THE EFFECTIVENESS OF COMPUTER-AIDED TEACHING ON THE QUALITY OF LEARNING GEOMETRIC CONCEPTS BY GRADE 7 LEARNERS AT A SELECTED PRIMARY SCHOOL IN KWAZULU-NATAL

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ABSTRACT

The emphasis of geometry is of such significance that the current National Curriculum Statement has included the learning of two-and three-dimensional shape from grade R. However it is observed that teachers rely on textbooks for their knowledge of a dynamic topic such as nets of solids. Learners do not have an opportunity to explore the different orientations of solid shapes. Not many teachers use technology to assist their pedagogy. Quality processes in mathematics education emphasise the use of technology in teaching and learning.

A study was performed using “Poly”, which is free open-source software for mathematics teaching and learning. This software was chosen because it was an easy to use application. It was able to show the different orientations of the solid shapes. Three-dimensional geometry can be explored using this software.

The research is based on a social constructivist view of learning and the methodology used is a case study. The Piagetian and van Hiele stages of development will be the basis of the researcher’s investigation. Piaget’s theory is based on age development whilst van Hiele alludes to the different stages of geometrical development. A control group was compared to the experimental group consisting of 20 learners each. The research was conducted in KwaZulu-Natal and involved a teacher with a class of 40 learners. Qualitative and quantitative data were collected and were analysed. The data consisted of classroom observations and learner questionnaires and interviews.

The findings of this study affirm that the use of technology in the teaching of geometry can enhance conceptual understanding. Classroom management breaks from routine while using Poly. Poly has the potential to improve learners' educational experiences; it can enable the effective application of constructive, cognitive and collaborative models of learning. Poly is not just a

mathematical tool but also a tool for thinking and helping to enhance learners’ learning. It can serve as a vehicle for helping learners to foster fundamental geometrical concepts. The assessment of the use of computers in mathematics by the learners of the experimental classes indicates that application of computers enables increasing the interest of the learners in mathematics and introduces more variety to the studies, making them more enjoyable and interesting.

DEDICATION

I dedicate this dissertation to my wife, Brenda Yegambaram, whose assistance, tolerance, patience, interest and support was instrumental in my completing this study.

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DECLARATION OF ORIGINALITY

I, Puvernentheran Yegambaram, declare that this dissertation on:

The effectiveness of computer-aided teaching on the quality of learning geometric concepts by grade seven learners at a selected primary school in KwaZulu-Natal

is my own work and that it has not been submitted for any degree at another Tertiary Institution.

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Statement by Supervisor

This dissertation is submitted with/ without my approval.

Signed ______________________

Professor R. Naidoo(PhD)

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List Of Acronyms And Abbreviations

APA- American Psychological Association

DoE- Department of Education

GDE- Gauteng Department of Education

HESA- Higher Education of South Africa

ICT – Information and Communications Technology

LAD- Language Acquisition Display

NAEP- National Assessment of Educational Progress

NCS- National Curriculum Statements

NCTM- National Council for Teachers of Mathematics

PC- Personal Computers

RNCS- Revised National Curriculum Statements

TI Interactive- Texas Instrument Interactive

TIMMS- The Third International Mathematics and Science Study Survey

TIMMS(R) - The Third International Mathematics and Science Study Repeat Survey

UNESCO - United Nations Educational, Scientific and Cultural Organisation

UNICEF - United Nations Children's Emergency Fund.

ZPD- Zone of Proximal Development

CHAPTER 1

1.1 Introduction to the Study

The emphasis on geometry is of such significance that the current Revised National Curriculum Statement has included the learning of two-and three-dimensional shapes from grade R. The motivation for this study arose out of interest in how geometry is being taught in primary schools. Since most research has been carried out in secondary schools, the researcher wanted to explore this in relation to the results in the primary school. Learning outcome 3 indicates that mastery of space and shape will be demonstrated by learners if they are able to describe and represent characteristics and relationships between two-dimensional shapes and three-dimensional objects in a variety of orientations, whilst also being able to analyse and explain properties of two-dimensional and three-dimensional shapes with justification. Learners are further required to investigate alternative definitions of various polygons.

Burger and Shaughnessy (1986:31) assert that many learners in the middle years of schooling have severe misconceptions concerning a number of important geometric ideas. De Villiers and Njisane (1987:117), in their studies of South African schools, indicate that grade 12 learners and high school learners in particular are still functioning more at concrete and visual levels than at an abstract level in geometry, despite the fact that the national school exit examination requires a clear understanding of underlying abstract processes. De Villiers (1997:16) notes that this and the transition from concrete to abstract levels of thinking poses “specific problems to second language speakers”. De Villiers (1997:16) also asserts that success in geometry involves the acquisition of technical terminology; hence there is little wonder that our learners perform so poorly. This view is supported by the cognitive theorists, Chomsky and Howard (1977:426), who assert that language is an important aspect in the learning arena.

The current classroom tends to resemble a one-person show with a captive, but often comatose, audience. Classes are usually driven by drill and kill, "teacher-talk" and depend heavily on textbooks. There is the idea that there is a fixed world of knowledge that the learner must come to know. Information is divided into parts and built into a whole concept. Educators serve as conduits of knowledge and seek to transfer their thoughts and meanings to the passive learner. There is little room for learner-initiated questions, independent thought or interaction between learners. The goal of the learner is to regurgitate the accepted explanation or methodology expostulated by the teacher (Adler and Davis, 2006a:272).

The findings of research studies support the use of computers in mathematics teaching and learning. Isiksal and Askar (2005:333) investigated the effect of spreadsheet and dynamic geometry software on mathematics achievement and mathematics self-efficacy. The results indicate that using technology effectively as a learning tool improves learners' mathematics achievement. Olkun, Altun and Smith (2005:317) found that learners who did not have computers at home initially had lower geometry scores. Therefore, Olkun *et al* (2005:325) suggest that in schools, it seems more effective to integrate mathematical content and technology in a manner that enables learners to do playful mathematical discoveries. Sinclair and Crespo (2006:438) found that using activities from a programme called the Geometer's Sketchpad, help learners notice geometric detail, explore relationships and develop reasoning skills related to geometric proof.

