aim to provide a solution (Rocco, Hatcher & Creswell, 2011; Trochim, 2006).

3.4.2. The Case Study Method

According to McMillan and Schumacher (2010), a research design is the plan and structure of an investigation used to gather information to answer research questions. Yin (2017) asserts that planning is necessary when researching the lives of individuals and learners because research is not purposeless but requires careful planning. Yin (2014) points out that the primary goal of planning is to avoid a situation where there is insufficient evidence to answer the research questions.

In this study, a case study design was used. A case study is a methodical and in-depth investigation of a specific case in its real-world setting (Buchholtz, 2019; Rule & John, 2011). Case studies aim to describe what it means to be in a particular situation, in

this case, the PCK of educators in teaching mathematics. However, they can also be used to make claims that can then be further tested (Cohen, Manion & Morrison, 2018). This study is concerned with the roles of educators′ PCK in the study of mathematics. I explored how these roles contribute to the teaching of mathematics in a school using a qualitative research approach in conjunction with the appropriate data generation tools.

3.5. The Qualitative Research Approach

The qualitative research method was used in this study. According to Yin (2014), qualitative research is defined by its objectives, which are related to understanding a particular aspect of public activity, and its procedures, which (generally) provide words rather than numbers as data for analysis. The qualitative approach will shape this research. According to MacDonald (2012), qualitative researchers seek deeper truths by attempting to understand or interpret events in terms of the meanings individuals assign to them as they examine things in their natural settings.

Qualitative research, according to Crossman (2017), is a sort of social science research that collects and works with non-numerical data to interpret meanings and assist people comprehend social life by examining a specific population in a specific location. Qualitative research is the creation of concepts that help understand social events in natural and non-experimental contexts, focusing on the meanings, experiences, and perspectives of participants (Simon, 2011).

Qualitative research was applied to determine what roles lesson study plays in educators' PCK in teaching mathematics. This study was conducted in the context of rural primary schools in KwaZulu-Natal. Qualitative research methods such as document analysis, observations and reflective journaling were used to understand the roles of lesson study on educators' PCK in teaching mathematics. In this qualitatively conducted research study, participants′ views were particularly important because they helped the study to assess the role of educators' PCK in the delivery of mathematics instruction. Crossman (2017), Lune and Berg (2017); and Merriam (2015) have identified three important elements in qualitative research studies that they believe define this research approach: naturalistic, emergent and purposeful.

3.5.1. Naturalistic

This explains the analysis of situations in their natural contexts, environments and the researcher's ability to not attempt to manipulate or control the situation in which the research is conducted (Lune & Berg, 2017; Merriam & Tisdell, 2015). This research was conducted in the work area where educators teach and in the corresponding mathematics subject area where the role of educators' content knowledge can be clearly perceived. From this, appropriate information emerged that opened the possibility of taking further action to alleviate the real challenges educators face in implementing content knowledge.

3.5.2. Emergent

Merriam (2015) believes that qualitative researchers should be able to use a design that is not too rigid, so that it does not limit opportunities to pursue new avenues of discovery. The researcher chose to use a case study because it is flexible and considers the thoughts of different participants. In addition, Lune and Berg (2017) made it clear that this means adapting the inquiry through a deep understanding of the changing situations. In this study, each participant was given ample opportunity to generously express their thoughts and opinions, as well as their suggestions about the role of lesson study on educators' PCK in teaching mathematics.

3.5.3. Purposeful

According to Merriam and Tisdell (2015), people offer functional interpretations of the phenomenon under study, promoting a more comprehensive insight. The researcher recognised that educators' PCK is a major barrier to teaching mathematics in a rural context. According to Crossman (2017), examples of research should be selected with care to provide rich and insightful information. In addition, qualitative research does not allow for situational empirical generalisations. In this study, all data were collected with a specific purpose in mind, and participants were not randomly selected; each had a specific role to play in the study.

The literature points out several demerits of the qualitative research approach. According to Mthiyane (2015), it begins with a universal challenge without specific questions to guide the research study. For this reason, at the beginning of this qualitative study, the researcher started from a general standpoint of observable fact, which led to the use of amorphous methods of data generation. According to Chinyoka (2013) and Chindanya (2011), qualitative research primarily offers the results in terminology without interpreting them. The researcher was guided by well-structured research questions that underpinned the study and led to the methodological decisions to refute these criticisms (Mthiyane, 2015). I also employed a variety of methods to collect data in its natural state, such as observations, document analysis, and reflective diaries, to make the study's conclusions more reliable and trustworthy.

3.6. Gaining Access

The Department of Basic Education (DBE) envisions a South Africa where everyone has access to lifelong learning, education and training to improve the quality of people′s lives and contribute to building a peaceful, prosperous and democratic nation. The department's strategic goals include improving educator supply, development and deployment, effective planning, coordination, information management, assessment and district support, improving social cohesion in schools and learner well-being (Government Gazette, 2017).

Undergraduate and postgraduate students (including DBE employees), academics, organisations, non-governmental organisations, universities, service providers, agencies, and research teams, among others, are required to complete the DBE Research Request Form prior to conducting research at Department institutions. The Research Request Form is designed to ensure that all applicants provide Department institutions with all important details about the research study to be conducted. Not all sections of the form are applicable to all applicants, and those deemed irrelevant are omitted unless otherwise specified (Government Gazette, 2017).

To gain access to schools in the Ilembe District, permission had to be obtained from the Durban University of Technology (DUT) and the KZN Provincial Education Department (PED). The researcher applied for permission to conduct the study from the DUT School of Education Departmental Research Committee (DRC) by

completing PG2a and presenting the proposal. The proposal was accepted by the DRC and then forwarded to the Faculty Research Committee (FRC). The FRC of the DUT Faculty of Arts and Design approved the conduct of the study for academic purposes with further approvals from the IREC of the DUT Research and Postgraduate Support Directorate. All documentation and approvals were then forwarded to the KZN DoE office. Permission to conduct a study in schools was granted by KZN DoE Head of Department office.

With the permission to conduct the study, schools were contacted, first the Ilembe District Director′s office, then the principals providing access to the schools and educators responsible for teaching mathematics. The researcher also asked selected educators from the selected PLCs to participate. In this regard, he explained the purpose of the study to all participants, which are Intermediate Phase mathematics educators (grades 4 to 6) and SMTs, including district officials who serve as mathematics subject advisors. The researcher also provided all participants with an informational letter. At the same time, all participants signed an informed consent form.

3.7. Population

According to Walliman (2017), a target population allows a study to be realistic, valid, manageable, feasible, and cost-effective while still achieving the necessary goals. According to Degu and Yigzaw (2016), a target population is a group of individuals within a larger population who are part of a particular study. Before a sample can be drawn, the target population must first be determined (Walliman, 2017; Degu & Yigzaw, 2016). Thus, the researcher selects specific segments of the population, particularly the South African population, and subjects them to a study that is representative of the population with the goal of achieving the objectives of the study.

The target population is mathematics educators in KwaZulu Natal, in South Africa, particularly in primary schools in the Ilembe District, who teach mathematics in grades 4 to 6. There are several primary schools in the Ilembe District with a range of mathematics educators for all primary grades. The study focused on 23 educators only in 7 schools teaching mathematics in grades 4 to 6, since a total of 8 educators withdrew from the study.

The study involved 2 Professional Learning Communities (PLCs) with a total of 7 schools. PLC 1 consists of 3 schools and 10 educators, PLC 2 consists of 4 schools and 13 educators, while the other PLCs have a smaller number of educators. These PLCs are the researcher's conscious decision in terms of distance and geographic location. Makrakis and Kostoulas-Makrakis (2016) states that a non-probability sample is a deliberate selection of a participant based on their characteristics. Since this is a qualitative study that uses small samples to generate in-depth subjective data (Cohen, Manion & Morrison, 2017), a minimum sample size of 23 educators was considered appropriate.

3.8. Sampling and Sampling methods

Convenience, non-probability sampling techniques were employed, as they best suited the purpose of the study. Non-probability sampling involves selecting individuals from the objective population who meet certain practical criteria, such as ease of access, topographical proximity, or accessibility at a particular time (Patton, 2002). Non-probability sampling was used to identify 2 of the 5 accessible PLCs in Ilembe District. The term PLC was coined by Shirley Hord in 1997 (Hord, 2009); however, it has only been in South African education since 2011 (Balyer, Karatas & Alci, 2015).

The criteria for recruiting research participants were that they had to be Intermediate Phase mathematics educators in 2 selected PLCs, regardless of their qualifications, teaching experience, or gender. During the February 2021 professional development, these participants were recruited, inducted and the annual plan for the 2021 school academic year was created. During the recruitment process, planning meeting, observation, and interview, participants were reminded of all COVID-19 pandemics social distancing and compliance with all safety requirements as per the Department of Health. During the lesson study cycles, we adhered to all regulations of Circular No. 18 of 2020 of the Ministry of Public Service and Administration.

This study was conducted during the COVID-19 pandemic over a three-year period, with three observation sessions per grade (grades 4 to 6). In order not to disrupt teaching and learning, the document analysis and reflection journals were conducted and reviewed after school hours and lasted $1\frac{1}{2}$ hours each.

3.9. Data Generation Method and Procedures

There are various data collection systems used as questionnaires, tests, interviews, classroom observations, journals, diaries, and more. However, I used document analysis, observations, reflective journals and interviews. These tools lend themselves to the collection of qualitative data consistent with my research paradigm, design, and approach. Using these methods together provides the opportunity for triangulation of data, which can help offset potential shortcomings that may be characteristic of a single method and provides an opportunity to examine the consistency of research findings (Cohen, Manion & Morrison, 2017).

3.9.1. Documents Analysis

The document analysis includes CAPS documents for grades 4 to 6, Mathematics Intermediate Phase Abridged Section 4, 2021 Revised Annual Teaching Plan (RATP) per grade, Lesson Plans for the topics presented (numeric and geometric patterns), and any other relevant teaching documents. The adoption of document analysis as a technique generating data is based on the notion that professional growth can be understood within a legitimate framework (Mukeredzi, 2009). Document analysis was conducted during the reflection meetings, to anticipate some pertinent questions that might arise during the lesson observation, as the observers did not give instructions or ask questions during the lesson presentation.

3.9.2 Observations

Observations were conducted during the lesson presentations after the research lesson planning meeting. During the lesson planning meeting, educators selected research lessons to implement the objectives and collaborative lesson planning. This took the form of lesson observations or in-class observations, with 3 lesson presentations observed per grade. However, 8 instead of 9 educators were observed due to certain constraints (observations were videotaped and transcribed). Each observed lesson lasted one hour in the presenter′s classroom. To maintain participant anonymity, participants' faces were obscured using Face Blur technology. However, where possible, the researcher attempted to balance the genders of the lesson presenters.

However, even if bias did occur, it would not affect the results because the focus was not on gender or the influence of gender attributions. Non-probability sampling, purposive sampling, and self-selected sampling were used. Non-probability sampling was used in selecting PLCs (individuals) from the objective population (mathematics educators) that meet specific practical criteria. The researcher targeted only Intermediate Phase mathematics educators for purposive sampling. In addition, self-selected sampling was used since the 2 PLCs were having more number of educators and easily reached by the researcher. This type of sampling often promotes engagement in the process and a willingness to provide more insight into the phenomenon being studied. The lesson observation instrument was used. This lesson observation instrument was developed by the researcher with assistance from his supervisors.

3.9.3. Reflection Meeting

The reflection meeting was conducted on the same day after each lesson observation (which was videotaped and transcribed after consent was obtained from the participants). A lesson observation instrument was used because it included reflection. The following aspects were considered: the facilitator of the lesson as the first person to reflect on how to improve a lesson, how to improve teaching skills, how to develop educators' PCK, and not to critique the presenter (Yurnetti, 2018). During lesson observations and reflections, meeting educators was according to the grades they teach.

3.9.4. Interviews

After each reflection session, a lesson presenter and educators who had observed a lesson were interviewed. Semi-structured interview questions were used to capture the motivations for presenters′ actions. The interview addressed Research Question 3 and was conducted in a semi-structured interview format. It lasted a minimum of 45 minutes and a maximum of 60 minutes. The interview was recorded to accurately capture responses and allow the interviewee to respond to the researcher and answer all questions rather than focusing on writing brief notes.

3.10. Piloting instruments

According to Israel (2016), a pilot study is a minor study conducted prior to a planned project to test components of the research design and allow for any adjustments before the design is fully implemented. As recommended by Groh (2018) for conducting pilot studies, the pilot study must be conducted with at least one participant or a representation of 10% of the expected sample size. Thus, the pilot study serves as a dress rehearsal for the full-scale data collection that will occur after the pilot study is completed. Pilot studies are in most cases critical to the outcome of the final study, so the researcher must approach participants with full confidence and attention. The key measures must be completed and the overall commitment must be maintained so that any gaps in the research can be addressed effectively before the actual research begins. This also prevents researchers from squandering too much time, effort, and resources (Groh, 2018; Israel, 2016).

As indicated earlier, a PLC is a group of professionals or experts who come together voluntarily to learn with and from each other about needs they have identified. For the actual research, 2 PLCs were selected with a total of 7 schools; PLC 1 consists of 3 schools and 13 educators, while PLC 2 consists of 4 schools and 18 educators. These PLCs were the researcher′s conscious decision in terms of distance and geographical location. This study was piloted in 1 PLC, excluding the participating PLCs. The pilot PLC consists of 3 schools and 10 mathematics educators for grades 4 – 6. The results and feedback from the pilot were used to prepare and conduct the actual study in more detail.

3.11. Data Analysis Approach and Procedures

Data analysis, according to Marshall and Rossman (2014), is the process of giving order, meaning and structure to a large amount of data. It is described as a convoluted, indeterminate, and lengthy process, but also as an imaginative and fascinating process. In general, it is an activity of giving meaning to, interpreting, and theorising data, which is a search for general assertions among categories of data, but it is not linear (Cohen, Manion & Morrison, 2018). It can be concluded that data analysis requires the use of logic for investigation. In this regard, Best and Khan (2010) unequivocally state that data analysis and interpretation requires the use of deductive and inductive logic in research. According to Bertram & Christiansen (2014), the

interpretive technique, which involves deduction from the data obtained, places a greater emphasis on how it feels to be a participant in the action being studied, which is part of qualitative research.

The analysis of this study was supported by the Thematic Analysis Approach. Video, audio, and sound recordings were transcribed, anonymised, coded, and organised into themes. Cohen, Manion and Morrison (2018) characterise Thematic Analysis as the search for themes that prove relevant to explaining the phenomenon. I read the transcribed lesson observation notes, as well as the transcripts of the audio, interview, and video recordings repeatedly to understand the participants′ views. In this way, the researcher was able to get a sense of why participants were using the lesson study approach.

3.12. Phases of Thematic Analysis Approach

The practice of discovering patterns or themes in qualitative data is known as thematic analysis. According to Braun and Clarke (2006, p.78), it is the first qualitative method and master because "… it provides essential skills that are useful for conducting many other types of analysis." Another advantage is that it is a method rather than a methodology, which is beneficial for learning and teaching (Clarke & Braun, 2014). Unlike many qualitative methods, it is not led by a particular epistemological or theoretical position. As a result, it is a very flexible strategy, which is a great advantage given the diversity of activities in learning and teaching.

In this study, Thematic Analysis was applied. It is often argued that the key to meaning lies in the interpretation (Bree & Gallagher, 2016; O'Leary, 2014), emphasising that the data obtained only make sense if they are carefully examined and understood. According to Ibrahim (2012), Thematic Analysis is the most appropriate method to discover through interpretation. In this study, I used Braun and Clarke's (2006) six-phase guide, which is a very effective paradigm for conducting Thematic Analysis.

3.12.1. Phase 1: Become Familiar with the Data

Reading and re-reading the transcripts is the first phase in any qualitative study. Before proceeding, the researcher should be familiar with the entire data, corpus (i.e., all interviews and any additional data that may be used). At this point, it is helpful to take notes and jot down initial impressions.

3.12.2. Phase 2: Generate Initial Codes

In this step, we begin to organize the data in a meaningful and methodical way. Coding breaks down large amounts of data into manageable fragments of information. There are several ways to code, and research goals and questions determine which one to use. The researcher coded openly, meaning that he did not use pre-written scripts, but created and modified them during the coding process.

3.12.3. Phase 3: Search for Themes

A theme is a pattern that says something intriguing or meaningful about the data and/or the Research Question. As Braun and Clarke (2006) illustrate, there are not set guidelines for what constitutes a theme. It is the relevance of a theme that makes it so. With a very small data set (e.g. a short focus group), there can be considerable overlap between the coding step and this stage of looking for early themes. In this example, the researcher examined the codes and found that some of them could obviously be assigned to another theme. The codes would then be organized into larger themes that would say something definite about the study topics at the end of this process.

3.12.4. Phase 4: Review of Themes

In this step, the researcher reviewed, revised, and expanded the preliminary ideas he had found earlier. This was to see if they were logical. It is a good idea to collect all relevant data on each theme at this stage. As reiterated by Bree and Gallagher, I can easily do this with the 'cut and paste' function in any word processing programme, with scissors in my transcripts, or with Microsoft Excel (Bree & Gallagher, 2016). Again, having access to qualitative data analysis software helps make this process faster and smoother, but it is not required.

3.12.5. Phase 5: Defining and Naming Themes

The goal of this final refinement of themes is to "...find the essence of what each theme is about" (Braun & Clarke, 2006). This makes it clear what the theme is saying and, if there are sub-themes, how they interact with and relate to the main topic and how the themes relate to each other. In this study, what learners want from their feedback is an overarching concept embedded in the other themes.

3.12.6. Phase 6: Producing the Report

A report, a journal article or thesis, is usually the result of a study. In this example, the researcher planned to write a thesis on the function of educators' PCK in teaching mathematics in rural primary schools in KwaZulu-Natal. Thematic Analysis of qualitative data can be difficult, especially for new researchers. The researcher applied Braun and Clarke's (2006) framework of Thematic Analysis to data from learning and teaching studies to make the 'how' of analysis transparent. Throughout the study, the researcher worked near a tape recorder that allowed him to record what was said or observed (with permission) and cross-referenced it with notes taken during the sessions (Cohen, Manion & Morrison, 2018). Tape recordings have the advantage of providing access to the entire record of a potentially rich data source. Transcription of the tape is required for data analysis.

3.13. Trustworthiness Issues

The trustworthiness of qualitative research can be established by applying these four strategies: audibility, confirmability, credibility and transferability (Buchholtz, 2019). As a non-participant observer, the researcher used an observation guide and wrote field notes to record not only what he saw and heard, but also reflections on what happened. This study considers these strategies to achieve trustworthiness. To assess the trustworthiness and transparency of qualitative research, the researcher used aspects of Guba and Lincoln's (2011) quality criteria for naturalistic inquiry, which are widely used in the social sciences. The process by which qualitative researchers achieve the criteria of i) dependability; ii) credibility; iii) transferability; and iv) confirmability is referred to as trustworthiness (Patton, 2002; Harrison, MacGibbon & Morton, 2001). To ensure that this study was trustworthy, the researcher used a multiple-case site and a variety of participants (educators) to collect data, validate, and verify the validity of his conclusions (Yin, 2014).

3.13.1. Dependability

The extent to which research techniques are documented and allow someone outside the research to observe, review, and evaluate the research process is referred to as dependability (Moon et al., 2016; Gunawan, 2015; Polit & Beck, 2014; Streubert & Carpenter, 2011). The researcher ensured that the results depended on the use of competent study designs which means that many triangulations of data collection methods led to the dependability of the research findings. According to Guba and Lincoln (2011), a reliable study must be accurate and consistent.

Stepwise replication and inquiry audit are two methods used to assess the dependability of data. In stepwise replication, a group of researchers is divided into two teams that conduct different studies to compare data and results. In an inquiry audit, an external reviewer examines the data and supporting documents. This is done to ensure that the results are accurate and that the data obtained support the conclusions. All interpretations and conclusions are reviewed to determine if they are supported by the data.

3.13.2. Credibility

Credibility of any research is achieved through a detailed "rich, dense" description of the setting, participants, and emerging themes (Creswell & Creswell, 2017). Credibility refers to the degree to which the research reflects the study participants' true meanings or the truth value (Moon et al., 2016; Gunawan, 2015). As a result, the appropriate and applicable theoretical framework is used in this study. In addition, triangulation was applied, which includes the analysis of observational data and a reflective notebook. It is described as the combination of data or methods to illuminate a topic from multiple perspectives (Moon et al., 2016). It is often though that data triangulation, or the mixing of types of data, helps validate statements that may emerge from an initial pilot study. Mixing approaches is a more informative form of triangulation.

The researcher offered thorough, detailed descriptions of the setting, participants, and themes that emerged from the data to increase trustworthiness. According to Gunawan (2015), researcher credibility is particularly important in qualitative research

because the researcher is a key instrument in data collection and processing. Consequently, credibility is defined as the congruence of how the researcher develops, evaluates, and presents research findings and the meanings and perspectives of research participants (Merriam & Tisdell, 2015).

3.13.3. Transferability

In qualitative research, the concept of transferability is used to determine the extent to which findings can be generalised (Creswell & Creswell, 2017). According to Gunawan (2015), transferability is achieved by describing settings point-by-point and completely to provide enough evidence to assess the relevance of the findings to different contexts. According to Moon et al. (2016), transferability refers to the extent to which the results of this study can be safely applied to a larger population of primary schools with a variety of classroom management styles. Therefore, the researcher demonstrated transferability by showing the reader that while the results of the study cannot be generalised, they are applicable to a variety of locations, situations, times, and populations.

In this study, transferability is achieved through a dense representation of the research sites. Anney (2014) notes that triangulation encourages the researcher to mitigate methodological bias and examine the honesty of participants' responses. To minimise bias, the researcher used note taking and observational tools to guide his data collection so that he could focus on the essential aspects relevant to this study.

3.13.4. Conformability

Confirmability involves the use of different techniques in a study so that data, findings, or phenomena confirm each other (Buchholtz, 2019). Gunawan (2015) believes that a critical criterion for confirmability is the extent to which researchers concede their own biases. Anney (2014) agrees that researchers should ensure that research findings come from their data. The final element of trustworthiness that a qualitative researcher must establish is conformity. Korstjens and Moser (2018) define this criterion as the degree of confidence that the conclusions of the research study are based on the narratives and words of the participants and not on likely biases of the researcher. Researchers must demonstrate that the data are clearly linked to the conclusions in a

way that can be replicated and duplicated as a process to achieve compliance (Moon et al., 2016; Gunawan, 2015).

The researcher used this to establish conformity in the findings of this study by conducting an audit trail explaining the details and processes of data generation. The researcher collated the codes and explained what the themes meant. To achieve reflexivity, I kept and maintained a reflexive journal throughout the data generation process. I achieved confirmability through 'members checking' where participants confirmed the verbatim transcriptions. The researcher honestly reported the data and findings without fabricating, distorting, or misrepresenting them.

3.14. Ethical Considerations

Research ethics is concerned with the interaction between researchers and the participants they study (Cohen, Manion & Morrison, 2017). In this study, research ethics were observed according to the Belmont report and Orb, Eisenhauer, and Wynaden (2001). Ethical considerations in research are crucial according to Swartling et al. (2014) and McMillan & Schumacher (2010). Ethics can be traced back to ancient Greek philosophical investigations into the moral life. It is a set of principles with the ability to radically modify prior decisions and actions.

According to Johnstone (2009), ethics is a discipline of philosophy that deals with the dynamics of decision making about what is right and wrong. Individual, community, and social values inform scientific study as well as other human endeavours. Research ethics includes criteria for daily work, maintaining the dignity of the individual, and disseminating research findings. Ethics are the rules or guidelines for behaviour that distinguish between right and wrong and help researchers distinguish between acceptable and inappropriate behaviour. The basic ideas are that researchers should treat participants with respect.

According to Best and Khan (2010), informed consent requires that the participant be fully informed about the procedures used, the risks involved, and the demands that may be placed on them. All participants signed a written informed consent form (Creswell & Creswell, 2017; Fouka & Mantzorou, 2011). Participants understood that their participation was voluntary and that there would be no consequences for refusing

to participate in the study or to answering or not answering certain questions. All participants had the right to privacy. This ethical principle requires that participants′ names or identities not be revealed (Kour, 2014; Best & Khan, 2010). Participants were assured that their right to anonymity would be fully honoured, both orally and in writing. To protect all participants and the selected schools, pseudonyms were used in this study.

Participants also had the right to confidentiality (Mugenda, 2011). This has to do with the individuals who will have access to the data collected. Creswell and Creswell (2017) advise researchers to protect the dignity and identity of participants by ensuring that the information collected remains confidential when the full thesis is published and when presentations are made at conferences and journal articles are published. The researcher informed the participants that they may benefit from the study by being given the opportunity to express their views and experiences about lesson study as a method of professional development, which could influence department officials. They will also be able to view the results at the Durban University of Technology library, where a copy will be available on the internet.

3.15. Limitations of the Study

When researchers conduct a study, there are several potential threats to the study. Simon (2011) explains that limitations are possible weaknesses in a study that are beyond one's control. Financial resources negatively impacted the study because the researcher was so driven to desperation. My efforts to conduct sound and comprehensive research have some cost implications. Therefore, the researcher only included the Ilembe District in South Africa, KwaZulu-Natal Province. These schools were chosen because they are not in his work jurisdiction and were deliberately chosen to ensure that the results of the study are authentic.

The researcher understands that data analysis takes time to conduct this study. Transcribing extensive qualitative data could be time consuming, so the researcher analysed the data as collected. The researcher used language with which the participants were familiar. Specifically, the researcher allowed the use of English and any other language recognised in South Africa so that participants could understand and communicate freely. The distance to the area where the researcher conducted

research was a challenge because it is not near. Therefore, the researcher had to plan effectively so that when he takes the time to travel to the research site, he should make sure that several planned activities were conducted. The researcher took adequate time to complete this research study.

3.16. Chapter Conclusion

The qualitative research design and methodology employed in this study were outlined and justified in this chapter. The research questions and objectives informed the design and methodology choices. The case study research design was chosen since it explained and discussed the techniques utilized to choose participants. Subsequently, a summary of the research context was given. The methodologies utilized to generate and analyze the data, as well as the efforts taken to ensure the study's reliability, were highlighted. The chapter came to a close with a discussion of the ethical considerations that were made, as well as the study's limitations and how they were addressed.

The data is analysed and interpreted in the following chapter.

CHAPTER 4 DATA PRESENTATION AND ANALYSIS OF FINDINGS

4.1. Introduction

The previous chapter discussed the study's research design and methodology, as well as the locations where the study was conducted. To answer the critical research questions, this chapter presented the findings, analysis, and evaluation of the data collected from the 2 PLCs. They were gathered through the participating educators' field notes and interviews about their participation in lesson study. The first session illustrates the background of the two PLCs and the participants, including information about their practices and perceptions of mathematics teaching. The aim was to gain an overview of the research context before the participants took part in the lesson study.

The second section chronologically describes the events and implementation of lesson study at the two PLCs. As mentioned earlier, the plan in this study was to conduct three lesson study cycles in each PLC within the allotted period of six months. Both PLCs were able to undertake 2 lesson study cycles by the end of the research. The smaller number of cycles was due to the constraints in conducting lesson study in schools, as well as the pandemic COVID-19. The events in the conduct of the lesson study cycles are discussed separately for the 2 PLCs as the context and circumstances were quite different. The first lesson study cycle in each PLC is described in detail, however, for subsequent cycles, only the main and relevant episodes are disclosed and discussed to provide the relevant data for analysis in the next section.

The third section highlights the findings and emerging themes of this study. Specifically, the themes of influence on participants' PCK and reflective practice which are central to the research questions, are discussed to conceptualise the influence of lesson study. This section also addresses the constraints and challenges of implementing the lesson study. Finally, in comparing the implementation of lesson study in the 2 PLCs, some supporting factors that emerge from the study are elaborated.

This study focused on one key question: What is the role of lesson study on educators' pedagogical content knowledge in teaching mathematics? To answer this key question, the following three sub-research questions needed to be answered:

1. What do educators need to enhance their pedagogical content knowledge of teaching numeric and geometric patterns?
2. How does educators' participation in lesson study enhance their pedagogical content knowledge about teaching of numeric and geometric patterns?
3. What are the educators' views about the lesson study when teaching numeric and geometric patterns?

4.2. Background of the Lesson Study PLCs

4.2.1. An Introduction to PLC 1

PLC 1 was formed by typical national primary schools located in a township area. As such, the area was easily accessible by public transport and pupil transport. Learners′ enrolment per school ranged from 700 to 1300, from grade R to grade 7 and were mainly drawn from the township and nearby informal settlement areas. In terms of infrastructure, the schools were equipped with basic facilities: classrooms, computer labs and toilets with running water.

School T consisted of 22 teaching staff: Principal, Acting Deputy Principal, 3 Departmental Heads (DHs) previously known as Head of Departments (HoDs) (1 acting and 2 permanent) and 17 Post Level 1 (PL1) educators; School L consisted of 28 teaching staff: Principal, Deputy Principal, 5 Departmental Heads (1 acting and 4 permanent) and 21 PL1 educators; and School S consisted of 46 teaching staff: Principal, 2 Deputy Principals, 8 DHs and 35 PL1 educators. Most of the educators lived in the township, while others lived in a nearby informal settlement and a town about 20 to 25 km from the school sites. Daily classes at the PLC 1 schools started at 08:00 and ended at 14:30, except for grades R and 1, which ended at 12:30. On Friday, however, classes ended earlier at 13:00.

The principals of School T and L were male, while the principal of School S was a female. The principals of these 3 schools from PLC 1 were dynamic and often motivate their staff to strive and improve their teaching. They often walked around to check if teaching and learning was being done properly by the educators in the classroom. In all 3 schools of PLC 1, the subject of English was taught as the English First Additional Language (EFAL). However, in School L, learners in grades 4 to 7 are encouraged to speak English even in the playing grounds.

When the researcher approached principals to introduce and explain the concept of lesson study for educators' professional development, they were excited and immediately gave him their consent for the study. During the interaction, the principal of School L told the researcher that he had instructed the DHs that educators should work together by subject and help each other to solve their teaching problems, however, he has not yet observed his educators following his advice. The principals of the PLC 1 schools, after their initial perception of lesson study, were convinced that such a non-workshop approach to professional development could help educators to enhance their teaching practices.

The researcher was then directed to consult with the Intermediate Phase DHs who were directly responsible for mathematics educators in grades 4 to 6 to discuss details about the lesson study research project. The DHs were as enthusiastic about the lesson study as the principals and they too hoped that all Intermediate Phase mathematics educators could be involved in the project. The researcher then sought advice from the DHs, who suggested that the lesson study meeting be held once a week (after school hours) and that the venues be rotated within the PLC 1 schools, taking into consideration the schools' changing schedules due to the COVID-19 pandemic, busy schedules and activities.

Participants in PLC 1 (see Table 3) were composed of novice and experienced educators, as measured by the number of years they had been teaching mathematics. 3 of the13 participants recruited from PLC 1 schools withdrew from the study. Of the 10 participants, 6 participants have a teaching qualification as their first qualification, while 4 others have specialised in areas other than teaching. All 6 participants with a teaching qualification as their initial qualification studied mathematics as a subject in

their teacher training. When the School Management Teams (SMTs) agreed to this research, the mathematics educators were not consulted by the researcher beforehand. Nevertheless, the DHs assured the researcher that the educators would comply and cooperate if the SMTs approved. One of the 10 participants had never taught mathematics in primary school as he had recently been promoted from secondary school to Deputy Principal educator.

