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Transition from Inflation to Dark Energy in Superfluid Vacuum Theory

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The laminar constant-velocity superflow of the physical vacuum modelled by logarithmic quantum Bose liquid is considered. We demonstrate that this three-dimensional non-relativistic quantum flow generates a four-dimensional relativistic quinton system, which comprises the dilaton and quintom (a combination of the quintessence and tachyonic phantom fields); all three fields are thus shown to be projections of the dynamical evolution of superfluid vacuum density and its fluctuations onto the measuring apparatus of a relativistic observer. The unified model describes the transition from the inflationary period in the early universe to the contemporary accelerating expansion of the universe, commonly referred to as the "dark energy" period. The quintessence and tachyonic scalar components of the derived model turn out to be non-minimally coupled, which is a hitherto unexplored generalization of cosmological phantom models.

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1. INTRODUCTION

The period of cosmological inflation, which occurred in the early universe, was characterized by the expansion of space at an exponential rate. Most popular theories of this inflation are constructed by non-minimally coupling of the scalar field called dilaton (inflaton, in cosmological terminology) to Einstein gravity [1]. It was believed that the accelerated expansion of the universe ended a long time ago and was replaced by the non-accelerated expansion, commonly referred to as the Big Bang, during which all known elementary particles, including baryons and cosmic microwave background photons, were formed.

However, it was discovered relatively recently that after the inflationary period has ended, and all known elementary particles have been formed, the universe is still continuing to expand with an acceleration, only at a slower rate [2, 3]. This re-acceleration period is commonly referred to as the dark energy (DE) epoch. Following this discovery, various theories were proposed to solve the dark energy and dark matter (DM) problems, ΛCDM model being probably the most popular approach among them.

A number of low-redshift observations later revealed that there are discrepancies between the values of the Hubble parameter at the present time from observations of Cepheids in the Large Magellanic Cloud, the gravitational lensing of quasars measurement, and the predicted value by the ΛCDM model using Planck CMB data [4]. This phenomenon, commonly referred to as Hubble tension, posed additional challenges for the ΛCDM model, and some of them have yet not been resolved, to the best of our knowledge.

This left the question of a complete cosmological model open once again, not to mention than the ΛCDM model alone does not explain the nature and origin of dark matter *per se* but serves as a phenomenological approach and a curve-fitting summary of astronomical data.

Another currently open question is how to explain the transition from inflation to the DE period, because they are usually described by very different theories: scalar-tensor gravities on the inflation side and a plethora of models on the DE side. If the dilaton/inflaton field did exist in the early universe then what happened to it at later times? If it was "used up" to produce the conventional matter then how did the fields, which generate the "dark energy" and "dark matter" phenomena we currently observe, appear? Is it possible to construct a cosmological model which would not only be a unified model of inflationary and dark epochs but also originated from a theory of quantum gravity itself?

In this paper, we propose to answer these questions by using the superfluid vacuum theory (SVT), which is the theory of physical vacuum, and a theory of quantum gravity at the same time.

The paper is organized as follows. In the next section, we give a brief review of the superfluid vacuum theory based on the logarithmic liquid model. In Section 3, we consider a cosmological model which arises from the logarithmic superfluid vacuum theory, assuming a simple superflow (laminar and constant-velocity) of the physical vacuum. In Section 4, we demonstrate a transition from the dilaton-driven inflation to the cosmological expansion driven by one of the candidates for the dark energy, quintom. We show that even the simplest laminar superflow generates the non-minimally coupled quinton system, which unifies the inflaton and quintom (quintessence field coupled to tachyonic phantom) models. In Section 5, we propose the generalized quinton model of dark energy and study its basic properties. Conclusions are drawn in Section 6.

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2. SUPERFLUID VACUUM THEORY

According to the SVT paradigm, physical vacuum is a quantum liquid with suppressed dissipative fluctuations (superfluid) “living” in three-dimensional Euclidean space, whereas four-dimensional curved spacetime and Lorentz symmetry are induced phenomena, occurring through so-called superfluid-spacetime correspondence. This theory is, in fact, a framework for constructing models of superfluid vacuum by assuming one or the other structure and dynamics thereof [5, 6].