Lehrer, Jenkins and Osana (1998:137) conducted a long term instructional experiment to examine the effects of LOGO on children's mathematics learning. LOGO is a computer programming language used for functional programming. It emerged that learning of geometry appeared to be enhanced. Using the Shape Makers, a computer programme providing learners with shape-making objects that can be manipulated on the screen, Battista (2001:105) showed that using interactive geometry software can foster learners’ understanding and reasoning about two-dimensional shapes. Olkun (2003a:43) found that using computer-based tangram puzzles can effectively enhance learners’ two-dimensional geometrical learning. The effects produced by virtual objects are comparable to those of physical manipulation.

A growing body of research is developing around the idea that there is great potential for teaching with technology. Pea (1985:167), for example, draws on a Vygotskian view of learning by stating that learning and reasoning should now be considered the activity of a system which involves minds, social contents and tools such as computers. It has been argued by Geiger (2005:246) that productive social interaction in mathematics classrooms can be mediated by technology. As the renowned mathematician Sir Michael Atiyah (2001:50) writes: “spatial intuition or spatial perception is an enormously powerful tool and that is why geometry is actually such a powerful part of mathematics - not only for things that are obviously geometrical, but even for things that are not. We try to put them into geometrical form because that enables us to use our intuition. Our intuition is our most powerful tool.”

1.2 Background and Purpose of Study

Much has been written about the poor performance in mathematics in South African schools (Roux, 2003:362). Generally, high school learners’ mathematical performance in South Africa appears to be unimpressive, but it is even more so in geometry. According to Roux (2003:362), “learners’ performance is even poorer when it comes to items involving understanding of features and properties of shapes.” Results from both national and international surveys of mathematical performance indicate that many “secondary learners cannot identify and name shapes like kite, rhombus, trapezium, parallelogram and triangle” (Roux, 2003:362).

According to Howie (2003:18), several studies have reported a number of shortcomings in the teaching and learning of mathematics and science in South Africa. One such study is the Third International Mathematics and Science Study (HSRC, 2005:8) conducted in 1995, in which South Africa participated with 41 other countries. Here it was reported that South African learners were placed last with a mean score of 351. This mean was significantly lower than the international benchmark of 513. Less than two percent of these learners reached or exceeded the international mean score (Beaton, Mullis, Martin, Gonzalez, Kelly and Smith:1997:25). The Third International Mathematics and Science Study Repeat Survey (TIMSS-R, 2003:25) conducted in 1999 revealed that grade eight learners once again performed poorly. Their mean score of 275 was significantly below the international mean of 487. It was also found that the South African mean of 275 was lower than that of Morocco, Tunisia and other developing countries such as Chile, Indonesia, Malaysia and the Philippines (Howie, 2004:155). TIMMS-R (2003:7) similarly indicated no improvement by South African mathematics and science learners (Reddy, Kanjee, Diedericks and Winnaar , 2007:13).

The systemic evaluation by the Department of Education (DoE, 2002a:8), targeting grade four learners, indicated that learners only obtained an average of 30 percent for numeracy. Another study was conducted by the Monitoring Learner Achievement project organised by United Nations Educational, Scientific and Cultural Organisation (UNESCO) and United Nations Children's Emergency Fund (UNICEF). The Monitoring Learner Achievement’s objectives were to continuously monitor the quality of basic educational programmes and assess learning outcomes (UNESCO, 2005:123). In this project, grade four learners from a number of African countries were assessed against a set of internationally defined numeracy and literacy learning competencies. Findings from countries including Tunisia, Mauritius, Malawi, Zambia and Senegal indicated that South African learners ranked fourth with an average literacy score of 48 percent and rated last with respect to numeracy, scoring 30 percent (DoE, 2001a:8).

More than 50 percent of all learners who wrote the matriculation mathematics in 2008 failed the examination (Paper delivered at the 3rd Annual Education Conference in Southern Africa – 4 March 2009). Some 13 000 students across seven campuses were tested in the five-year project commissioned by Higher Education South Africa (HESA) after universities began to express concern at extremely high failure rates. A staggeringly low seven percent of the students who wrote the mathematics tests were found to be proficient. About 73 percent had intermediate skills. The rest, 20 percent, had only the most basic skills and would need long-term consistent attention. (Extract from article by Janet Smith *in Saturday Star* 15 August 2009). Of the nearly 300 000 pupils who wrote mathematics, only 30 percent managed to get more than 40 percent in the final matriculation examination. (South African Institute of Race Relations website:23)

It has been reported that grade 12 learners' performance in school geometry has also been inadequate. Examiners Reports (House of Delegates and Gauteng Department of Education (GDE) 1995, 2001, 2002, 2003, as well as the Mathematics, Science and Technology Report 2003) all comment on learners' poor performance in Euclidean Geometry**.** The inability of learners to conceptualise geometric concepts begins at primary school. Intervention is required to eradicate this problem. The researcher believes that teaching geometry with technology will assist learners to gain a better insight to three - dimensional shapes. In education, the primary focus is improving the instructional processes related to teaching and learning so that quality in education improves. In 1997, the Geometry Working Group of a South African Mathematics Non-Governmental Organisation attempted to re-conceptualise the teaching and learning of Geometry (Bennie, 1998b: 2).

A comparison of the assessment standards of the South African mathematics curriculum with van Hiele descriptors suggests that in terms of geometry South African learners who have completed primary school should have reached van Hiele's thinking level two; that is, they should be "able to describe and represent the characteristics and relationships between two-dimensional shapes and three-dimensional objects in a variety of orientations" (Department of Education, 2002a: 6). The TIMMS-R (2003:8) report showed that the learners from South Africa performed extremely poorly in geometry. Evidence from matriculation results of past examinations bears testimony to the fact that our learners do not perform well in geometry because most of the learners are not exposed to geometry (Howie, 2001: 18).

