

Do we need a non-perturbative theory of Bose-Einstein condensation?

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Abstract. We recall the experimental data of one-dimensional axial propagation of sound near the center of the Bose-Einstein condensate cloud, which used the optical dipole force method of a focused laser beam and rapid sequencing of nondestructive phase-contrast images. We reanalyze these data within the general quantum fluid framework but without model-specific theoretical assumptions; using the standard best fit techniques. We demonstrate that some of their features cannot be explained by means of the perturbative two-body approximation and Gross-Pitaevskii model, and conjecture possible solutions.

1. Introduction

In a seminal experimental work by Andrews *et al.* [1, 2] (where the second paper reported the corrected data), sound propagation was studied in a magnetically trapped dilute Bose-Einstein condensate, with localized excitations being induced by modifying the trapping potential by means of the optical dipole force of a focused laser beam. The measurements were done in the vicinity of the center of the Bose-Einstein condensate cloud where the axial density varied slowly. The resulting propagation of sound was observed using a rapid sequencing of nondestructive phase-contrast images, and the speed of sound was determined as a function of condensate peak density.

In order to explain their data, authors invoked the most popular model at that time, the Gross-Pitaevskii (GP) one, which was historically first proposed for describing the Bose-Einstein condensation phenomena [3, 4]. This model can be formulated as a quantum-mechanical fluid version of the quantum many-body model with the condensate particles' interaction potential being truncated at the two-body contact term, in spirit of the perturbation theory paradigm [5], resulting in the Gross-Pitaevskii (a.k.a. cubic Schrödinger) equation. This approach thus assumes that two-body interactions contribute predominantly when it comes to the Bose-Einstein condensation, whereas other terms can be safely neglected. This assumption seems robust for diluted condensates, such as clouds of cold alkali atoms, in which interparticle distances are sufficiently large compared to particles' thermal de Broglie wavelengths.

On the other hand, over the last few decades, perturbative approaches have shown their limitations, especially when it comes to strong coupling regimes, lower-dimensional systems or vacuum effects. Therefore, the comprehensive theory of quantum liquids, including Bose-Einstein condensates of cold gases, is far from its completion. For example, the question remains whether polynomial functions of condensate density in wave equations are exact, or they perhaps represent first terms of an infinite series expansion of some non-polynomial function. Another



related question is how to take into account non-perturbative quantum effects of physical vacuum, for they must be substantial in low-temperature systems. The continuously growing set of quantum liquid models, which go beyond the Gross-Pitaevskii approximation, currently includes models based on the nonlinear wave equations with higher-order polynomials of density [6, 7, 8] or even non-polynomial functions of density [9, 10, 11].

In this paper, we analyze experimental data of the work [1, 2] within the general quantum fluid framework but without imposing any specific model; instead, we will be using the standard best fit techniques. We demonstrate their features which cannot be explained in the perturbative two-body approach, and conjecture possible solutions.

2. The perturbative theory

In the Gross-Pitaevskii model, one starts with the quantum-mechanical interparticle potential of the two-body contact (Dirac delta singular) type [5]. Computations result in the occurrence of the wave equation of a cubic nonlinear Schrödinger type:

$$i\hbar\partial_t\Psi = \left[-\frac{1}{2}\hbar\mathcal{D}\nabla^2 + V(\mathbf{x}) - F_{\text{GP}}(|\Psi|^2) \right] \Psi, \quad (1)$$

$$F_{\text{GP}}(n) \equiv -U_0n, \quad (2)$$

where $\mathcal{D} = \hbar/m$, $V(\mathbf{x})$ being an external or trapping potential, and the parameter U_0 defines the strength of the two-body interaction, which can be expressed in terms of the scattering length a of atom-atom collisions:

$$U_0 = 4\pi\hbar^2a/m, \quad (3)$$

whereas

$$n = \rho/m = |\Psi|^2 \quad (4)$$

is the particle density, where $\Psi = \Psi(\mathbf{x}, t)$ being the condensate's function. This wavefunction is normalized:

$$\int_V |\Psi|^2 dV = \int_V n dV = N, \quad (5)$$

where N is a number of condensate particles of mass m .

Furthermore, using the Madelung ansatz the wave equation (1) can be rewritten in the hydrodynamic form, from which one can deduce the equation of state and related values. In the leading order approximation with respect to Planck constant, one can neglect second derivatives of density (which is robust as long as we do not consider shock waves and other fluctuations with a non-smooth density profile). Then the hydrodynamic equations acquire a perfect-fluid form, from which we obtain the equation of state $p = p(n)$ and speed of sound c , respectively:

$$p(n) \approx - \int n F'_{\text{GP}}(n) dn = n^2 U_0 / 2, \quad (6)$$

$$c(n) \approx \sqrt{-n F'_{\text{GP}}(n) / m} = \sqrt{n U_0 / m}, \quad (7)$$

where prime denotes an ordinary derivative.

3. Data analysis

The experimental setup of [1, 2] for creating Bose-Einstein condensates can be briefly described as follows.