Logarithmic nonlinearity naturally occurs in the theory of laboratory quantum liquids, such as Bose-Einstein condensates of alkali atoms and helium superfluid, where it provides a more accurate fitting of experimental curves and even resolves certain puzzles [7–9]. This motivates us to use this nonlinearity to describe the background superfluid as well, some landmark works being [10–12].

Let us introduce the state vector $|\Psi\rangle$ and the wavefunction in a position representation, $\Psi = \Psi(\boldsymbol{x}, t) = \langle \boldsymbol{x}|\Psi\rangle$, which obeys the normalization condition

$$\int_{\mathcal{V}} |\Psi|^2 d\mathcal{V} = \int_{\mathcal{V}} \rho d\mathcal{V} = \mathcal{M} = m\mathcal{N} > 0, \qquad (1)$$

where $\mathcal{M}$ and $\mathcal{V}$ are the total mass and volume of the system, and m and $\mathcal{N}$ are mass and number of constituent particles; here and in what follows, angle brackets indicate Dirac’s bra-ket notation.

We assume that the liquid is described by the vector $|\Psi\rangle$ whose dynamics obeys the logarithmic Schrödinger equation, which reads in a position representation

$$i\partial_t \Psi = \left[-\frac{\mathcal{K}}{2} \boldsymbol{\nabla}^2 + \frac{1}{\hbar} V_{\text{ext}}(\boldsymbol{x}, t) - b \ln(|\Psi|^2/\rho_c) \right] \Psi, \qquad (2)$$

where b, ρ_c and $\mathcal{K} = \hbar/m$ are real-valued parameters, and $V_{\text{ext}}(\boldsymbol{x}, t)$ is external potential. For brevity, we assume $V_{\text{ext}} \equiv 0$.

One can show that the inviscid flow of the logarithmic liquid (2) is observed as four-dimensional curved spacetime by an observer who can operate only with the small-amplitude low-momentum fluctuations of this fluid (in what follows referred to as small fluctuations). In other words, Lorentz symmetry is not an exact symmetry in superfluid vacuum theory, but an induced effect and approximation. The mapping which relates these two pictures, quantum three-dimensional Euclidean and classical four-dimensional relativistic, is the superfluid-spacetime correspondence mentioned at the beginning of this section, see [11] for technical details.

These small fluctuations are observed by the above-mentioned observer as relativistic particles, transforming according to the irreducible representations of the Poincaré group, for which reason this observer is referred to as the R(elativistic)-observer. The approach thus has two types of observers: the F(ull)-observer who can “see” the vacuum as three-dimensional quantum fluid, and the R-observer who is unable to observe any underlying Euclidean objects or processes, but observes four-dimensional relativistic phenomena instead.

In particular, the matter observed by the R-observer is defined, up to an overall factor, by the induced stress-energy tensor [10]:

$$T^{(\text{ind})}_{\mu\nu} \sim R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \qquad (3)$$

where $R_{\mu\nu}$ and R are, respectively, the Ricci tensor and scalar curvature derived from the induced metric $g_{\mu\nu}$, whereas the latter comes about as a result of superfluid dynamics via the superfluid-spacetime correspondence. The right hand side of the definition (3) depends on a theory of gravity which one assumes; here we adhere to the canonical GR-type one, without higher-order Riemannian terms, topological invariants, torsion, et cetera.

Apart from explaining the occurrence of the Lorentz symmetry and relativistic phenomena in nature, superfluid vacuum theory is a well-defined quantum theory (with respect to the F-observer), therefore it can be regarded as a theory of quantum gravity observed by the R-observer. The consistent workflow is to regard Lorentz-covariant gravity as an effective theory for macroscopic measurements by the R-observer, but to quantize the underlying Euclidean superfluid, then use the above-mentioned superfluid-spacetime correspondence as a “dictionary” to translate the outcomes into the R-observer’s language.

In most cases, however, one does not know the wavefunction of the vacuum but does know the energy-momentum tensor, therefore, one must reverse-engineer it from energy-momentum tensor. In this reverse workflow, the inverse superfluid-spacetime correspondence acts as a gravity quantization procedure, because it delivers a transition from the (classical) metric to (quantum-mechanical) wavefunction.