With the introduction of information technology in South African primary schools, it has become necessary to explore avenues to maximise the use of computer-assisted learning. The use of manipulatives as part of mathematics lessons has long been advocated as part of a comprehensive mathematics learning experience. Recent developments such as virtual manipulatives, along with research, have caused some to question the role that manipulatives play in learning mathematics. There exists an abundance of research into the effectiveness of this learning medium and the manner in which it can be implemented (Karuppan, 2001:187). An area that can assist learners and improve their visualisation skills is computer-aided instruction. Most public primary schools are bound by financial constraints and the disadvantaged socio-economic circumstances of learners. The International Society of Technology in Education members have monitored research on the effectiveness of education technology on learner outcomes for more than 20 years and one convincing trend has emerged: when implemented appropriately, the integration of technology into instruction has a strong, positive impact on learner achievement.

Computer-assisted teaching and learning has come to occupy a significant role in classrooms around the world, with positive learning outcomes being reported by various researchers (Sivin-Kachela, 1996:29). This is especially true in relation to the use of this technology to teach mathematics (Papert, 1990:45). Research into mathematics classrooms shows that computer technology can support problem-solving skills (Fey and Hirsch,1992:240); decrease the amount of time required to master skills, allowing for more time to be spent on developing conceptual understanding (Wagner and Parker, 1993:119) and facilitate the development of deeper understanding of algebraic ideas (Kaput, 1989:7).

There is a tendency in current thinking to embrace a broader view of geometry. The post-1994 curriculum for mathematics in South Africa, sees space and shape within the context of social experiences. One of the specific outcomes of the mathematics curriculum suggests that learners need to be able to “describe and represent experiences with shape, space, time and motion, using all available senses” (DoE, 1997d:3). This apparent shift is consistent with a global epistemological paradigm move towards recognising that cognition is an active and complex process of social interaction.

As South Africa currently faces a crisis in mathematics education, which has seen it placed last in the TIMMS-R (2003:28), this research will re-affirm that computers are indeed able to impact positively on the mathematical performance of the learners. The assumption underlying the implementation of computer-based technology into schools in South Africa is that the technology will help to develop autonomous learners, who are both mathematically and technologically literate (DoE, 1996:10). Indeed it would appear, given the right circumstances, that the computer can facilitate the development of autonomous learners capable of exploring their world and constructing knowledge. While policy documents (DoE, 2002a:7) point to the desired outcomes of the current curriculum (such as qualified, competent, dedicated, caring , confident , independent, literate, numerate, multi-skilled, compassionate educators and lifelong learners who are active citizens) ,there is little indication about what must be transformed in order to meet these outcomes. One thing seems clear and that is the need to develop mathematically literate learners who are capable of engaging in the global marketplace. The use of computers as a teaching and learning tool offers, at least, the hope of meeting this outcome (Hardman, 2004:258).

In a study entitled “The impact of Information and Communications Technology (ICT) in schools: a landscape review”, Condie (2007:5) gave a detailed analysis of the benefits of the use of ICT in primary and secondary education. With the support of empirical data, the study presented many positive outcomes of ICT-enhanced education, among which are:

- the motivation and engagement of learners
- independent learning and autonomy of learners
- the development of core skills such as collaborative learning and communication among learners

These are visible effects of deploying technologies in education which can contribute to better educational outcomes. Specifically, ICTs are found to have improved learners’ attainment in different subject areas. Such improvement has been observed in core subjects such as English language, mathematics and science (Condie, 2007:4). As Haddad and Jurich (2002:25) explain, these technologies make it possible for educators and learners to master educational materials by “endlessly going over the same material in a variety of forms and media, layering in additional information and nuances of understanding while re-enforcing the learning objectives”. Furthermore, technologies enable more interaction and collaboration among educators and learners who may be separated in both time and space.

According to van Hiele’s model (Mayberry, 1981:59), educators should take cognisance of the level at which their learners are functioning, so that the gap between educators’ dissemination of knowledge and learners knowledge can be narrowed. Educators need to understand and be able to use technology to improve their pedagogy (Robinson*,* Robinson and Maceli, 2000:123). One needs to be mindful that technology affects what learners learn and how learning is accomplished.

At present, geometry education in South Africa is in a state of turmoil (Mogari, 1998:52). For example, de Villiers (1997:42) states that in South Africa, “it is well known that on the average, pupils’ performance in matriculation geometry is far worse than in algebra.” When describing what they see as the impoverished nature of school geometry at the primary level in the United States of America, Battista and Clements (1988:11) note that the poor performance of primary school learners in geometry “is due, in part, to the current elementary school geometry curriculum, which focuses on recognising and naming geometric shapes and learning to write the proper symbolism for simple geometric concepts.”

In KwaZulu-Natal and the Eastern Cape, for example, research indicates that the majority of high school learners have a poor understanding of many geometric concepts (de Villiers, 1997:15). A large number of learners perform poorly in Euclidean Geometry (Mogari, 1998:52). National as well as international research has shown that the majority of learners in schools tend to have a backlog in their intuitive understanding of space in comparison with their intuitive number knowledge (van Niekerk, 1999:70). Learners have a poor conceptual understanding in the higher levels of study when a deeper knowledge of geometric concepts is expected. This view is corroborated by Pegg and Davey (1998:110) who are of the opinion that "…there is increasing evidence that many learners in the middle years of schooling have severe misconceptions concerning a number of important geometric ideas".

Providing immediate graphing and calculation devices to learners will benefit mathematics teaching and learning (Papert, 1996:23; Hoyles and Jones, 1998:121). Wright, Thompson, Ganis, Newcombe and Kosslyn, (2008:763) assert that Information and Communications Technology, particularly mathematical software, helps to provide better visual and dynamic representations of abstract ideas and the links between symbols, variables and graphs. Oldknow (2005:174) gives an account of an innovative project in the application of Information and Communications Technology in the mathematics classroom in 2000 and 2001.The ‘Mathematics Alive’ project focused on the use of Information and Communications Technology in twenty Year 7 (11-12 year-olds) mathematics classes in England. This project developed the hardware (Personal Computers and interactive whiteboards) and software (The Geometer’s Sketchpad, Texas Instrument InterActive!) infrastructure necessary to integrate Information and Communications Technology into the mathematics classrooms.