4.2.2. An Introduction to PLC 2

Unlike PLC 1, PLC 2 schools were situated within a semi-urban vicinity and located 5 km away from town. The accessibility of transport in PLC 2 is similar to PLC 1. As such, the area was easily accessible by public and pupil transport. Learners′ enrolment per school ranged from 350 to 1350, from grade R to grade 7 and were mainly drawn from the semi-urban and nearby informal settlement or rural areas. In terms of infrastructure, the schools were equipped with basic facilities: classrooms, multi-purpose class and toilets with running water. All 4 schools within PLC 2 do not provide hostel facilities to all learners.

School H consisted of 32 teaching staff: Acting Principal, 2 Deputy Principals (1 acting and 1 permanent), 6 DHs and 23 PL1 educators; School D consisted of 30 teaching staff: Principal, Deputy Principal (other Deputy Principal post is vacant), 5 Departmental Heads and 23 PL1 educators; School M consisted of 48 teaching staff: Principal, 2 Deputy Principals, 8 Departmental Heads (DHs) and 37 PL1 educators; and School P consisted of 9 teaching staff: Principal, 2 DHs (1 acting and 1 permanent) and 6 PL1 educators. Most of the educators lived in the nearby town, while others lived around the school sites. Daily classes at the PLC 2 schools started at 07:40 and ended at 14:00, except for grades R and 1, which ended at 12:00. On Friday, classes ended earlier at 13:00 similar to PLC 1 schools.

All principals of PLC 2 schools were female. The principal of School P teaches all grades in the Foundation Phase, while the other principals teach only one class in the Intermediate Phase / Senior Phase. In schools′ D and M of PLC 2, the subject of English was taught as EFAL, while in the other two Schools H and P, English was taught as the mother tongue / Home Language (HL).

The principals of PLC 2 gave the researcher their consent for the study after the researcher approached the principals to introduce and explain the concept of lesson study for educators' professional development. The researcher was then directed to consult with the Intermediate Phase DHs who were directly responsible for the mathematics educators in grades 4 to 6 to discuss details about the lesson study research project. However, 3 DHs did not teach mathematics as a subject. The researcher then sought advice from the DHs who suggested that the lesson study should be conducted at least twice a week (after school hours) if possible and that the venue should be at School P as it had free classrooms, bearing in mind that furniture and classrooms from other schools were not available due to the pandemic COVID-19. The DH of School P later became the coordinator of the lesson study and was fully supportive and committed to the implementation of the lesson study. On many occasions during the study, he was helpful in planning and making arrangements for participants to engage in the lesson study process.

4.2.3. An Introduction to PLC 2

To safeguard the identity of the schools and participants (educators) involved, codes were used when referring to the participating schools and participants. The pseudo names were used for both schools and participants (educators). Schools are named using the first letter of the school name such as School M. The participants are named using the first letter of the school, first two initials and the first letter of the surname, such as educator LNSM, this educator is from School L.

Table 3: Distribution of Participants by Academic Qualification

No.	Educator	Gender	Qualification(s)	Total Teaching Experience	Mathematics Teaching Experience	Grade(s) Taught	LoLT
1	LNSM	Male	Advanced Cert in Education	20 years	20 years	Grades 6 - 12	EFAL
2	SBPM	Female	Bachelor of Education (Hons)	10 years	10 years	Grade 6 and 7	EFAL
3	TTZN	Female	Bachelor of Education (Hons)	7 years	7 years	Grade 4 and 5	EFAL
4	SPMM	Female	Bachelor of Education (Hons)	7 years	6 years	Grades 5 - 7	EFAL
5	LPSK	Male	Post Grad Dip in Education	4 years	4 years	Grade 6, 7 and 9	EFAL
6	LPFM	Female	Advanced Cert in Education	15 years	15 years	Grade 6 and 7	EFAL
7	SLFM	Female	Bachelor of Education (IP)	6 years	2 years	Grade 6	EFAL
8	LSTM	Female	National Primary Dip in Educa	13 years	3 years	Grade 6 and 7	EFAL
9	SBSM	Male	Post Grad Dip in Education	5 years	5 years	Grade 6 and 7	EFAL
10	TMBN	Male	Post Grad Dip in Education	7 years	7 years	Grade 6 and 7	EFAL
11	PSPM	Male	Bachelor of Education (Hons)	14 years	11 years	Grade 6 and 7	EHL
12	DSSZ	Male	Post Grad Dip in Education	8 years	7 years	Grade 6 and 7	EFAL
13	MTAM	Female	Advanced Cert in Education	18 years	18 years	Grade 6 and 7	EFAL
14	HMSB	Male	Bachelor of Education (InterSen)	6 years	5 years	Grade 6 and 7	EHL
15	MGPT	Female	Bachelor of Education (FP)	1 year	1 year	Grade 4	EFAL
16	MMNM	Female	Bachelor of Education (Hons)	15 years	13 years	Grade 6 and 7	EHL
17	HPNM	Female	Bachelor of Education (InterSen)	9 years	9 years	Grade 5 and 6	EHL
18	PTBM	Male	Bachelor of Education (IP)	5 years	5 years	Grades 4 - 7	EHL
19	MSCM	Male	Bachelor of Education (FP & IP)	8 years	8 years	Grade 5	EFAL
20	HTSM	Female	Further Dip in Education	24 years	16 years	Grade 4, 6 and 7	EHL
21	DSZM	Female	Post Grad Dip in Education	6 years	4 years	Grade 4 and 7	EFAL
22	DNNM	Female	Bachelor of Education (Hons)	27 years	20 years	Grades 4 - 7	EHL
23	HATG	Female	Advanced Cert in Education	16 years	16 years	Grades 3 - 5	EHL

In August 2021, educator MMTD joined the lesson study PLC 2 as a participant when she was temporary appointed as a substitute for educator MSCM to School M. Compared to PLC 1, the participants of PLC 2 had relatively many years of mathematics teaching experience, except educator MGPT, who had only 1 year of teaching experience. 5 of the18 participants recruited from PLC 2 schools withdrew from the study. Of the 13 participants, 11 participants have a teaching qualification as their first qualification while 2 others have specialised in areas other than teaching. All 11 participants with a teaching qualification as their first qualification studied mathematics as a subject in their teacher training, but educator MGPT majored in mathematics at the Foundation Phase level. Educator MGPT said,

> "*I am an educator in the Foundation Phase. The pandemic caused educators to take long leaves and no substitutes were employed, so there were reshuffling in duty loads*."

4.3. Response Analysis

A total of 23 educators were eventually able to participate in the study; 8 educators were unable to participate for various reasons. Although the schools are located in the same district (Ilembe District), the reason for conducting two separate PLCs was to obtain reliable and valid data since these clusters of schools are far apart (almost 100 km apart). This contributed a better understanding that was not limited to one geographic location or community and contributed to a more meaningful understanding during the observations as the observations were conducted in schools in different communities and therefore had different backgrounds and experiences. The researcher spread the participants across different schools as much as possible to meet the requirements of the study. These observations also highlighted the impact that the environment, communities and backgrounds have on PLCs and teaching practice as a whole. Fugard and Potts (2015) stress the importance of sample size and its careful selection as the whole research depends on the sample. Although it is not the focus of the study, the researcher also considers the gender and age of the participants.

Table 4: Distribution of Participants by Academic Qualification

Qualification	Number of Participants
Diploma (*including other certificates***)**	**11**
Degree (*or higher***)**	**12**

Knowledge of qualifications helped the researcher determine whether different academic qualifications had an impact on the educators' contribution to PLC and how this ultimately affected their pedagogical content knowledge. Boekaerts (2002) asserts that practical understanding and skills are built through classroom interactions and dialogues by professional educators whose goal is not only to teach or present new knowledge or content, but also to ensure that individual learners learn and progress at a deeper level. To say that this can be done by any unqualified person with expertise, or to compare it to child rearing, is condescending and ridiculous.

Table 5: Distribution of Participants by Years of Experience

Years of Experience	Number of participants
1 – 5	**8**
6 – 10	**7**
11 – 15	**3**
16 – 20	**5**

The researcher collected data over the years of the participants' experience. The goal was to gain more insight into the impact of years of experience on the content of pedagogical content knowledge and how educators with different years of experience relate to each other during PLC meetings and lesson study. According to Ingersoll, Merrill and Stuckey (2014), support from educators′ professional work environment affects their progress over time. Educators with previous expertise in the same grade level, subject, or district also had a higher return experience than those with less relevant prior experience.

Of the 23 participants: 8 participants have taught mathematics for 1 to 5 years; 7 participants have taught mathematics for 6 to 10 years; 3 participants have taught mathematics for 11 to 15 years; and 5 participants have taught mathematics for 16 to 20 years. The average number of years of teaching experience among the 23 participants was 9 years, with the lowest number of years of experience being 1 year and the highest number of years of experience being 20 years.

4.4. Process of Implementing Lesson Study Cycles

In this section, the chronology of events resulting from the participants' involvement in the lesson study process was explained and discussed. The aim was to explore and investigate the role of lesson study on the participants, particularly pedagogical content knowledge and reflective practice. This study also aimed to explore the constraints and challenges of implementing lesson study in the education context of KZN DoE Ilembe District.

Within the time allotted for this study, both PLCs conducted two lesson study cycles. In general, you should plan for approximately 12 hours of work for a lesson study cycle. A typical cycle programme might include 8 one-hour weekly sessions, 3 hours for the research or planning lesson and post-lesson discussion, and one hour for overall reflection. However, only one cycle could be conducted in each term (an academic year has four terms). It was found that each lesson study cycle extended over a period of more than two months due to various constraints in the implementation and consequently, reducing the number of lesson study cycles planned for this study.

Participants indicated that they did not have enough time for lesson study because they planned as a Phase rather than a grade, that they were behind on RATPs, and that they were overloaded due to COVID-19 regulations. All the schools that participated in the study introduced an alternating time-table for learners during the pandemic. They also lamented about the heavy workload in marking learners' activities in the exercise books and DBE workbooks given to them when they were not in school. Therefore, only one lesson study cycle was conducted in each term.

4.4.1. PLC 1: First Lesson Study Cycle

In addition to participating in the lesson study, participants also attended a workshop aimed at introducing and explaining the concept and practice of the lesson study as an innovative form of teacher professional development. This workshop was conducted by the subject advisors for the mathematics curriculum in Ilembe District at the request of the researcher. The PLC coordinator (educator LPFM) was also elected in this workshop.

Although this was the first attempt by the participating educators to engage in lesson study, it was conducted systematically following the model of lesson study advocated by Alamri (2020).

The decision to conduct the lesson study weekly after school hours was well received by all participants, who preferred not to disrupt teaching and learning, however, they preferred not to change the meeting location. Generally, educators are allowed to leave the school after dismissal unless there is a meeting or school activity that compels them to stay in the school. Consequently, all the participants behaved well and cooperated fully in participating in the lesson study as they were first consulted before gatekeepers which is the SMTs.

This first lesson study cycle of PLC 1 was implemented with success. A major reason was the initiative and strong commitment of the lesson study PLC coordinator. She took a proactive approach and adopted a positive attitude towards the implementation of the lesson study. She not only actively participated in the lesson study, but also volunteered to be the first to conduct the lesson in this cycle. Her actions, and positive attitude motivated and encouraged the other participants to some extent to participate in the lesson study. Therefore, the smooth implementation of this lesson study cycle is partly due to the work, initiative and enthusiasm of all the participants.

4.4.1.1. First PLC 1 Planning Meeting

All 10 participants attended the planning meeting, which was held in the school boardroom after school dismissal. At the beginning, the participants were sceptical and unsure about how they were going to be involved in the lesson study despite the workshop that had been conducted earlier. The researcher briefly explained to them what the lesson study was all about as it was important that the participants fully understood the aims and rationale of the study. The educator LPFM then complied with the request to lead the group but was hesitant about the steps of the lesson study process. This indicated that the lesson study workshop conducted may have been inadequate as the concept and practice of lesson study was relatively alien to the teaching culture of the school. In certain circumstances, the researcher supported educator LPFM in the lead coordination role to lead the lesson study PLC meeting. Educator LPFM never dominated the discussion but fully involved the other participants in the discussion as lesson study is a highly collaborative group work effort.

The researcher informed the participants that the study proposed the topic of numeric and geometric patterns in the mathematics curriculum for grades 4 to 6 for the lesson study discussion and the participants collectively agreed after considering the lesson study timetable

and RATPs for this phase. However, participants agreed to focus on numeric patterns during the first lesson study cycle and present grades 5 and 6. Prior to discussing the content, the researcher inquired from the participants about the purpose of the lesson study as suggested in the lesson study model, and they seemed to understand it. Therefore, after a brief discussion, the participants collectively agreed that the aim of the lesson study was to *increase learner engagement or to adopt a learner-centred approach*.

After analysing the RATPs with the contents of the textbook and reference books, the participants agreed that some learners have problems in understanding the concept of numeric patterns. Although the discussion was fruitful, some of the participants behaved passively, especially the grade 4 educators. To engage the participants, the researcher had to ask them appropriate questions, such as:

> "*Do they have problems with LoLT, especially in grade 4, because the learners were taught in IsiZulu in the Foundation Phase?"*
>
> "*What prior knowledge do the learners have?*" and
>
> "*What are the problems in teaching this topic?*"

Educator TTNZ, who teaches only grade 4, indicated that the learners encountered language challenges and educators SBSM and LPSK agreed with her, and they said even in grade 5, learners encountered language challenges. Gradually, the participants acted as they discussed and shared teaching problems with their colleagues.

Educator TTNZ said,

> "*I alternate between English and IsiZulu in teaching grade 4 because if I use only English all the time, most learners will not understand anything."*

According to educator TMBN, learners should know what a numeric pattern is. He further explained that it is

> "… *a sequence of numbers that follow a certain order."*

According to the participants, learners should be able to count forward (adding) or backward (subtracting) using a constant difference. Educator SBSM reiterated,

> "In g*rade 4, a constant difference or ratio is used of 1-digit number, but in grades 5 and 6, 2-digit or multi- numbers can also use a constant difference or ratio."*

Educator LNSM added,

> "*Learners need to know that in patterns there is a certain number that needs to be added or subtracted to get the next number. This number is called the common difference."*

Participants explained that it is challenging for them to teach learners how to determine or extend the pattern by multiplying the previous number by a certain term.

The educator LPFM was assigned the task of facilitating the discussion as the coordinator of the PLC. She effectively involved the participants in the planning session. The aim of this session was to provoke participants to think about alternative or effective teaching strategies. This provocation was intended to get participants to reflect on their knowledge and instructional practices, as is the case in the lesson study model. At the end of the discussion, participants teaching grades 5 and 6 were asked to prepare a draft lesson plan for the next discussion.

4.4.1.2. Second PLC 1 Planning Meeting

Nine participants attended this second planning meeting, except the educator LNSM who served as the principal that day. The planning meeting for the research lessons to be presented by educators SBPM and LPFM, who volunteered, continued as planned. Attendance reflected the positive attitudes of the participants and their positive perceptions of the practice of lesson study. The PLC coordinator said,

> "*I really hope that my colleagues will continue to be engaged in the lesson study and enhance their pedagogical content knowledge."*

At the beginning, participants were asked to briefly present their drafted lesson plans. Educator LNSM was absent yet submitted his draft lesson plan for grade 6. Educators LPFM and LSTM from School L used the same lesson plan template. The following items were addressed in all draft lesson plans presented during the planning meeting:

- Lesson objective/s
- Prior knowledge of the learners
- Mental Maths activity
- Educator and learner activities
- Resources

Educators SBPM and LPFM volunteered to teach the research lessons to grade 5 and 6, respectively. According to the PLC coordinator, the main purpose of this meeting was to review and refine the lesson plans designed for the research lessons. No changes were made to the sequence of steps in the lesson plans to be presented; however, participants spent a great deal of time reviewing the introductions, teaching models, and activities/worksheets. The rationale was to vary the questions with different cognitive levels and difficulty to suit the learners' abilities. Overall, the participants were active and the discussion was lively, the timing of the research lesson was also well chosen. In the end, the participants were happy and satisfied with the lesson plans as it was their first product or outcome after two planning meetings, however, the finalization of lesson plans to be presented will be done by the presenters.

4.4.1.3. Grade 5B1 PLC 1 Research Lesson Teaching

The lessons for the research lesson were planned and delivered in a multi-purpose class. The main reason was that the lesson observers had to maintain the COVID-19 social distance of 1.5 metres. The educator SBPM seemed a little nervous at the beginning of the research lesson. However, as the lesson progressed he seemed relaxed and excited, probably because this is the first time his lesson has been observed by a group of educators, as opposed to the Integrated Quality Management System (IQMS) where only supervisors and colleagues observe the lesson. All participants were present and on time.

Class began at 10:30; thus, all participants were seated 10 minutes before class began, as the first recess was 10:00 – 10:30. The learners were also on time as the bell rang 5 minutes earlier and they were informed about the research lesson taking place. The educator SBPM handed out the lesson plan and worksheet to all the observers while the researcher ensured that all the observers had the observation instrument.

The lesson plan was handwritten, but the worksheet was typed. The Content Area: *Patterns, Functions and Algebra* and sub-topic: *Numeric Patterns* were clearly indicated. The educator SBPM demonstrated a misconception of Language Across the Curriculum (LAC) and inclusivity. The educator stated five lesson objectives, learners should be able to:

- discover and create patterns.
- transfer their visual representations to numerical patterns.
- relate non-numerical patterns to the numerical patterns.

- identify the rule and the common difference; and
- identify the term in the pattern, whether it is an increasing or decreasing pattern.

In the introductory phase, the educator SBPM introduced the research lesson with an observation exercise. Learners had to investigate by how many beads each group had increased and then predict by how many beads the next groups would be composed. Then they were asked to transfer their visual representation to a numerical pattern. The educator instructions were clear and specific with the beads so that they should be easily understood by the learners. The introduction to the research lesson covered all the lesson objectives.

Next, learners were given an individual task (Figure 5) as learners cannot be grouped due to the provisions of COVID-19. The following questions were given to the learners to answer:

i. Find the common difference.
ii. Copy and complete the missing sequence of numbers.
iii. Find the rule of the pattern.
iv. Indicate whether the pattern is increasing or decreasing.

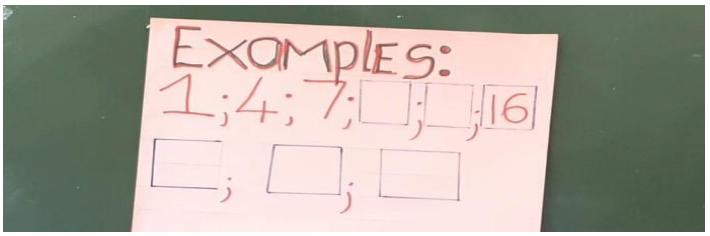


Figure 5: Grade 5B1 Whole Class Activity

Learners worked individually to answer the class work using their class workbook. However, some learners seemed to have difficulty in answering the questions, so the participants, especially educators SPMM and LSTM, helped the learners even though they had been reminded not to do so. It was then the learners' turn to present their answers on the board and some were able to give the correct answers. In short, this lesson introduced two strategies: *linking non-numeric patterns to the numeric pattern and determining the sequence number when certain position numbers are skipped*. Judging from the results, learners seemed to be able to understand the content of the research lesson, even though some could not give all the answers correctly.

Overall, the educator SBPM′s teaching was clear and he seemed confident. The teaching was systematic as he executed the lesson plan accordingly, however, the research lesson was not consolidated due to the length of time. While working with the learners, the participants moved around to observe the learners. The participants took their lesson observation seriously and wrote down the details. In terms of the aim of lesson study, the lesson was considered successful as the learners were actively engaged in the lesson.

4.4.1.4. Grade 6E PLC 1 Research Lesson Teaching

The second research lesson for PLC 1 was designed and implemented in the grade 6E classroom. The classroom had space for the participants (lesson observers) and learners so that the protocols of COVID-19 could be followed. The educator LPFM seemed confident in what she was doing and the lesson went smoothly as planned. Despite being observed by her Departmental Head as one of the participants, educator LPFM showed no signs of nervousness. All participants were present and on time.

Class began at 13:15, the last lesson for the day. The educator LPFM handed out the lesson plan and worksheet to all observers before the research lesson, while the researcher made sure that all observers had the observation instrument. The steps in the lesson plan were done systematically and more time was given to the learners (learner-centred). It was discovered that the learners took an active role in the research lesson. However, she was not able to complete the entire lesson as planned because she had given learners too much time to show their answers on the board.

The lesson plan was handwritten, but the appendices were typed. The Content Area: *Patterns, Functions and Algebra* and sub-topic: *Numeric Patterns* were clearly stated. The educator stated four lesson objectives, learners should be able to:

- Explain numeric patterns and the relationship between objects and patterns;
- Describe the elements of the flowchart or diagram;
- Find the input or output using the output or input and rule; and
- Describe inverse operations.

In the induction phase, the educator LPFM introduced the research lesson with a non-numerical pattern to a numerical pattern as shown in Figure 6. Learners had to investigate and find out how many squares the next three position numbers will have. Then they were asked to transfer their visual representation to a numerical pattern to increase by how many and then find the general rule of the pattern. The educator instructions were clear and specific so that they should be easily understood by the learners.

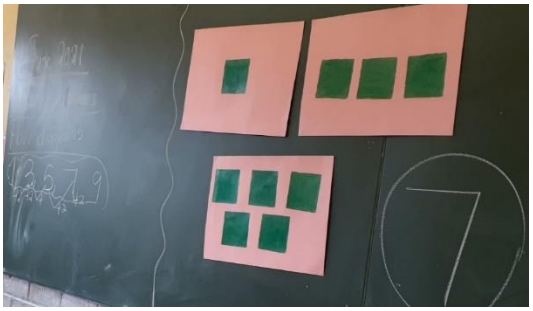


Figure 6: Grade 6E Sequence of the Squares in the Charts

Participants observed the lesson carefully and took notes. The learners were more prepared for the educator LPFM's lesson which was shown in the charts (see Figure 7) and covered lesson objectives 2 to 4.

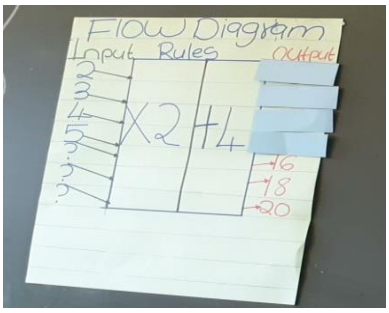


Figure 7: Grade 6E Flow Diagram

4.4.1.5. PLC 1: First Cycle Reflection of Research Lessons

The participants, both presenters and observers, stayed behind for a short reflection session on the same day that the research lesson presentations took place after school. The educator LTSM had previously informed the researcher and the PLC coordinator that she was unable to attend the lesson reflection as she had to attend a parent meeting. Under these circumstances, the participant was asked to submit her written comments on the lesson to the PLC coordinator.

Since the PLC coordinator was presenting a lesson that day, educator TMBN was chosen to lead the reflection session. Educator TMBN allowed the research lesson presenters (educators SBPM and LPFM) to reflect first, followed by the observers. However, the participants will reflect on the lesson of grade 5B1 first and then the lesson of grade 6E. Educator LTSM confirmed that the content of the lesson plans was carried out coherently and systematically. She praised the use of charts or teaching models to avoid spending a lot of time writing on the board. The educator SBSM added that:

> "*The lesson activities were projected on the charts but educator SBPM rewrote the activities on the board."*

During the reflection session the chair reminded participants "*not to engage in the research lesson they were observing*," without mentioning names.

The educator LPFM expressed satisfaction with her teaching, although she was not able to complete the research lesson as planned. In particular, she felt that although she was happy with her teaching, too many aspects were covered:

> "*I am happy and satisfied because the learners were able to follow the lesson, but the treatment of inverse operations confused the learners and resulted in too many aspects being covered in one research lesson."*

However, the participants argued differently and explained the importance of covering both (inputs and outputs) in a flow diagram. They felt that educator LPFM's lesson was clear and should be easily understood by the learners.

Moreover, educator SPMM emphasised that:

> "*As early as 4th grade, learners begin to calculate inputs from outputs. So, the concept of inverse operations of the rule should be understood by the learners in grade 6*."

Educator SLFM stated that:

> "*The presenter confused the learners about inverse operation, she only focused on the rule without considering the output, see Figure 8*."

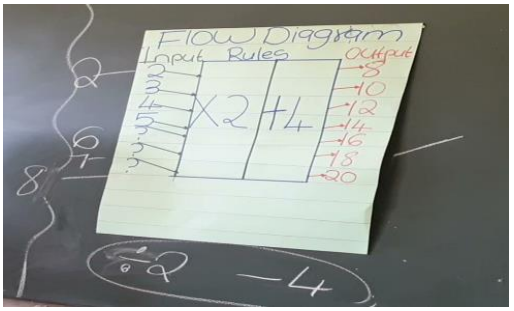


Figure 8: Grade 6E Incomplete Inverse Operation

Overall, there were not many comments from the participants. It was probably because this was their first lesson study cycle.

4.4.2. PLC 1: Second Lesson Study Cycle

For the second lesson study cycle, the school principals allowed the participants to rearrange the schedule so that they could start the planning session earlier at 13:00 and twice a week. The reason for the time change was School Governing Bodies (SGBs) election week when election meetings were held starting at 13:00. This was made possible by the fact that the participating educators were exempted from any teaching work from 12:40 as the teaching hours were shortened and reduced to five periods per day. Participants thus agreed to hold both planning meetings during this election week.

During the first lesson study cycle, the researcher observed that some participants felt tired and exhausted at the end of the day, but they never complained about it. The changes in session times alleviated the constraints of conducting the lesson study. The principals' move reaffirmed their support for the lesson study research project and likely influenced the outcome of this second cycle. Participants were pleased that planning meetings could begin earlier and it was noted that there was much positive engagement and participation.

According to Nurtanto et al. (2021), Tay, Lee and Ramachandran (2021), and Liptak (2005), one of the critical elements to the success of lesson study implementation is adequate time for educators. The educator SLFM felt that the second lesson study cycle went smoothly. As she explained,

> "The l*esson study process was actually very enjoyable maybe because we had already completed the first cycle."*

Educator TMBN completely agreed with educator SLFM about how smoothly the second lesson study cycle went. However, educator TMBN noted that the educators had become aware of the reasons for undertaking the lesson study. As he explained,

> "*After the first cycle, I realised that observation is not about whether the teacher is good or bad at teaching."*

Moreover, the educator SLFM explained,

> "*We actually observe teaching so I feel a little more comfortable because I was presenting and no one is criticising the presenters and the lesson plan. Our planning for the second cycle went well, so there was nothing to worry about."*

The second lesson study cycle took place from the 26th of July 2021 until the 3rd of September 2021.

For this cycle, participants chose the concept and content of *geometric patterns*. Two lesson planning meetings were held consecutively on Monday and Thursday (dated 2nd of August 2021 and 5th of August 2021). The presenters for the research lessons were educator TTNZ for grade 4 and educator SLFM for grade 6. The reflection for both lessons took place on the same day after school hours.

4.4.2.1. Third PLC 1 Planning Meeting

During the second lesson study cycle (third planning meeting for PLC1), participants select subtopics or concepts for discussion: *Investigating and extending patterns; from tables to flow diagrams; more pictures and tables (especially in grade 6); flow diagram and calculation plan*. After experiencing a complete lesson study cycle, the participants seemed to have understood and grasped the basic idea and practice of lesson study. This helped the participants to effectively conduct the lesson study in this cycle.

Although the participants chose *geometric patterns* as the topic that focused on the above five subtopics, they emphasised the *calculation plan* as a difficult and challenging concept. During this planning meeting and lesson study discussions, participants struggled with their content knowledge as most of them lacked effective teaching experience on the selected subtopics. Only educators SLFM, LNSM, and LPFM seemed confident in presenting the topic, with educator LNSM admitting that he had taught the topic at high school level before.

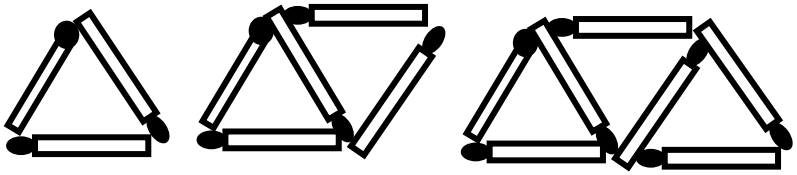
This suggests that some participants are not confident about their content. Lack of content knowledge and confidence in teaching was a constraint in the discussions as this could undermine educator's credibility. As highlighted by Henriksen et al. (2022) and Liptak (2005), an educator's fear and anxieties can arise due to cultural and personal factors which became an obstacle in conducting the lesson study. At the conclusion of the planning meeting, educators TTZN and SLFM agreed to present for grades 4 and 6 respectively.

4.4.2.2. Fourth PLC 1 Planning Meeting

In this planning meeting, participants collaboratively designed lesson plans for the research after they had an opportunity to research geometric patterns. Educator LSTM lamented that learners have difficulty performing inverse operations and finding the rule or calculation plan, especially in a flow diagram. From the tendency and nature of the discussion, it appears that the participants perceived the lesson study as a place where they can discuss their teaching problems. In fact, Boonsongsak (2022), Richit, da Ponte and Tomasi, and Takahashi (2005) suggested that one of the approaches to beginning a lesson study is to discuss mathematical concepts that are difficult for learners to understand.

During the first planning meeting where participants were introduced to the study, they thought patterns were simple content. Although the content of *patterns* was perceived as *easy* by the participants, the lesson study discussions made them think seriously about their knowledge of content. It undermined their confidence and knowledge of content when they could not provide convincing arguments and explanations to their peers about the concepts of *rule, calculation plan and inverse operations*.

Participants agreed to work together to complete an activity that would cover the teaching objectives of the next research lesson. Then the following activity was done by the participants together. The educator TTNZ distributed the box of matches to the participants:



Stage 1 Stage 2 Stage 3

(a) How many matchsticks are there in each stage?
(b) Predict the number of matchsticks for stage 4.
(c) Construct stage 4 with matchsticks and count the number of matchsticks used.
(d) Explain in your own words how many matchsticks will be used for stage 5 and give a reason for your answer.
(e) Classify the geometric pattern formed as either growing or repeating pattern.
(f) What is the difference of the number of matchsticks from each stage?
(g) Use a flow diagram to represent the relationship between the stage (input) and the number of matchsticks (output).

Participants were able to answer questions (a) to (f) easily, but (g) was challenging for most participants. Teacher LPSK emphasised that:

"We need to agree on the rule or calculation plan first because the flow diagram consist of inputs, rule and outputs."

Nevertheless, most of the participants were confused about the rule as they said that *we add +2 matchsticks from the previous stage to get the next stage* but adding +2 could not get any other outputs except the 1st output. Educator LNSM used the trial and error method to determine the rule. He concluded that the rule was $stage\ number/input \times 2 + 1$ and educator TMBS agreed with him.