One of the predictions of the logarithmic SVT is the formula for speed of light:

$$c^2_{(0)} \sim \frac{\hbar}{2m}(\omega - b), \qquad (4)$$

where ω is the eigenfrequency of a given quantum state of the physical vacuum (notations can be found in section 4 of [11]). Logarithmic nonlinearity plays a crucial role here, because if one starts not with Equation (2), but with any other nonlinear Schrödinger equation from the class $i\partial_t\Psi + \left[\frac{\hbar}{2m}\boldsymbol{\nabla}^2 + F(|\Psi|^2)\right]\Psi = 0$, then the speed of light would no longer have a constant (independent of density) limit because function $\rho F'(\rho)$, which occurs in the derivation of the speed of light, is constant if and only if $F(\rho)$ is a logarithm, see [11] and section 3 of [13] for technical details. Such a limit is necessary for the compatibility of SVT with relativity postulates in the small-fluctuations regime, and also for defining the fundamental constant of the speed of light in vacuum, $c \approx c_{(0)}$.

3. SVT COSMOLOGY

Within the framework of superfluid vacuum theory, let us consider the global flow of superfluid vacuum absent any distortions, and its observational consequences for R-observers.

In the simplest possible case, the phase of a superfluid wavefunction is a linear function with respect to spatial coordinates and time. This corresponds to a laminar constant-velocity flow in Euclidean space along one of directions, if viewed by F-observers. The R-observers, however, would see a totally different picture, according to superfluid-spacetime correspondence. They will discover themselves, by measuring trajectories of probe particles, as "living" in a conformally flat four-dimensional spacetime. In the leading-order approximation with respect to $\mathcal{K}$, their metric can be written as

$$g_{\mu\nu} \sim \rho\, \eta_{\mu\nu} \sim |\Psi|^2\, \eta_{\mu\nu}, \tag{5}$$

where $\eta_{\mu\nu}$ is a metric of Minkowski spacetime, and the conformal factor depends on the superfluid wavefunction squared. For the R-observer, this metric is defined up to an overall factor, which determines the choice of physical frame, but here we assume for simplicity that this factor is one. The logarithmic nonlinearity is crucial for this conformal flatness – basically, due to a constant value of the velocity $c_{(0)}$ from Equation (4).

According to the Petrov classification, the class of conformally flat spacetimes includes all universes with acceleration, where they differ from each other by their conformal factors; there is an approach to the study of cosmology in these coordinates alone [15–18]. For example, de Sitter spacetime can be written in the form (5) with

$$\rho = \rho_{\text{dS}} \sim (\tau - \tau_0)^{-2}, \tag{6}$$

where τ is conformal time (in this case coinciding with the Euclidean time of the F-observer).

In this case, R-observers find themselves inside a four-dimensional de Sitter spacetime which expands exponentially, whereas the F-observer "sees" the homogeneous three-dimensional superfluid with quadratically decreasing density as time passes. Note that singularity exists for the R-observer, when the metric's conformal factor vanishes or diverges, but not for the F-observer because the infinite value of superfluid density is disallowed by the normalization condition (1), whereas the zero value is regular and asymptotic. This illustrates our earlier remarks about superfluid vacuum theory being well-defined as a theory of gravity: spacetime singularities menace its small-fluctuation (relativistic) limit, but not the full underlying theory.

From the metric (5), using definition (3) and formula (A10) in one of the intermediate steps, one can reverse-engineer the basic Lorentz-invariant action functional describing the gravitational interaction experienced by R-observers, see [10, 11] for details. One then obtains the following action (in Planck units):

$$S_d \;=\; \frac{1}{2}\int d^D x \sqrt{-g}\, \mathrm{e}^{\tilde{D}\Phi} \left[R + \tilde{D}(\tilde{D}+1)(\partial\Phi)^2\right] - \int d^D x \sqrt{-g} V_0 = \frac{1}{2\rho_c^2}\int d^4 x \sqrt{-g}\left[\rho^2 R + 3!(\partial\rho)^2 - 2\rho_c^2 V_0\right], \tag{7}$$

where $(\partial f)^2 \equiv g^{\mu\nu}\partial_\mu f\, \partial_\nu f$, R is the Ricci scalar with respect to the metric $g_{\mu\nu}$, $\Phi = \ln(\rho/\rho_c)$ is a function of superfluid background density $\rho = |\Psi|^2$, $V_0 = \text{const}$, and $\tilde{D} = D - 2 = 2$ in four dimensions. The topological term with the constant V_0 can always be added to a scalar field action; the physical meaning of this constant is the reference value for counting energy of scalar field.