Oldknow (2005: 182-3) refers to the indicators of success of this pilot project as follows:

• Educators have been extremely positive about the values and usefulness of the resources throughout the project and have wanted to continue with the project beyond the period of the pilot;

• Educators have felt that the resources and the training offered have enabled them to implement the objectives and needs of the National Numeracy Strategy, using technology to support their teaching;

• Educators have felt that the technology has added to their teaching strategies and approaches;

• Learners have reported positively throughout the period of the project on the value of the resources and the impact it has had on their learning and on their positive attitudes towards mathematics;

• Resources have been shown in practice to support both teaching and learning.

In a study of grade eight Princeton, New Jersey learners who used simulation and higher order thinking software, the results showed gains in mathematics scores of up to 15 weeks above grade level as measured by the National

Assessment of Educational Progress (NAEP) (Wenglinsky, 1998:4) showed mathematics achievement gains that were significantly greater than the control groups who did not use the computer. The new curriculum has set the stage for a move in South African pedagogy from a predominantly transmission model of learning, where skilled educators view learners as having limited knowledge and easily filled with knowledge, to an understanding of learning that appreciates that learners are much more actively involved in constructing knowledge. In their search for alternative models to explain learning, many researchers have turned their attention to Vygotsky's (1978:4) notion of mediation, where a more competent peer or adult is viewed as assisting performance, bridging the gap between what the learners know and can do and what the learners need to know. Vygotsky (1978:4) conceptualises this gap between unassisted and assisted performance as the zone of proximal development (ZPD), that 'space' where learning leads to development (Vygotsky,1978:4). Crucially, for Vygotsky (1978:4) and indeed for all activity theorists who have followed in his footsteps, human consciousness is social.

1.3 Research Aim

The aim of this study is to evaluate the effectiveness of computer-aided teaching on the quality of learning geometric concepts in mathematics by grade seven learners.

1.3.1 To identify the shortcomings of the traditional method of teaching geometry at primary schools and to recommend a strategy that may improve the quality of teaching and learning.

1.3.2 To investigate how learners' understanding of three-dimensional shapes can be improved through the use of computer programmes and thus improve the quality of primary school mathematics teaching programmes.

1.4 Research Questions

For this study, the following questions have been identified as focal:

- Can computer-generated instruction enhance the conceptual understanding of primary school learners?
- To what extent are educators willing to adopt a modified teaching strategy in pursuit of improved learner achievement?

1.4.1 Research Objectives

- To investigate whether learners' mathematical skills can be improved through the use of computer-supported materials;
- To explore and describe learners' experiences and perceptions of computer-supported learning;
- To explore and describe learners' experiences of their use of computers and to get a better understanding of three-dimensional shapes. These experiences may help in the management of a computer mathematics laboratory.

1.4.2 Hypothesis

It is hypothesised that, if computer-assisted teaching and learning take place, then the learners will be able to improve their understanding of two-and three-dimensional geometrical concepts.

This study, in addition, tried to answer the following question: What are the qualitative and quantitative differences in learners' understanding of two-and three-dimensional geometry concepts between traditional and computer-aided teaching?

1.5 Problem Statement

The problem to be investigated is the effectiveness of computer-aided teaching on the quality of learning two-and three-dimensional geometrical concepts. The reason for two-and three-dimensional geometry being chosen is that educators often find learners experiencing difficulty with these concepts at primary school level. The research was based on the idea that a clear understanding of the relationship between two-and three-dimensional geometry has proven to be an invaluable aid for future mathematics learning. Currently, learners have little exposure to hands-on methods for understanding these relationships. Textbooks are two-dimensional representations of three-dimensional concepts. This may be due to poor visualisation techniques, confusing nets and not understanding two-and three-dimensional shapes. Technology facilitates the observance of pattern and relationships. It creates a virtual environment for exploration and conjecturing via simulations.

Many educators teach three-dimensional shapes using available resources namely the text book and the chalkboard. Worksheets of three-dimensional shapes are displayed as a two-dimensional shape. The apparent lack of depth of teaching three-dimensional shapes has a lasting impression on the minds of the learners. The learner's perception of three-dimensional shapes is that of it being a two-dimensional shape. The approach of the teacher in teaching three-dimensional figures using two-dimensional media has a negative impact on the geometrical understanding of learners in the later grades.

1.6 Structure of the study

This study was structured so as to inform practising educators about the difficulties learners face in understanding three-dimensional shapes and how technology will enhance their conceptual understanding of space and shape. The study proposes the use of technology that will benefit learners and educators at primary schools through the teaching and learning of three-dimensional shapes, using a free software Poly.

Chapter 1 has highlighted the background and purpose of the study, conceptual and theoretical location of the study, the research questions and the research method employed in the study.

Chapter 2 reviews pertinent literature and provides a theoretical and conceptual framework for the study.

Chapter 3 provides an account of how the study was designed and conducted. It describes the research method employed, the research instruments, ethical issues, data analysis and limitation of the study.

Chapter 4 presents and discusses the findings acquired from the questionnaires, activities on nets of solids and focus group interview.

Chapter 5 summarises the main findings of the study. This chapter also outlines recommendations based on the findings and theories discussed in chapter two. Finally, recommendations for future research are considered.

1.7 Conclusion

This chapter provided the introduction to the study. The background, rationale, aims and objectives of the study have been set forth.

The next chapter will present the literature review that forms the basis of the study. The theoretical and conceptual framework regarding technology in mathematics education will be discussed in detail. Various theories and approaches relating to mathematical thinking will be reviewed.