Educator LPSK argued and disputed educator LNSM's rule on the grounds that:

"*In the previous lessons, the rule was formulated that we add the common difference of the previous sequence to get the next sequence"*

Other educators or participants completely agreed with him. He became defensive and other participants were also confused by the argument. The educator SLFM exclaimed,

"I am confused now! But I will discuss this with our subject advisors so that they can help us, if necessary they will come to the next session."

Such academic discourse could encourage participants to think deeply and reflect on their content knowledge. Although no final outcome was reached at the end of this planning meeting, engaging in the lesson study process has enhanced the participants' content knowledge.

In the second lesson study cycle of PLC 1, there were considerable events that warranted improvement in reflective practice among participants. Although the discussions did not lead to common conclusions regarding lesson plans for grades 4 and 6, the process encouraged participants to be more reflective. For example, an extraordinary suggestion was made by the educator LPFM,

> *"Whenever we teach patterns, whether numeric or geometric, the rule should be tested using a flow diagram."*

Participants fully agreed with this suggestion and felt that the calculation plan or rule still needs to be addressed in the research lessons teaching. The educators TTNZ and SLFM *accepted the suggestion and will conduct further research before teaching.* Even though the concept to be taught brought further challenges or confusion, it made the participants reflect on their content and PCK.

4.4.2.3. Grade 4A PLC 1 Research Lesson Teaching

The third research hour lesson for PLC 1 was planned and conducted in the grade 4A classroom. The classroom accommodated nine lesson observers, one researcher, and twenty learners to adhere to the protocols of COVID-19. The educator TTNZ seemed fully prepared for her lesson and the lesson went smoothly as planned. All the participants were present and on time, except educator TMBN who went to his class to give assignments to his learners.

The lesson began at 11:30, the last lesson before second recess. The lesson plan and observation tool were handed out before the lesson began. The steps in the lesson plan were done systematically and time was allocated to each aspect, however, all aspects took more time. The research lesson was learner-centred, more like a hands-on activity. Learners were found to be actively engaged in the research lesson although some could not follow the educator's instructions.

The lesson plan and appendices were handwritten. The Content Area: *Patterns, Functions and Algebra* and subtopic: *Geometric Patterns* were clearly stated. In the introductory phase, the educator TTZN initiated the research lesson by asking the learners to come to the board and extend the pattern as shown in Figure 9. The learners had to figure out which colour circle would be next.

Figure 9: Grade 4A Introduction Activity

Each learner was given a box of matches to complete a hands-on activity and then a worksheet to answer. Learners were asked the following questions in different applications:

- make stage 4 and stage 5.
- draw the sixth stage.
- write down the number of matchsticks in each stage.
- explain by how many matchsticks each stage increases or decreases; and
- find the rule.

4.4.2.4. Grade 6C2 PLC 1 Research Lesson Teaching

The research lesson was planned and conducted in a multi-purpose class as this was the only class that could accommodate 25 people (learners and observers). All participants were present and on time. The lesson started at 13:30, the last lesson of the day. The lesson plan was distributed earlier along with grade 4A. Educator SLFM as well as educator SBPM showed a misunderstanding of EAL and inclusion, although this was explained during the reflection of the first PLC 1 research lesson.

The CA 2: *Patterns, Functions, and Algebra* and subtopic: *Geometric Patterns* were clearly stated. The aim of the research lesson she stated was for learners to be able to:

- investigate and extend geometric patterns by looking for rules or relationships for patterns.

When introducing the lesson, educator SLFM asked learners to distinguish between 2-D shapes and 3-D objects. Then learners had to investigate and extend the given geometric patterns using shapes and objects. The introduction to the research lesson encompassed the objective of the lesson. Although the lesson was learner-centred, no individual task was given to the learners except to come to the board and answer questions asked. In Figure 10 below, learners were asked to extend the pattern with stages 4 and 5.

Figure 10: A Grade 6C2 Learner's Response

Overall, the educator SLFM teaching was clear and she appeared confident. The teaching was systematic, however, she was not able to execute the lesson plan appropriately due to the class work and consolidation of the research due to time duration. The participants were serious in their lesson observation and wrote down the details. In terms of the objectives of the lesson study, the lesson was considered a success as the learners were actively involved in the lesson.

4.4.2.5. PLC 1: Second Cycle Reflection on the Research Lessons

On the same day that the research lessons were presented after the lesson, the participants, both presenters and observers stayed behind for a reflection meeting. After reflecting on the lesson presenters, participants choose to discuss the rule or calculation plan of the sequence. After experiencing a complete lesson study cycle, participants seemed to have understood and conceptualised the rationale and practice of lesson study. This helped the participants to conduct the lesson study smoothly in this cycle.

The participants chose to discuss *the calculation plan* as a concept and content for patterns that were not taught by the presenters and seemed to be challenging for the educators and even the learners. During the reflection discussion, educator TMBN lamented to his colleagues,

> "It is v*ery difficult to teach calculation plan. Sometimes we have headaches about it too.*"

Educator LNSM supported educator TMBN's view regarding the *calculation plan.* He further said,

> "*Our learners do not understand it; they are used to saying that we add a certain number to the previous number to get the next number."*

The educator SPMM also expressed concerns about the *flow diagram, which* was only partially covered in both research lessons, while it was discussed in-depth during the planning meetings.

Such expressions from the participants indicated that they were interested in discussing the content and mathematical concepts that were causing some difficulties for the learners. This was quite reasonable because choosing a difficult concept helps the participants to deepen their understanding of the content and find ways to help the learners understand and learn the concept (Richit et al., 2021; Weaver et al., 2021; Takahashi, 2005). In short, participants seemed to view the lesson study as a place where they could discuss and solve their teaching problems.

4.4.3. PLC 2: First Lesson Study Cycle

PLC 2 participants also attended a workshop aimed at introducing and explaining the concept and practice of lesson study as an innovative form of educator professional development. The workshop was conducted by the mathematics curriculum subject advisors in Ilembe District as this PLC is one of the piloting projects for lesson study in 2021. The educator PSPM, former cluster coordinator, was elected as the coordinator of the PLC in this workshop.

The first lesson study cycle in PLC 2 lasted about four weeks (six weeks of the workshop), during which the revised lesson plans were not taught. The duration was considered moderate because there were no constraints in conducting the lesson study implementation. Although the lesson study research project was endorsed by all four principals of the schools that formed PLC 2, the educators at School H were never present 100% of the time and did not present a research lesson. Educator PSPM took the lead and responsibility for conducting the PLC 2 lesson study implementation.

Although this was the first attempt by the participating educators to conduct a lesson study, it was conducted systematically following the lesson study model recommended during the workshop. However, the participants who also teach grade 7 considered lesson study as a 1+9 model. The participants decided to conduct the lesson study weekly after school as they did not want to disrupt teaching and learning. Therefore, they preferred not to change the meeting location. This first lesson study cycle of PLC 2 was implemented with success. The PLC coordinator took a proactive approach and adopted a positive attitude towards implementing the lesson study. His actions, positive attitude, and demeanour motivated and to some extent encouraged the other participants to participate in the lesson study.

4.4.3.1. First PLC 2 Planning Meeting

11 of the 13 participating educators were present, except educators HATG and HMSB who called in sick, and met in the staff room reserved for lesson study meetings. Initially, although the participants seemed well informed about the implementation of the lesson study, the PCL 2 coordinator gave a brief overview of the lesson study process so that they could have a good understanding of their role in this study. He also pointed out the goals of the lesson study as provided in the lesson study model. In addition, the researcher briefly explained the background of the study to the participants as it was important that they fully understood the aims and rationale of the study.

The participants agreed to focus on numeric patterns in this lesson study cycle and grades 4 and 5 will present them. Educator DSSZ suggested this,

> "*Colleagues, I suggest that we plan by grade."*

Educator MGPT disagrees,

> "Mr. *Chairman, I would like to disagree with the sir and suggest joint planning as Intermediate Phase educators*."

As a result, the participants agree to plan together and the educators presenting the research lesson would be responsible for finalizing the lesson plans. In addition, educator PTBM suggested the following,

> "*Since our PLC consist of four schools, two of which use EFAL and the two use EHL as LoLT, I suggest that we try to balance LoLT during the lesson plan presentation."*

Participants fully agreed with this suggestion, although educators at School H indicated that their classrooms are *overcrowded. Our learners are overcrowded and there is no social distance, so we cannot accommodate lesson observers in our classrooms.*

The educators agreed to focus on *lesson objectives* at this planning meeting so that both lessons will have the *same objectives with different activities*. Educator PSPM led the discussion as the PLC coordinator and effectively engaged the participants, although some participated passively in the planning meeting. At the end of the discussion, educators DSZM and PTBM were nominated to present research lessons for grades 4 and 5, respectively. These educators will prepare draft lesson plans and present them in the next planning meeting.

4.4.3.2. Second PLC 2 Planning Meeting

All participants were present for the discussion aimed at helping educators DSZM and PTBM develop their plans for the research lesson. Educator PSPM led the planning meeting,

> *"Dear colleagues, I suggest that we first have educator DSZM give a presentation, then educator PTBM, and only after that, we will start discussing the lesson plans. I submit."*

The participants fully agreed with him, *we agree sir*.

Educator DSZM said the following,

> "*I will first assess the learners' prior knowledge of whole numbers in terms of counting forward and backward, addition, and multiplication."*

She will introduce the lesson with a practical example that will engage learners. As this is a first numeric patterns lesson for grade 4s in 2021, the educator DSZM stated that:

> "*As an introduction, I would use the sequence to explain the terminology and notation used in the sequence. Yes, they may know these but their LoLT is now English from IsiZulu for four years of schooling including grade R."*

After consolidating her lesson outline, the participants started discussing it through the coordinator. Educator MGPT advised her,

> *"I suggest that you allow learners to use IsiZulu if they cannot express the terminology in English, and then use grade 4 LoLT."*

The educators could not agree with this suggestion, but in the end said this was the professional decision of the presenter. Participants could not discuss much about the lesson plan presented

by educator DSZM because she presented a narrative lesson plan and no mathematical activities. No changes were made to the sequence of steps in the lesson outline presented. However, participants spent a lot of time helping her develop activities/worksheets. The rationale was to vary the questions to balance the cognitive levels and level of difficulty to match the learners' abilities.

The educator PTBM commented,

> "*I will first check the learners' prior knowledge of whole numbers in terms of multiplication and division as well as the inverse operations."*

For the lesson introduction,10 minutes were allocated for learners to focus on the table below and answer the following questions:

- Complete the table.
- Which method(s) did you use to complete the table? Discuss.
- Discuss what patterns you see in the table and say how that helps you "remember" the tables.

Table 6: The Sequence

▬	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15					
4	4	8	12							
5	5	10	15				35			
6	6	12	18					48		
7	7	14	21							

He went on to emphasise,

> "*The focus in grade 5 on number sequences is on three types of sequences, namely: sequences with a constant difference, sequences with a constant ratio, and sequences without a constant difference or ratio."*

His lesson presentation/development consisted of various applications where learners were expected to:

- Investigate and extend numeric patterns, looking for relationships or rules of patterns (sequences with a common difference and ratio; and sequences without a constant difference or ratio); and
- Describe the observed relationships or rules in the learners own words.

There were no major changes to the draft research lesson, but minor details were added or changed, such as decreasing sequences having a minus or division sign. After reviewing the draft research lesson, the participants were happy and satisfied with the outcome of their discussion during this planning meeting.

4.4.3.3. Grade 4AZ PLC 2 Research Lesson Teaching

The lesson for the research lesson was planned and delivered in the grade 4AZ classroom. This classroom had space for the learners and nine observers including the researcher. All four educators from School H were absent during the observation of the research lesson. The educator DSZM and her learners seemed a little nervous at the beginning of the research lesson, however, as the lesson progressed they were all relaxed. All COVID-19 rules were followed and adhered too. The lesson started off at 07:30 so all participants were seated in the classroom 15 minutes before. The learners were also punctual as the morning bell rings at 07:20 and the first lesson starts at 07:30.

The lesson plan was typed up and handed out to the participants in the morning when they arrived. The Content Area: *Patterns, Functions and Algebra* and sub-topic: *Numeric Patterns* were clearly stated. Educator DSZM stated the lesson objectives as they had been discussed in the PLC 2 planning session. In the introductory phase, she introduced the research lesson with Mental Maths which was followed by the observation exercise. Educator DSZM asked the following learners to come forward, "*Six boys and six girls will come and line up. They line up in the following order: two boys; two girls; two boys; two girls; two boys; two girls."* Then the learners were given the following question: *What is the same and what is different in the line where the learners are in front?*

Instead of asking the learners to explain the terminology and notation used in the sequences, educator DSZM explained them to the learners using the following illustrations.

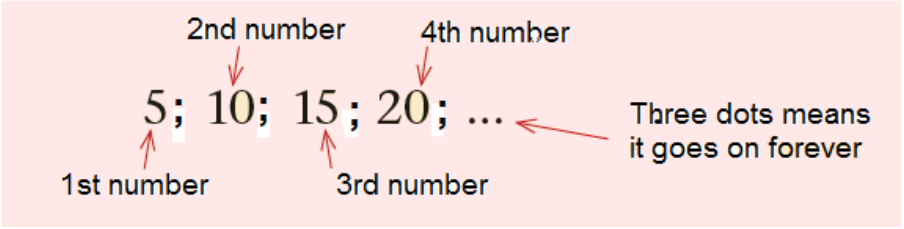


Figure 11: Mathematics Sequence Representation

Position number	1	2	3	4	5	6	7	100
Sequence	5	10	15	20	25			

Figure 12: Mathematics Sequence in a Table

Learners were then given a 15-minute individual task in which they were asked to answer the following questions:

i. Investigate and extend numeric patterns.
ii. What is the rule of the pattern?
iii. Indicate whether the pattern involves a constant difference or ratio.
iv. Describe the observational relationships or rules in your own words.

After grading each activity, the lesson was consolidated and the class dismissed.

4.4.3.4. Grade 5E PLC 2 Research Lesson Teaching

The lesson for the research lesson was planned and conducted in the grade 5E class. All participants were present and on time, including the educators from School H who had not participated in the grade 4 research lesson presentation that morning. The lesson began at 13:15 the last lesson of the day, and ended at 14:15. The research lesson plan for educator PTBM were handed out earlier, he only gave it to those who were absent in the morning while the researcher ensured that everyone had an observation tool.

Educator PTBM admitted that he was nervous because it was the first time his lesson was observed by multiple educators,

> "*When it's something where others are observing me, like IQMS, I am forced to be prepared. So I feel pressured.*"

He began the research lesson with a Mental Maths activity. Overall, the educator PTBM lesson went clearly and smoothly. The planned research lesson plan was implemented as discussed in the planning meetings. The research lesson discussions had an impact on educator PTBM's pedagogical content knowledge.

Due to the limited time, educator PTBM had to interrupt some learners to reflect on their work, which was very important for the learners to enhance their mathematical thinking as it was the goal of the lesson study. During the lesson observation, educator MTAM seemed to gain confidence that the learners were indeed able to complete the table without prior demonstration by educator PTBM. Towards the end of the lesson, he rushed the activities due to time constraints and was unable to consolidate them.

4.4.3.5. PLC 2: First Cycle Reflection of Research Lessons

After the presentation of grade 5E research lesson presentation, School P was dismissed. All participants stayed behind for a reflection session for about $1\frac{1}{2}$ hours. According to educator DSZM, *the research lesson was successfully implemented and was learner-centred.*

> *"Sir (referring to educator PSPM), I observed that my grade 4 learners had difficulty in distinguishing between constant difference and ratio, even describing their observational relationships to use English as LoLT."*

Educator PTBM felt that his presentation of the research lesson to grade 5E was a success. He said the following,

> "*The lesson plan was well presented and the objectives were met. The collaborative planning enhanced my pedagogical content knowledge but the activities took more time. I suggest that in future we consider both the learners' pace and the educator explanations."*

Educator PSPM altered the participants regarding the Mental Maths approach that was incorrectly performed, *in both research lesson presentations.* Educators MMNM and DNNM agreed with him, *yes, it was done incorrectly. Whenever we teach Mental Math, it is advisable to use flashcards or shouting instead of writing it on the board.* Educator MSCM emphasised the importance of explaining or assessing learners' knowledge of terms used in a lesson. He said,

> "*Learners may have content knowledge but they encounter language barriers. From my observation, learners were able to extend and state the rule of a pattern, but they could not tell if it was a constant difference or a ratio."*

Educators discussed the challenges of LoLT transaction at the Foundation Phase and Intermediate Phase for about 10 minutes.

All participants agreed that the learners were able to complete the table correctly. The l*earners completed the table as required but in future we need to allocate time for the activity, suggested* by educator HMSB. The learners said,

> "T*hey filled in the table using the multiplication method.* However, when they talked about the patterns in the table, the learners said that *we added the same number each time, which is called constant difference.*"

Although the educator PTBM tried to ask them about the *vertical pattern*, *a pattern of multiplying by the same number each time*, they seemed unconvinced. The participants spent some time discussing the teaching challenges they face as Intermediate Phase educators.

4.4.4. PLC 2: Second Lesson Study Cycle

The second lesson study cycle in PLC 2 was conducted shortly after the school reopened for a third term school year. However, there were two constraints at the beginning. The first constraint involved COVID-19 cases in the PLC 2 participants' schools, and the second involved all learners returning to school, so a new timetable was to be developed. The second lesson study cycle for PLC 2 was delayed by two weeks due to the COVID-19 cases. The second constraint was that participant felt they could not stay after school hours because their teaching will be overstretched with the return of all learners. The issue was resolved amicably as PLC 2 SMTs continued with the alternating time-table due to space constraints and KZN Circular No. 77 of 2021.

This second lesson study cycle of PLC 2 was implemented with success, although there were some constraints. A major reason for this was the initiative and strong commitment of the participants in the lesson study, including the educators at School H. The educator PSPM (PLC 2 coordinator) took a proactive approach and adopted a positive attitude towards the implementation of this lesson study cycle. Not only did he actively participate in the lesson study, but he also volunteered to conduct the grade 6 lesson in this cycle. His actions, positive attitude and attitude motivated and encouraged the other participants to some extent to participate in the lesson study. Therefore, the smooth implementation of this lesson study cycle is partly due to the work, initiative and enthusiasm of all the participants.

4.4.4.1. PLC 2: Second Lesson Study Cycle Planning Meeting

Three planning meetings took place during this lesson study cycle prior to lesson presentations. However, the first planning meeting was mainly to refresh and create the schedule for the second lesson study cycle. Despite the COVID-19 cases directly or indirectly impacting participants, there were some constraints as participants were often away from school due to official duties such as attending School Based Assessment Moderation, in-service workshops and HRM 35 of 2021 interviews. Considering the school context and the participants' perspectives, the researcher did not postpone any of the planned planning meetings as the participants themselves did not know how to proceed in the following weeks.

During the discussion on research lessons, it was easy to view the participants' lesson as curriculum driven/oriented. In the refresher session, educator DSSZ reminded participants that,

> "*Geometric patterns had been merged, were no longer taught in more than one term and the number of hours had been reduced, particularly in grades 5 and 6.*"

Educator MMNM also added the issue of contact sessions with learners:

> "*Yes, they have been merged and the number of hours reduced, but learners no longer come from Monday to Friday, so we cover more in one hour to coincide with the RATP.*"

These comments clearly show that participants are rushing to complete the curriculum even if learner attendance is not 100% in the academic year 2021 due to the pandemic COVID-19. This aspect is highlighted because the teaching concept for the purpose of the curriculum could indirectly complicate the implementation of the lesson study. Moreover, it influenced and shaped educators' teaching towards educator-centredness by the educator PTBM during the

first lesson study cycle. The practice of lesson study did not match the participants' conception of teaching, which was probably due to the constraints in the implementation of lesson study.

Educators agreed to focus on the following concepts: *from tables to flow diagram; more pictures and tables; and flow diagrams without calculation plan* during planning meetings. The educator conducting the research lesson may choose one of the above concepts and, if possible, cover all of them in his/her lesson. Although the educator PSPM, as the coordinator of the PLC, facilitated the discussions, he effectively involved the participants, but some were still passive, such as during the planning meetings of the first lesson study cycle. At the end of the third planning meeting, educators MGPT, MSCM, and PTBM volunteered to present the research lessons for grade 4, 5, and 6 respectively.

4.4.4.2. Grade 4B PLC 2 Research Lesson Teaching

The third research lesson for PLC 2 was planned and conducted in the grade 4B classroom. All participants except educators HPNM, HATG and MSCM (was on sick leave) attended the research lesson which was delivered by educator MGPT. She appeared nervous throughout the duration of the research lesson presentation. This is her first lesson observed as an in-service educator and was appointed in September 2020. She admitted that she was nervous because it was her first time being observed by multiple educators, "*When I was observed, I felt pressured. I have never been observed as an in-service educator before, most recently observed as a student-teacher*." Her whole lesson was learner-centred.

Educator MGPT introduced the research lesson by asking learners to count forward and backward using multiples of 2s; 3s and 7s. Then learners were asked to come forward to extend the pattern as shown in Figure 13 below.

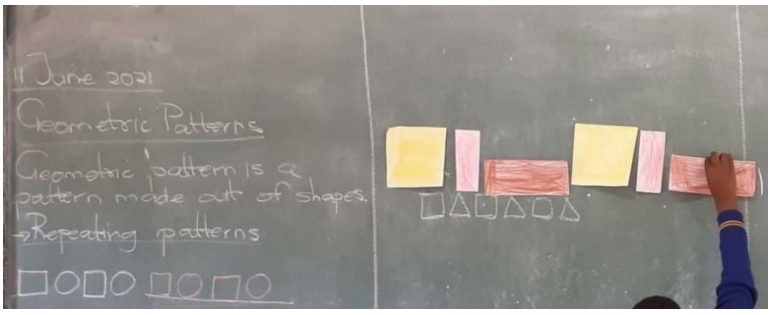


Figure 13: Grade 4B Repeating Pattern

The whole research lesson of educator MGPT was about repeating patterns and extending patterns, and learners were asked to come to the board to demonstrate their answers. All her explanations were accompanied by examples, as can be seen in Figure 14 below. She used more IsiZulu than English as LoLT for grade 4 learners.

Figure 14: Grade 4B Explanations and Examples

4.4.4.3. Grade 5C PLC 2 Research Lesson Teaching

The grade 5C research lesson did not take place on the scheduled date because educator MSCM was ill. Educator MSCM's illness was another constraint in implementing the lesson study, his leave lasted almost two weeks. Another constraint hindered the implementation of lesson study was that the other grade 5 educators were not ready to present.

4.4.4.4. Grade 6PM PLC 2 Research Lesson Teaching

Lesson study is a research process for educators, and focusing on the research aspect helps to create more meaningful and deeper opportunities for educator learning (Murata & Lee, 2020). The participating educators had formulated the goal of the lesson study only to achieve the actual objective of the teaching. The educator PSPM seemed to overcome the constraint associated with the *curriculum-based* education system that influenced the participants' teaching beliefs and conceptions. Accordingly, the lesson study's goal was achieved by the educator PSPM which reflects that it is relevant to the context of primary schools in Ilembe District.

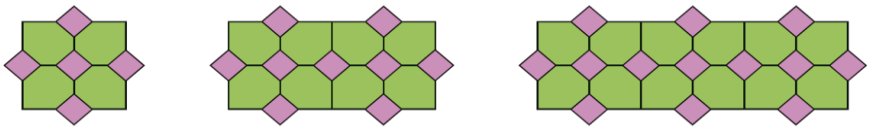
The presentation of the draft research plan by the educator PSPM *enhanced the content knowledge of educators.* While reviewing the syllabus and Intermediate Phase textbook during lesson study discussion he led, the participants were surprised to see the proposed method used for *geometric patterns.* After a revision of mathematics ATPs in 2020 due to the COVID-19 pandemic, *geometric patterns* were moved to the second term and lessons were reduced from 11 to 9 hours. The participants were not aware of this because last year patterns were taught in the fourth term according to the RATPs. The content knowledge of *tables to flow diagram; more pictures and tables; and flow diagrams without calculation plan* was unknown to most of the participants, especially those teaching grades 4 and 5 before the lesson study discussions, but by the end they had gained some content knowledge. Clearly, the collaboration of colleagues in the lesson study has enhanced the content knowledge of the participants.

This final cycle of PLC 2 has introduced participants to the strategy of cooperative learning in the lesson study. In the teaching of the research lesson, learners were engaged one-on-one to work on and solve problems about *geometric patterns* in their exercise books. Initially, they worked alone to solve the problems, and later they compared their solutions with other learners through cooperative learning strategies. Before the lesson study, the participants had little knowledge about the cooperative learning strategy and they seemed enthusiastic about the pedagogical approach in the lesson plan.

The educator PSPM who taught the grade 6PM lesson admitted that the process of the lesson study influenced his PCK,

> "*This is the first time I have conducted cooperative learning. Although I am a former lead educator of the 1+9 model, I have never done it or even thought about it before."*

Based on the results in this cycle, participants indicated that the lesson study impacted their PCK. The following activity was one of the activities given to grade 6PM learners:



1. In this above pattern, Size 1 is made of 4 green tiles and 5 smaller purple tiles. The pattern is then continued as shown:
   (a) Complete the table below and describe your methods.

Size	1	2	3	4	5	30
No. of green tiles	4					
No. of purple tiles	5					

(b) Use a flow diagram to represent the relationship between the size(input) and the number of purple tiles(output) as shown above.

4.4.4.5. PLC 2: Second Cycle Reflection of Research Lessons

In this cycle, participants collaboratively designed research lesson plans to introduce the concept of *geometric patterns*. From the tendency and nature of the discussion, it was evident that the participants perceived the lesson study as a place to discuss their teaching problems. In fact, Takahashi (2005) suggested that one of the approaches to beginning a lesson study is to discuss mathematical concepts that are difficult for learners to grasp. Although participants did not select *patterns,* learners content difficulties were revealed by the participants, and the timing of the lesson study cycle was also an important consideration.

During the discussion on the 10th of August 2021, educator MTAM said,

> "*The content of patterns is not difficult, but learners have difficulty grasping the concept of rule in both numeric and geometric patterns. Learners could not distinguish between a common difference or ratio and calculation plan or rule.*"

Educator MMNM gave another reason for the topic,

> "*Inverse operations; and more pictures and tables, learners have problems*."

Based on these expressions during the discussion, participants seemed interested in discussing mathematical content that learners found problematic. This reinforced the notion that the lesson study is seen as a place where teaching problems can be discussed.

At the beginning of the reflection discussion, the educator MGPT apologised for not teaching according to the planning of the meeting research lesson.

> "*I was tense and observed that learners were struggling with English, so I ended up using IsiZulu a lot."*

*The e*ducator MGPT felt that the content of the lesson was far too much for her and she needed more time to adapt it.

> "*Since I specialise in the Foundation Phase, I need more professional development sessions for the Intermediate Phase content*."

Although criticism of teaching is part of the lesson study process, the participants seemed cautious and reserved about making negative comments to their colleagues. Observers praised her for this: "*Your teaching was clear and easy for learners to understand."*

After a brief reflection by the educator PSPM, the observers expressed satisfaction with the teaching of the research lesson from the planning meeting. After the participants' reflection and lesson observation, they were satisfied that more time was given to the learners' activities: *inverse operations; and more pictures and tables, learners have problems.* According to educator MTAM,

> "*The first two activities were sufficient for the lesson presentation, so educator PSPM would have more time to emphasise critical aspects."*

Therefore, all participants agreed that four activities were too much for one hour.

4.5. Emerging Themes

Creswell (2013) argues that data analysis is conducted to develop a detailed knowledge of the phenomenon under study. The aim of this study was to explore the role of lesson study on educators' pedagogical content knowledge in teaching mathematics, particularly in primary schools. Data coding, according to Braun and Clarke (2006), entails familiarising oneself with the data, producing initial codes, searching for themes, reviewing themes, defining and labelling themes, and constructing a record. They also claim that this can be achieved by taking notes in the margins of the transcribed interviews.

Relevant text, patterns or themes can also be highlighted. A similar coding approach was used in this study. The data obtained from the lesson observations and the video based, semi-structured individual interviews were transcribed manually. Braun and Clarke's (2006) characteristics were used, focusing on pedagogical content knowledge. The audios of the semi-structured individual interviews and observations were listened to repeatedly and read sequentially to gain a clear and deep understanding of the educators' PCK in teaching *numeric and geometric patterns.* Notes were also taken in the margins. During the observations of the research lessons, notes were also taken on each activity that the educator had done. This was done to identify and discuss the component of PCK that was used (see observation schedule).

The total of four lesson study cycles (two cycles per PLC) conducted in this study yield credible data on the role of lesson study in educators' pedagogical content knowledge in mathematics teaching. Through data analysis, the following themes were identified: *knowledge of content and learners (KCL); knowledge of content teaching (KCT); and knowledge of content and curriculum (KCC).* This section argues that the findings of this research point to the possibility of further developing and enriching this model by providing empirical evidence of *KCL in educators' lesson study practices and a greater understanding of KCT and KCC incorporated within lesson study.* It is important to reiterate that in this proposed model (see Figure 15), the elements of teacher knowledge are not assumed to be separate from each other but, rather overlap and complement each other in educator learning.

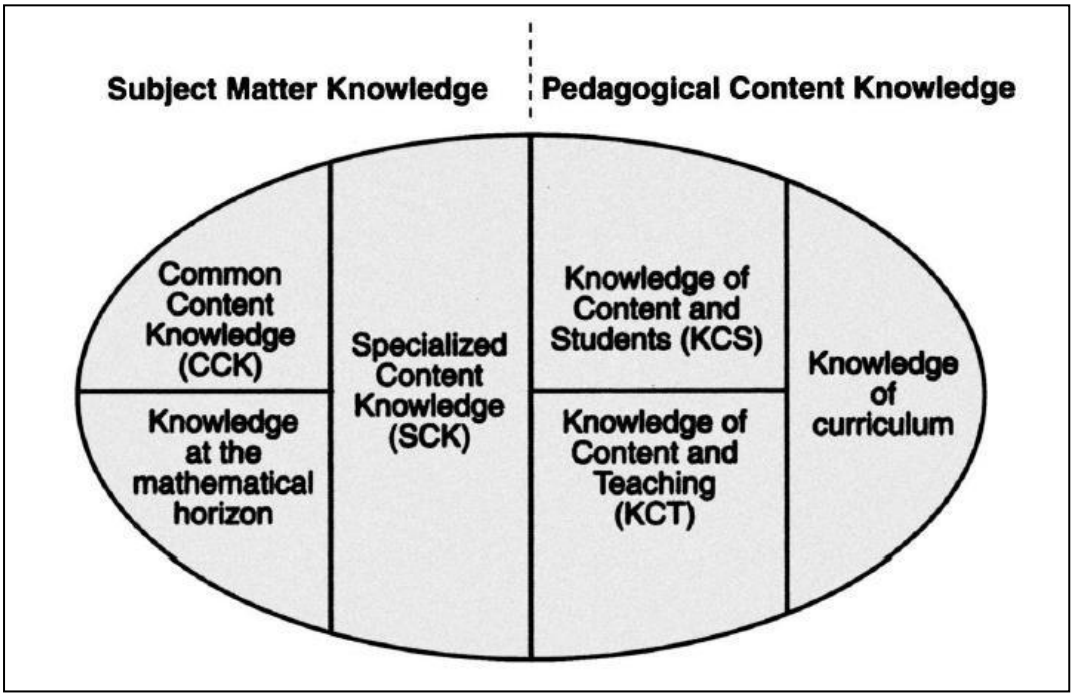


Figure 15: Mathematical Pedagogical Content Knowledge (Kristano, Panuluh & Atmajati; 2020, p.2)

4.5.1. Knowledge of Content and Learners

Generally, knowledge of content and learners refers to educators' understanding of how learners learn a particular content (Ball, Thames & Phelps, 2008). KCL includes educators' knowledge of common learner errors and conceptions or misconceptions about mathematical topics (Hill, Schilling & Ball, 2008). KCL has been explained as what can be expressed about mathematics and perceived about learners (Ball, Thames & Phelps, 2008).