One can see that the background superfluid induces, in the R-observer's picture, not only spacetime but also the scalar field.

4. THE TRANSITION

Model (7) can be directly applied to early-universe cosmology because it can describe exponential expansion during the inflationary epoch, for instance of the de Sitter type (6), it also explains the origin of the dilatonic inflaton field from superfluid vacuum density.

Let us assume the R-observer being in the middle of the inflation era, say, described by the de Sitter universe. Then, recalling the remark after Equation (6), the F-observer observes during the same period of time that the background density of the superfluid vacuum decreases as time passes.

This means that at some stage of evolution, fluctuations of density (i.e., the wavefunction's amplitude), which are always present in a quantum realm, become no longer negligible, although still small if compared to the background value. From the viewpoint of the R-observer, it means that the Lagrangian (7) acquires small corrections, which can break the original symmetry (5).

We thus assume the perturbation

$$\rho = \bar{\rho} + \delta\rho, \quad |\delta\rho| \ll \bar{\rho}, \quad |\partial\, \delta\rho| / |\partial\bar{\rho}| \sim 1, \tag{8}$$

which means that fluctuations $\delta\rho$ are much smaller than the background value $\bar{\rho}$, but derivatives $\partial\,\delta\rho$ and $\partial\bar{\rho}$ have the same order of magnitude. In this approximation, in the leading order, the perturbation of the model (7) yields

$$S_q = \frac{1}{2\rho_c^2}\int d^4x\sqrt{-g}\left[\bar\rho^2 R + 3!(\partial\rho)^2 - 2\rho_c^2 V_0\right] = \frac{1}{2}\int d^4x\sqrt{-g}\left[\mathrm{e}^{\sqrt{2/3}\,\phi}R + (\partial\sigma)^2 - 2V_0\right], \qquad (9)$$

where we denoted

$$\phi = \sqrt{3!}\,\ln(\bar\rho/\rho_c), \quad \sigma = \sqrt{3!}\,\rho/\rho_c = \sqrt{3!}\,(\bar\rho + \delta\rho)/\rho_c. \qquad (10)$$

Due to the explicitly covariant form of the action (9), values ϕ and σ can be interpreted as a new set of relativistic scalar fields.

Furthermore, to extract more physical information from action (9), let us rewrite it in the Einstein frame. Under the conformal transformation

$$g_{\mu\nu} = (\bar\rho/\rho_c)^{-2}\breve g_{\mu\nu} = \mathrm{e}^{-\sqrt{2/3}\,\phi}\breve g_{\mu\nu}, \qquad (11)$$

and using the formula (A15), action functional (9) transforms into

$$S_q = \frac{1}{2}\int d^4x\sqrt{-\breve g}\left[\breve R - (\breve\partial\phi)^2 + \mathrm{e}^{-\sqrt{2/3}\,\phi}(\breve\partial\sigma)^2 - 2V_0\mathrm{e}^{-2\sqrt{2/3}\,\phi}\right], \qquad (12)$$

where $(\breve\partial f)^2 = \breve g^{\mu\nu}\partial_\mu f\,\partial_\nu f$ and other checked values are computed with respect to the Einstein-frame metric $\breve g_{\mu\nu}$.

In this form, induced gravity action reveals an important feature of the model: the kinetic couplings for scalars ϕ and σ have opposite signs, indicating that if one of them is bradyonic then the other one must be tachyonic.

The tachyon occurrence cannot be explained within the framework of the "orthodox" theory of relativity, because it would require drastic changes of its postulates and mathematical structure [19, 20]. Within the SVT framework, the explanation is rather natural: at a certain stage of evolution, when superfluid vacuum fluctuations became sufficiently large, the original scalar field Φ decayed into the quintessence ϕ and phantom σ fields with the latter being tachyonic.

Whereas the original dilaton field is instrumental in the inflationary models of the early universe, the combination of the quintessence and phantom fields, often referred to as the quintom, can be used to describe the accelerated expansion occurring nowadays, and is regarded as a form of "dark energy" [21–24]. Quintom cosmology is also instrumental in explaining the Hubble tension mentioned in the Introduction, cf. [25].