CHAPTER TWO : LITERATURE REVIEW

2.1. Introduction

Van Hiele's, Piaget's, Constructivism and Skemp's theories have been used to expound the teaching of geometry in this study. Of the range of theoretical work concerned with geometrical ideas, that of Piaget (1970:25) and van Hiele (1986:16) are probably the most well known. Other researchers have studied, analysed and validated the levels of geometric thought proposed by Pierre and Dina van Hiele (Burger and Shaughnessy, 1986:38). However, it is suggested that within this hierarchical structure of levels, not all people use a single level of reasoning at one time. Sometimes, several levels are evident at the same time. The notion was reported that people do not behave in a simple linear manner, which the van Hiele (1986:16) model of geometric thinking suggests.

Significantly, there is research evidence indicating that the South African national curriculum prescriptions for geometry in the intermediate phase are consistent with the van Hiele theory (Feza and Webb, 2005:38). van Hiele (1986:16) hypothesised five sequential levels of geometric reasoning. They are visualisation, analysis, informal deduction, formal deduction and rigour (Burger and Shaughnessy, 1986:39). A major purpose for distinguishing learners' levels of understanding is to recognise obstacles that they may experience in the learning process and to allow educators to develop strategies which will enable learners to progress in terms of conceptual development (Bishop, 1997:50). van Hiele (1986:56) stated that "the transition from one level to the following is not a natural process; it takes place under influence of a teaching-learning program."

With respect to the hierarchical levels of geometric thinking proposed by Piaget and van Hiele, this study found that the learner can simultaneously construct geometric meaning when given constructivist opportunities for collaborative engagement, discourse and reflection. This finding is supported by Clements and Battista (1992:430) who claimed that “it may be that the learner does not construct first topological and later projective and Euclidean ideas. Rather, it may be that ideas of all types develop over time, becoming increasingly integrated and synthesised”. Space and shape involve connections with various other areas of mathematics. An understanding of measurement, proportional reasoning, algebra and integers, among others, is necessary to develop an understanding of space and shape. Van de Walle (2004:347) defines spatial sense as an intuition about shapes and the relationships among shapes. He argues that although 'a feel' for geometric aspects is implied in the definition, experiences with space and shape can develop spatial sense. This belief is consistent with research which states that understanding is built in geometry across the grades, from informal to more formal thinking (NCTM, 2000:41).

Children tend to move through different levels in thinking as they learn about geometric shapes (Clements and Battista, 1992:425; van Hiele, 1986:59). They have an innate, implicit ability to recognise and match shapes. However, at the earliest, pre-recognition level, they are not explicitly able to reliably distinguish circles, triangles and squares from other shapes. Children at this level are just starting to form unconscious visual schemes for the shapes, drawing on some basic competencies.

At the next level, children think visually or holistically about shapes and have formed schemes, or mental patterns, for shape categories. When first built, such schemes are holistic, unanalysed and visual. At this visual/holistic step, children can recognise shapes as wholes but may have difficulty forming separate mental images that are not supported by perceptual input. A given figure is a rectangle, for example, because it looks like a door. They do not think about shapes in terms of their attributes, or properties. Children at this level of geometric thinking can construct shapes from parts, but they have difficulty integrating those parts into a coherent whole. Next, children learn to describe and then analyse geometric figures.

The culmination of learning at this descriptive/analytic level is the ability to recognise and characterise shapes by their properties. Initially, they learn about the parts of shapes for example, the boundaries of two-and three-dimensional shapes and how to combine them to create geometric shapes. For example, they may explicitly understand that a closed shape with three straight sides is a triangle. In this sense, the data in this study suggested that learners' thinking occurred across levels.

2.2 Epistemology

Epistemology refers to the theory of knowledge embedded in the theoretical perspective. This study is based on a social constructivist view of learning: pupils learn mathematics through active construction of their own knowledge and this can be facilitated in a computer environment through the interactive process of conjecture, feedback, critical thinking, discovery and collaboration. Constructivism is considered to be the socially collective generation and construction of meanings rather than a meaning-making activity of the individual mind.

The research was based on a social constructivist nature of knowledge in which, "The meanings are negotiated socially and historically. In other words, they are not simply imprinted on individuals but are formed through interaction with others (hence social constructivism) and through historical and cultural norms that operate in individuals' lives" (Creswell, 2003:8). Social constructivism claims that rather than being transmitted, knowledge is created or constructed by each learner; there is no knowledge independent of the meaning attributed to experience constructed by the learner.

According to certain cognitive theories, learning does not involve a passive reception of information; instead, the learning process can be regarded as an active construction of knowledge in a learner centred instruction. Constructivism claims that learners cannot be given knowledge; learners learn best when they discover things, build their own theories and try them out rather than when they are simply told or instructed. Vygotsky (1962:82-83) argues that ‘‘Direct teaching of concepts is impossible and fruitless; a teacher who tries to do this accomplishes nothing but empty verbalism, a parrot-like repetition of words by the child, simulating a knowledge of the corresponding concepts but actually covering up a vacuum’’.

By participating in social constructivism activity, learners have the opportunity not only to learn mathematical skills and procedures, but also to explain and justify their own thinking and discuss their observations. From a social constructivist perspective, computer-assisted instruction offers educators a powerful pedagogical tool-kit. In mathematics lessons involving computers, learning is achieved through social interaction for three reasons: the social nature of mathematics; the collaboration that computer-based activities include and the basis for viewing the computer as one of the partners of the discourse.

2.3 Theoretical perspective

The theoretical perspective refers to the philosophical stance informing the methodology, providing a context for the process followed and justifying its logic. Recognising the fact that there are multiple socially constructed realities, this study adopts the interpretive paradigm and, more specifically symbolic inter-actionism as the primary theoretical perspective. The interpretive paradigm is to understand “the subjective world of human experience” (Cohen , Manion and Morrison, 2007:21).