Focusing on learners' mathematical thinking provides educators with the opportunity to enhance their knowledge of how best to develop content to enhance learners' understanding (Ni Shúilleabháin, 2015). Moreover, Gulpric (2018) asserts that this knowledge includes the educator's ability to plan lessons that motivate learners and deal with the misconceptions they bring from a particular concept. He also believes that knowing what is helpful to learners and what might hinder them in understanding a concept also represents a high level of competence in this category of knowledge on the part of the educator.

The four lesson study cycles conducted on the *patterns* did indeed enhance the participating educators' knowledge of content and learners. However, the knowledge gained by everyone was highly dependent on the concept of the research lesson discussed and the participants' background, attitude, and commitment to the lesson study as professional development. The data analysis for this theme, identified the following sub-themes: i) objectives of the research lesson, ii) resources used in a research lesson, iii) prior knowledge of learners, and iv) examples used in a research lesson.

Separate literature emphasising learners' mathematical thinking (Jacobs, Lamb, & Philipp, 2010) and educator knowledge (Jacobs, Lamb, & Philipp, 2010) supports these sub-themes (Krauss et al., 2008). While this is not an exhaustive list of all the elements included when focusing on learners' mathematical thinking, it reflects what emerged from qualitative data obtained from educators' discussions as part of their participation in this study and is especially relevant for examining changes in educators' knowledge when participating in lesson studies.

i) Objectives of the Research Lesson

Mathematics as a subject has seven specific aims and six specific skills (DBE, 2011). The specific aims focus on what should be developed through the teaching and learning of mathematics, while the specific skills focus on what essential mathematical skills a learner should develop. Therefore, some educators state lesson objectives as concepts and skills to be achieved. The lesson objectives are achieved by the learners at the end of the lesson, they should know and be able to do.

The participants, both research lesson presenters and observers, demonstrated an adequate understanding of their subject as educators through the ability to formulate lesson objectives that are appropriate for quality transmission of knowledge to learners. When asked if their teaching had specific objectives, educator SBPM replied:

> *"I had set objectives, namely, to develop learners' critical thinking skills, to prepare them for more complex computations, and to develop learners' generalisation skills."*

Educator PSPM agreed with how to teach to achieve set objectives. He opined that:

> "*…every educator knows that teaching begins with the setting of objectives and the ability to achieve those objectives. As for me, I had the objectives for my lesson that I wanted to achieve. These are: table and flow diagram."*

The ability to disclose the lesson objectives as the purpose of the lesson of the day by the educator is the merit of having the knowledge of the content to be taught as well as knowing the recipient of the educational package in this case, the learners. When asked if the objectives set were achieved, the educator DSZM stated:

> *"I think I achieved the objectives of the lesson as most of the learners were able to respond to the questions asked and extend the numeric patterns even though they were struggling to identify the rule."*

The participants agreed that the objectives had been achieved. The educator LPFM cited the following statement as the reason for achieving the set objectives:

> *"It was achieved because it was something that was observed on teaching aids and clearly explained using diagrams."*

This was also confirmed by educators TTZN and MGPT who said,

> "… *yes, we have achieved the objectives of the lesson. Learners were able to independently differentiate between repetition and extension of geometric patterns."*

ii) Resources Used in the Research Lesson

Participants agreed that the teaching models/resources and media used had an impact on the achievement of the objectives. When teaching, it is important that the educator shows expertise in selecting the teaching resources used. The selection of good teaching resources assists the educator in teaching and achieving lesson objectives, whereas poor teaching resources affect

the overall lesson. When asked about the reasons for selecting the teaching resources used, educator PTBM responded as follows:

> "*When preparing the lesson, I used cards with slots as resources, although I did not use them at the end...the selection of these resources was to give my learners more clarity about numeric patterns."*

Educator MGPT added that,

> *"…the resources were easy for learners to use and a chart to observe."* Educator SLFM added, *"I used these resources because the learners were familiar with them (shapes and objects) so it was easy for them to relate and master the lesson while using these resources."*

In addition, educator PSPM said that,

> "*I used the projector to clearly display the activities because they were colours and printing a colour worksheet would have been too expensive. In addition, the use of black and white worksheet would confuse the learners and make it difficult to achieve the lesson objectives. In addition, the use of ICT attracts learners' attention and focuses on the topic/concept to be taught, but the use of ICT as a resource also requires a certain level of competence."*

The responses of the above participants are a clear indication that an educator should use both knowledge of content and skills of the learners who constitute the class. The educator should base the selection of teaching resources on the assumed knowledge of the learners. These learning models/aids, according to the participants, should enhance the understanding of the concepts being taught.

iii) Prior Knowledge of Learners

This sub-theme looks at how educators recognise learners' prior knowledge as a component of KCL that may be used and improved through educators' participation in lesson study. What students learn in class is heavily influenced by what they already know, whether correct or incorrect. Learners can be challenged in mathematics by drawing on existing knowledge and elaborating on assumptions they have already made (Baumert et al., 2010).

The fact is that not all participants in this study were able to identify learners' prior knowledge as an integral part of their decision on how to begin and develop a research lesson. Grade 4 educators were unsure of the learners' mathematical knowledge when they arrived at the beginning of Intermediate Phase education. Educator TTNZ said,

> "*I would assume that they know. If you introduce the concept of patterns, whether it's numerical or geometric, they are going to know it regardless of LoLT. In the research lesson I presented, I tested learners' prior knowledge of number sentences, patterns, and properties of 2-D shapes."*

Educator DSZM said,

> "*When I teach the concept for the first time, I remind the learners that they have already learned it at an earlier stage. Then in the second lesson, I assess their prior knowledge based on the previous lesson I presented. I did not assess the learners' prior knowledge, but I told them what they should know."*

When educator SLFM was asked what concepts or knowledge she thought learners should have before beginning lessons on geometric patterns, she replied,

> "… *I think learners should first be reminded of the number sentence and be able to complete simple number patterns."*

The above suggestion from almost all educators is an indication that educators should consider the connection between topics when preparing or planning lessons and see which topic should start and which topic should follow the other. This is important for teaching as other topics are in the foreground and prepare the teaching of other lessons. In this case, for example, they have stated that teaching number sentences and patterns is very important and should be done before teaching geometric patterns.

In short, each grade is an extension of the previous grade level. Educators should therefore know that when preparing their lessons, they should first do some revision to revive what the learners already have in their minds so that the learning is coherent, linked and meaningful.

iv) Examples Used in Research Lesson

Based on the data collected, the participants explained the examples they use when conducting a lesson. Educator SBPM said:

> "*I used beads so that learners could easily explore and discover how the sequence grows. This allowed the learners to transfer the visual to the numerical representation."*

This was supported by educator LPFM who commented,

> "*I transferred the non-numerical to numerical representation as learners were able to count the number of squares in each diagram. The example enhanced learners' understanding and connection of non-numerical to numerical patterns and developed understanding of identifying the rule and using it to find the next sequence."*

The examples used during the presentation in the lesson enhanced and enabled understanding of the concepts of patterns. The above responses from the participants indicate that educators should ensure that they use examples to reinforce their teaching lessons. However, the examples should relate to the content and be friendly and familiar to the learners so that they become the catalysts for understanding.

The word "*understanding the patterns*" was used by the participants when they explained what they thought examples do for learning. This therefore implies that the lesson study for teaching of mathematics in school should aim to examine the nature and context of the examples selected that are appropriate for the delivery of the lesson. It also emerged from the semi-structured interviews that the examples should be understandable and simple for most or all learners (if possible) as they involve everyday applications. According to the participants, the *examples should also encourage learners to participate without requiring much effort from the educator*.

4.5.2. Knowledge of Content and Teaching

Ball, Thames and Phelps (2008) argue that this group of PKC, known as KCT, combines knowledge of mathematics and knowledge of how to build learners understanding of this mathematical idea. This means that the educator must have knowledge of the teaching strategies, resources, and materials used to successfully teach a concept, as well as a comprehensive knowledge of the subject matter. Ball, Thames and Phelps (2008) further said that designing mathematical questions, sequencing content, and acting as facilitator of learning

are all elements of KCT – content knowledge that is specifically relevant to mathematics educators. According to Ni Shúilleabháin (2015), KCT belongs to this model of educators' knowledge that can be enhanced by educators' participation in iterative cycles of lesson study using a critical learner lens in their practice.

Educators exhibit characteristics of KCT as they use relevant and effective teaching materials. The following analysis presents the evidence obtained through data generation using semi-structured individual interviews, observations and reflective meetings or focus group interviews. The data analysis for this theme, identified the following sub-themes: i) *learning sequence*, ii) *contextualised questions*, and iii) *mathematical tasks*.

i) Learning Sequence

Sequencing of mathematics material requires educators to make connections between the mathematics being taught and the way learners are to be introduced to the topic (Ni Shúilleabháin, 2015). She further said that the mathematics content learning sequence combines all themes of this study (KCL, KCT and KCC) because educators need to incorporate into the sequence: learners' prior knowledge of the topic to be taught and the objectives of the assigned curriculum. Therefore, knowledge of how to sequence learning for learners is a key dimension of PCK and is linked to this theme of KCT.

Learners' prior knowledge provided educators with the opportunity to plan research lessons as they participated in the lesson study and beyond. Educator LSPK said,

> "*I only observed the research lessons, but I can now plan the sequence of activities in the lesson plans in more detail. Sequence of learning is critical to achieving objectives and meaningful teaching."*

Educator LPFM agreed with educator LSPK about the importance of learning sequence,

> "*Eish, I now know the logic between specification of content and ATPs sequencing. Geometric patterns involve 2-D shapes and 3-D objects, which is why we teach these topics beforehand to prepare learners for geometric patterns. I have always wondered why some Content Area 3 topics are taught before we exhaust Content Area 2, it's the reason for the learning sequencing*."

In the research lesson planning sessions, educators planned to gradually build learners' mathematical understanding of patterns. Previously, educators had taught this topic using the textbook without considering other resources and examples of sequencing learning. The participants of PLC 1 advised educator TNNZ to use only matchsticks as a teaching model and she should consider sequencing the topic for the benefit of the learners because *with matchsticks as a teaching model, one can make any pattern, either a growing or a repeating one.* They also advised the educator LPFM the importance of learning sequencing. B*efore doing the inverse operations in a follow diagram, learners should learn how to get an output using the input and the rule.*

During the PLC 2 planning meetings, educator MMNM realised that it would be far more beneficial for the development of learner's mathematical thinking and understanding to design the lesson from the learners' perspective and focus on slowly building learners' understanding of the concept being taught. Educator DNNM said,

> "*I looked at the patterns in the classroom from the learners' perspective. This learning experience built my KCT through sequencing the lesson and also enhanced my knowledge.*"

In addition, the participants had intensive discussions about the teaching approach of the topic while conducting the learning sequence. The educator PSPM demonstrated approaches that can be used when teaching numeric and geometric patterns considering the learning sequence to give more meaning to the learners. In conclusion, participants agreed to change their teaching methods to move *from context to abstraction rather than the other way around.* Educator HMSB said,

> "*I do not want to focus on the rule anymore, I want my learners to develop a deeper understanding of what the rule represents."*

ii) Contextualised Questions

Ball, Thames and Phelps (2008) consider the selection of appropriate questions as an important element of teaching to highlight the intended learning objectives or specific aims within a question. The educators created a set of contextualised questions for the research lessons during the planning meetings and discussed the sequence of these questions within the research lesson to best promote learners′ mathematical thinking whilst supporting their learning.

In both PLCs, the educators decided that the lesson should begin with a question that either asked for prior knowledge or introduced concepts for the day.

Educator PTBM responded to the contextual questions portion as follows,

> *"The questions are designed to test learners, understanding and check their prior knowledge of the topic. I also wanted them to help learners develop problem-solving skills. The questions are asked so that learners can think creatively and analytically as they engage in answering the questions.*"

The data obtained showed that the questions used in the presenting of a lesson on exploring geometric patterns helped learners to become familiar with the appropriate vocabulary for patterns. It was also found to help if learners identify the core of the pattern and get a clear picture of what they should do. The educators worked together to uncover some of the misconceptions that learners often have when learning about geometric patterns. Together, they found that when given a position number and the sequence number, most learners focus on the rule and use it as a count, ignoring the fact that some position numbers are skipped, as shown in Figure 16 below. According to educator LPFM,

> *"This is a misconception that needs to be corrected by asking questions. To correct this, I usually repeat a similar activity to teach the correct method."*

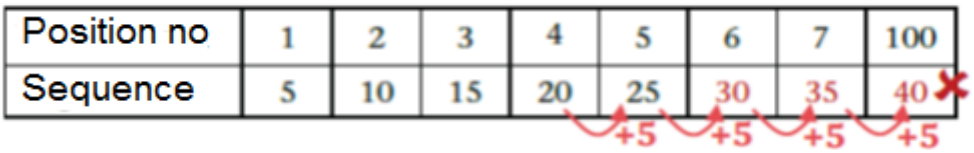


Figure 16: Grades 4 – 6 Learners' Misconception

If learners do not see the relevance of mathematics in their real lives, they are less likely to continue with the subject (Brown, Brown & Bibby, 2008). According to the educators, *the use of beads, matchsticks and other models that can be found anywhere should connect mathematics to real life and get learners enthusiastic about the subject.* In addition, the educator SLFM emphasised the importance of sequencing questions,

> *"Learners must master the flow diagram, before they are asked to represent the table sequence as a flow diagram."*

Educator PSPM gradually assigned the more difficult questions to the scaffolding questions, such as,

"Use the flow diagram to show the relationship between the inputs and the outputs."

Holten and Thomas (2001) state that questions that are not difficult for learners are not a reason for them to bother with them. However, if the questions are too difficult, learners are unlikely to be able to engage with them in a meaningful way. As the study progressed, educators often preferred to develop their own questions rather than adopt them from other curriculum documents. When developing contextual questions, educators were able to enhance their KCT by designing the questions to build on learners' mathematical thinking.

iii) Mathematical Tasks

This is the third and final sub-theme of KCT that will be elaborated on in this proposed analytical framework. Contextual questions are a sub-theme that is linked or integrated with mathematical tasks and learners. The questions should be aligned between the task and the learner (Holten & Thomas, 2001). Mathematical tasks should be evaluated before asking learners to complete these tasks in class. This helps educators to anticipate learners' strategies and identify points that may be unclear to learners (Stein et al., 2008). Educator TTNZ's worksheet had numerous errors that needed to be corrected during the presentation of the research lesson and participants did not collaborate on her worksheet.

"*I did not have much time to check the errors after administrative assistant typed up the worksheet. There were many activities going on in our school so I did not have the time to fully prepare,"* said educator TTNZ.

Mason (2009) said that in practicing mathematics questions, educators strengthen and deepen their understanding of the ideas and they become aware of their own learners' difficulties in solving mathematics problems.

Asking questions in mathematics, of course, means that educators use and develop their subject content knowledge. When they see other strategies to answer a question, they also develop their KCT (Ni Shúilleabháin, 2015). During the focus group interview, educator LPSK reflected on his time as a student,

"… *The lecturer of general didactics always emphasised that a teacher should go through all the activities he/she gives to learners first. She even said, imagine a learner*

who has a misconception, who will answer the question incorrectly, etc., and plan how to address that challenge in your teaching."

In addition, educator SBPM said:

"*It is so embarrassing to struggle to answer a task in front of learners, an educator should always be well prepared. Lesson study has developed my KCT and PCK. I am refreshed by what I have learned and have discovered how effective collaborative planning is for the benefit of educators and learners."*

Through participating in the lesson study and as part of the planning process exploring different mathematical strategies of patterns, educator PSPM enhanced his KCT by recognising more ways he could teach this topic especially the *general rules* concept. Educator PSPM said,

"*Since the last reflection meeting, I have thought of more ways to simplify and enhancing the rule concept (either in numerically or geometrically) for my colleagues. My observation was that most textbooks mislead us by not considering the 1st sequence and not finding a sequence without counting all the previous sequences."*

By engaging in mathematics as part of lesson study planning, educators developed their KCT, which in turn impacted their KCL as educators had more strategies to choose from in deciding how best to build on and anticipate learners' mathematical thinking. Mathematical task assessment also helped educators develop more mathematical questions because it allowed educators to view tasks from the learners' perspective. Educators learned how their peers interpreted the same mathematical concepts and thus developed a better sense of how other learners might answer the same or similar types of questions.

4.5.3. Knowledge of Content and Curriculum

Knowledge of Content and Curriculum refers to knowledge of the curriculum requirements and the materials that can be used to teach the appropriate content (Chikiwa, Westaway & Graven, 2019). Gulpric (2018) describes KCC as knowledge that "demonstrates an educator's broader knowledge of how topics apply to others within the same subject matter but across grade levels." This means that an educator who is able to show how one topic is relevant to another topic within the same subject actually possesses characteristics of KCC.

Content Area 2 consists of three topics, numeric patterns and geometric patterns are the first two topics in this Content Area respectively. The weighting of Patterns, Functions and Algebra as Content Area 2 is 10% from grade 1 to grade 6, but 25% in grade 7. Although the weighting is the same at the Foundation and Intermediate Phases, there is progression in the concepts taught. The educators who participated in this study demonstrated their knowledge of KCC when they located the patterns in CAPS. Location was followed by the concepts and skills to be taught in each grade according to CAPS and RATP per grade. Therefore, educators knew how each lesson plan should be structured based on KCC and were able to identify appropriate resources such as Learner and Teacher Support Material (LTSM).

The KCC is considered important by educators as it shows understanding of the curriculum. Educator PTBM said:

> "… *The purpose of the lesson was to teach learners what numeric patterns are, teach them how to investigate numeric patterns, and enable them to extend numeric patterns."*

In support of her colleague, educator TTNZ elicits the following:

> *"The purpose of the lesson is to observe how things change, the aspect of geometric patterns…"*

Participants related the use of resources to the repetition of taught concepts. They strongly believed that concept mastery and comprehension are enhanced when teaching is delivered using teaching and learning aids, media, and other relevant resources. Educator DSZM commented:

> *"…I think if learners had been reminded of the number sentence, it would have been easier for them to grasp the concept of numeric patterns. …The resources I used in my teaching were concrete, not costly and manageable by the learners and also familiar to the learners."*

When asked about the selection of resources and the use of examples in class, educator LPFM responded that:

> *"The resources I use are aligned with the syllabus. The worksheets are taken from the Platinum textbook. As an educator, I have reviewed them and found that they are cognitively balanced for learners and will help them develop good mathematical skills over time as they complete them with greater competence and confidence. And the*

DBE Workbook, also known as the blue book, is a Department of Education mandated resource. It helps develop well-informed and well-adjusted learners."

The data collected shows that the participants are in favour of the use of teaching and learning resources in the teaching. When doing lesson study, one should consider the resources included in the lesson presentation and the importance that these resources have in understanding the concept being taught. All participants agreed that the lesson presented with learning resources will help learners in the next grade to solve more complex patterns challenges.

The discussions that the educators had in their consultations showed a satisfactory understanding of the KCC that they were teaching to the learners. It is highly advisable to point out that educators are the masters of the content they impart to learners. Moreover, it is a great disadvantage if they only know the content but not the curriculum they teach. Therefore, echoing this, educator SLFM said in this context,

"The lesson I presented helped learners to make predictions. It helped them to make logical connections and apply logical thinking. Patterns are everywhere, whether at home or in the classroom……..This lesson has enhanced learners to design, architecture and visual interest. They can look at spaces, identify patterns and make meanings and interpretations."

The participants had their views in relation to PCK and KCT. When asked about the concepts and knowledge that learners must have to understand the topic of elaborating geometric patterns, the educator MGPT stated that,

"The use of symbolic expressions, shapes and objects lays the foundation for a better understanding of patterns. Learners need to know the concepts of investigating and extending patterns and look for relationships or rules in them."

This exclamation was supported by educator PSPM who added that knowing the concepts and knowledge learners should have before elaborating geometric patterns is meritocratic in the achieving the set objectives. He said:

"His lesson was a success because based on the observation through assessment, the learners were able to extend the decreasing and increasing pattern with or without the

use of concrete material, they are able to recognise the rule and use it to find the next terms in the sequence."

It becomes clear that it is crucial to have the knowledge that provides the background of a topic before that topic is introduced. In fact, the PCK of the educator should be sufficient to interpret the curriculum for the grade level he/she is teaching. Lessons of the lower grades should be simple at first and then complex regarding a higher grade level. The data collected shows that learners in the lower grades can represent patterns in a table, while in the higher grades they must represent the patterns graphically. The responses reflect that knowing the previous, current grade and next grade enhance the mathematical language in the current grade so that learners can normalise the mathematical concepts. It also enhanced the methods used to teach the concept of the day.

4.6. School Constraints Influencing Lesson Study Implementation

Anyone who has worked on a project has had to face certain constraints in its execution or implementation. Therefore, the interpretation and implementation of lesson study in different contexts is associated with various school constraints (Kihwele & Guoyuan, 2020). These constraints result from misinterpretation of lesson study cycles and the correct starting point. The results of this study suggest that there are 5 main constraints in the school that influenced the implementation of the lesson study in the two PLCs. These constraints resulted from: i) time, ii) duty load, iii) work beyond school hours, iv) educators′ attitude towards professional development, and v) lack of research skills.

4.6.1. Time

Time is an important resource in faciliating and achieving the implementation of lesson study (Coe, Carl & Frick, 2010; Meyer & Wilkerson, 2011). Time plays an important role in understanding educators′ pedagogical choices.Time pressure is a term that defines various factors that place time constraints on lesson study. Educators must implement a cycle of lesson study, which in some cases includes a second or multiple phases (Schipper et al., 2017). According to Hart and Carriere (2011), in most contexts, it is difficult for educators to achieve the second or third phase of the lesson study cycle because time is limited.

Time constraints include deadlines, work management, and allocated resources. Educators who feel less pressure in their work are more likely to be self-directed in their teaching and learning (Huang, Takahashi & da Ponte, 2019). Given the complexity of the lesson study as professional development, the time available to implement it is critical. According to Gero (2014), a small amount of time to conduct lesson study limits the analysis of planned and implemented lessons, thus hindering educators' ability to reform instruction. Analysis of participants' reflections during the lesson study discussions and the semi-structured interview revealed that lack of time is a practical limitation.

The lack of time in conducting lesson study was highlighted as a constraint in this study. As the educator LPFM said,

> *"As I said before, lesson study is a good model for professional development. The reason I said good is that educators are the driving force behind this model, but implementing it on our own will be difficult because we do not have time."*

In addition, educator LPSK said,

> *"I believe time will be the biggest obstacle (constraint) to implementing the lesson study model at the school level. Because of the alternating attandance of learners due to the pandemic, what we normally teach in one week sometimes takes us two weeks or more, even though the mathematics curriculum is full because of the pandemic."*

In addition, the educator MTAM said,

> *"Time is an important factor in implementing lesson study, sometimes we had to reschedule because time was a challenge."*

Thus, the lesson study cycle has a set time frame and a date by which it must be completed. Each individual lesson study cycle cannot last indefinitely. As mentioned earlier, the cycle should take a maximum of 4 weeks to implement and complete. In fact, some of the PLC 2 lesson study sessions had to be necessarily rescheduled because educators were not available. The schedule for daily activities was usually very tight for educators, so they participated after school hours and had little time. Thus, time plays an important role in PCK decisions and educators' professional development.

4.6.2. Duty Load

Education Labour Relations Council (ELRC) provided guidance to SMTs in 2003 on the duty load of educators in primary and secondary schools. Brunton (2003) identifies both teaching and non-teaching core duties and responsibilities of an educator. However, due to the limited number of educators, educators in schools are overloaded with both teaching and non-teaching duties and responsibilities. Due to the tight and heavy schedule of school activities and departmental programmes, educators often do not have time to manage the extensive tasks and duties.

Educators complained that non-teaching duties in the school stifled classroom teaching. Educator LNSM commented thus:

> *"As a member of the SMT, I have other administrative duties that sometimes compete with teaching. Even though my duty load meets the guidelines, administrative work takes up a lot of my time at school."*

Educator PSPM said,

> *"As the only departmental head in my school without an administrative assistant, I have taken on all the duties of an administrative assistant. In reality, I perform the duties of two positions, educator and non-teaching staff, so administrative work takes up too much of my teaching time."*

In addition, it has become the norm for educators to provide tutoring before or after regular school hours, especially for grades 6 and 7, which has increased educators' duty load by requiring them to spend additional time preparing and teaching. As a result, the time they have to spend attending lesson study becomes a contstraint.

4.6.3. Working Beyond School Hours

Efforts were made to accommodate lesson study meetings such as planning and reflection within school hours. However, this was not possible due to various constraints in the school context, such as COVID-19′s impact on teaching and learning and educators′ travel to the PLC meeting location. Consequently, participants had to stay behind beyond school hours to carry out lesson study.

In both PLCs, lesson study discussions were largely conducted after school hours, and in both cycles per PLC. During the second lesson study cycle, although the sessions in both PLCs were held 1 hour 30 minutes before normal school dismissal, it was after school hours. In other words, the participants in this study had to stay behind in school after dismissal. From the data collected, it was inferred that one of the constraints was that some participants were very tired, exhausted and passive during the sessions, especially in the sessions that were held after normal working hours.

The educators SPMM and DNNM were obviously uncomfortable staying until 16:00 for the lesson study discussions. They openly expessed their reluctance, and it was evident from their actions and gestures, even when the discussions seemed long. As a researcher, I was aware of their problem, so transportation arrangements were made when discussions went beyond 16:00. Educator DNNM lamented about the following during the semi-structured interview,

> "*Transportation after 4:00 p.m. is a challenge for me, not that I am not willing to stay past school hours. I am very greatful for the arrangements that were made to overcome my challenge and allow me to fully participate."*

Although, working beyond school hours was a constraint, most participants never expressed any negative comments about implementing lesson study during this study. Participants teaching grade 6 also said the following:

> "*We have to sacrifice extra classes to participate in lesson study. Our time is very limited since the schools have implemented alternate time-tabling for all grades except grade 7."*

This statement indicated participants' displeasure in participating in the lesson study beyond school hours.

However, educators stated that working beyond school hours is not a major problem if only the affected educators are informed in advance. In addition, educator PSPM stated that:

> "The schedule for the l*esson study should be included in the school year planner, but 3 days before, the chairperson should remind us*."

According to educator LPFM:

> "*This professional development requires extra time from us as educators, unlike workshops and other developmental sessions that happens within working hours. I feel*

that, its (lesson study) implementation may experience resistance from educators since it will take up their extra time."

Although the participating educators were willing to sacrifice their personal time for lesson study, working beyond hours is a constraint. Therefore, this study assumes that it will be challenging to implement lesson study in Ilembe District schools unless some support is provided by the administrators.

4.6.4. Educators' Attitude Towards Professional Development

A professional demeanour shows that an educator takes his or her responsibilities seriously. An educator who does not behave professionally sets a poor example and may lose the trust and respect of learners as a result. Therefore, a professional demeanour in all aspects of an educator's career provides learners with the best possible education. One of the basic requirements for lesson study is that educators have a positive attitude toward collaboration and working in teams to achieve common goals.

In South Africa and other countries, some educators have a culture of working in isolation. Consequently, they (educators) do not want to be observed preparing a lesson and even presenting in class (Saito & Sato, 2012; Hart & Carriere, 2011; Coe, Carl & Frick, 2010). In addition, educators believe that teaching is an individual practise, so they perceive lesson study as an activity that adds responsibility to their task (Gero, 2014; Hart & Carriere, 2011).

Lesson study as a model of professional development encourages educators to be active participants in their own learning. Thus, lesson study is the educator-led form of professional development. According to Jhang (2020), an educator's attitude is very important. Attitude affects an educator in many ways and can shape his/her knowledge of pedagogical content knowledge. Educators need to have a positive attitude towards professional development because it benefits not only themselves but also the learners (Gundogdu & Silman, 2007). According to Inprasitha (2015), the culture of isolation among educators develops a negative attitude toward professional development, such as lesson study. Therefore, it is difficult to implement lesson study successfully (Saito et al., 2006).

This study acknowldges that educators′ attitudes are an important factor as constraints to implementing lesson study were mainly associated with the participants′ attitude. Most educators who passively participated in the study and exhibited negative attitudes towards lesson study withdrew from the study for various reasons, resulting in a total of 8 participants withdrawing. As mentioned earlier, some participants did not contribute much during the lesson study cycles, especially in PLC 2. In some cases, participants missed or were late for lesson presentation observations. The consequences of educators′ attitudes toward professional development are fragmented learning, as some educators develop new knowledge that improves their PCK, while others did not learn new knowledge during this study.

4.6.5. Lack of Research Skills

Although the lesson study is a model for professional development, action research requires mastery of research procedures that lead to successful educator professional development (Akiba & Wilkinson, 2016; Shingphachanh, 2018; Elliott, 2019). The process of collecting data through the lesson study cycle and sharing the findings requires strong research skills (Schipper et al., 2017). Unfortunately, educators' lack of research skills hindered effective implementation of the lesson study (Seleznyov, 2019). During the reflection sessions, some educators were passive or repeated the findings of others.

To compensate for the lack of research skills among educators participating in lesson study, all educational stakeholders such as DBE, PDE, districts and schools should collaborate with research experts such as university institutes. This collaboration will help educators improve the conduct of lesson study that requires research experts. Guests or knowledgeable individuals are welcome to attend lesson study sessions. Experienced researchers from universities, for example, can serve as coaches to enhance educators' research skills in conducting the lesson study (Hart & Carriere, 2011). Since the research experts are not fully involved in the lesson study, they can slowly withdraw to give full autonomy to the educators in the schools. The withdrawal process will empower the educators and make them independent researchers in their PLCs. However, occasional follow-ups can be conducted to monitor progress.

4.6.6. Summary of Constraints

The findings have shown that constraints and challenges are inevitable if lesson study is to be introduced or implemented in the South African school context, especially in Ilemebe District. Despite the initial workshop on lesson study, some participants continue to infer that lesson study was only conducted for the benefit of this research and cannot be implemented thereafter. The questions and doubts raised by the participants clearly showed that there is still a lot of work to be done for educators to understand and conceptualise the meaning of lesson study.

The constraint factors of time and duty load are closely related. The findings of this study indicate that educators′ attitudes toward professional development can be attributed to the school context. As mentioned earlier in this chapter, only one school places importance on educators having the opportunity to receive professional development within the school, apart from external training such as workshops and others. Despite these constraints, the impact of the lesson study in this study was positive and encouraging. This shows that there is a sign of hope and potential for lesson study to flourish in schools in the Ilembe District of KwaZulu-Natal Province of South Africa.