One can notice the crucial difference between the conventional quintom cosmology, where kinetic couplings of the scalars are postulated to be constant, and the action (12), where the kinetic coupling of the phantom field is a function of the quintessence field. This coupling ensures that at large positive values of quintessence (in the F-observer's picture, it corresponds to the background density $\bar\rho$ becoming much larger than the critical value ρ_c), phantom decouples from the system, thus making the latter purely quintessential. At small values of ϕ, one recovers the plain quintom cosmological model.

Another effect is the transition between the conformal and Einstein frames during the dilaton-quintom transition. In scalar-tensor theories of gravity, the question of which physical frame is physical is always a good one. In this model, the Einstein-frame metric $\breve g_{\mu\nu}$ minimizing action (12) is more instrumental because it explicitly takes into account the tachyonic nature of σ; whereas the conformal-frame metric $g_{\mu\nu}$ minimizing the action (7) is more suitable when dealing with the dilaton-driven inflationary period. Both frames momentarily coincide in the instant point $\Phi = \phi = 0$ where induced spacetime becomes empty in the R-observer picture; in the F-observer picture, this corresponds to the non-perturbed superfluid density reaching a critical value ρ_c.

5. QUINTON MODEL OF DARK ENERGY

Using action (12) as a starting point, let us formulate a general model of a current-epoch cosmology motivated by superfluid vacuum theory. Restoring the Einstein gravitational constant and omitting the checked notations for brevity (while remembering that we are working in a different frame from that in the inflation phase, as discussed in the previous section), we write

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}\mathrm{e}^{-\lambda\phi}(\partial\sigma)^2 - V_0\mathrm{e}^{-2\lambda\phi} - \Delta V(\phi,\sigma)\right] + S^{(M)}, \qquad (13)$$

where the term $S^{(M)} = \int d^4x\sqrt{-g}\mathcal{L}_M$ is added to account for the other matter and radiation content of the universe which was generated during the inflaton-quinton transition. In this action, we added the scalar potential perturbation $\Delta V(\phi,\sigma)$, which can be chosen *ad hoc*, as is commonly done in cosmological models involving scalar fields. We also relaxed the non-minimal coupling's rate constant $\sqrt{2/3}$ to λ, a free scale parameter of the model. These generalizations are expected to account for self- and mutual interactions of scalar fields and spacetime geometry with quantum matter and radiation, which inevitably occur. For the same reason, we also assume that Lagrangian density $\mathcal{L}_M$ can depend, in general, not only on the matter's fields but also on the quinton fields ϕ and σ.

Note that the quintessence and phantom fields are now non-minimally coupled, for which reason their effects cannot be separated from each other as clearly as before. In the limit $\phi \to 0$, one obtains the conventional quintom cosmology action, but otherwise the system's dynamics becomes more complicated. To begin with, scalar field equations turn out to be significantly entangled:

$$\Box\,\phi - \frac{1}{2}\lambda \mathrm{e}^{-\lambda\phi}(\partial\sigma)^2 - \frac{\partial V}{\partial\phi} = -\frac{\partial \mathcal{L}_M}{\partial\phi}, \qquad (14)$$

$$\Box\,\sigma - \lambda\partial_\mu\phi\,\partial^\mu\sigma + \mathrm{e}^{\lambda\phi}\frac{\partial V}{\partial\sigma} = \mathrm{e}^{\lambda\phi}\frac{\partial \mathcal{L}_M}{\partial\sigma}, \qquad (15)$$

where we denoted

$$V = V_0 \mathrm{e}^{-2\lambda\phi} + \Delta V(\phi,\sigma). \qquad (16)$$

Notice that the non-minimal coupling couples the quintessence field component to the non-quinton matter if the latter interacts with the tachyonic component, cf. the right-hand side of Equation (15).

Furthermore, by varying the action with respect to the metric, we obtain the equations of motion

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa^2\left(T^{(M)}_{\mu\nu} + T^{(Q)}_{\mu\nu}\right), \qquad (17)$$

where $T^{(M)}_{\mu\nu}$ is the stress-energy tensor of the non-quinton matter and radiation, and

$$\begin{aligned} T^{(Q)}_{\mu\nu} &= \partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}(\partial\phi)^2 \\ &\quad -\mathrm{e}^{-\lambda\phi}\left[\partial_\mu\sigma\,\partial_\nu\sigma - \frac{1}{2}g_{\mu\nu}(\partial\sigma)^2\right] - g_{\mu\nu}V \end{aligned} \qquad (18)$$

is the stress-energy tensor of the quinton.