The interpretivist approach looks for “culturally derived and historically situated interpretations of the social life” (Creswell, 2007:21), while symbolic inter-actionism “explores the understanding and meanings in culture as the meaningful matrix that guides our lives” (Crotty, 1998:71). This is directly linked to the purpose of the research: to get inside the classroom and see how the computer software could be used as an aid in learners' understanding of mathematics.

2.4 Computers in the classroom

Prior to the advent of the computer, a common philosophy of teaching was based on what the French call the "didactic triangle" between the pupil, the teacher and the mathematics (figure 1).This model indicates that the learner interacts with the teacher and the mathematical content knowledge. The teacher interacts with the learner and the mathematical content knowledge.

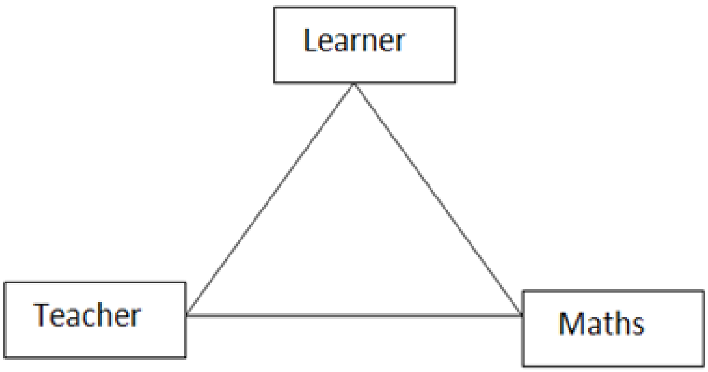


Figure 1: The Didactic Triangle Model (Kansanen and Meri, 1999:107)

The mathematics is part of a shared knowledge system, shared by those who have already learnt to understand it. The representative of this culture in the classroom is the teacher. The mathematics is in the mind of the teacher and the only externalised physical representations are usually in a text book. Here, the mathematics is static in fixed words and pictures. The only dynamic representation is through the verbal explanation of the teacher and any diagrams that may be drawn. The introduction of the computer brings a new dimension into the learning situation. There are now four major components, which may be viewed as forming a tetrahedron in a suitable educational context (figure 2).In this model, we are introduced to technology as a significant role-player. The pedagogy needs to be informed about this interaction. The learner now has to interact with the teacher, the mathematical knowledge and the computer inputs.

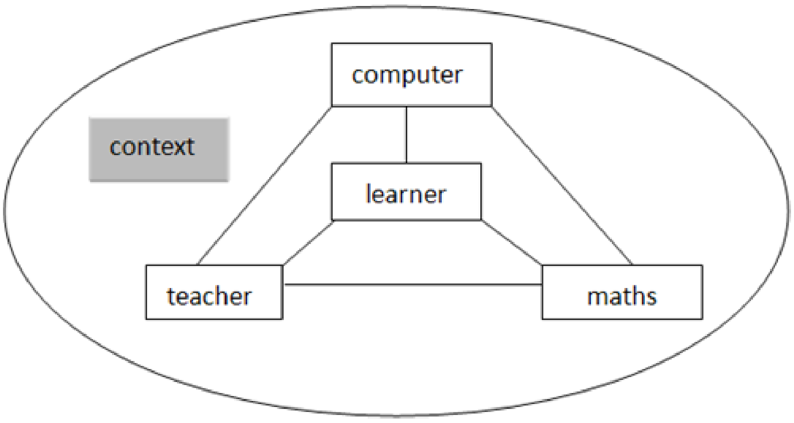


Figure 2: The Didactic Tetrahedron Model by Tall and Thomas (1986:60)

It is assumed that the computer has appropriate software available to represent the mathematics and that this software is designed in a manner that makes the mathematics as explicit as possible. It must show the processes of the mathematics as well as giving the final results of any calculation.

2.4.1 Computer-enhanced learning: Guiding Perspectives

Current literature on the role of technology in education shows that computers are seen as a particular cognitive technology that is media that help transcend the limitations of the mind in thinking, learning and problem-solving activities (Pea, 1987:91). In the specific case of mathematics, they act not only as amplifiers of the intellect; but are also re-organisers of thinking processes (Pea, 1987:93*).* Following this approach on the use of technology in the mathematics classroom, Poly might be considered as a support for learners to transcend cognitive limitations and construct a new relation to knowledge. Poly is a freeware programme for investigating polyhedral shapes. It can display polyhedral shapes in three main ways: as a three-dimensional image, as a flattened two-dimensional net and as a topological embedding in the plane. The three-dimensional images may be interactively rotated and folded/unfolded. Physical models may be produced by printing out the flattened two-dimensional net, cutting around its perimeter, folding along the edges and finally attaching together neighbouring faces.

2.5 Constructivism

Research study by Glaserfeld (1992:169) supports a social constructivist approach to teaching and learning, and suggests that children can negotiate meaning for complex systems of geometric concepts in a culture that values curiosity, wonder, exploration, reflection, provocation and conversation. Educators can provide opportunities for young children to perceive structure, form and space through first-hand observation and action (Davis, 1990:31). This research study demonstrates that educators can help children make sense of complex mathematical concepts and ideas by providing rich environments, encouraging conversation, presenting interesting provocations and materials to embrace and explore. Collaborative activity (Glaserfeld, 1992:173) in which children question each other and themselves, debate their ideas and try to understand different points of view were helpful as the children negotiated meaning for complex geometric representations to construct their maps.

Fosnot (1996: ix) refers to constructivism as “a theory about knowledge and learning.” According to the constructivist theory, knowledge is “temporary, developmental, non-objective, internally constructed and socially and culturally mediated”. Fosnot (1996:32) presents learning as “a self-regulatory process of struggling with the conflict between existing personal models of the world and discrepant new insights”. Learners construct new models which are refined through “cooperative social activity, discourse, and debate” (Fosnot 1996: ix). Such a theory of knowledge and learning has significant implications for teaching. It changes the dynamics of the traditional classroom by empowering the learner as the focus and architect of the learning process while redefining the role of the instructor to be a guide and helper rather than the source and conduit of knowledge.