4.7. Conclusion

The emerging themes discussed here have highlighted the role of the lesson study in this research. Despite the differences in the education system and teaching culture, the role of lesson study is similar in many ways to the findings reported in related lesson study research from local, African countries such as Malawi and Zambia and from abroad such as Japan and the United States of America. Although there were constraints and challenges in this study, the overall findings were positive and encouraging in terms of educators' learning and professional development. The participating educators expressed confidence that the practice of lesson study could influence their teaching practices in the long run.

CHAPTER 5 DISCUSSION, RECOMMENDATIONS AND CONCLUSION

5.1. Introduction

The main purpose of this phenomenographic study is to explore the role of lesson study for primary school educators′ PCK in teaching numeric and geometric patterns in grades 4 to 6 in Ilembe District of KwaZulu-Natal province, South Africa. This chapter includes a summary of the study's findings, recommendations based on the findings, and a conclusion.

This chapter is organised into 4 categories with nine primary sections, according to the table of contents. The first category consists of the introduction and revisiting the research questions. The second category includes the discussion of findings and implications of the study. The recommendations for the promotion and potential main features of lesson study follow. Finally, this chapter concludes with the category that addresses the limitations and conclusion of the study.

5.2. Revisiting Research Questions

The purpose of this study was to explore the role of how lesson study affected educators' PCK in the teaching of mathematics. To date, professional development approaches and courses conducted for educators′ professional development focus more on curriculum and policy, and are said to have little impact on educators′ teaching practice.

Lesson study as a kind of educator-led and bottom-up approach of innovative professional development was conducted in two PLCs from seven primary schools to engage mathematics educators in grades 4 to 6 in the process of lesson study. To explore the role of lesson study, a qualitative case study was used as the design. By revisiting the research questions stated in section 1.6, the findings of this study are summarised. Below are the research questions, as well as a selection of the research outcomes to show how the research topics were addressed.

5.2.1. What do educators need to enhance their pedagogical content knowledge of teaching numeric and geometric patterns?

This section explores what educators need to enhance their PCK about teaching numeric and geometric patterns. Not surprisingly, it has become clear that educators' pedagogical knowledge, content knowledge, and contextual knowledge are critical to good mathematics teaching and learner understanding (Sekao & Engelbrecht, 2021; Stols & Ono, 2016). PCK plays an important role in classroom instruction (Awanbor, 2019; Kathirveloo, Puteh & Matematik, 2014). PCK is knowledge that is unique to educators, according to Shulman (1986), and is based on how educators tie their PK (what they know about teaching) to their SMK (what they know about learning).

In South African education, the design and preparation of a lesson is usually the responsibility of an individual educator. It is the responsibility of the individual educator to decide how the lesson will be designed, what materials will be used and how learners will be assessed. Educators have a significant impact on the academic and lifelong success of their learners (Takahashi & McDougal, 2019). Hourigan and Leavy (2021) agreed and stated that an educator as an aspect of mathematics teaching environment is related to learners' achievement. Therefore, enhancing educators PCK is crucial to improving learner achievement.

Most professional development activities focus on building educators' expertise (Takahashi & McDougal, 2019; 2016). Shulman (1986) also emphasised the need for educators to know learners' mind-sets. Therefore, in this study, educators assessed learners' prior knowledge of numeric and geometric patterns and whether they understood the objectives of the lessons presented. The literature in Chapter 2 shows the importance and the need for PCK among educators. Therefore, a lack of PCK will affect educators' effective teaching and learning in the classroom. When asked what educators need to enhance their PCK of teaching numeric and geometric patterns, educators responded as follows:

> *"We need professional development that focuses on developing our subject knowledge in the classroom. We need to be able to apply in our knowledge in the classroom, especially during this pandemic COVID-19."*

They went on to say:

> *"We need lesson study to enhance our PCK of teaching, not just patterns but the whole topics. We strongly believe that all subjects need this model, not just mathematics alone."*

According to this study, participants need lesson study to enhance their PCK of teaching during numeric and geometric patterns. *This professional development, called lesson study, has enhanced our knowledge that makes us mathematics educators rather than mathematicians.* According to Kaput (2017), educators differ from mathematicians not necessarily in quality or quantity, but in how that knowledge is organised and used. The participants in this study who are mathematics educators demonstrated knowledge organised from a pedagogical perspective and used as a basis for helping learners understand patterns. In addition, they exhibited a firmer structure that presumably resulted from the constraints of the curriculum, using lesson study as a vehicle to enhance their PCK.

The findings are consistent with the literature on lesson study (for example Boz & Belge-Can, 2020; Lampley, Gardner & Barlow, 2018; Juhler, 2016) which demonstrates the positive role of lesson study on educators' PCK. The findings of this study show that educators need lesson study to enhance their PCK of teaching numeric and geometric patterns: their mathematical PK, their mathematical CK, and their contextual knowledge. Educators shared their knowledge at the lesson study meetings, such as their understanding of mathematical concepts that should be taught for numeric and geometric patterns.

Learning and mastering patterns is still a big problem for learners in primary schools, and therefore, it is considered as one of the problematic topics to teach and understand, especially in the Intermediate Phase. According to Scott, Stone and Dinham (2001), educators' difficulties with patterns are well documented in many countries, and many authors consider patterns to be the most problematic concept of mathematics covered in primary schools. Educators needed a lesson study to collaboratively plan their research lessons and enhance their PCK of teaching numeric and geometric patterns. As the educator TTNZ explained:

> *"For me, the way the lesson study was conducted enhanced my PCK and I believe that of the other educators as well. I learnt that one model can be used for different activities and be meaningful to the learners, while many models or teaching aids in one*

lesson can be a destruction. In fact, it (the lesson study) was a tool I needed to enhance my PCK about teaching patterns in general."

Educator PSPM agreed with educator TTNZ, however, added another insight:

"The lesson study enhanced our PCK on mathematics teaching and attitude towards professional development. The majority of us (participants/educators) have a positive attitude towards the lesson study and were willing to participate in the lesson study, with the exception of few individuals who showed a negative attitude and no willingness. It has changed our teaching which was weak before. So lesson study is a model we need to enhance our PCK in teaching general."

Chiew, Dali and Lim (2016); Chiew (2009); and Yamnitzky (2010) found that the role of lesson study can impact primary school mathematics educators' PCK. Despite the limited time in which this study was formally conducted, lesson study was determined to have the potential to enhance educators' PCK of teaching numeric and geometric patterns in this study. Furthermore, this study confirms that lesson study is exactly what educators need to enhance their PCK. In group meetings, educators shared their PCK of numeric and geometric patterns. As a result of the total examination of the data, educators' PCK of teaching patterns is enhanced by lesson study.

5.2.2. How does educators′ participation in lesson study enhance their pedagogical content knowledge in teaching numeric and geometric patterns?

The overall analysis of the data showed that participation in lesson study has enhanced educators′ PCK. This finding concurred with other related lesson study researchers, in Japan, where lesson study has been established as a trend to improve educators′ PCK (Coenders & Verhoef, 2019; Lucenario et al., 2016). Hill, Schilling and Ball (2008; 2004) said that PCK is an essential and crucial component for an educator′s success in mastering the learning and teaching process, in addition to facilitating effective teaching. According to Isoda and Olfos (2021), lesson study is understood as enhancement of a school based approach to teaching improvement and professional development. Sekao (2019); and Stols and Ono (2016), the lesson study is not only for the development of mathematical content knowledge but also mathematical PCK.

In this study, PCK was found to be deeply personal, highly contextual, and influenced by teaching interactions and experiences. Therefore, the lesson study approach to teaching of numeric and geometric patterns was a timely attempt to enhance educators′ PCK, especially in mathematics, since it was highly structured and involved collaborative interaction among educators based on actual teaching. Thus, as a lesson competency-based approach to teaching, lesson study attracted much attention from participants to enhance their teaching competencies for patterns.

In this study, the participating educators expressed how participation in lesson study enhanced their PCK in teaching of numeric and geometric patterns. The educators mainly refer to the lesson study as an 'eye opener' to enhance their PCK in teaching patterns. This is illustrated by the following response from educator MGPT about participating in the lesson study that enhanced her PCK in teaching:

> *"My participation in this study enhanced my PCK in teaching geometric patterns as a concept I presented. I was able to design lesson accordingly and analyse my learners' abilities, which led me to not follow the prepared lesson plan."*

Although educators claim that participating in the lesson study has enhanced their PCK in the teaching, some of the educators in both PLCs emphasise that it is still difficult to adapt their teaching to the new strategies they learned in this study. This is exemplified by educator LNSM:

> *"Applying new strategies I learned during lesson study research is still difficult, especially for me since all of my experience is high school based and I was recently appointed at a primary school. My PCK has enhanced, not only in terms of teaching patterns, but in terms of the entire subject of mathematics."*

In addition, educator HSTM said:

> *"I still find it difficult to teach learners the concept of the rule from the new teaching strategy we discussed as a PLC. I have tried but I still find it difficult to make the learners understand and grasp the concept."*

In both PLCs during the initial planning meetings, both experienced and novice educators demonstrated a traditional teaching approach, however, their participation in this study enhanced their PCK in teaching numeric and geometric patterns. The PLCs agreed to focus on making learners more active in teaching mathematics, especially grade 4 learners, by having

them do hands-on activities, especially in teaching geometric patterns. The educators felt that it would be beneficial to give learners time to work in pairs or groups, however, the regulations of COVID-19 did not provide for such classroom design.

Educator TMBN commented:

> *"My PCK in terms of teaching patterns was enhanced by participating in the lesson study. I didn't teach a lesson, but I was able to use the new strategies I learned in the PLC in my teaching. Since I teach grade 6, I allow learners time to research and work on their own in most lessons and only explain content to learners who have specific questions."*

Educator TTNZ said the following:

> *"Before participating in the lesson study, I always relied on the textbook for my teaching. Finally, I learned new teaching strategies by participating in the lesson study research."*

Educators also said that *participating in lesson study does not automatically lead to change.* Some educators also said that *this aspect of teaching is not yet part of our regular classroom*. Although educators' participation in the lesson study enhances their PCK in teaching numeric and geometric patterns, the time factor was cited as a limitation. Using this model to prepare and present lessons requires *additional effort and time*, however, educators strongly believed that this in-service training would enhance their PCK.

Participation in lesson study enhanced PCK in terms of the teaching patterns of both presenters and observers. As mentioned earlier, educators had the opportunity to observe their colleagues presenting lessons in their classrooms. The classroom observations were the vehicle to enhance educators' PCK in teaching numeric and geometric patterns and school culture. According to Flynn, Hedges and Bruce (2009), classroom observation is a central feature of lesson study. The lesson observers in this study benefited by using their observations while participating to enhance their PCK in teaching patterns. Participants valued the opportunity to observe other educators in action, and presenters valued the opportunity to be observed. Educator SPMM commented:

"It was really a great experience to participate in the lesson study. The lessons presented were in line with the Annual Teaching Plans, so I implemented everything we discussed during the meeting in my grade 5 lessons."

The lesson study cycle's phases contain major elements of educator professional development related to mathematics curriculum reform (Ni Shúilleabháin & Seery, 2017), and it has been proved to encourage reform in countries such as Japan (Yalcin, 2019). Participants stated that taking part in the lesson study enhanced not just their PCK but also their grasp of the curriculum. In South African education, the curriculum is a top-down approach, meaning that educators are the 'implementers' of the curriculum. In addition, the lesson study enhanced their PCK to adapt to the changes in the curriculum as the patterns were merged in the Revised Annual Teaching Plans for 2021, whereas in 2020 they were not taught in the 2nd and 3rd school terms. Thus, the role of lesson study affected their professional practice and attitude towards teaching and learning numeric and geometric patterns.

All participants endorsed lesson study as an approach of professional development that enhanced their PCK for teaching numeric and geometric patterns. Educators believe that *lesson study is more valuable than professional workshops because they provide leadership*. Although the impact of the role of lesson study on educators and learners was not formally assessed through re-teaching and assessment, educators expressed their professional opinion that the quality of their PCK in teaching improved because of participating in lesson study.

Educator DSZM, for example, was convinced: *"… because of the improvement in my PCK, the learners' performance has also improved, unlike the previous years before I participated in the lesson study*." Educator LPSK commented as follows: *"Participating in lesson study has not only enhanced my PCK in terms of teaching patterns, but has also built good relationships with educators who are not part of my school. I think this was beneficial because without the lesson study, I would not have had the opportunity to learn from their expertise."* Therefore, lesson study positively enhanced the PCK of the educators who participated in this study positively in terms of teaching practice and collegiality.

In addition, the degree of enhancement in educators' PCK also varied depending on the concepts discussed in the different lesson study cycles or planning meetings. Educators' PCK was enhanced to varying degrees by the individual PLCs in each lesson study cycle. Prior to this study, it was rare for educators to collaborate and observe each other, except in IQMS where a senior and peer observe an educator teaching, however, they do not plan the lesson together. In the lesson study, on the other hand educators discuss and plan a lesson collaboratively and observe each other teaching and reflect on one another's teaching. In short, educators' participation in the lesson study enhanced their PCK in teaching numeric and geometric patterns.

5.2.3. What are the educators′ views about the lesson study when teaching numeric and geometric patterns?

There is a growing amount of research that advocates lesson study (Stokes, Suh & Curby, 2020; Alder & Alshwaikh, 2019; Mhakure, 2019). Since the beginning of the Japanese public education system, lesson study has been the major method of professional development for both aspiring and active educators (Moquin, 2019; Lewis, 2006; Lewis, Perry & Murata, 2006). Educators' perceptions are articulated in their own terms and language about lesson study when teaching numeric and geometric patterns.

The educators (both presenters and observers) viewed the process of lesson study as an enriching and useful experience, and saw it as an aspect that was of value to them professionally. They felt that the dual role as presenters and observers in the research cycles broadened their perceptions and knowledge of practice. The sharing of teaching practice of numeric and geometric patterns through lesson study was found to be enriching and inspiring by the educators in both PLCs. They expressed the view that for the first time they were able to observe lessons they were involved in planning and see how learners learn with the approach presented. Because of the collegial attitude, the lesson study model was considered a successful method to professional development. Traditionally, the norm has been an individual rather than a collaborative approach to professional practice.

Educators viewed the sympathetic relationship as beneficial and enjoyable as evidenced by a comment from educator SLFM:

> *"By lesson study, we have the time to exchange ideas with our colleagues and make valuable contacts with colleagues from other schools who teach mathematics."*

In addition, educator MSCM stated that:

> *"Undoubtedly, I view this as a way forward in terms of teaching and learning as I have had the opportunity to plan, observe and reflect alongside educators from my school and neighbouring schools who are at a different stage in their teaching careers to me."*

Furthermore, the educator MTAM said:

> *"I have been inspired by certain strategies that educators have used in their classoroom when teaching numeric or geometric patterns, especailly the educator PSPM. Lesson study has given me the opportunity to learn from other professionals who teach Intermediate Phase mathemtaics and continue to grow and develop."*

Educators also viewed the lesson study as a change agent in the way they think about lesson planning. Educator PTBM responded:

> *"I view lesson study as a tool or a change agent that makes me think about each learner and his/her learning when planning lessons."* Furthermore, educator DSSZ said, *"I view it (lesson study) as a tool that makes me view lessons as dynamic and should be revised, however, on my own it is difficult to revise or reflect on the lessons I present."*

The following findings are the educators responses:

> *"When learners' answers are correct, they are correct. I never bothered to find out how they arrived at the answers,"* said educator SBPM.

Educator LPFM who served as the PLC coordinator in this study, participated in lesson planning, observation and reflection, and facilitated a lesson,said the following:

> *"Before the lesson study, I always explained to my learners what I had prepared and what I knew and never bothered to let them express their ideas. My teaching always consisted of explaining with the help of textbooks. However, my view about the lesson study is that learners are not empty shells, but have knowledge of every concept that is to be taught."*

It can be concluded that educators regard lesson study in the teaching of numeric and geometric patterns as an approach that allows them to give learners the opportunity to better express themselves through their answers, which has shaped a new type of belief among educators about teaching practice.

In addition to views about lesson study, educators indicated that it was beneficial to participate in the lesson study process. Collaboration, especially in planning research lessons, allowed all educators to share their thoughts and encouraged reflection, which in turn led to discussion of learner challenges and practice of teaching methods. One novice to teaching, educator MGPT particularly welcomed this:

> *"Planning research lessons with others was definitely very helpful for me."*

Educator PTBM welcomed collaborative lesson study planning:

> *"Sitting around the table and going over lessons to be presented with other educators was very helpful. I even enjoyed observing educator PSPM, he is such a good educator."*

Furthermore, educator LNSM, who has more experience than any of the other participants, said the following:

> *"In the teaching profession, you don't have that much time to collaborate, especially when you are teaching mathematics because it is taught every day. I view lesson study as the best professional development in teaching patterns because it gave us the opportunity to gather and reflect."*

According to Huang, Takahashi and Ponte (2019), lesson study is not just a "*nice to have",* but a *"must have*." He went on to say that lesson study gives educators access to outside specialists, or educated others, so that they can discover new ideas for enhancing teaching and learning through actual examples. The educators who participarted in this study agreed with him. They believe that there is a need to involve mathematics subject advisors in lesson study PLCs. In addition, they viewed lesson study as a fundamental driver of professional development that enables educators not only to learn new ideas for enhancing the teaching of numeric and geometric patterns, but also to increase their expertise in teaching mathematics. When examining the role of lesson study in enhancing primary school mathematics educators' PCK, it is found that lesson study is the vehicle that educators need when teaching numeric and geoemtric patterns.

5.3. Discussion of Findings

Lesson study appears to be an innovative model of professional development that effectively enhances educators' PCK through PLC, according to the findings of this study. Educators who participate in a professional development programme to gain knowledge of content learners, knowledge of content and teaching, and knowledge of content and curriculum are more likely to change their PCK (Koehler, Mishira & Cain, 2013).

Increasing educators' knowledge through the role of lesson study is the only intermediate outcome that would, in due time, translate into enhancing their PCK of teaching as a long-term outcome (Murata, Lewis & Perry, 2004). According to the literature, the outcomes of lesson study are able to enhance and sustain educators' motivation and attitudes toward professional development, which eventually leads to a change in their teaching practise (Baki & Arslan, 2022; Vermunt et al., 2019). According to educator DSSZ:

> *"This research on lesson study has helped me to enhance my PCK in teaching patterns and other topics, even in other subjects I teach."*

Educator TMBN agreed with educator DSSZ, saying,

> *"I have benefited greatly from this study on lesson study. I believe we have all benefited by enhancing our PCK in teaching numeric and geometric patterns. I would like to believe that this is not the end of our PLC but we will continue after this study."*

Prior to the implementation of this study, participating educators worked in isolation because there were no structural professional development activities for primary schools in the school or within schools where they could collaborate and share ideas. The lesson study role provided educators with the context and venue to enhance their PCK for teaching numeric and geometric patterns by actively planning and reflecting. The process of the lesson study process provides educators with the opportunity to share different opinions and ideas about teaching numeric and geometric patterns. This study demonstrated that educators who collaborate with each other can enhance their teaching practices. As educator HPNM said:

> *"A lot has been learned from this process called the lesson study. I never thought that we as educators could plan a lesson together and also discover ideas during the observation on how to improve that lesson when it is taught again in the future."*

Education has become one of the primary drivers of the change agenda in South Africa since 1994, when the country became democratic (DBE, 2018). South Africa continues to struggle to enhance educators' PCK in mathematics education, despite the excellent intentions represented in the curriculum reforms. The CAPS is equivalent in quality, breadth, and depth to mathematics curricula in other nations, according to DBE (2013), the current South African curriculum declaration. When educators design, present, and reflect on lessons, they should keep 4 fundamental characteristics of mathematics in mind: conceptual comprehension, mathematical procedures, strategic competency, and reasoning, all of which are supported by learner-centred classrooms (DBE, 2018).

Research has shown that learners are able to solve pattern problems involving symbols, procedures, graphs and tables, but have difficulty generalising patterns (Nurrahmi, Suryadi & Fatimah, 2017; Barbosa & Vale, 2015). In the previous chapter, educator MTAM agreed with the research findings, "*I observed that learners have difficulty grasping the concept of rule in both numeric and geometric patterns*." At the Intermediate Phase, learners are expected to describe the general rules of patterns in their own words rather than in algebraic language.

Although this study explored the role of lesson study for educators' PCK in teaching mathematics, it also discovered that educators are also having difficulty generalising patterns in both numeric and geometric patterns. The goal of teaching numeric and geometric patterns is to develop the concepts of variables, relationships and functions. Understanding these relationships allows educators to describe the rules that create the patterns. Thus, if the lesson study enhances educators' PCK when teaching mathematics, their SMK and content knowledge will also be developed.

5.4. Implications of the Study

The conclusions of this study have consequences for the lesson study as an internal professional development technique in the future. In addition, this research implies that lesson study could be a novel and alternative paradigm for educator professional development in South Africa. Lesson study has been hailed as an effective model for professional development in the literature, (Lertdechapat & Faikhamta 2021; Murata & Lee , 2020; Shingphachanh, 2018; Lampley, Gardner & Barlow, 2018), and thus has the potential to influence educators' teaching,

as evidenced by the findings and results of this study. The following 4 implications for educational institutions and educators may arise from the efficient application of lesson study.

Firstly, lesson study is educator-directed and reilies on a bottom-up approach to professional development, Lewis (2002), which requires a great deal of time and commitment from the educators, such as a positive attitude towards the teaching profession. In a lesson study, educators might examine problematic concepts or topics, lesson plans, teaching tactics, problems that may occur during the class, and the use of teaching models as part of a PLC. They can learn from each other's opinions and guidance when they review teachings in the reflection session. This helps educators in honing and enhancing their PCK.

Secondly, primary schools in Ilembe District seem to place very little emphasis on promoting and enhancing the professional development of educators, the emphasis being more on the district officials. As observed in both PLCs, there was no specific programme or planned professional development activities by the SMTs. However, only School L had unstructured professional development sessions that sometimes took place once a term. Although pre-pandemic COVID-19 educators were required to participate in in-service professional development activities outside of school, this was generally a top-down instruction. Some educators attend such in-service training because they are labeled “present”. However, they do not implement what they have learnt. In other words, this study suggests that edcuators’ awareness or desire to pursue professional development is very weak, so studying teaching will improve educators′ professionalism.

Thirdly, in contrast to Japanese practise, lesson study is considered as formal meetings where activities are planned as part of the academic school programme (Fernandez & Yoshida, 2012). This indicates that in Japanese classroom culture, lesson study is viewed by both SMTs and educators as an important aspect of educators’ professional development. Although in South Africa some mathematics subject advisors travelled to Japan in 2018 to learn more about lesson study, there has been little emphasis placed on this aspect. Currently, not all schools in the KZN DoE are implementing lesson study in mathematics and some are not even aware of this professional development called lesson study. Therefore, for a smooth implementation of lesson study, it is crucial that all stakeholders in the education sector, including educators, are made aware of the importance of lesson study in their professional development.

Finally, the association between educators' mathematical PCK and their ability to differentiate mathematics instruction in the normal classroom is the most important conclusion drawn from the data of this study. This knowledge can help educators in better understanding their learners and the classroom environment, allowing them to construct the most appropriate lesson plans and teaching practices to meet the needs of their learners.

5.5. Recommendations for the Promotion of Lesson Study in South Africa

This study has shown that lesson study offers an alternative, innovative and effective approach to educator professional development that ultimately impacts on teaching. In view of the positive outcome, this section attempts to make some suggestions for the promotion of lesson study in the South African educational context. Given the constraints and challenges faced in implementing lesson study in the South African educational context, it is imperative that the various stakeholders: policy makers, school management team, organised labour and educators understand the rationale for lesson study and work together to improve educator learning.

5.5.1. Involving Policy Makers

Although lesson study is a model of educator-led professional development, the involvement of policy makers to promote lesson study in the early stages is still important and relevant. A directive from the Minister of DBE, Mrs. Angie Motshekga, and members of the Provincial Education Executive Councils (MECs) would go a long way in overcoming the initial barriers, especially the resistance of practicing educators to engage in lesson study. Therefore, the top-down approach of education authorities is sometimes unavoidable, even though it is not encouraged in the context of lesson study. Currently, lesson study is implemented top-down especially in the KZN DoE, with district officials taking the lead to ensure smooth implementation. Therefore, policy makers (including educators' unions) should provide clear guidelines for the implementation of lesson study in schools, which include the rationale, objectives and procedures for the implementation of lesson study.

Given the South African education scenario, the involvement of education authorities is imperative to support and promote the implementation of lesson study. If lesson study is not guided by higher authorities, it may be neglected during implementation due to the contextual constraints and challenges identified in this study. Policy makers could provide support by assigning officials from the directorate of Teacher Development, circuit managers and curriculum officials to monitor progress in implementing lesson study in schools.

This may include documentation of lesson study reports and the number of lesson study cycles to be conducted in an academic year, although such matters should be left to the discretion of SMTs and educators. Each school should maintain a progress report on the conduct of lesson study implementation and should be reviewed regularly by the education officials. To support and sustain lesson study, policy makers could establish a network of collaboration between different parties: educators, educators' unions, higher institutions of learning and education officials. The results of the lesson study implementation should then be shared and discussed in seminars at district, provincial or/ and national level to promote educators' learning and professional development.

5.5.2. Collaboration with Curriculum Specialists

The relevance and importance of curriculum knowledgeable officials, especially subject advisors, in lesson study is important. The participants of Chiew, Dali and Lim (2016) indicated that the presence of external observers enhanced their confidence and gave them some insights and ideas for their lesson plans. Overall, they appreciated the involvement of the lecturers as they were sometimes indecisive about their teaching strategies and approaches.

Indeed, the presence of curriculum experts may encourage educators to seriously participate in the lesson study. For example, the participating educators suggested that they should consult the subject advisors on certain issues for more clarity. This study finding suggests that the involvement of curriculum experts officials can overcome the limitations of teaching knowledge in lesson study PLC as noted by González, Villafañe-Cepeda and Hernández-Rodríguez (2021). Therefore, the involvement of knowledgeable others would also be of great help to support the lesson study process Seino & Foster (2021).

5.5.3. Support from the School Management Team

As highlighted in this study, support from the SMTs is crucial and important in the implementation of lesson study. Akiba, Murata and Howard (2019) have emphasised the importance of school leadership support in the implementation of lesson study in schools. Indeed, SMTs can provide their support in several ways. They could set up a schedule for lesson study to reduce time pressure as cited in this study.

To implement lesson study, the SMT could include lesson study in the school's annual programme. This shows that the SMT is indeed serious about implementing lesson study. SMT could establish a lesson study committee to oversee the implementation of lesson study at the school level. The Departmental Heads could not only provide motivational and moral support but also participate in the discussions on lesson study as they are subject educators themselves.

Through the lesson study, the SMTs listen to the educators and understand their teaching problems, which gives them an opportunity to offer some constructive suggestions to the educators. Their presence in the lesson study would certainly motivate the educators to seriously participate in their lesson study. In short, the support of SMTs is important to promote lesson study as innovative school-based professional development for educators.

5.5.4. Promoting the Culture of Lesson Study

It was found that there is lack of opportunities and support for educator learning in the South African school context, especially in primary schools. The study concludes that educators' lack of awareness of professional development hinders the implementation of a lesson. This aspect is indeed seen as a difficult task as it involves changing the culture of teaching, that is, the mind-set and attitude of educators towards their profession, which have long been embedded in the culture and the education system. For example, some educators believe that the time after school hours is private. Even though this issue is controversial, the practicing educators should discard this mind-set and change their attitude and commitment to the teaching profession in the context of educator learning and professional development.

Although time is an obstacle, this would not be a problem if the educators and SMTs are serious about implementing lesson study. For a long-term strategy, educators' motivation and commitment should be the driving forces in implementing lesson study. However, until such a culture and attitude is nurtured and promoted among practicing educators, it would be a difficult task to convince educators of the relevance of lesson study practice. This study has shown that it is imperative to create a context for lesson study that is highly rewarding for educators. Educators who had experiences that were beneficial to them would be motivated to participate in further lesson study cycles. Educators would even be more satisfied if lesson study experience gave them the opportunity to take charge of their own professional development Chokshi & Fernandez (2005).

5.6. Potential Main Features of Lesson Study as a Form of Professional Development

Effective professional development engages educators in learning opportunities that are supportive, in-service, instructional, collaborative, and ongoing (Hourigan & Leavy, 2021). They further said that SMTs who are guided by these features can develop meangful learning experiences for all educators. Various studies have examined the features of lesson study. For example, Willems and Van de Bossche (2019) stated that lesson study helps educators improve by focusing on collaborative planning and revision of a single lesson. According to McMillan and Jess (2021), the features of lesson study can be modified by researchers depending on their background and studies. In this study, lesson study as a form of professional development was found to elicit a number of notable features in educators as described in this section. As this study involved two cycles within a six-months period, caution is warranted as some of these findings may not be transferable to longer cycles or in other settings. The notable feaures of this study were: i) synergy, ii) autonomy, iii) permissiveness, iv) reflection, and v) continuity.

5.6.1. Synergy

The planning meetings of this study in both PLCs revealed that the edcuators collaborated in developing lesson plans for the research. Clear evidence of synergy was seen in the lesson study PLCs, where educators discussed the strategies they would use to teach numeric or geometric patterns. PLC 1 had two planning meetings per cycle, while PLC 2 had two during the first cycle and three during the second cycle planning meetings, therefore, a total of nine planning meetings were held in this study. Lesson study PLC 1 produced 4 research lessons (1 – grade 4, 1 – grade 5 and 2 – grade 6). While lesson study PLC 2 produced 5 research

lessons (2 – grade 4, 2 – grade 5 and 1 – grade 6), four research lessons were presented in PLC 2.

Educators in both lesson study PLCs shared responsibilities, with one educator presenting a lesson and the other educators observing, while the lesson study PLC coordinator presented a lesson outline in the planing meeting and the other educator stepped in as coordinator, as also noted by Akiba, Murata and Howard (2019). Lesson study is intended to be a way for educators to work together to improve the quality of teaching Cajkler et al.(2013). These findings are consistent with previous research showing that lesson study can create synergy between educators in planning, teaching, and reflecting Mills & Harrison (2020).

5.6.2. Autonomy

The educators who participated in this study were able to work alone, without supervision or time for meetings. They were able to solve most problems on their own without consulting experts or mathematics subject advisors. The educators managed the activities of the teaching PLCs independently, including their meetings, observations, reflections, and time, without consulting the SMTs. The educators' lesson plans were appropriate for the curriculum as a whole and aligned with the DBE lesson plans. The lesson study process was conducted by the edcuators as participants in the professional developemnt; the mathematics subject advisors only led the advocacy workshop.

5.6.3. Permissiveness

The educators in this study were permissive when their colleagues, as observers criticised them. The educators recognised that the observers had the advantage of seeing deficiencies in teaching that they themselves, as presenters, did not see. For example, when educator SBPM was asked to avoid choral responses during the first cycle reflection meeting of PLC 1, he gladly took the advice and promised to refine the strategy by asking learners to raise their hands and respond individually. Another example: when it was pointed out to educator TTNZ that she had not given clear instructions for the class activity and that the activity had errors, she took the feedback positively and promised to avoid such errors in the future.