Let us resort now to a special case of the spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) geometry

$$ds^2 = -dt^2 + a^2(t)d\boldsymbol{x}^2,\ \phi = \phi(t),\ \sigma = \sigma(t), \qquad (19)$$

for which equations of motion (14)-(17) reduce to the following system of ordinary differential equations:

$$H^2 = \frac{1}{3}\kappa^2(\rho_M + \rho_Q), \qquad (20)$$

$$\dot{H} = -\frac{1}{2}\kappa^2(\rho_M + P_M + \rho_Q + P_Q), \qquad (21)$$

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{2}\lambda\mathrm{e}^{-\lambda\phi}\dot{\sigma}^2 + \frac{\partial V}{\partial\phi} = \frac{\partial\mathcal{L}_M}{\partial\phi}, \qquad (22)$$

$$\ddot{\sigma} + 3H\dot{\sigma} - \lambda\dot{\phi}\dot{\sigma} - \mathrm{e}^{\lambda\phi}\frac{\partial V}{\partial\sigma} = -\mathrm{e}^{\lambda\phi}\frac{\partial\mathcal{L}_M}{\partial\sigma}, \qquad (23)$$

where the dot denotes a derivative with respect to time, densities ρ and pressures P are defined via diagonal components of their respective stress-energy tensors, such that

$$\begin{aligned} \rho_Q &= \frac{1}{2}\left(\dot{\phi}^2 - \mathrm{e}^{-\lambda\phi}\dot{\sigma}^2\right) + V \\ &= \frac{1}{2}\left(\dot{\phi}^2 - \mathrm{e}^{-\lambda\phi}\dot{\sigma}^2\right) + V_0\mathrm{e}^{-2\lambda\phi} + \Delta V, \end{aligned} \qquad (24)$$

$$\begin{aligned} P_Q &= \frac{1}{2}\left(\dot{\phi}^2 - \mathrm{e}^{-\lambda\phi}\dot{\sigma}^2\right) - V \\ &= \frac{1}{2}\left(\dot{\phi}^2 - \mathrm{e}^{-\lambda\phi}\dot{\sigma}^2\right) - V_0\mathrm{e}^{-2\lambda\phi} - \Delta V, \end{aligned} \qquad (25)$$

and $H = \dot{a}/a$ is the Hubble parameter per usual.

In this model, dark energy is attributed to the quinton:

$$\rho_{DE} = \rho_Q,\ P_{DE} = P_Q \qquad (26)$$

and its equation of state is given by

$$w_{DE} = \frac{P_{DE}}{\rho_{DE}} = \frac{\frac{1}{2}\left(\dot{\phi}^2 - \mathrm{e}^{-\lambda\phi}\dot{\sigma}^2\right) - V_0\mathrm{e}^{-2\lambda\phi} - \Delta V}{\frac{1}{2}\left(\dot{\phi}^2 - \mathrm{e}^{-\lambda\phi}\dot{\sigma}^2\right) + V_0\mathrm{e}^{-2\lambda\phi} + \Delta V}, \qquad (27)$$

in the perfect-fluid approximation $\partial P/\partial\rho \approx P/\rho$; an exact form of the equation of state $f(P_{DE},\, \rho_{DE}) = 0$ can be obtained by substituting the found solutions (19) into formulae (26) and eliminating the time variable from the resulting equations.

Further study of this model depends on specifying properties of the non-quinton matter in $\mathcal{L}_M$ and its interaction with quinton; which is an extensive topic on its own.

6. DISCUSSION AND CONCLUSIONS

In this report, we considered the laminar flow with a constant velocity of the physical vacuum modelled by logarithmic superfluid. We demonstrated that this three-dimensional non-relativistic quantum flow generates a four-dimensional Lorentz-symmetric "quinton" system, which consists of the dilaton and the quintom, a combination of the quintessence and tachyonic phantom fields; and explains a transition between them.

All three fields were shown to be projections of the Euclidean dynamical evolution of superfluid vacuum density and its fluctuations onto the measuring apparatus of a relativistic observer; their four-dimensional action functionals were not postulated but derived from a single quantum mechanical theory. This unified cosmological model describes the transition from the inflationary period in the early universe to the contemporary accelerating expansion of the universe, commonly referred to as the "dark energy" period.