A critical factor that relates to the learning process and construction of knowledge is socio-cultural development. Constructivists view socio-cultural development as one of the significant factors that contribute to the construction of knowledge. Vygotsky (1978:67) states that cognitive development is dependent on social interaction and that cultural development has two levels: social and interpersonal. During social interaction, learners recognise the new knowledge and then internalise it. For effective learning, learners have to cooperate in an environment where social interaction is taken into account (Bonk and Reynolds, 1997:172).

There is substantial literature in which relations between factors of spatial ability, such as visualisation, mental imagery and mathematical performance have been investigated (Bishop, 1997:175; Presmeg, 2006:34). Though there are some differences in the literature, the importance of spatial ability to the development of mathematical thinking is supported by many researchers (Bishop, 1997:176; Tartre, 1990:45; Gutiérrez, 1996:3). Moursund (2005:8) created a mathematics cognitive development scale which represents his current insights into a six-level Piagetian-type model.

What are the underpinnings for a constructivist learning setting and how do they differ from a classroom based on the traditional model? Although the literature expounds many types of constructivism (Ernest, 1995:459), all embrace the basic principle that "learning is not a passive receiving of ready-made knowledge but a process of construction in which the learners have to be the primary actors" (Glasersfeld, 1992:180). Rather than passively receiving and recording information, the learner actively interprets and imposes meaning through the lenses of his or her existing knowledge structures.

Constructivists believe that learning involves the generation of knowledge and learning strategies. According to this view, learning in schools has to emphasise the use of intentional processes which learners can use to construct meaning from information, experiences and their own thoughts and beliefs. Glasersfeld (1992:174) argues that, “From the constructivist perspective, learning is not a stimulus-response phenomenon. It requires self-regulation and the building of conceptual structures through reflection and abstraction”.

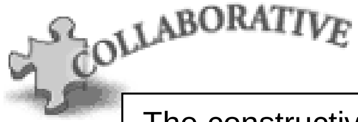
The central point of constructivism is that learning involves more than just the transfer of information from the educator to a learner; instead, each learner plays an active role in working with and integrating the information according to his or her own background or experience. This integration involves applying personal study and learning skills and monitoring one’s own comprehension (Gordon, 1996:47). Therefore, to construct or reconstruct his or her knowledge, the learner needs to employ certain techniques regarding managing his or her thinking such as thinking about thinking, planning, monitoring and evaluation. In other words, for learners to reach the equilibrium case and then assimilate or accommodate their knowledge, they should employ various meta-cognitive strategies. Meta-cognitive strategies are techniques that learners use to plan, monitor, control and evaluate their own cognitive processes.

In a constructivist classroom, learning is . . .

One may look at constructivism as a spiral. When learners continuously reflect on their experiences, they find their ideas gaining in complexity and power. They develop increasingly strong abilities to integrate new information. One of the teacher's main roles is to encourage this learning and reflect on this process. Learners are not blank slates upon which knowledge is etched. They come to learning situations with already formulated knowledge, ideas and understandings. This previous knowledge is the raw material for the new knowledge they will create (Figure 3).

The learner is the person who creates new understanding for him/herself. The teacher coaches, moderates and suggests, but allows the learners room to experiment, ask questions, tries things that don't work. Learning activities require the learners' full participation.

Learners control their own learning process, and they lead the way by reflecting on their experiences. This process makes them experts of their own learning among learners.



The constructivist classroom relies heavily on collaboration among learners.

The main activity in a constructivist classroom is solving problems. Learners use inquiry methods to ask questions, investigate a topic, and use a variety of resources to find solutions and answers.

Learners have ideas that they may later see were invalid, incorrect, or insufficient to explain new experiences. These ideas are temporary steps in the integration of knowledge.

Figure 3: A constructivist classroom (adapted from Glasersfeld: 1990:14)

The psychological theoretical base for constructivism comes from Piaget. He uses the term schemata to describe mental or cognitive structures that allow one to think about, organise and make sense of experiences (Borich and Tombari, 1997:5). The individual continuously constructs his or her schemata. Cognitive development is the lifelong process by which the learner constructs and modifies his or her own personal schemata. Mugny and Doise (1978:1) maintain that “Educational research has shown that learners tend to comprehend complex concepts much better and to retain them as part of their body of knowledge much longer when they become actively involved in their learning process”.

Mathematics can be actively learned by involving learners in their learning process. Ahmed, Clark-Jeavons and Oldnow (1987:315) assert that “Mathematics can be effectively learned only by involving learners in experimenting, questioning, reflecting, discovering, inventing and discussing. Mathematics should be a kind of learning which requires a minimum of factual knowledge and a great deal of experience in dealing with situations using particular kinds of thinking skills”. Carpenter and Lehrer (1999:23) indicate that the critical learning of mathematics by learners occurs as a consequence of building on prior knowledge via purposeful engagement in activities and by discourse with other learners and educators in classrooms. So learners must engage in activities that encourage their mathematical understanding.

This view of learning mathematics leads to the characteristics of learning mathematics with understanding. Hiebert and Carpenter (1992:71) assert that learning mathematics with understanding implies learners not only must learn the concepts and procedures of mathematics, but they must learn to use such ideas to solve non-routine problems and learn to utilise mathematics in a variety of situations. Therefore, the concentration should shift from judging learner learning in terms of mastery of concepts and procedures to making judgments about learners’ deep understanding of the concepts and procedures and their ability to apply them to mathematics problem situations.

Language plays a pivotal role in the understanding of mathematics. The medium of language in the teaching of mathematics affects how the content is understood by the learners. Chomsky and Howard (1977: 37) pioneered the idea that each human child has an innate capacity to master the grammar and deep structure of language. His insight was based on the observation that children learn grammar at a rate far greater than can be explained by their extrapolating from examples given to them. They must therefore have an innate capacity not only to learn language but also to understand how it works. Because language acquisition is universal, all languages must share the same fundamental structure or "depth grammar".