Sodium atoms were optically cooled and transferred into a magnetic trap where they were further cooled by rf-induced evaporation. The thermal components of the condensate clouds of about 5×10^6 atoms in the $F = 1$, $m_F = -1$ ground state were removed by performing

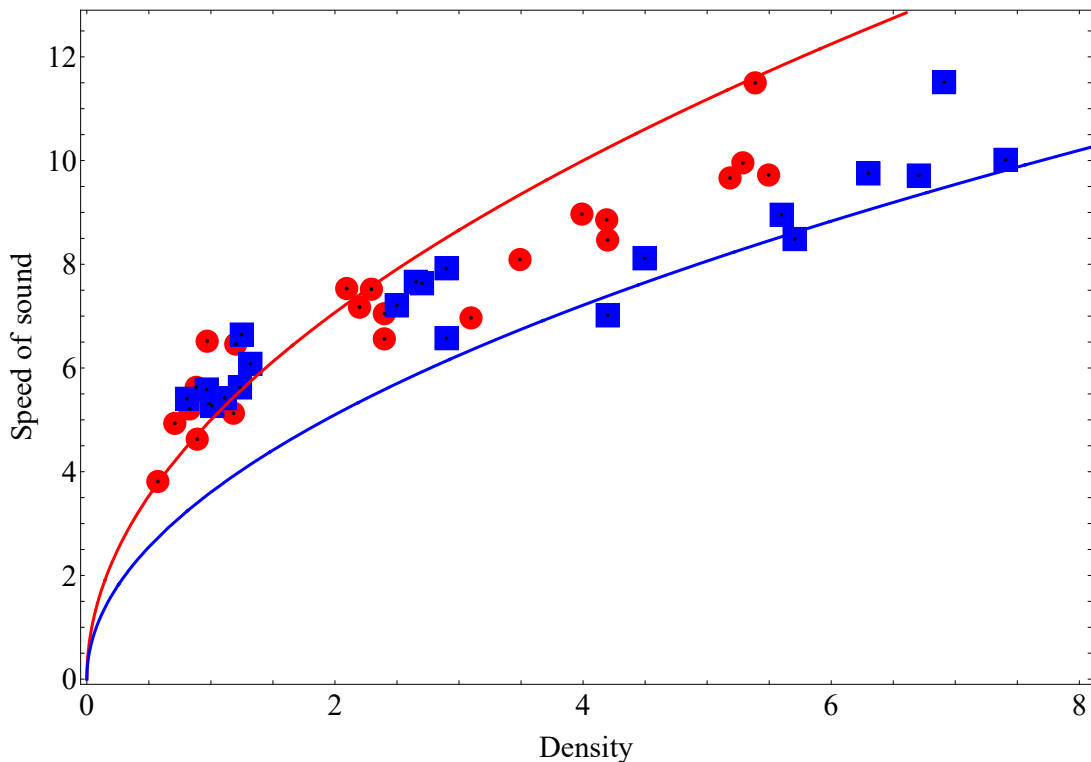


Figure 1. Speed of sound (in mm/s) versus condensate peak density (in 10^{14} cm^{-3}), for the data reported in ref. [1] (circles) and corrected in ref. [2] (squares). The curves are the corresponding theoretical predictions based on eq. (7), as drawn in the above-mentioned references.

the evaporative cooling significantly below the transition temperature. The condensates were confined to a cloverleaf magnetic trap, determined by the axial curvature of the magnetic field of up to 125 G cm^{-2} , radial gradient 120 G cm^{-1} , and bias field 1.5 G . The clouds were cigar-shaped, with the long axis horizontal, which was directly observed by nondestructive phase-contrast imaging [12].

Furthermore, localized density perturbations were generated by using the repulsive optical dipole force of a focused blue-detuned far-off-resonant laser beam, which was focused into the center of the trap. The $1/e$ half-widths were about $12 \mu\text{m}$ and $100 \mu\text{m}$, which created a light shift of about 70 nK per 1 mW of laser power. Localized density surges were created by abruptly switching on the laser beam after the condensate was formed. The repulsive optical dipole force expelled atoms from the center of the condensate, creating two density peaks propagating symmetrically outward.

Finally, the density dependence of the speed of sound was studied using adiabatically expanded condensates. The weakest trap was formed when the field curvature was reduced to 20 G/cm^2 and the bias field increased to 4 G . The critical temperatures in the strongest and weakest traps were estimated to be 1.6 and $0.4 \mu\text{K}$, respectively.

In the figure 1, we plot the results of measurement of speed of sound, along with theoretical prediction curves from the Gross-Pitaevskii model. At first look, the agreement seems good. However, the first issue immediately occurs: most of experimental dots fall quite far outside the curves: either in the higher range or lower range of density values, depending on whether one tries to fit the lower and higher density ranges, respectively.