It should be emphasized that the model (7), and even its "perturbed" generalization (12), is obviously the simplest possible one, because it neglects any large distortions of the laminar flow of the background superfluid (in the F-observer picture), which would otherwise induce and introduce additional fields and terms in a Lagrangian. Nevertheless, even such a simple kind of flow can already capable to resolve, in a unified way, at least three important problems in the modern theory of gravity and cosmology: the emergence of spacetimes with large-scale accelerated expansion leading to the occurrence of the inflationary period in the early universe, a generation mechanism for long-range scalar fields (which are not otherwise predicted by the Standard Model of particle physics), and the transition from the inflationary era to the current "dark energy" epoch.

One can also recall that the logarithmic model in the weak-gravity limit with inhomogeneous and rotationally-symmetric superfluid density (which is another limit of SVT different from a homogeneous laminar flow) quantitatively explains the non-Keplerian behaviour of rotating curves in galaxies: the fittings closely correspond with observational data, even for those galaxies whose rotation velocity profiles do not have flat asymptotics [14]. These effects are usually attributed to the phenomenon known as "dark matter".

To conclude, we showed that the dilaton-driven inflation and the effects attributed to "dark energy" can be viewed as different manifestations of the same object and a kind of matter, superfluid vacuum.

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APPENDIX A: CONFORMAL TRANSFORMATIONS ET CETERA

Here we present various formulae useful when dealing with the induced spacetimes occurring in the small-fluctuation (relativistic) limit of the logarithmic superfluid vacuum theory.

Conformal transformations. Let us transform the metric tensor by multiplying it with a function of coordinates $\Omega = \Omega(x)$:

$$d\bar{s}^2 = \Omega^2 ds^2, \ \bar{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \ \bar{g}^{\mu\nu} = \Omega^{-2} g^{\mu\nu}, \ \bar{g} \equiv \det(\bar{g}_{\mu\nu}) = \Omega^{2D} \det(g_{\mu\nu}), \tag{A1}$$

where D is the number of manifold dimensions. The Christoffel symbols, assuming the definition $\Gamma^\alpha_{\mu\nu} = \frac{1}{2} g^{\alpha\beta}(\partial_\mu g_{\nu\beta} + \partial_\nu g_{\mu\beta} - \partial_\beta g_{\mu\nu})$, transform as

$$\bar{\Gamma}^\alpha_{\mu\nu} = \Gamma^\alpha_{\mu\nu} + \Big(2\delta^\alpha_{(\mu} \nabla_{\nu)} - g_{\mu\nu} g^{\alpha\beta} \nabla_\beta \Big) \ln\Omega, \tag{A2}$$

the Ricci tensor, assuming the conventions $R_{\mu\nu} = R^\alpha_{\mu\alpha\nu}$ and $R^\alpha_{\mu\beta\nu} = \partial_\beta \Gamma^\alpha_{\mu\nu} - \partial_\nu \Gamma^\alpha_{\mu\beta} + \Gamma^\alpha_{\beta\sigma}\Gamma^\sigma_{\mu\nu} - \Gamma^\alpha_{\nu\sigma}\Gamma^\sigma_{\beta\mu}$, transforms as

$$\bar{R}_{\mu\nu} = R_{\mu\nu} - \Big(\tilde{D} \nabla_\mu \nabla_\nu + g_{\mu\nu} \Box \Big) \ln\Omega + \tilde{D} \nabla_\mu \ln\Omega \nabla_\nu \ln\Omega - \tilde{D} g_{\mu\nu} (\nabla \ln\Omega)^2, \tag{A3}$$

the scalar curvature transforms as

$$\bar{R} = \bar{g}^{\mu\nu} \bar{R}_{\mu\nu} \Omega^{-2} \Big[R - 2(\tilde{D}+1) \Box \ln\Omega - \tilde{D}(\tilde{D}+1)(\nabla \ln\Omega)^2 \Big], \tag{A4}$$

and the Einstein tensor transforms as

$$\bar{G}_{\mu\nu} = G_{\mu\nu} + \tilde{D} \left\{ \nabla_\mu \ln\Omega \nabla_\nu \ln\Omega - \nabla_\mu \nabla_\nu \ln\Omega + g_{\mu\nu} \left[\Box \ln\Omega + \frac{1}{2}(\tilde{D}-1)(\nabla \ln\Omega)^2 \right] \right\}, \tag{A5}$$

where $\tilde{D} = D - 2$.