The permissiveness helped educators to refine their teaching practises. The educators realised that their beliefs might be wrong Lewis & Perry (2017); therefore, they needed advice from others. It is also important to note that observers need to consider ethical issues when providing feedback to an educator giving a presentation. The ethical aspect was highlighted in the advocacy session at the beginning of this study, so observers were given a lesson study cycle observation tool to guide them. Their feedback focused on the educator's PCK related to the teaching process of the day and avoided commenting on the educator's personality. Through lesson study, educators can learn to observe and be lenient in giving objective feedback to each other.

5.6.4. Reflection

In this study, the presenters were the first to reflect on their teaching before the observers. For example, in the lesson study PLC 2, the educator DSZM reflected on her lesson and said that the objectives of the lesson were met and the implementation went as planned in the planning meetings and according to the lesson plan for grade 4AZ. In another example, educator PSPM reflected that the use of a data projector when teaching geometric patterns helped learners to see all the colours appropriately. He thought this was a good idea as learners could easily engage with activities during the lesson presentation.

According to Çimer, Çimer and Vekli (2013), reflection is an important way to improve professionalism in teaching. They further assert that almost all educators reflect regularly and thoroughly on their teaching practise, which is why the KZN DoE lesson plan template includes reflection. The self-reflection could help educators to correct their own inaccuracies in teaching. In addition, Clarke (2000) claims that reflection can help educators construct their understanding based on their teaching experience.

5.6.5. Continuity

The analysis of the lesson study activities demonstrated that this type of professional development might potentially run throughout the school year. However, lesson study was only completed in two cycles per PLC. Educators can continue these cycles of activities, which is in line with Özdemir′s (2019; 2013) results that educators can learn continually while studying a lesson as a model of professional development. Educators can enhance their teaching after talking about it in class. Lewis et al. (2013) found that lesson study can help educators enhance

their PCK in mathematics teaching over time in a longer-term professional development programme for educators.

5.7. Suggestions for Future Research

This study encountered some challenges in conducting the lesson study, whcih may have affected the role of lesson study for the participating educators. Due to the limitations in this study, future research is suggested. The limitations of this study, such as a lack of time, physical distance between the researcher and the participants, and the number of lesson study cycles completed, may be sufficient to influence or change the educators' teaching.

Although the outcomes were evident in terms of educators' PCK and reflective practice, the impact on educators' teaching as a whole is inconclusive. Collet and Greiner (2020) emphasised that the knowledge gained in the context of lesson study applies beyond the particular study. Thus, it was not known exactly how the role of lesson study might influence educators' teaching beyond the lessons discussed. Therefore, more time is needed; probably three years or more, to examine the role of lesson study on educators' PCK. Longitudinal studies with multiple lesson study cycles would be useful to examine the role of lesson study on mathematics educators' PCK.

Some educators were apathetic to engage in lesson study, which to some extent affected the role of lesson study, which is the focus of this study. Therefore, it is suggested that future research on the role of lesson study should focus on educators who volunteer and engage. This study has identified several factors and constraints that influence the implementation of lesson study. These factors should be explored in detail through case studies to provide further information and suggest guidelines for the implementation of lesson study in the South African educational context.

As it is a relatively new type of professional development for educators in the South African context, further research on lesson study is needed. Lesson study has a strong potential to motivate educators to develop themselves. Therefore, research should identify the factors that would lead to the growth and continuation of lesson study. PLCs have encouraged educators to take charge of their own professional development, and perhaps the lesson study offers an innovative and effective model. Therefore, it is worthwhile exploring how the lesson study

process of studying teaching might inspire, motivate, and encourage educators to grow in the teaching profession.

As a mathematics curriculum subject advisor, the researcher advises that the expert researcher might undertake the lesson study as an outsider. This could assist educators in overcoming their challenges and provide them with the opportunity to communicate with a professional in any field. Educators in the lesson study PLC had not only to discuss and share their knowledge and concerns with their colleagues, but they also had to learn knowledge or abilities that none of them had and that only an expert could supply.

5.8. Limitations

It is vital to acknowledge the limits of this study when presenting the findings, contributions, comments, recommendations, and conclusion. There were a few limitations in the study.

The study's sample population was one of the study's limitations. To begin with, this study only included 2 PLCs, out of a total of 463 primary schools. These two PLCs cannot be considered representative of all PLCs in Ilembe District, KZN Province, or the entire country. Although these PLCs are more than 100 km apart, they are both located in Ilembe District and represent public schools. Further research may be needed to investigate how participation in iterative lesson study cycles might affect mathematics educators in larger or more rural schools.

The study's second limitation is that educators were not evaluated conclusively on how their CK changed as a result of their participation. The study's focus on educators' PCK was one explanation for the lack of summative data. To avoid unncessary pressure on participants, no summative assessment was conducted. In addition, instruments have not yet been developed to assess the SMK and PCK of primary school mathematics educators (Howell, 2014). The evidence for educator learning is the community of educators who participate in lesson study and their self-report and learning in semi-structured interviews.

The third limitation was the study's duration, which was limited to 2 cycles and occurred during the COVID-19 epidemic. It was not possible to see if lesson study could be done consistently throughout time. According to Smit and Humpet (2012), it takes more than two years for educators' teaching practices to alter. As a result, it is proposed that a longer-term study be conducted to better understand the impact of lesson study on educators' PCK in mathematics education. A long-term study would have the advantage of being able to identify changes in educators' PCK that would be difficult to detect in a short-term study. This study, on the other hand, serves as a beginning point for future research on educators' teaching and PCK in mathematics education, which should be conducted over a longer period of time and with more cycles.

The physical distance between the researcher and the participants was the final limitation of this study. This limitation had a slight impact on the study because I had to be present for all sessions of all lesson study cycles. At the end of the first cycles in both PLCs, participants were unable to submit their lesson study cycle observation tools, so they had to send them to me via WhatsApp. When further clarity was requested or some tools were not clear, it took some time for communication to receive a response. This is the stage in the research that the physical distance between the researcher and the participants becomes a limitation. This limitation did not effect the accuracy of the study because it was meant to have minimal interaction between the researcher and the participants.

5.9. Conclusion

Lesson study encompasses many of the most important facets of educator professional development: collaboration, observation, and reflection. By implementing lesson study in two different PLCs at the time of the COVID-19 pandemic, this study found how educators enhance their PCK in mathematics teaching by participating in iteractive cycles of lesson study as part of a professional learning community in Ilembe District primary schools. The impact on educators' PCK was evident, showing that lesson study has great potential to enhance educators' mathematics teaching. Indeed, there is much more to explore about the role of lesson study in South Africa. Lesson study is educator-led and bottom-up approach that is driven by educators themselves. Lesson study differs from conventional educator professional development practises mandated by higher level authorities.

Educators are the most important asset in the education of learners (Ugbe, Bessong & Agah, 2010). Therefore, educators should continuously develop their professionalism to help learners achieve their learning goals. This study concluded that time, duty load and lack of research skills are major constraints in professional development as these factors have contributed significantly to the limitations and challenges in conducting lesson study. The results of the study indicate that educators who are open to lesson study derive greater benefit from it compared to others who show little or no engagement. Athough lesson study shows promise in enhancing educators' PCK in mathematics teaching, it also depends on educators' attitudes and personalities.

Furthermore, this research adds to the literature on lesson study in a unique South African context where no previous research in primary schools involving mathematics educators has been conducted. I took part in this study as a subject advisor in mathematics for grades 4 to 9, as well as someone who wished to help mathematics educators deal with the lack of PCK in their classrooms. Working with educators to produce qualitative data and analyse that data taught me a lot as a researcher.

Finally, research that duplicates and expands on this study is required. Although further research on the impact of lesson study on educators' PCK when teaching mathematics is needed, the findings so far imply that educators' mathematical PCK may be the most essential component in enabling them to teach mathematics when addressing learners' mathematical learning needs. Promoting lesson study as an innovative school-based professional development for educators is a valuable long-term goal. SMTs play a critical role in encouraging and motivating educators to continue their education. The lesson study, if implemented in the South African educational context, would represent a significant paradigm shift for educator professional development.

REFERENCES

Alamri, N.M. 2020. The Implementation of the Lesson Study Strategy in Teaching Mathematics: Teachers′ Perspectives. Education Research International.

Adler, P.S. and Borys, B. 1996. Two types of bureaucracy: Enabling and coercive. Administrative science quarterly, 43(3), pp. 61-89.

Admiraal, W., Schenke, W., De Jong, L., Emmelot, Y. and Sligte, H. 2021. Schools as professional learning communities: what can schools do to support professional development of their educators?. Professional development in education, 47(4), pp. 684-698.

Akiba, M. and Wilkinson, B. 2016. Adopting an international innovation for teacher professional development: State and district approaches to lesson study in Florida. Journal of Teacher Education, 67(1), pp. 74-93.

Akiba, M., Murata, A., Howard, C.C. and Wilkinson, B. 2019. Lesson study design features for supporting collaborative teacher learning. Teaching and Teacher Education, 77, pp. 352-365.

Akturk, A.O. and Ozturk, H.S. 2019. Educators′ TPACK Levels and Students′ Self-Efficacy as Predictors of Students′ Academic Achievement. International Journal of Research in Education and Science, 5(1), pp. 283-294.

Akyeampong, K., Pryor, J., Westbrook, J. and Lussier, K. 2011. Teacher preparation and continuing professional development in Africa. Brighton, Centre for International Education, University of Sussex.

Akyuz, D. 2018. Measuring technological pedagogical content knowledge (TPACK) through performance assessment. Computers & Education, 125, pp. 212-225.

Al Riyami, T. 2015. Main approaches to educational research. International Journal of Innovation and Research in Educational Sciences, 2(5), pp. 412-416.

Alder, J. and Alshwaikh, J. 2019. A case of lesson study in South Africa. In Theory and Practice of Lesson Study in Mathematics. Springer, Cham, 32(2), pp. 317-342.

Anderson, J.G. 1966. Bureaucratic rules: Bearers of organisational authority. Educational Administration Quarterly, 2(1), pp. 7-34.

Anderson, J.G. 1968. Bureaucracy in education. Baltimore: Johns Hopkins Press.

Anney, V.N. 2014. Ensuring the quality of the findings of qualitative research: Looking at trustworthiness criteria. Journal of Emerging Trends in Educational Research and Policy Studies (JETERAPS), 5(2), pp. 272-281.

Assembly, C. 1996. Constitution of the Republic of South Africa. Cape Town, 230(38), pp. 1241-1331.

Awanbor, J.E. 2019. French language teacher trainees perception of pedagogical content knowledge in French language. Journal of Educational and Social Research, 9(3), p. 246. South Africa. In Theory and Practice of Lesson Study in Mathematics. Springer, Cham, pp. 317-342.

Aykan, A. and Yildirim, B. 2021. The Integration of a lesson study model into distance STEM education during the covid-19 pandemic: Educators′ view and practice. Technology, Knowledge and Learning, 21(2), pp. 1-29.

Bakhurst, D. 2009. Reflections on activity theory. Educational Review, 61(2), pp. 197-210.

Baki, M. and Arslan, S. 2022. Impact of lesson study on pre-service primary educators′ mathematical pedagogical content knowledge. International Journal of Mathematical Education in Science and Technology, 3(2), pp. 1-19.

Ball, D.L., Thames, M.H. and Phelps, G. 2008. Content knowledge for teaching: What makes it special? Journal of Teacher Education, 59(5), pp. 389-407.

Balyer, A., Karatas, H. and Alci, B. 2015. School principals' role in establishing collaborative professional learning communities at schools. Procedia-social and behavioural sciences, 197, pp. 1340-1347.

Bantwini, B.D. 2019. Developing a culture of collaboration and learning among natural science educators as a continuous professional development approach in a province in South Africa. Teacher Development, 23(2), pp. 213-232.

Barbosa, A. and Vale, L. 2015. Visualization in pattern generalization: Potential and Challenges. Journal of the European Teacher Education Network, 10, pp. 57-70.

Barab, S.A., Barnett, M., Yamagata-Lynch, L., Squire, K. and Keating, T. 2002. Using activity theory to understand the systemic tensions characterising a technology-rich introductory astronomy course. Mind, Culture, and Activity, 9(2), pp. 76-107.

Barrichello, A., Morano, R.S., Feldmann, P.R. and Jacomossi, R.R. 2020. The importance of education in the context of innovation and competitiveness of nations. International Journal of Education Economics and Development, 11(2), pp. 204-224.

Barwell, R. 2016. Formal and informal mathematics discourses: Bakhtin and Vygotsky, dialogue and dialectic. Educational Studies in Mathematics, 92(3), pp. 331-345.

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., Klusmann, U., Krauss, S., Neubrand, M. and Tsai, Y.M. 2010. Educators' mathematical knowledge, cognitive activation in the classroom, and student progress. American Education Research Journal, 47(1), 133-180.

Bedny, G.Z., Seglin, M.H. and Meister, D. 2000. Activity theory: History, research and application. Theoretical Issues in Ergonomics Science, 1(2), pp. 168-206.

Bernadine, G.G.K. 2019. Challenges faced by educators in the implementation of Continuing Professional Teacher Development (CPTD): Gauteng Province. In Teacher Education in the 21st Century. IntechOpen.

Bertram, C. and Christiansen, I. 2014. Understanding research: An introduction to reading research. Pretoria: Van Schaik Publisher.

Best, J.W. and Khan, J.V. 2010. Research in Education. 9th ed. Boston: Allyn & Bacon.

Beswick, K., Watson, A. and DeGeest, E. 2010. Comparing theoretical perspectives in describing mathematics department: complexity and activity. Journal of Educational studies in Mathematics, 75(2), pp. 153-170.

Bissessar, C.S. 2021. Social learning and collaborative professional learning among Caribbean educators: implications for Praxis. Teacher Learning and Professional Development, 6(1), pp. 13-32.

Blömeke, S. and Delaney, S. 2012. Assessment of teacher knowledge across countries: A review of the state of research. ZDM Mathematics Education, 44, 223-247.

Blunden, A. 2007. Modernity, the individual, and the foundations of cultural-historical activity theory. Mind, Culture, and Activity, 14(4), pp. 253-265.

Bjuland, R. and Mosvold, R. 2015. Lesson study in teacher education: Learning from a challenging case. Teaching and teacher education, 52, pp. 83-90.

Boekaerts, M. 2002. Motivation to Learn. Educational Practices. International Academy of Education.

Boonsongsak, C. 2022. An Investigation of Anxiety of Thai Teaching in Chinese Universities. Turkish Journal of Computer and Mathematics Education (TURCOMAT), 13(2), pp. 583-590.

Bourke, R. and McGee, A. 2012. The challenge of change: Using activity theory to understand to understand a cultural innovation. Journal of Education Change, 13(2), pp. 217-233.

Bourdieu, P., Calhoun, C.J., Gerteis, J., Moody, J., Pfaff, S. and Virk, I. 2007. Contemporary sociological theory.

Boz, Y. and Belge-Can, H. 2020. Do Pre-Service Chemistry Teachers′ Collective Pedagogical Content Knowledge Regarding Solubility Concepts Enhance after Participating in a Microteaching Lesson Study?. Science Education International, 31(1), pp. 29-40.

Braun, V. and Clarke, V. 2006. Using thematic analysis in psychology. Qualitative research in psychology, 3(2), pp. 77-101.

Bree, R.T. and Gallagher, G. 2016. Using Microsoft Excel to code and thematically analyse qualitative data: a simple cost-effective approach. All Ireland Journal of Higher Education, 8(2), pp. 106-139.

Breen, S., McCluskey, A., Meeham, M., O'Donovan, J. and O'Shea, A. 2014. A year of engaging with the discipline of noticing: five mathematics lecturer's reflections. Teaching in Higher Education, 19(3), pp. 289-300.

Brown, M., Brown, P. and Bibby, T. 2008. "I would rather die": Reasons given by 16-year-olds for not continuing their study of mathematics. Research in Mathematics Education, 10(1), pp. 3-18. doi: 10.1080/1479480080915814.

Brown, B.D., Horn, R.S. and King, G. 2018. The effective implementation of professional learning communities. Alabama Journal of Educational Leadership, 5, pp. 53-59.

Brunton, C. 2003. Education Labour Relations Council and Education Labour Relations Council, 2003. Policy Handbook for Educators. Pretoria: Universal Print Groups.

Buchholtz, N. 2019. Planning and conducting mixed methods studies in mathematics educational research. Compendium for early career researchers in mathematics education, 11(2), pp. 131-152.

Burney, D. 2004. Craft Knowledge: The Road to Transforming Schools. Phi Delta Kappa International, 85(7), pp. 526-531.

Chan, K.K.H. and Hume, A. 2019. Towards a consensus model: Literature review of how science educators′ pedagogical content knowledge is investigated in empirical studies. Repositioning pedagogical content knowledge in educators′ knowledge for teaching science, pp. 3-76.

Cajkler, W., Wood, P. Norton, J. and Peddar, D. 2013. Lesson Study: towards a collaborative approach to learning in Initial Teacher Education? Cambridge Journal of Education, 43(4), pp. 537-554.

Carlsen, W. 1999. Domains of teacher knowledge. In Examining pedagogical content knowledge. Springer, Dordrecht, pp. 133-144.

Cerbin, B. 2011. Lesson study: Using classroom inquiry to improve teaching and learning in higher education. Sterling, VA: Stylus.

Cerbin, W. and Kopp, B. 2011. Lesson study guide.

Chai, C.S. 2019. Teacher professional development for science, technology, engineering and mathematics (STEM) education: A review from the perspectives of technological pedagogical content (TPACK). The Asia-Pacific Education Researcher, 28(1), pp. 5-13.

Cheng, L.P. and Yee, P.Y. 2011. A Singapore case of lesson study. The Mathematics Educator, 21(2), pp. 34-57.

Chichibu, T. and Kihara, T. 2013. How Japanese schools build a professional learning community by lesson study. International Journal for Lesson and Learning Studies, 2(1), 12-25.

Chikiwa, S., Westaway, L. and Graven, M. 2019. What mathematics knowledge for teaching is used by a Grade 2 teachers when teaching counting? South African Journal of Childhood Education, 9(1), a567. https://doi.org/10.4102/sajce.v9i1.56

Chindanya, A. 2011. Parental involvement in primary schools: a case study of the Zaka district of Zimbabwe (Doctoral dissertation, University of South Africa).

Chinyoka, K. 2013. Psychosocial effects of poverty on the academic performance of the girl child in Zimbabwe (Doctoral dissertation, University of South Africa).

Chiva-Bartoll, O. and Fernández-Rio, J. 2021. Advocating for Service-Learning as a pedagogical model in Physical Education: towards an activist and transformative approach. Physical Education and Sport Pedagogy, pp 1-14.

Christie, P. 1991. The right to learn. Johannesburg: SACHED/Raven.

Chiew, C.M. 2009. Implementation of lesson study as an innovative professional development model among mathematics teachers (Doctoral dissertation, University Sains Malaysia).

Chiew, C.M., Dali, M.H. and Lim, C.S. 2016. Implementation of lesson study as an innovative professional development model among Malaysian school teachers. Malaysian Journal of Learning and Instruction (MJLI), 13, pp. 83-111

Chokshi, S. and Fernandez, C. 2005. Reaping the systemic benefits of lesson study: Insights from the U.S. Phi Delta Kappan, 86(9), pp. 674-680.

Çimer, A., Çimer, S.O. and Vekli, G.S. 2013. How does reflection help teachers to become effective teachers. International Journal of Education Research, 1(4), pp. 133-149.

Clarke, D. 2000. Time to reflect. Journal of Mathematics Teachers Education, 3, pp. 201-203.

Clarke, V. and Braun, V. 2014. Thematic analysis. In Encyclopaedia of critical psychology (pp. 1947-1952). Springer, New York, NY.

Coe, K.L. 2010. The process of lesson study as a strategy for the development of teaching in primary schools: A case study in Western Cape Province, South Africa (Doctoral dissertation, Stellenbosch: University of Stellenbosch).

Coe, K., Carl, A. and Frick, L. 2010. Lesson study in continuing professional teacher development: A South African case study. Acta Academica, 42(4), pp. 260-280.

Coenders, F. and Verhoef, N. 2019. Lesson study: professional development (PD) for beginning and experienced educators. Professional development in education, 45(2), pp. 217-230.

Cohen, L., Manion, L. and Morrison, K. 2017. Research methods in education. Routledge.

Cohen, L., Manion, L. and Morrison, K. 2018. Research Methods in Education. London: Taylor Francis.

Collet, V.S. and Greiner, A.C. 2020. Revisioning grammar instruction through collaborative lesson study: A new apprenticeship of observation. Literacy Research and Instruction, 59(2), pp. 95-120.

Corcoran, D. 2009. Mathematics Teaching as a Shared Enterprise: Learning by ‘Doing’. In D. Corcoran, T. Dooley, S. Close & R. Ward (Eds.), Third National Conference on Research in Mathematics Education (MEI) (pp. 34-52). St. Patrick’s College, Dublin.

Creswell, J.W. 2013. Educational research: Planning, conducting, and evaluating quantitative and qualitative research (2nd ed.). Upper Saddle River, NJ: Pearson Education.

Creswell, J.W. and Creswell, J.D. 2017. Research design: Qualitative, quantitative, and mixed methods approaches. Sage publications.

Crossman, A. 2017. An overview of qualitative research methods. Available at https://www.thoughtco.com (Accessed: 04 February 2021).

Daniels, H. 2004. Activity theory, discourse and Bernstein. Educational Review, 56(2), pp. 121-132.

Daniels, H. 2006. The “social” in post-Vygotskian theory. Theory & Psychology, 16(1), pp. 37-49.

Daniels, H., Cole, M. and Wertsch, J.V. (2007). Activity theory, discourse and Bernstein. The Cambridge Companion to Vygotsky. Cambridge: Cambridge University Press.

Degu, G. and Yigzaw, T. 2016. Research Methodology: lecture notes for health science students. Addis Ababa: The Carter Centre (Ethiopian Public Health Training Initiative), pp. 45-50.

Depaepe, F., Verschaffel, L. and Kelchtermans, G. 2013. Pedagogical content knowledge: A systematic review of the way in which the concept has pervaded mathematics educational research. Teaching and teacher education, 34, pp. 12-25.

Department of Basic Education. 2011. Curriculum and Assessment Policy Statement Grades 4-6: Mathematics. Pretoria: National Department of Education.

Department of Basic Education. 2014. National Guidelines on the Employment of Foreign Educators in South Africa. Pretoria: National Department of Education.

Department of Basic Education. 2017. Circular D2 of 2017: Identification, Management and Support of Underperforming Schools. Pretoria: National Department of Education.

Department of Basic Education. 2017. Circular E12 of 2017: Appointment of Task Team to Observe Lesson Study in Japan as a Professional Development Model. Pretoria: National Department of Education.

Department of Basic Education. 2017. Government Gazette No. 39583. Approval to conduct research in public institutions. Pretoria: National Department of Education.

Department of Basic Education. 2017. Professional learning communities of school principals. Pretoria: National Department of Education.

Department of Basic Education. 2021. 2020 National Senior Certificate Results: Schools Subject Report 23 February 2021. Pretoria: National Department of Education.

Department of Basic Education. 2022. 2021 National Senior Certificate Results: Schools Subject Report 20 January 2022. Pretoria: National Department of Education.

Department of Education. 2000a. Norms and Standards for Educators. Government Gazette, 415 (20844). Pretoria: DoE.

Doig, B., Groves, S. and Fujii, T. 2011. The Critical Role of Task Development in Lesson Study. In L.C. Hart, A.S. Alston & A. Murata (Eds.), Lesson Study Research and Practice in Mathematics Education (pp. 181-200). USA: Springer.

Dudley, P. 2012. Lesson Study development in England: from school network to national policy. International Journal for Lesson Studies, 1(1), pp. 85-100.

Dudley, P. 2014. Lesson Study: a handbook. UK: Cambridge.

Elliott, J. 2019. What is lesson study?. European Journal of Education, 54(2), pp. 175-188.

Engeström, Y. 1987. Learning by expanding: An activity-theoretical approach to developmental research. Helsinki: Orienta-Konsultit.

Engeström, Y. 1993. Developmental studies of work as a testbench of activity theory: The case of primary care medical practice. In S. Chaiklin & J. Lave (Eds.), Understanding practice:

Perspectives on activity and context (pp. 64-103). Cambridge: Cambridge University Press.

Engeström, Y. 1999. Expansive visibilisation of work: An activity-theoretical approach. Computer Supported Cooperative Work, 8, pp. 63-93.

Engeström, Y. 2001. Expansive learning at work: toward an activity theoretical reconceptualization. Journal of Education and Work, 14(1), pp. 133-156.

Engeström, Y. 2007. Enriching the theory of expansive learning: Lessons from journeys towards co-configuration. Mind, Culture, and Activity, 14(1), pp. 23-39. Expansive learning at work: toward an activity theoretical reconceptualization. Journal of Education and Work, 14(1), pp. 133-156.

Engeström, Y. 2009. From learning environments and implementation to activity systems and expansive learning. Actio: An International Journal of Human Activity Theory, 2, pp. 17-33.

Engeström, Y., Engeström, R. and Suntio, A. 2002a. Can a school community learn to master its own future? An activity-theoretical study of expansive learning among middle-school educators. In G. Wells & G. Claxton (Eds.), Learning for life in the 21st century: Sociocultural perspective on the future of education (pp. 211-224). Malden, MA: Blackwell.

Engeström, Y. and Sannino, A, 2010. Studies of expansive learning: Foundations, findings and future challenges. Educational Research Review, 5(10), pp. 1-24.

Ernest, P. 2018. The ethics of mathematics: Is mathematics harmful?. In The philosophy of mathematics education today (pp. 187-216). Springer, Cham.

Ewens, T. 2014. Reflective Primary Teaching. Critical Publishing.

Fang, Y. and Lee, C.K.E. 2009. Lesson Study in Mathematics: Three cases from Singapore. In K.Y. Wong, P.Y. Lee, B. Kaur, P.Y. Foong & S.F. Ng (Eds.), Series on Mathematics

Education: Vol. 2. Mathematics education: The Singapore journey (pp. 104-129). Singapore: World Scientific.

Fatima, R. 2012. Role of Mathematics in the Development of Society. National Meet on Celebration of National Year of Mathematics. Organized by NCERT, New Delhi, pp. 1-12.

Feldman, A. and Weiss, T. 2010. Understanding change in educators' way of being through collaborative action research: A cultural-historical activity theory analysis. Educational Action Research, 18(1), pp. 29-55.

Fernandez, C. 2002. Learning from Japanese approaches to professional development: the case of lesson study. Journal of teacher education, 53(5), pp. 393-405.

Fernandez, C. 2005. Lesson study: A means for elementary educators to develop the knowledge of mathematics needed for reform-minded teaching? Mathematical Thinking and Learning, 7(4), pp. 265-289.

Fernandez, C. and Chokshi, S. 2002. A Practical Guide to Translating Lesson Study for U.S. Setting. Phi Delta Kappa International, 84(2), pp. 128-134.

Fernandez, C. and Yoshida, M. 2012. Lesson study: A Japanese approach to improving mathematics teaching and learning. Routledge.

Fishbein, B., Foy, P. and Yin, L. 2021. TIMSS 2019 user guide for the international database. Hentet fra https://timssandpirls.bc.edu/timss2019/international-database (Accessed: 19 October 2021).

Fiske, E.B. and Ladd, H.F. 2004. Elusive equity: Education reform in post-apartheid South Africa. Washington, D.C.: Brookings Institution Press.

Fleer, M. 2010. Early Learning and Development: Cultural-historical concepts in play. Cambridge, England: Cambridge University Press.

Flynn, T., Hedges, H. and Bruce, C. 2009. The ripple effect of mathematics lesson study: One school story. In annual meeting of the Canadian Society for the Study of Education, Ottawa, ON.

Forsten, C., Grant, J. and Hollas, B. 2002. Differentiated Instruction: Different Strategies for Different Learners. Crystal Springs Books, Peterborough, NH 03458-0050.

Fouka, G. and Mantzorou, M. 2011. What are the major ethical issues in conducting research? Is there a conflict between the research ethics and the nature of nursing? Health science journal, 5(1), pp. 3-14.

Freshwater, D. and Cahill, J. 2013. Paradigms lost and paradigms regained. Journal of Mixed Methods Research, 7(3), pp. 3-5.

Friedman, R.E. 2005. An examination of lesson study as a teaching tool in US public schools (Doctoral dissertation, Ashland University).

Fugard, A.J.B. and Potts, H.W.W. 2015. Supporting thinking on sample sizes for thematic analyses. International Journal of Social Research Methodology.

Fujii, T. 2018. Lesson study and teaching mathematics through problem solving: The two wheels of a cart. In Mathematics lesson study around the world (pp. 1-21). Springer, Cham.

Fujii, T. 2019. Designing and adapting tasks in lesson planning: A critical process of lesson study. In Theory and practice of lesson study in mathematics (pp. 681-704). Springer, Cham.

Furlong, G.T. 2020. The conflict resolution toolbox: Models and maps analysing, diagnosing, and resolving conflict. John Wiley & Sons.

Gabriel, J. and Farmer, P. 2016. Developing a Vision and a Mission. Available at http://www.ascd.org/publications/books/107042/chapters/developing-a-visionand (Accessed: 24 August 2020).

Gao, S., Damico, N. and Gelfuso, A. 2021. Mapping and reflecting on integration of the components of pedagogical content knowledge (PCK) for teaching natural selection. A case study of an experienced middle-school science teacher. Teaching and Teacher Education, 107, p.103473.

Gasteiger, H., Bruns, J., Benz, C., Brunner, E. and Sprenger, P. 2020. Mathematical pedagogical content knowledge of early childhood educators: A standardized situation-related measurement approach. ZDM, 52(2), pp. 193-205.

Gero, G. 2014. The prospects of lesson study in the US. International Journal for Lesson and Learning Studies, 4(1), pp. 7-25. Translation beyond Japan. International Journal for Lesson and Learning Studies.

Gess-Newsome, J. 1999a. Pedagogical content knowledge: An introduction and orientation. In J. Gess-Newsome & N.G. Lederman (Eds.), Examining pedagogical content knowledge: The construct and its implications for science education (pp. 3-17). Dordrecht: Kluwer Academic Publishers.

González, G., Villafañe-Cepeda, W. and Hernández-Rodríguez, O. 2021. Leveraging prospective teachers′ knowledge through their participation in lesson study. Journal of Mathematics Teacher Education, pp. 1-24.

Groh, A. 2018. Research Methods in Indigenous Contexts. New York: Springer. ISBN 978-3-319-72774-5.

Gronn, P. 2002. Distributed leadership as a unit of analysis. The Leadership Quarterly, 13(4), pp. 423-451.

Grossman, P.L. 1990. The making of a teacher: Teacher knowledge and teacher education. Educators College Press, Educators College, Columbia University.

Grossman, P., Wineburg, S. and Woolworth, S. 2001. Toward a theory of teacher community. Educators College Record, 103(6), pp. 942-1012.