But the most striking issue reveals when one plots not the velocity of sound but the velocity

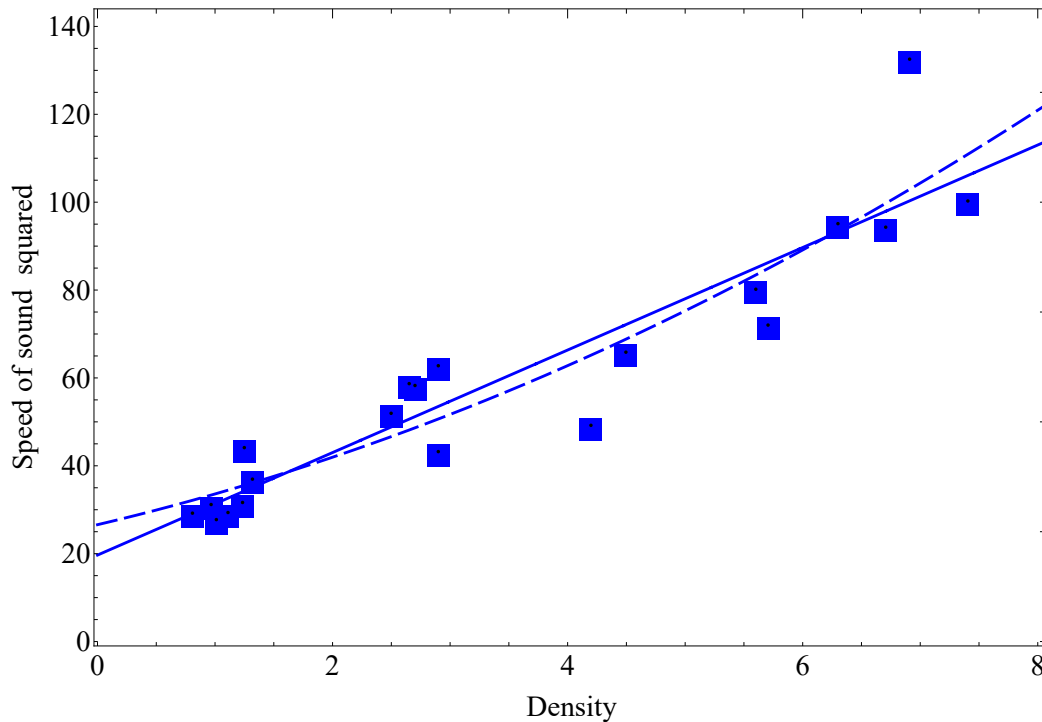


Figure 2. Speed of sound squared (in mm^2/s^2) versus condensate peak density (in 10^{14} cm^{-3}), for data [2]. The data are marked by squares, the curves are their best fits: linear (solid line) and quadratic (dashed curve).

squared for the data [2], cf. circles in figure 2. According to the Gross-Pitaevskii model, cf. eq. (7), the velocity squared must be proportional to density, $c^2 \propto n$. Instead, the data in figure 2 suggests that the velocity squared is a linear function of density, $c^2 \sim n$; and also that it approaches a non-zero value as $n \rightarrow 0$, according to the two-parameter best fit represented by a solid line in figure 2.

When considering a quadratic three-parameter best fit trial function, see a dashed curve in figure 2, the asymptotic behaviour at $n \rightarrow 0$ still persists. Besides, the quadratic-order parameter turns out to be one order of magnitude smaller than the linear and constant ones, which indicates that the linear and constant terms are predominant.

To summarize, we suggest that the best fit for data [2] is not the formula (7), but the function of the form:

$$c(n) = \bar{c} \sqrt{1 + knU_0/m + \mathcal{O}(n^2)}, \quad (8)$$

where \bar{c} and k are fitting parameters, and $\mathcal{O}(n^2)$ indicate terms of power n^2 and higher, to be disregarded for the system [1, 2]. One can check by analogy that similar pattern takes place for the sound propagation data from ref. [1].

While at first look all this seems to be just a slight deviation from the the Gross-Pitaevskii model, it has profound foundations and implications, as will be discussed below.

4. Discussion and conclusion

At second glance, formula (8) appears to be counter-intuitive and even somewhat unphysical, because the speed of sound does not vanish as the condensate density approaches zero. It is

however possible to explain it by the presence of physical vacuum - a quantum medium playing a role of non-removable background underlying the observable physical systems. The perturbative theory is unable to predict this behaviour, but another candidate model exists which has the desired features.

This model of quantum Bose liquid or condensate, defined by a wave equation with the non-polynomial (logarithmic) nonlinearity, was proposed in ref. [9] and the formalism was further developed in the works [10, 14, 15, 16]. More recently, it was shown that the logarithmic nonlinearity can be derived directly from quantum-informational and quantum-mechanical foundations of condensate-like systems, *i. e.*, the systems with the following main properties [11]: (i) they allow a hydrodynamic description in terms of the fluid wavefunction (cf. ref. [13]), and (ii) their characteristic interparticle potentials are substantially larger than kinetic energies. Our studies suggest that the logarithmic models is a key to understanding the nature of physical vacuum as a quantum matter; a profound relation between logarithmic models and quantum information entropy should be mentioned as well [10].

The application of the logarithmic fluid model to the Bose-Einstein condensates of alkali atoms specifically is an ongoing work which will be reported elsewhere.

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