Conformally flat spacetime. The metric tensor for the conformally flat pseudo-Riemannian manifold has the form

$$g_{\mu\nu} = \Omega^{-2}\eta_{\mu\nu}, \tag{A6}$$

where $\eta_{\mu\nu}$ is a metric of Minkowski spacetime and Ω is a function of coordinates.

To calculate the tensors necessary to write Einstein equations and the stress-energy tensor for such a metric, one can use the conformal transformation formulae above where assuming

$$\bar{g}_{\mu\nu} \equiv \eta_{\mu\nu}, \tag{A7}$$

hence $\bar{R}_{\mu\nu} = 0$ and $\bar{R} = 0$. We then immediately obtain the Ricci tensor

$$R_{\mu\nu} = \left(\tilde{D}\nabla_\mu\nabla_\nu + g_{\mu\nu}\Box\right)\ln\Omega - \tilde{D}\nabla_\mu \ln\Omega\nabla_\nu \ln\Omega + \tilde{D}g_{\mu\nu}(\nabla\ln\Omega)^2, \tag{A8}$$

scalar curvature

$$R = 2(\tilde{D}+1)\Box\ln\Omega + \tilde{D}(\tilde{D}+1)(\nabla\ln\Omega)^2, \tag{A9}$$

and the Einstein tensor

$$G_{\mu\nu} = \tilde{D}\left\{\nabla_\mu\nabla_\nu \ln\Omega - \nabla_\mu \ln\Omega\nabla_\nu \ln\Omega - g_{\mu\nu}\left[\Box\ln\Omega + \frac{1}{2}(\tilde{D}-1)(\nabla\ln\Omega)^2\right]\right\}, \tag{A10}$$

which is equal to the stress-energy tensor up to a multiplicative constant, cf. Equation (3).

The last formula can be interpreted as Einstein equations in which the function Ω is postulated at the beginning; then their solutions can always be written in the form (A7).

Scalar-tensor theories of gravity. Absent additional matter, their action is written in the form

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}A\left[R - B(\partial\Phi)^2 - 2W\right], \tag{A11}$$

where we use the shorthand notation $(\partial\Phi)^2 = g^{\mu\nu}\partial_\mu\Phi\,\partial_\nu\Phi$. Here $A = A(\Phi)$ and $B = B(\Phi)$ define, respectively, the nonminimal coupling between metric and scalar field Φ and the kinetic coupling, and $W = W(\Phi)$ describes self-interaction of the scalar field in the frame with the $g_{\mu\nu}$ metric.

Varying this action with respect to the metric tensor, one obtains Einstein equations [26]:

$$G_{\mu\nu} = \left(B + \frac{A''}{A}\right)\nabla_\mu\Phi\nabla_\nu\Phi - \frac{A'}{A}(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\Phi - g_{\mu\nu}\left[\left(\frac{B}{2} + \frac{A''}{A}\right)(\partial\Phi)^2 - W\right], \tag{A12}$$

whereas the variation with respect to Φ yields

$$\Box\Phi + \frac{A'}{2AB}R + \frac{1}{2}\left(\frac{A'}{A} + \frac{B'}{B}\right)(\partial\Phi)^2 - \frac{1}{B}\left(W' + \frac{A'}{A}W\right) = 0, \tag{A13}$$

where prime denotes a derivative with respect to Φ.

Furthermore, using the conformal transformation

$$\check{g}_{\mu\nu} = Ag_{\mu\nu}, \tag{A14}$$

the action (A11) can be written in the Einstein frame:

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-\check{g}}\left\{\check{R} - \left[B + \frac{3(A')^2}{2A^2}\right](\check{\partial}\Phi)^2 - 2\frac{W}{A}\right\}, \tag{A15}$$

where $(\check{\partial}\Phi)^2 = \check{g}^{\mu\nu}\partial_\mu\Phi\,\partial_\nu\Phi$ and other checked values are those computed with respect to the Einstein-frame metric $\check{g}_{\mu\nu}$; we also omitted terms which can be absorbed into a divergence and transformed into boundary terms.

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