Guba, E.G. and Lincoln, Y.S. 2011. Competing paradigms in qualitative research. Oaks, CA: SAGE.

Gulpric, V.C. 2018. Mathematical knowledge for teaching: A literature review on ideology, instrumentation and investigations. Published online 15 August 2018.

Gunawan, J. 2015. Ensuring trustworthiness in qualitative research. Belitung Nursing Journal, 1(1), pp. 10-11.

Gundogdu, K. and Silman, F. 2007. Teaching as a professional and effective teaching. Introduction to Education: Handbook of Basic Concepts.

Gurl, T. 2011. A model for incorporating lesson study into the student teaching placement: what worked and what did not? Educational studies, 37(5), pp. 523-528.

Halla, K. 2015. Deeper Learning Through Technology: Using the Cloud to Individualize Instruction. Corwin Press.

Halverson, R. 2004. Accessing, documenting, and communicating practical wisdom: The phronesis of school leadership practice. American Journal of Education, 111(1), pp. 90-121.

Halverson, R. 2006. A Distributed Leadership Perspective on How Leaders Use Artefacts to Create Professional Community in Schools. WCER Working Paper No. 2006-4. Wisconsin Centre for Education Research (NJ1).

Hardman, J. 2005. An exploratory case study of computer use in a primary school mathematics classroom: New technology, new pedagogy? Research: Information and communication technologies. Perspectives in Education, 23(1), pp. 99-111.

Harrison, J., MacGibbon, L. and Morton, M. 2001. Regimes of trustworthiness in qualitative research: The rigors of reciprocity. Qualitative inquiry, 7(3), pp. 323-345.

Harsono, M. 2016. The impact of lesson study on primary school educators' knowledge and skills in differentiating primary school mathematics instruction. A digital mixed methods approach (Doctoral dissertation, Curtin University).

Hart, L. and Carriere, J. 2011. Developing the Habits of Mind for a Successful Lesson Study Community. In L.C. Hart, A. Alston & A. Murata (Eds.), Lesson Study Research and Practice in Mathematics Education: Learning Together. New York: Springer.

Hasan, H. and Kazlauskas, A. 2014. Activity theory: who is doing what, why and how. Wollongong, Australia: University of Wollongong.

Hatcher, R. 2005. The distribution of leadership and power in school. British Journal of Sociology of Education, 26(2), pp. 253-267.

Helmbold, E., Venketsamy, R. and Van Heerden, J. 2021. Implementing Lesson Study as a professional development approach of early grade educators: A South African case study. Perspectives in Education, 39(3), pp. 183-196.

Henriksen, D., Richardson, C., Gruber, N. and Mishra, P. 2022. The uncertainty of creativity: opening possibilities and reducing restrictions through mindfulness. In Uncertainty: A catalyst for Creativity, Learning and Development (pp. 103-124). Springer, Cham.

Hervas, G. and Medina, J.L. 2022. Higher education educators′ perception and use of Content Representations in lesson study. International Journal for Lesson & Learning Studies.

Hiles, J.B. 2019. Teacher Learning Through the Dialogic Space of a Lesson Study Cycle (Doctoral dissertation, University of Pittsburgh).

Hill, H.C., Ball, D.L. and Schilling, S.G. 2004. Developing measures of teachers′ mathematics knowledge for teaching. The Elementary School Journal, 105(1), pp. 11-30.

Hill, H.C., Ball, D.L. and Schilling, S.G. 2008. Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers′ topic-specific knowledge of students. Journal for Research in Mathematics Education, 39(4), pp. 372-400.

Hilliard, A. T. (2012). Practices and value of a Professional Learning Community in higher education. *Contemporary Issues in Education Research (CIER)*, *5*(2), 71. doi:10.19030/cier.v5i2.6922

Hix, S.L. 2008. Learning in lesson study: A professional development model for middle school Mathematics educators (Doctoral thesis). Atlanta, Ga.: University of Georgia.

Holten, D. and Thomas, G. 2001. Mathematical Interactions and Their Influence on Learning. In D. Clarke (Ed.), Perspectives on Practice and Meaning in Mathematics and Science Classroom. Netherlands: Kluwer Academic Publishers.

Hord, S.M. 2008. Evolution of the professional learning community. The Learning Professional, 29(3), p.10.

Hord, S.M. 2009. Professional learning communities: Educators work together toward a shared purpose. Journal of Staff Development, 30(1), pp. 40-43.

Hourigan, M. and Leavy, A.M. 2021. The complexities of assuming the ‘teacher of teacher’ role during lesson study. Professional Development in Education, pp. 1-14.

Howell, H.E. 2014. Extending Understanding and Measuring of Secondary Mathematical Knowledge for Teaching. Paper presented at the EARA, Philadelphia.

Huat, C.P. 2009. Learning communities: Roles of educators' network and zone activities. In Mathematics Education: The Singapore Journey, pp. 85-103.

Huang, R. and Shimizu, Y. 2016. Improving teaching, developing educators and teacher educators, and linking theory and practice through lesson study in mathematics: An international process. ZDM Mathematics Education, 48, pp. 393-409.

Huang, R., Takahashi, A. and da Ponte, J.P. 2019. Theory and practice of lesson study in mathematics. New York: Springer International Publishing.

Hudson, R. 2020. Mathematics in Language. Cognitive Semantics, 6(2), pp. 243-278.

Huitt, W. 2011. Self and self-view. Educational psychology interactive.

Ibrahim, A.M. 2012. Thematic Analysis: A Critical Review of its Process and Evaluation. Croatia: Zagreb.

Ingersoll, R.M., Merrill, L. and Stuckey, D. 2014. Seven Trends: The Transformation of Teaching Force. Consortium for Policy Research in Education.

Inprasitha, M. 2015. Transforming Education Through Lesson Study: Thailand′s Decade-Long Journey. In S.J. Cho (Ed.), Selected Regular Lectures from the 12th International Congress on Mathematical Education. New York: Springer.

Isoda, M. and Olfos, R. 2021. Teaching Multiplication with Lesson Study: Japanese and Ibero-American Theories for International Mathematics Education (p. 296). Springer Nature.

Israel, M. 2016. *Research ethics and integrity for social scientists: Beyond regulatory compliance* (Second ed.). Los Angeles, CA: SAGE.

Jackson, L. and Rothmann, S. 2005. Work-related well-being of educators in a district of North-West Province. Perspectives in education, 23(1), pp. 107-122.

Jacob, J.S. and Gwany, D.M. 2020. Educators′ pedagogical content knowledge and students′ academic achievement: A theoretical overview. Journal of Global Research in Education and Social Science, 14(2), pp. 14-44.

Jacobs, V.R., Lamb, L.L.C. and Philipp, R.A. 2010. Professional noticing of children′s mathematical thinking. Journal for Research in Mathematics Education, 41(2), pp. 169-202.

Jaworski, B. 2010. The practice of (university) mathematics teaching: Mediational inquiry in a community of practice or an activity system. CERME 6 – WORKING GROUP 9, pp. 1585-1594.

Jhang, F.H. 2020. Teachers′ attitude towards lesson study, perceived competence, and involvement in lesson study: evidence from junior high school teachers. Professional Development in Education, 46(1), pp. 82-96.

Johnson, J. 2017. Comparing the major definitions of mathematics pedagogical content knowledge. JMETC 8, pp. 19-28.

Johnson, R.B. and Onwuegbuzie, A.J. 2004. Mixed Methods Research: A Research Paradigm Whose Time Has Come. Educational Researcher, 33(7), pp. 14-26.

Johnstone, M. 2009. Bioethics: a nursing perspective. 5th ed. Chatswood, NSW: Churchill Livingstone.

Jolejole-Caube, C., Dumlao, A.B. and Abocejo, F.T. 2019. Anxiety towards mathematics and mathematics performance of grade 7 learners. European Journal of Education Studies.

Juhler, M.V. 2016. The use of lesson study combined with content representation in the planning of physics lessons during field practice to develop pedagogical content knowledge. Journal of Science Teacher Education, 27(5), pp. 533-553.

Kabutu-Njekwa, C. 2019. Grade 9 educators' and learners' perceptions of the causes of poor mathematics performance and possible interventions in Durban's Ethusini Circuit (Doctoral dissertation).

Kaiser, G. 2020. Mathematical modelling and applications in education. Encyclopaedia of mathematics education, pp. 553-561.

Kandjinga, J.J. 2018. Investigating the grade 12 educators' subject and pedagogical content knowledge of Mathematics in some selected public schools in the Khomas education region, Namibia (Doctoral dissertation, University of Namibia).

Kaptelinin, V. 2005. The object of activity: making sense of the sense-maker. Mind, Culture, and Activity, 12(1), pp.4-18.

Kaput, J.J. 2017. What is Algebra? What is Algebraic Reasoning?. In Algebra in the early grades (pp. 5-18). Routledge.

Kathirveloo, P., Puteh, M. and Matematik, F.S. 2014. Effective Teaching: Pedagogical Content Knowledge. In Proceeding of International Joint Seminar Garut. Indonesia.

Kenedi, A.K., Helsa, Y., Ariani, Y., Zainil, M. and Hendri, S. 2019. Mathematical Connection of Elementary School Students to Solve Mathematical Problems. Journal on Mathematics Education, 10(1), pp. 69-80.

Kgothego, N.N. 2020. An investigation into the role of lesson study in developing educators′ mathematics content and pedagogical content knowledge (Doctoral dissertation, Rhodes University).

Kihwele, J.E. and Guoyuan, S. 2020. Affordances and Constraints on Implementing Lesson Study for Teachers′ Professional Development: A Review. African Journal of Teacher Education, 9, pp. 49-69.

Kioupi, V. and Voulvoulis, N. 2019. Education for sustainable development: A systemic framework for connecting the SDGs to educational outcomes. Sustainability, 11(21), p.6104.

Khan, L. 2015. What is mathematics-an overview. International Journal of Mathematics and Computational Science, 1(3), pp. 98-101.

Khoza, S.B. and Biyela, A.T. 2020. Decolonising technological pedagogical content knowledge of first year mathematics student. Education and Information Technologies, 25(4), pp. 2665-2679.

Kloser, M., Edelman, A., Floyd, C., Martínez, J.F., Stecher, B., Srinivasan, J. and Lavin, E. 2021. Interrogating practice or show and tell?: Using a digital portfolio to anchor a professional learning community of science educators. Journal of Science Teacher Education, 32(2), pp. 210-241.

Koehler, M.J., Mishira, P. and Cain, W. 2013. What is technological pedagogical content knowledge (TPACK)?. Journal of Education, 193(3), pp. 13-19.

Korstjens, I. and Moser, A. 2018. Series: Practical guidance to qualitative research. Part 4: Trustworthiness and publishing. European Journal of General Practice, 24(1), pp. 120-124.

Kour, S. 2014. Ethical and Legal issues in Educational Research. Indian Journal of Applied Research, 4(6), pp. 23-45.

Krauss, S., Brunner, M., Kunter, M., Baumert, J., Blum, W., Neubrand, M. and Jordan, A. 2008. Pedagogical content knowledge and content knowledge of secondary mathematics teachers. Journal of Educational Psychology, 100(3), pp. 716-725. doi: 10.103/0022-0663.100.3.716.

Kruse, S., Louis, K.S. and Bryk, A. 1994. Building professional community in schools. Issues in Restructuring Schools, 6, pp. 3-6.

Kuramoto, T. and Shi, H. 2012. Summary of “Lesson Study” and “Curriculum Management” in Japan. J. Fac. Edu. Saga Univ, 17(1), pp. 133-147.

Kutluca, A.Y. 2021. An investigation of elementary educators′ pedagogical content knowledge for socioscientific argumentation: The effect of a learning and teaching experience. Science Education, 105(4), pp. 743-775.

Kuutti, K. 1995. Activity theory as a potential framework for human-computer interaction research. Published in B.A. Nardi (ed.): Context and Consciousness: Activity Theory and Human Computer Interaction.

Lai, T.L. and Lin, H.F. 2018. An investigation of the relationship of beliefs, values and technological pedagogical content knowledge among educators. Technology, Pedagogy and Education, 27(4), pp. 445-458.

Lampley, S.A., Gardner, G.E. and Barlow, A.T. 2018. Exploring pedagogical content knowledge of biology graduate teaching assistants through their participation in lesson study. Teaching in Higher Education, 23(4), pp. 468-487.

Le Grange, L. 2021. Covid-19 pandemic and the prospects of education in South Africa. Prospects, 51(1), pp. 425-436.

Leavy, A.M. and Hourigan, M. 2016. Using Lesson Study to support knowledge development in initial teacher education: insights from early number classrooms. Teaching and Teacher Education, 57, 161-175.

Lee, J.F.K. 2008. A Hong Kong case of lesson study – Benefits and concerns. Teaching and Teacher Education, 24(5), pp. 1115-1124.

Lee, J. 2010. Exploring kindergarten educators′ pedagogical content knowledge of mathematics. International Journal of Early Childhood, 42(1), pp. 27-41.

Lee, Y., Capraro, R.M. and Capraro, M.M. 2018. Mathematics educators′ subject matter knowledge and pedagogical content knowledge in problem posing. International Electronic Journal of Mathematics Education, 13(2), pp. 75-90.

Leontiev, A.N. 1978. Activity, consciousness and personality. Englewood Cliffs, NJ: Prentice Hall.

Leontiev, A.N. 1981. Problems of the development of the mind. Moscow: Progress Publishers.

Leung, F.K.S., Stillman, G.A., Kaiser, G. and Wong, K.L. 2021.Mathematical modelling education in east and west. Springer.

Lertdechapat, K. and Faikhamta, C. 2021. Enhancing pedagogical content knowledge for STEM teaching of teacher candidates through lesson study. International Journal for Lesson & Learning Studies

Lewis, C. 2002. Does Lesson Study Have a Future in the United States? Nagoya Journal of Education and Human Development.

Lewis, C. 2006. Lesson study in North America: Progress and challenges. Lesson study: international perspective on policy and practice. Beijing: Educational Science Publishing House.

Lewis, C., Perry, R. and Murata, A. 2006. How should research contribute to instructional improvement? The case of lesson study. Educational Research, 35(3), pp. 3-14.

Lewis, C.C., Perry, R.R. and Hurd, J. 2009. Improving mathematics instruction through lesson study: A theoretical model and North America case. Journal of mathematics teacher education, 12(4), pp. 285-304.

Lewis, C. and Hurd, J. 2011. Lesson Study Step by Step: How Teacher Learning Communities Improve Instruction. USA: Library of Congress Cataloging-in- Publication Data.

Lewis, C., Jennifer, F.D., Riggs, I. and Wasserman, K. 2013. Teacher learning in lesson study. Mathematics Enthusiast, 10(3), pp. 583-620.

Lewis, C. and Perry, R. 2017. Lesson study to Scale up Research-Based Knowledge: A Randomized, Controlled Trial of Fractions Learning. Journal for Research in Mathematics Education, 48(3), pp. 261-299.

Lieberman, J. 2009. Reinventing teacher professional norms and identities: the role of lesson study and learning communities. Professional Development in Education, 35(1), pp. 83-99. doi: 10.1080/13674580802264688.

Likando, S. 2018. The implementation of lesson study in mathematics: the case of Zambia (No. PhD). Deakin University.

Lin, W., Lee, M. and Riordan, G. 2018. The role of teacher leadership in professional learning community (PLC) in International Baccalaureate (IB) schools: A social network approach. Peabody Journal of Education, 93(5), pp. 534-550.

Ling, L.M., Man, P.C.P., Yuk, K.P., Lun, A.L.Y., Yan, P.W., Wah, P.L.-F.Y. and Ping, D.N.F. 2005. Learning studies: Development and impact. In L.M. Ling, P.W. Yan & P.C.P. Man (Eds.), For each and everyone: Catering for individual differences through learning studies (pp. 41-740). Hong Kong: Hong Kong University Press.

Liptak, L. 2005. For Principals: Critical Elements. In P. Wang-Iverson & M. Yoshida (Eds.), Building Our Understanding of Lesson Study. Philadelphia: Research for Better Schools.

Loughran, J. 2020. Pedagogical content knowledge. In Science Education for Australian Students (pp. 205-232). Routledge.

Lucenario, J.L.S., Yangco, R.T., Punzalan, A.E. and Espinosa, A.A. 2016. Pedagogical content knowledge-guided lesson study: Effects on teacher competence and students′ achievement in chemistry. Education Research International, 2016.

Luft, J.A., Hanuscin, D., Hobbs, L. and Törner, G. 2020. Out-of-field teaching in science: An overlooked problem. Journal of Science Teacher Education, 31(7), pp. 719-724.

Lune, H. and Berg, B.R. 2017. Qualitative research methods for the social sciences. Pearson.

Lynch, B. 2011. Towards an Instructional Model to support Teaching and Learning in Mathematics. Paper presented at the Fourth Conference on Research in Mathematics Education MEI4. St. Patrick's College, Drumcondra, Dublin 9.

Maass, K., Geiger, V., Ariza, M.R. and Goos, M. 2019. The role of mathematics in interdisciplinary STEM education. ZDM, 51(6), pp. 869-884.

MacDonald, C. 2012. Understanding participatory action research: A qualitative research methodology option. The Canadian Journal of Action Research, 13(2), pp. 34-50.

Makinae, N. 2019. The origin and development of lesson study in Japan. In Theory and Practice of Lesson Study in Mathematics (pp. 169-181). Springer, Cham.

Makrakis, V. and Kostoulas-Makrakis, N. 2016. Bridging the qualitative-quantitative divide: Experiences from conducting a mixed methods evaluation in the RUCAS programme. Evaluation and Program Planning, 54, pp. 144-151.

Malcolm, S.A., Mavhunga, E. and Rollnick, M. 2019. The validity and reliability of an instrument to measure physical science educators′ topic specific pedagogical content knowledge in stoichiometry. African Journal of Research in Mathematics, Science and Technology, 23(2), pp. 181-194.

Magnusson, S., Krajcik, J. and Borko, H. 1999. Nature, sources, and development of pedagogical content knowledge for science teaching. In Examining pedagogical content knowledge (pp. 95-132). Springer, Dordrecht.

Marshall, C. and Rossman, G.B. 2014. Designing qualitative research. Sage Publications.

Mason, J. 2009. Learning from listening to yourself. In J. Houssart & J. mason (Eds.), Listening Counts: listening to young learners of mathematics. Staffordshire, England: Trentham Books Limited.

McCulloch, A.W., Hollebrands, K., Lee, H., Harrison, T. and Mutlu, A. 2018. Factors that influence secondary mathematics educators′ integration of technology in mathematics lessons. Computers & Education, 123, pp. 26-40.

McMillan, P. and Jess, M. 2021. Embracing complex adaptive practice: the potential of lesson study. Professional Development in Education, 47(2-3), pp. 273-288.

McMillan, J.H. and Schumacher, S. 2010. Research in Education: Evidence-Based Inquiry, MyEducationLab Series. Pearson.

Mehrtash, S. 2021. Designing, implementing, and evaluating a professional learning community for group exercise instructors.

Meng, C.C. and Sam, L.C. 2013. Developing Pre-Service Educators′ Technological Pedagogical Content Knowledge for Teaching Mathematics with Geometer′s Sketchpad through Lesson Study. Journal of Education and Learning, 2(1), pp. 1-8.

Merriam, S.B. 2015. Qualitative research: Designing, implementing, and publishing a study. In Handbook of research on scholarly publishing and research methods (pp. 125-140). IGI Global.

Merriam, S.B. and Tisdell, E.J. 2015. Qualitative research: A guide to design and implement. John Wiley & Sons.

Mertens, D. 2015. An Introduction Research. In research and evaluation in education and psychology. Integrating diversity with quantitative, qualitative and mixed methods. 4th ed. Los Angeles, California: Sage Publications.

Meyer, R.D. and Wilkerson, T.L. 2011. Lesson Study: The Impact on Educators' Knowledge for Teaching Mathematics. In L. Hart, A. Murata & A.S. Alston (Eds.), Lesson Study Research and Practice in Mathematics Education: Learning Together (pp. 15-27). New York: Springer.

Mhakure, D. 2019. School-based mathematics teacher professional learning: A theoretical position on the lesson study approach. South African Journal of Education, 39.

Mills, V.L. and Harrison, C. 2020. Intentional professional learning design: Models, tools, and the synergies they produce supporting teacher growth. Educational Assessment, 25(4), pp. 331-354.

Mills, A. and Murgatroid, S. 1991. Organisational rules: A framework for understanding organisational action. Philadelphia, (England): Open University Press.

MoGE and JICA. 2016. Lesson Study in Zambia for Effective Teacher Professional Growth & Improvement of Students' Learning. Teaching Skills Book. Volume 2. Lusaka, Zambia.

Monteiro, S., Isusi-Fagoaga, R., Almeida, L. and García-Aracil, A. 2021. Contribution of higher education institutions to social innovation: practices in two southern European universities. Sustainability, 13(7), p.3594.

Moon, K., Brewer, T.D., Januchowski-Hartley, S.R., Adams, V.M. and Blackman, D.A. 2016. A guideline to improve qualitative social science publishing in ecology and conservation journals. Ecology and Society, 21(3).

Moquin, F.K. 2019. How Do Teachers Experience Lesson Study (Doctoral dissertations, Syracuse University).

Mthiyane, N.P. 2015. Chronicles of the experiences of orphaned students in a higher education institution in KwaZulu-Natal (Doctoral dissertation, University of the Free State).

Mugenda, A.G. 2011. Social science research methods: Theory and practice. Nairobi: ARTS Press.

Mukeredzi, T.G. 2009. Exploring professional development experiences of the professionally unqualified teachers in rural secondary schools. (Unpublished doctoral dissertation). University of KwaZulu-Natal, Pietermaritzburg, South Africa.

Murata, A. 2011. Introduction: Conceptual overview of lesson study. In L.C. Hart, A. Alston & A. Murata (Eds.), Lesson study research and practice in mathematics education: Learning together (pp. 1-12). New York, NY: Springer.

Murata, A. and Lee, C.K. 2020. Stepping Up Lesson Study: An Educator's Guide to Deeper Learning. Routledge.

Murata, A., Bofferding, L., Pothen, B.E., Taylor, M.W. and Wischnia, S. 2012. Making connections among student learning, content, and teaching: Teacher talk paths in elementary mathematics lesson study. Journal for Research in Mathematics Education, 43(5), pp. 616-650.

Murata, A., Lewis, C. and Perry, R. 2004. Teacher learning in lesson study: Developing professional capacity and resources. North American Chapter of the International Group for the Psychology of Mathematics Education. Toronto, Ontario: Canada. content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. Journal for Research in Mathematics Education, 39(4), pp. 372-400.

Murtafiah, W. and Lukitasari, M. 2019. Developing Pedagogical Content Knowledge of Mathematics Pre-Service Teacher through Microteaching Lesson Study. Online Submission, 13(2), pp. 201-218.

Mwanza-Simwami, D., Engeström, Y. and Amon, T. 2009. Methods for evaluating learner activities with new technologies: Guidelines for the Lab @ Future Project. International Journal on E-Learning, 8(3), pp. 361-384.

Ndlovu, M. 2021. Paradigms in Mathematics Teacher Professional Learning Research: A Review of South Africa′s Literature for 2006-2015. Mathematics Teaching and Professional Learning in sub-Sahara Africa, pp. 167-187.

Nelson, M.J., Voithofer, R. and Cheng, S.L. 2019. Mediating factors that influence the technology integration practices of teacher educators. Computers & Education, 128, pp. 330-344.

Ni Shúilleabháin, A. 2015. Developing mathematics educators' pedagogical content knowledge through lesson study: A multiple case study at a time of curriculum change (Doctoral dissertation, Trinity College Dublin).

Ni Shúilleabháin, A. and Seery, A. 2017. Enacting curriculum reform through lesson study: a case study of mathematics teacher learning. Professional Development in Education, pp. 1-15.

Ni Shúilleabháin, A. and PDST. 2018. Lesson Study Introductory Booklet. PDST, Dublin.

Ni Shúilleabháin, A. and Bjuland, R. 2019. Incorporating lesson study in ITE: organisational structures to support student teacher learning. Journal of Education for Teaching, 45(4), pp. 434-445.

Nkabinde, Z.P. 1997. An analysis of educational challenges in the new South Africa. University Press of Amer.

Nkengbeza, D. and Heystek, J. 2017. Professional learning communities: A comparative study of three educational areas in the North West Province of South Africa. Open Journal of Social Sciences, 5(4), pp. 98-119.

Noel, L.A. 2016. Promoting an emancipatory research paradigm in design education and practice.

Nurrahmi, H., Suryadi, D. and Fatimah, S. 2017. Students′ algebraic thinking process in context of point and line properties. In Journal of Physics: Conference Series, 895(1), pp pedagogical content knowledge: Conceptualizing and measuring teachers′ topic-specific knowledge of students. Journal for Research in Mathematics Education. IOP Publishing.

Nurtanto, M., Kholifah, N., Masek, A., Sudira, P. and Samsudin, A. 2021. Crucial Problems in Arranged the Lesson Plan of Vocational Teacher. International Journal of Evaluation and Research in Education, 10(1), pp. 345-354.

Odumosu, O. and Fisayi, A. 2018. Educators′ content and pedagogical knowledge on students′ achievement in algebra. International Journal of Education and Research, 6(3), pp. 83-94.

Ogegbo, A.A., Gaigher, E. and Salagaram, T. 2019. Benefits and challenges of lesson study: A case of teaching Physical Sciences in South Africa. South African Journal of Education, 39(1), pp. 1-9.

Olson, J.C., White, P. and Sparrow, L. 2011. Influence of lesson study on educators' mathematics pedagogy. In lesson study research and practice in mathematics education (pp. 39-57). Springer, Dordrecht.

Ono, Y and Ferreira, J. 2010. A case study of continuing teacher professional development through lesson study in South Africa. South African Journal of Education, 30(1), pp. 57-74.

Orb, A., Eisenhauer, L. and Wynaden, D. 2001. Ethics in qualitative research. Journal of nursing scholarship, 33(1), pp. 93-96.

Ostovar-Nameghi, S. A. and Sheikhahmadi, M. (2016). From teacher isolation to teacher collaboration: Theoretical perspectives and empirical findings.

Ozawa, H. 2009. Lesson Study in Mpumalanga Province, South Africa.

Ozawa, H., Ono, Y. and Chikamori, K. 2010. Change of a South African natural science teacher through the training focused on lesson study in Japan. Journal for the Science for Schooling, 11, pp. 129-140.

Özdemir, S.M. 2013. Exploring the Turkish Teachers' Professional Development Experiences and Their Needs for Professional Development. Online Submission, 3(4), pp. 250-264.

Özdemir, S.M. 2019. Implementation of the lesson study as a tool to improve students' learning and professional development of teachers. Participatory Education Research, 6(1), pp. 36-53.

Paine, L.W. and Fang, Y. 2006. Reform as hybrid model of teaching and teacher development in China. International Journal of Educational Research, 45, pp. 279-289.

Patton, M.Q. 2002. Qualitative research and evaluation methods. Thousand Oaks: 3rd Sage.

Perry, R.R. and Lewis, C.C. 2009. What is successful adaptation of lesson study in the US? Journal of educational change, 10(4), pp. 365-391.

Pirtle, S. and Tobia, E. 2014. Implementing Effective Professional Learning Communities. SEDL Insights. Vol. 2, No. 3, Winter 2014. SEDL. Available at: http://www.sedl.org/insights/23/implementing_effective_professional_learning_communities.pdf (Accessed: 29 September 2020).

Pirozzo, R. 2014. Differentiating the curriculum: Supporting teachers to thrive in mixed-ability classroom. Cheltenham: Hawker Brownlow Education.

Polit, D.F. and Beck, C.T. 2014. Essential of nursing research. Appraising evidence for. New York: Lippincott.

Postholm, M.B. and Vennebo, K.F. 2019. Applying Cultural Historical Activity Theory in Educational Settings: Learning, Development and Research. Routledge.

Posthuma, A.B. 2012. Mathematics educators' reflective practice within the context of adapted lesson study. Pythagoras, 33(3), pp. 1-9.

Powell, W. (n.d.). Chapter 5: Collaboration. *U.S. Department of State.* Available at: http://www.state.gov/m/a/os/43980.htm (Accessed: 26 September 2020).

Pratama, G.S. and Retnawati, H. 2018, September. Urgency of higher order thinking skills (HOTS) content analysis in mathematics textbook. In Journal of Physics: Conference Series (Vol. 1097, No. 1, p. 012147). IOP Publishing.

Provini, C. 2012. Best practices for professional learning communities. Education World. Available at: http://educationworld.com/a_admin/bestpractice-for-professional-learning-communities.shtml (Accessed: 10 October 2020).

Puchner, L.D. and Taylor, A.R. 2006. Lesson study, collaboration and teacher efficacy: Stories from two school-based math lesson study groups. Teaching and Teacher Education, 22(7), pp. 922-934.

Punch, K.F. and Oancea, A. 2014. Introduction to research methods in education. London: SAGE.

Purwaningsih, E., Sari, A.M., Yuliati, L., Masjkur, K., Kurniawan, B.R. and Zahiri, M.A. 2020. Improving the problem-solving skills through the development of teaching materials with STEM-PjBL (science, technology, engineering, and mathematics-project based learning) model integrated with TPACK (technological pedagogical content knowledge). In Journal of Physics: Conference Series (Vol 1481, No. 1, p. 012133). IOP Publishing.

Putri, I and Ilma, R. 2011. Professional development of mathematics primary school educators in Indonesia using lesson study and realistic mathematics education approach. Proceeding of ICSEI 2011, Lymasol Cyprus.

Ramírez, E.N.A., Quintana, Z.M., Sanhueza, A.O. and Valenzuela, S.S.V. 2013. The emancipatory paradigm and its influence on the development of nursing knowledge. Enfermería Global, (30), pp. 422-433.

Rapanta, C., Botturi, L., Goodyear, P., Guárdia, L. and Koole, M. 2020. Online university teaching during and after the Covid-19 crisis: Refocusing teacher presence and learning activity. Postdigital science and education, 2(3), pp. 923-945.

Redmond, P. and Lock, J. 2019. Secondary pre-service educators′ perceptions of technological pedagogical content knowledge (TPACK): What do they really think?. Australasian Journal of Educational Technology, 35(3).

Remillard, J.T. 2005. Examining Key Concepts in Research on Educators' Use of Mathematics Curricula. Review of Educational Research, 75(2), pp. 211-246.

Remillard, J.T. and Brynas, M.B. 2004. Educators' Orientations toward Mathematics Curriculum Materials: Implications for Teacher Learning. Journal for Research in Mathematics Education, 35(5), pp. 352-388. doi: 10.2307/30034820

Richit, A., da Ponte, J.P. and Tomasi, A.P. 2021. Aspects of Professional Collaboration in a Lesson Study. International Electronic Journal of Mathematics Education, 16(2), p.em0637.

Rocco, T.S., Hatcher, T. and Creswell, J.W. 2011. *The handbook of scholarly writing and publishing.* San Francisco, CA: John Wiley & Sons. 2011 ISBN 978-0-470-39335-2

Rochintaniawati, D., Riandi, R., Kestianty, J., Kindy, N. and Rukayadi, Y. 2019. The Analysis of Biology Educators Technological Pedagogical Content Knowledge Development in Lesson Study in West Java Indonesia. Jurnal Pendidikan IPA Indonesia, 8(2), pp. 201-210.

Rock, P. 1998. Rules, boundaries and the courts: Some problems in the neo-durkheimina sociology of deviance. The British Journal of Sociology, 49(4), pp.241-256.

Roth, W. and Lee, Y. 2007. Vygotsky's neglected legacy: Cultural-historical activity theory. Review of Educational Research, 77(2), pp. 186-232.

Roy, S. and Roy, R. 2015. Pedagogical content knowledge: An emerging issue in preparing educators. Scholarly Research Journal for Interdisciplinary Studies.

Rozakis, L. 2007. Schaum's quick guide to writing great research papers. McGraw Hill Professional.

Rule, P. and John, V. 2011. The guide to case study research. Pretoria: Van Schaik.

Russel, D. 1997. Writing and genre in higher education and workplaces. Mind, Culture and Activity, 4, pp. 224-237.

Russell, B. 2020. The principles of mathematics. Routledge.

Rowland, T., Huckstep, P. and Thwaites, A. 2005. Elementary educators' mathematics subject knowledge: The knowledge quartet and the case of Naomi. Journal of mathematics teacher education, 8(3), pp. 255-281.

Saka, Y. Southerland, S. and Brooks, J. 2009. Becoming a member of a school community while working toward science education reform: Teacher induction from a cultural-historical activity theory perspective. Science Education, 93(6), pp. 997-1025.

Saito, E., Harun, I., Kuboki, I. and Tachibana, H. 2006. Indonesian lesson study in practice: Case study of Indonesian mathematics and science teacher education project. Journal of In-Service Education, 32(2), pp. 171-184.

Saito, E. and Sato, M 2012. Lesson study as an instrument for school reform: A case of Japanese practices. Management in Education, 26(4), pp. 181-186.

Schipper, T., Goei, S.L., de Vries, S. and van Veen, K. 2017. Professional growth in adaptive teaching competence as a result of Lesson Study. Teaching and Teacher Education, 68, pp. 289-303.

Seino, T. and Foster, C. 2021. Analysis of the final comments provided by a knowledgeable other in lesson study. Journal of Mathematics Teacher Education, 24(5), pp. 507-528.

Sekao, R.D. 2019. The lesson study cycle. Available at: https://www.up.ac.za/lesson-study (Accessed: 21 May 2020).

Sekao, D. and Engelbrecht, J. 2021. South African Primary Mathematics Educators' Experiences and Perspectives About Lesson Study. International Journal of Science and Mathematics Education, 3(2), pp. 1-23.

Seleznyov, S. 2018. Lesson study: An exploration of its translation beyond Japan. International Journal for Lesson and Learning Studies, 7(3), pp. 217-229.

Seleznyov, S. 2019. Lesson study beyond Japan: evaluating impact. International Journal for Lesson and Learning Studies, 8(1), pp. 2-18.

Scheiner, T., Montes, M.A., Godino, J.D., Carrillo, J. and Pino-Fan, L.R. 2019. What makes mathematics teacher knowledge specialized? Offering alternative views. International Journal of Science and Mathematics Education, 17(1), pp.153-172.

Schmidt, W.H, Cogan, L., & Houang, R. 2011. The role of opportunity to learn in teacher preparation: An international context. Journal of Teacher Education, 62(2), pp. 138-153.

Schmid, M., Brianza, E. and Petko, D. 2020. Developing a short assessment instrument for Technological Pedagogical Content Knowledge (TPACK. xs) and comparing the factor structure of an integrative and transformative model. Computers & Education, 157, p.103967.

Schoenfeld, A.H. and Klipatrick, J. 2008. Towards a theory of proficiency in teaching Mathematics. In International handbook of Mathematics teacher education, v.2: Tools and processes in Mathematics teacher education (pp. 321-354). Rotterdam: Sense Publishers.

Schofer, E., Ramirez, F.O. and Meyer, J.W. 2021. The societal consequences of higher education. Sociology of Education, 94(1), pp. 1-19.

Schwab, M. and Borgdorff, H. 2014. *The Exposition of Artistic Research*: Publishing Art in Academia, Leiden: Leiden University Press.

Scott, C., Stone, B. and Dinham, S. 2001. International patterns of teacher discontent.

Serrao, O. 2019. The Global Competitiveness Index 4.0 2019 Rankings. WEF.

Shaw, J 2022. Emerging paradigms for ethical review of research using artificial intelligence. The American Journal of Bioethics, 22(5), pp. 42-44.

Shields, P.M. and Rangarjan, N. 2013. *A Playbook for Research Methods: Integrating Conceptual Frameworks and Project Management*. Stillwater, OK: New Forums Press.

Shimizu, Y. 2022. Lesson study in mathematics education. Encyclopaedia of mathematics education, 12(2), pp. 472-474.

Shingphachanh, S. 2018. Teachers′ understanding and concerns about the practices of lesson study in suburb schools in Laos. International Journal for Lesson and Learning Studies.

Shulman, L. 1986. Those who understand: knowledge growth in teaching. Educational research, 15, pp. 4-14.

Shulman, S.L. 1987. Knowledge and Teaching: Foundations of the new Reform. Harvard Educational Review, 57(1), pp. 1-21. Available at: http://people.ucsc.edu/~ktellez/shulman.pdf (Accessed: 07 July 2020).

Sibuyi, C.D. 2012. Effective educators' pedagogical content knowledge in teaching quadratic functions in mathematics (Doctoral dissertation, University of Pretoria).

Simon, M.K. 2011. Dissertation and scholarly research: Recipes for success. Seattle, WA: Dissertation Success, LLC.

Smit, R. and Humpet, W. 2012. Differentiated instruction in small schools. Teaching and Teacher Education, 28(8), pp. 1152-1162.

Snyders, E.M. 2017. Professional Learning Communities' role in high school educators' professional learning at a private school in the Western Cape (Doctoral dissertation, Stellenbosch: Stellenbosch University).

Spaull, N. 2013. Poverty & privilege. Primary school inequality in South Africa. International Journal of Educational Development, 33(5), pp. 436-447.

Spiteri, M. and Chang Rundgren, S.N. 2020. Literature review on the factors affecting primary educators′ use of digital technology. Technology, Knowledge and Learning, 25(1), pp. 115-128.

Spillane, J., Halverson, R. and Diamond, J. 2004. Towards a theory of leadership practice: A distributed perspective. Journal of Curriculum Studies, 36(1), pp.3-34.

Stein, M.K., Engle, R.A., Smith, M.S. and Hughes, E.K. 2008. Orchestrating Productive Mathematical Discussions: Five Practices for Helping Teachers Move Beyond Show and Tell. Mathematical Thinking and Learning, 10(4), pp. 313-340. doi: 10.1080/10986060802229675.

Stewart, R.A. and Brendefur, J.L. 2005. Fusing lesson study and authentic achievement: A model for teacher collaboration. The Phi Delta Kappan, 86(9), pp. 681-687.

Stigler, J.W. and Hiebert, J. 1999. The teaching gap: best ideas from the world's educators for improving education in the classroom. New York: Free Press.

Stokes, L.R., Suh, J.M. and Curby, T.W. 2020. Examining the nature of teacher support during different iterations and modalities of lesson study implementation. Professional Development in Education, 46(1), pp. 97-111.

Stols, G., Ferreira, R., Pelser, A., Olivier, W.A., Van der Merwe, A., De Villiers, C. and Venter, S. 2015. Perceptions and needs of South African Mathematics educators concerning their use of technology for instruction. South African Journal of Education, 35(4), pp. 1-13.

Stols, G. and Ono, Y. 2016. Lesson Study: An Implementation Manual.

Streubert, H.J. and Carpenter, D. 2011. Designing data generation and management strategies. Qualitative research in nursing: Advancing the humanistic imperative.

Stryker, R. 1994. Rules, resources, and legitimacy processes: Some implications for social conflict, order, and change. American Journal of Sociology, 99(4), pp. 847-910.

Swartling, U., Helgesson, G., Ludvigsson, J., Hansson, M.G. and Nordgren, A. 2014. Children's view on long-term screening for type 1 diabetes. Journal of Empirical Research on Human Research Ethics, 9(4), pp. 1-9.

Takahashi, A. 2005. For Teachers: Planning and Writing a Research Lesson. In P. Wang-Iverson & M. Yoshida (Eds.), Building Our Understanding of Lesson Study. Philadelphia: Research for Better Schools.

Takahashi, A. and McDougal, T. 2016. Collaborative lesson research: Maximizing the impact of lesson study. Zdm, 48(4), pp. 513-526.

Takahashi, A. and McDougal, T. 2019. Using school-wide collaborative lesson research to implement standards and improve student learning: Models and preliminary results. In Theory and Practice of Lesson Study in Mathematics (pp. 263-284). Springer, Cham.

Tan, S., Smith, B.A. and O′Halloran, K.L. 2015. Online leadership discourse in higher education: A digital multimodal discourse perspective. Discourse & Communication, 9(5), pp. 559-584.

Tay, L.Y., Lee, S.S. and Ramachandran, K. 2021. Implementation of online home-based learning and students' engagement during the COVID-19 pandemic: A case study of Singapore mathematics teachers. The Asia-Pacific Education Researcher, 30(3), pp. 299-310. .

Taylor, P.C. and Medina, M.N.D. 2013. Educational research paradigms: From positivism to multi-paradigmatic. *Journal for Meaning Centered Education*, 1(1), pp. 1-16.

Tepylo, D.H. and Moss, J. 2011. Examining change in teacher mathematical knowledge through lesson study. In Lesson study research and practice in mathematics education (pp. 59-77). Springer, Dordrecht.

Trochim, W.M.K. 2006. Research Methods Knowledge Base. Atomic Dog Pub.

Tuithof, H., Van Driel, J., Bronkhorst, L., Dorsman, L. and Van Tartwijk, J. 2021. Educators′ pedagogical content knowledge of two specific historical contexts captured and compared. Educational Studies, 23(5), pp. 1-26.

Ugbe, A.U., Bessong, F.E. and Agah, J.J. 2010. The mathematics/science teachers′ factor in achievement of the goal of Universal Basic Education (UBE). Global Journal of Educational Research, 9(1), pp. 35-37.

Uştuk, Ö. and Çomoğlu, İ. 2021. Reflexive professional development in reflective practice: what lesson study can offer. International Journal for Lesson & Learning Studies.

Van de Pol, J., Volman, M. and Beishuizen, J. 2010. Scaffolding in teacher-student interaction: A decade of research. Educational psychology review, 22(3), pp. 271-296.

Van der Walt, M. and Maree, K. 2007. Do Mathematics implement metacognitive strategies? South African journal of education, 27(2), pp. 223-241.

Van Driel, J. 2021. The development of preservice chemistry educators′ pedagogical content knowledge (pp. 157-191). Brill.2011.

Van Putten, S., Stols, G. and Howie, S. 2014. Do pre-service mathematics educators teach who they say they are? Journal of Mathematics Teacher Education, 17, pp. 369-392.

Venkat, H. and Alder, J. 2008. Expanding the foci of activity theory: Accessing the broader context and experiences of mathematics education reform. Educational Review, 60(2), pp. 127-140.

Vermunt, J.D., Vrikki, M., Van Halem, N., Warwick, P. and Mercer, N. 2019. The impact of Lesson Study professional development on the quality of teacher learning. Teaching and Teacher Education, 81, pp. 61-73.

Virkkunen, J. and Kuutti, K. 2000. Understanding organisational learning by focusing on “activity systems”. Accounting, Management and Information Technologies, 10(4), pp. 291-319.

Volmink, J.D. 2020. Maths needs to be taught differently if pupils’ marks are going to improve. Available at: http://www.umalusi.org.za (Accessed: 03 May 2020).

Voss, T., Kunter, M., & Baumert, J. 2011. Assessing teacher candidates’ general pedagogical/ psychological knowledge: Test construction and validation. Journal of Educational Psychology, 103(4), pp. 952-969.

Wallace, J. 1998. Collegiality and educators' work in the context of peer supervision. The Elementary School Journal, 99(1), pp. 81-98.

Walliman, N. 2017. Mixed Methods and Interdisciplinary Research. In Research Methods (pp. 167-177). Routledge.

Weaver, J.C., Matney, G., Goedde, A.M., Nadler, J.R. and Patterson, N. 2021. Digital tools to promote remote lesson study. International Journal for Lesson & Learning Studies.

Weber, S. 2011. Five dysfunctions of a professional learning community. The Whole Child. Available at: http://www.wholechildeducation.org/blog/five-dysfunctions-of-a-professionallearning-community (Accessed: 10 September 2020).

Wei, G. 2019. How Could Cultural-Historical Activity Theory Inspire Lesson Study? In Theory and Practice of Lesson Study in Mathematics (pp. 39-58). Springer, Cham.

Weick, K.E. 1969. Laboratory organisations and unnoticed causes. Administrative Science Quarterly, 14(2), pp. 294-303.

Werner, K. and Woessmann, L. 2021. The legacy of Covid-19 in education.

Willems, I. and Van den Bossche, P. 2019. Lesson Study effectiveness for educators′ professional learning: a best evidence synthesis. International Journal for Lesson and Learning Studies.

Wiseman, P.P., Arroyo, H. and Richter, N. 2013. Reviving professional learning communities: Strength through diversity, conflict, teamwork, and structure. New York, NY: Rowman & Littlefield.

Widjaja, W., Groves, S. and Ersozlu, Z. 2021. Designing and delivering an online lesson study unit in mathematics to pre-service primary educators: opportunities and challenges. International Journal for Lesson & Learning Studies.

Wood, K. 2021. On the development of pedagogical content knowledge through lesson study. International Journal for Lesson & Learning Studies.

Yalcin, A.F. 2019. The role of lesson study in teacher learning and professional development of EFL teachers in Turkey: A case study. TESOL Journal, 10(2), p. e00409.

Yamagata-Lynch, L. 2010. Activity systems analysis methods. New York: Springer.

Yamnitzky, G.S. 2010. Elementary teachers′ perspectives on the impact that lesson study participation had on their mathematical content and pedagogical-content knowledge. University of Pittsburgh.

Yamping, F. and Lee, C.K.-E. 2010. Lesson study and instructional improvement in Singapore. Research Brief, 10-001. Available at: http://www.nie.edu.sg/files/NIEresearch_brief_10_001.pdf (Accessed: 14 May 2020).

Yarema, C.H. 2010. Mathematics Educators' View of Accountability Testing Revealed through Lesson Study. Mathematics Teacher Education and Development, 12 (1), pp. 3-18.

Yin, R.K. 2014. Case Study Research: Design and Methods. 5th ed. USA: Sage.

Yin, R.K. 2017. Case study research and applications: Design and methods. Sage Publications.

Yoshida, M. 2002. Framing lesson study for U.S. participants. In M. Yoshida (Ed.), Studying classroom teaching as a medium for professional development (pp. 53-57). Washington, D.C.: National Academy Press.

Yoshida, M. 2012. Mathematics lesson study in the United States: Current status and ideas for conducting high quality and effective lesson study. International Journal for Lesson and Learning Studies, 1(2), pp. 140-152.

Yoshida, M., Gorman, J., Shimizu, Y., Wang-Iverson, P., Rosenbau, P., Nakamura, T. and Lewis C. 2003. Lesson study U.S.-Japan panel discussion 1. In Learning across

boundaries: U.S.-Japan collaboration in Mathematics, Science and Technology education. Arlington, VA: National Science Foundation.

Younus, S.B. 2021. Building Professional Learning Communities to Enhance Professional Teacher Development in Primary Schools.

Yurnetti, Y. 2018. Implementation of lesson study in physics teaching by group of educators in Solok West Sumatera. In Journal of Physics. Conference Series, 1006(1), p. 12-32. IOP Publishing.

Zvobwo, E.C. 2013. The mathematics genius in you. Johannesburg: Maths Genius.

Appendix A: Permission to Conduct Research

education

Department:
Education
PROVINCE OF KWAZULU-NATAL

Enquiries: Phindile Duma/Buyi Ntuli **Tel:** 033 392 1063/51 **Ref.**:2/4/8/4057

Mr Sikhumbuzo Sithembiso Dhlamini
P.O. Box 424
EMONDLO
3105

Dear Mr Dhlamini

PERMISSION TO CONDUCT RESEARCH IN THE KZN DoE INSTITUTIONS

Your application to conduct research entitled: **"EXPLORING THE IMPACT OF LESSON STUDY ON TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE IN TEACHING MATHEMATICS"**, in the KwaZulu-Natal Department of Education Institutions has been approved. The conditions of the approval are as follows:

1. The researcher will make all the arrangements concerning the research and interviews.
2. The researcher must ensure that Educator and learning programmes are not interrupted.
3. Interviews are not conducted during the time of writing examinations in schools.
4. Learners, Educators, Schools and Institutions are not identifiable in any way from the results of the research.
5. A copy of this letter is submitted to District Managers, Principals and Heads of Institutions where the Intended research and interviews are to be conducted.
6. The period of investigation is limited to the period from 06 February 2020 to 10 January 2022.
7. Your research and interviews will be limited to the schools you have proposed and approved by the Head of Department. Please note that Principals, Educators, Departmental Officials and Learners are under no obligation to participate or assist you in your investigation.
8. Should you wish to extend the period of your survey at the school(s), please contact Miss Phindile Duma /Mrs Buyi Ntuli at the contact numbers above.
9. Upon completion of the research, a brief summary of the findings, recommendations or a full report/dissertation/thesis must be submitted to the research office of the Department. Please address it to The Office of the HOD, Private Bag X9137, Pietermaritzburg, 3200.
10. Please note that your research and interviews will be limited to schools and institutions in KwaZulu-Natal Department of Education.

Dr. EV Nzama
Head of Department: Education
Date: 06 February 2020

...Championing Quality Education - Creating and Securing a Brighter Future

KWAZULU-NATAL DEPARTMENT OF EDUCATION
Postal Address: Private Bag X9137 • Pietermaritzburg • 3200 • Republic of South Africa
Physical Address: 247 Burger Street • Anton Lembede Building • Pietermaritzburg • 3201
Tel.: +27 33 392 1063 • **Fax.:** +27 033 392 1203• **Email:**Phindile.Duma@kzndoe.gov.za •**Web:**www.kzneducation.gov.za
Facebook: KZNDOE....Twitter: @DBE_KZN....Instagram: kzn_education....Youtube:kzndoe

Appendix B: Ethical Clearance Letter

Institutional Research Ethics Committee
Research and Postgraduate Support Directorate
2nd Floor, Berwyn Court
Gate 1, Steve Biko Campus
Durban University of Technology

P O Box 1334, Durban, South Africa, 4001

Tel: 031 373 2375
Email: lavishad@dut.ac.za
http://www.dut.ac.za/research/institutional_research_ethics

www.dut.ac.za

13 April 2021

Mr S S Dhlamini
P O Box 424
eMondlo
3105

Dear Mr Dhlamini

Exploring the role of lesson study on teachers' pedagogical content knowledge in teaching mathematics
Ethical Clearance number IREC 156/20

The Institutional Research Ethics Committee acknowledges receipt of your final data collection tool for review.

We are pleased to inform you that the data collection tool has been approved. Kindly ensure that participants used for the pilot study are not part of the main study.

In addition, the IREC acknowledges receipt of your gatekeeper permission letter.

Please note that FULL APPROVAL is granted to your research proposal. You may proceed with data collection.

Any adverse events [serious or minor] which occur in connection with this study and/or which may alter its ethical consideration must be reported to the IREC according to the IREC Standard Operating Procedures (SOP's).

Please note that any deviations from the approved proposal require the approval of the IREC as outlined in the IREC SOP's.

Yours Sincerely.

Dr K Padayachy
Chairperson: IREC

Appendix C: Informed Content Form

LETTER OF INFORMATION

.

Title of the Research Study:

Exploring the role of lesson study on educators' pedagogical content knowledge in teaching mathematics

Principal Investigator/s/researcher:

Sikhumbuzo Sithembiso Dhlamini – Bachelor of Education, Honours Bachelor of Education and Master of Education.

Co-Investigator/s/supervisor/s:

Dr. J.P. Abraham (PhD) and Dr. V.Z. Masuku (PhD)

Brief Introduction and Purpose of the Study:

This study will explore the role of lesson study on educators' pedagogical content knowledge in teaching mathematics. Therefore, there is a need to explore what role and how participating in lesson study may develop educators pedagogical content knowledge, especially in their learning within collaborative planning environment. I think the lesson study approach might help in resolving educators lack of pedagogical content knowledge in teaching mathematics, since is a model of professional development which supports educators learning through collaboration.

The lesson study approach was first established in Japan over 100 years ago and is increasingly being used in other countries, particularly Singapore, but also in the United States, African countries and now in South Africa. Lesson study is different from other forms of professional development because it takes place in the moment of teaching and learning. Participating in lesson study can reduce educators' sense of isolation as a group of teachers introduce themselves to norms of working together as part of an educator community.

The vision and mission of the Department of Basic Education (DBE) is to provide quality education for all. Mrs. Angie Motshekga, Minister of Department of Basic Education (DBE), moving from the stated premise appointed a team of officials in 2016 and 2017 to go and observe Japanese lesson study; a professional development

approach. As mathematics subject advisor, I have to ensure the implementation of lesson study and its effectiveness to educators. Lesson study consists of a cycle of phases, where a group of educators work together to plan, conduct, observe and reflect on a research lesson. This study will use Cultural Historical Activity Theory (CHAT) as a theoretical framework.

Outline of the Procedures:

The participants of this proposed study are primary school mathematics educators in Ilembe District teaching grades 4 – 6 mathematics. Convenience, non-probability sampling techniques will be employed, it best suits the purpose. Non-probability sampling will be used to identify 2 accessible Professional Learning Communities (PLCs) out of 5 PLCs within Ilembe District. These 2 PLCs will make a total of 7 schools and 31 educators. Given that this will be a qualitative study which works with small samples to generate in-depth subjective data, a minimum sample size of 31 educators is regarded as appropriate.

I will use documents analysis, meetings, lesson presentations and observations, and interviews. These instruments are suitable for collecting qualitative data in line with my research paradigm, design and approach. The document analysis will involve CAPS Documents for Grades 4 – 6, Annual Teaching Plans, Lesson Plans for presented topics and any other relevant teaching documents. The observation will be carried out during lesson presentations after research lesson planning meeting. The reflection meeting will be carried out on the same day after each and every lesson observation. After each reflection meeting a lesson presenter will be interviewed on a semi-structured interview to capture rationale behind presenter actions and will take approximately 45 minutes to one hour. While all other 22 educators will be interviewed on focus or group interview where a PLC will be put together at the end of 3 lesson study cycles per grade to minimise number of meetings.

Risks or Discomforts to the Participant: (Description of foreseeable risks or discomforts to for participants if applicable e.g. Transient muscle pain, VBAI, post-needle soreness, other adverse reactions, etc.)

Benefits:

Teachers, curriculum planners, curriculum developers and subject advisors to interpret, practice curriculum. I will proceed to inform the participants that they will benefit from the study by way of getting the opportunity to air their views and experience on lesson study as a professional development method which may influence department officials. They will also access the results of the study from Durban University of Technology library were a copy will be available on internet.

Reason/s why the Participant May Be Withdrawn from the Study:

The core principles are that researcher should respect participants. I will provide all participants with letters of information, at the same time, all participants will sign consent forms. I will inform them all participants that participation is voluntary and that one can withdraw at any time without any prejudice to them.

Remuneration:

No remuneration will be received by participants.

Costs of the Study:

Participants will not cover any costs towards the study.

Confidentiality:

I will not put anyone of the participants at risk, real names of schools and participants will be hidden. Instead, I will use pseudonyms throughout the study. This is done to protect participants' identities and to protect participants from disclosure of information. I will keep all data confidential and anonymous.

Research-related Injury:

No injuries will take place in this study since participants will be conducting teaching.

Persons to Contact in the Event of Any Problems or Queries:

Institutional Research Ethics Administrator on 031 373 2375. Complaints can be reported to the Director: Research and Postgraduate Support, Prof C E Napier on 031 373 2577 or carinn@dut.ac.za.

CONSENT

Statement of Agreement to Participate in the Research Study:

- I hereby confirm that I have been informed by the researcher, Sikhumbuzo Sithembiso Dhlamini, about the nature, conduct, benefits and risks of this study - Research Ethics Clearance Number: _____,
- I have also received, read and understood the above written information (Participant Letter of Information) regarding the study.
- I am aware that the results of the study, including personal details regarding my sex, age, date of birth, initials and diagnosis will be anonymously processed into a study report.
- In view of the requirements of research, I agree that the data collected during this study can be processed in a computerised system by the researcher.
- I may, at any stage, without prejudice, withdraw my consent and participation in the study.
- I have had sufficient opportunity to ask questions and (of my own free will) declare myself prepared to participate in the study.
- I understand that significant new findings developed during the course of this research which may relate to my participation will be made available to me.

_____	_____	_____	_____
Full Name of Participant	**Date**	**Time**	**Signature / Right Thumbprint**

I, Dhlamini S.S. (name of researcher) herewith confirm that the above participant has been fully informed about the nature, conduct and risks of the above study.

Sikhumbuzo Sithembiso Dhlamini	28 April 2020	
Full Name of Researcher	**Date**	**Signature**

Full Name of Witness (If applicable)	**Date**	**Signature**
Full Name of Legal Guardian (If applicable)	**Date**	**Signature**

Appendix D: Lesson Study Cycle Observation Tool

Mathematics Lesson Study Cycle Observation Tool
for
The Lesson Study Research

Researcher : Mr. Dhlamini S.S.

Institution : Durban University of Technology

Department : School of Education

Qualification : Doctor of Education

Research Topic :

Exploring the role of lesson study on educators' pedagogical content knowledge in teaching mathematics.

Before using this tool, it is important to reach agreement about:

- the purpose of the observation
- the curriculum context of the lesson / learning situation
- the profile of the learners
- who have access to the data collected

This observation tool was used to collect data to respond to the following research questions:

1. What do the educators need to enhance their pedagogical content knowledge of teaching numeric and geometric patterns?

2. How do educators' participation in lesson study enhance their pedagogical content knowledge in the teaching of numeric and geometric patterns?

3. What are the educators' view about the lesson study when teaching numeric and geometric patterns?

LESSON OBSERVATION TOOL

Name of School	Grade	Name of observer	Date							
							2	0	2	1

	Focus of identifying problematic concept(s)	Comments
DIAGNOSTIC ANALYSIS	Was the common assessment administered in all schools that forms a PLC?	
	Was the assessment administered to a certain group or all learners in a grade?	
	Were all the scripts marked?	
	What informed problematic concepts to be tackled?	
	Which concepts were identified as posing a challenge to learners?	
	Were there any concepts identified by teachers as posing a challenge to learners, not being informed by diagnostic analysis?	
	Focus of lesson planning	**Comments**
ATTENDANCE	Was the PLC facilitator democratically elected or appointed?	
	Were all PLC members present in all planning meetings and how many sessions?	
	Were there any guests presents? State their position in the PLC	

BRAINSTORMING	Were lesson presenters identified before or after brainstorming sessions and what method was used to select them?	
	How individual data gathering process on the best approach to teach?	
	Who and how was the construction of research lesson plan cooperated?	
	Were research lessons goals and unit goals clearly identified?	
PLAN THE LESSON	How many planning meetings took place before final write up of a lesson plan?	
	Was the final write up of the lesson plan done collaboratively or by a lesson presenter?	
	When was the final lesson plan made available to observers?	
	Describe any specific Information Computer Technology (ICT) needed.	
	Which documents were used to develop a research lessons?	
	Focus of lesson presentation and observation	**Comments**
OBJECTIVES	Clearly presented and achievable in the stipulated duration of the lesson.	
	Aligned to curriculum.	
	Grade appropriateness of the content (depth and scope of the content).	

INTRODUCTION	Does the lesson start well and in an organised way?	
	Does the teacher engage the learners' interest in the topic effectively?	
	Is prior knowledge connected to new knowledge?	
TEACHER'S PRESANTION	Conceptual development.	
	Were learners actively involved in the lesson?	
	Strategies/opportunities used helped learners to understand the concept well.	
	Sufficient time was given to learners to think, discuss and respond.	
	Learners were asked to justify their answers where necessary.	
	The teacher checked how learners worked in their class work book/workbook.	
	When learners worked, the teacher moved around to offer support.	

LEARNER PARTICIPATION	Individual.	
	Collaborative/cooperative groups.	
	Do the learners get started with the activities quickly?	
ASSESSMENT	Does the teacher give a clear demonstration?	
	Kinds of question asked.	
	Questions contribution to conceptual understanding.	
	Are the learners clear (purposefulness) about what they are doing and why?	
RESOURCES	Are the resources to hand?	
	Are materials for learners to use distributed effectively and/or readily available for them to access?	
	Relevance to the topic.	
	Contribute to the achievement of the objectives.	
	ICT used to expand curriculum learning opportunities?	

OBSER VERS	Did the observers checked how learners worked in their class work book/workbook?	
	When learners worked, did the observers moved around to offer support?	
	Was there any contribution from observers during lesson presentation?	
CONCLUSION	The lesson was wrapped up by emphasising key issues related to the objective(s).	
	Achievement of the lesson objectives by learners.	
	Focus of lesson reflection	**Comments**
REFLECTION	Highlights of the lesson.	
	Area of improvement.	

Appendix E: Turn-it-in Report

Exploring the role of lesson study on educators' pedagogical content knowledge in teaching mathematics4

ORIGINALITY REPORT

11%	10%	4%	%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	researchspace.ukzn.ac.za Internet Source	1%
2	mafiadoc.com Internet Source	1%
3	espace.curtin.edu.au Internet Source	1%
4	eprints.qut.edu.au Internet Source	1%
5	scholar.sun.ac.za Internet Source	1%
6	hdl.handle.net Internet Source	<1%
7	ujcontent.uj.ac.za Internet Source	<1%
8	"Theory and Practice of Lesson Study in Mathematics", Springer Science and Business Media LLC, 2019 Publication	<1%

281	Candice Bocala. "From Experience to Expertise", Journal of Teacher Education, 2015 Publication	<1 %
282	Fang Hua Jhang. "Teachers' attitudes towards lesson study, perceived competence, and involvement in lesson study: evidence from junior high school teachers", Professional Development in Education, 2019 Publication	<1 %
283	Gwendolyn M. Lloyd, Olive Chapman. "International Handbook of Mathematics Teacher Education: Volume 3", Brill, 2020 Publication	<1 %
284	Willig, Carla. "EBOOK: Introducing Qualitative Research in Psychology", EBOOK: Introducing Qualitative Research in Psychology, 2013 Publication	<1 %
285	rgu-repository.worktribe.com Internet Source	<1 %

Exclude quotes	On	Exclude matches	< 5 words
Exclude bibliography	On		

Appendix F: Editor's Report

Angela Bryan & Associates

6 Martin Crescent
Westville

Date: 27 March 2022

To whom it may concern

This is to certify that the Doctoral Thesis: Exploring the Role of Lesson Study on Educators ' Pedagogical Content Knowledge in Teaching Mathematics written by Sikhumbuzo Dhlamini been edited by me for language.

Please contact me should you require any further information.

Kind Regards

Angela Bryan

angelakirbybryan@gmail.com

0832983312