Chomsky’s and Howard’s (1977:38) revolutionary work on grammar and language deeply influenced not only linguistics, but also cognitive science in general. He focused particularly on the impoverished language input children receive. Adults do not typically speak in grammatically complete sentences. In addition, what the child hears is only a small sample of language. Chomsky and Howard (1977:39) concluded that children must have an inborn faculty for language acquisition. According to this theory, the process is biologically determined and the human species has evolved a brain whose neural circuits contain linguistic information at birth.

The child's natural predisposition to learn language is triggered by hearing speech and the child's brain is able to interpret what s/he hears according to the underlying principles or structures it already contains. This natural faculty has become known as the Language Acquisition Device (LAD). Chomsky and Howard (1977:38) did not suggest that an English child is born knowing anything specific about English. He stated that all human languages share common principles. It is the child's task to establish how the specific language he or she hears expresses these underlying principles.

The construction of relationships is one of the important forms of mental activity where mathematical understanding emerges. For learners to learn mathematics with understanding, new ideas take on meaning by the ways they are related to other ideas. Learners construct meaning for a new idea or process by relating it to ideas or processes that they have already understood. Although learning with understanding entails forging connections between what the learners already know and the knowledge they are learning, it is not sufficient to think of developing understanding simply as appending new concepts and processes to existing knowledge.

The teacher's role should be shifted from being an orator to a learning manager and facilitator who manages, directs and encourages learners' creations or from sage on the stage to guides on the side where he provides learners with opportunities to test the adequacy of their current understandings. Doyle (1988:167) argues that educators should be especially attentive to the extent to which meaning is emphasised and the extent to which learners are explicitly expected to demonstrate understanding of the mathematics underlying the activities in which they are engaged.

Such an emphasis can be maintained if explicit connections between the mathematical ideas and the activities in which learners engage are frequently drawn. Also, Carpenter and Lehrer (1999:20) assert that connections with what learners already know and understand what they are learning plays an important role in engaging learners in high level thought processes. The mathematical activities should therefore be selected to encourage the learners to link the knowledge that they have already learned with the new knowledge.

The learners' role should also be changed from obtaining knowledge from the teacher to assimilating or accommodating their own knowledge by connecting the relationships between what they have known and what they are learning. The learners' role should also be shifted to confront their understanding in light of what they encounter in the new learning situation (Cohen, Manion and Morrison ,2000:259). If what the learners encounter is inconsistent with their current understanding, their understanding can change to accommodate the new experience.

The teacher has to keep in mind that learners come to the learning situations with knowledge gained from previous experience and that prior knowledge influences what new or modified knowledge they will construct from the new learning experiences. A critical factor underlying unsuccessful task implementation is a lack of alignment between tasks and learners' prior knowledge, interests and motivation. Such mis-matches may cause learners to fail to engage with the task in ways that will maintain a high level of cognitive activity.

The role of the teacher is to organise information around conceptual clusters of problems, questions and discrepant situations in order to engage the learner's interest. Educators assist the learners in developing new insights and connecting them with their previous learning. Ideas are presented holistically as broad concepts and then broken down into parts. The activities are learner centred and learners are encouraged to ask their own questions, carry out their own experiments, make their own analogies and come to their own conclusions.

The following represents a summary of some suggested characteristics of a constructivist teacher (Brooks and Brooks, 1993:7):

- Become one of many resources that the learner may learn from, not the primary source of information;
- Engage learners in experiences that challenge previous conceptions of their existing knowledge;

- Allow the learner response to drive lessons and seek elaboration of learners' initial responses, allow the learner some thinking time after posing questions;
- Encourage the spirit of questioning by asking thoughtful, open-ended questions;
- Encourage thoughtful discussion among learners;
- Use cognitive terminology such as "classify," "analyse" and "create" when framing tasks;
- Encourage and accept learner autonomy and initiative;
- Be willing to let go of classroom control;
- Use raw data and primary sources, along with manipulative, interactive physical materials;
- Do not separate knowing from the process of finding out;
- Insist on clear expression from learners;
- When learners can communicate their understanding, then they have truly learned.

From the constructivist perspective, learners must necessarily construct their own knowledge, irrespective of how they are taught. Even in the case of direct teaching, learners cannot absorb an idea exactly as it is taught, but must interpret it and give meaning to what the teacher says in terms of their existing knowledge. Let us look a little closer at the process of communication. However, when we talk to someone else, we must necessarily try to express those concepts in words, that is, in symbols. So what B receives is not A's meanings and feelings, but A's symbols representing those meanings. B must now interpret these symbols in terms of his own concepts and meanings of these symbols. Because B has a different background and different life experiences than A, B's meanings of these symbols will invariably be different from A's meanings of the same symbols. This situation is depicted in the following diagram (Skemp, 1976: 23).

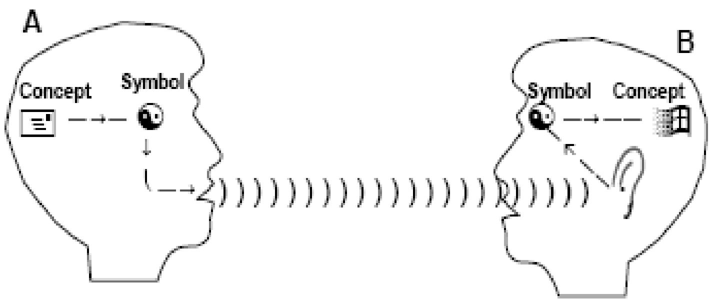


Figure 4: The process of communication (Skemp, 1976: 23)

It should be clear that communication and in particular the transfer of knowledge, is problematic. One-way communication and the transfer of knowledge can only be successful if A and B have nearly the same meanings for the symbols; that is, if their concepts are nearly the same (Figure 4). However, if they have vastly different concepts for the symbols, one of three things can happen:

Either there is a total breakdown of understanding, or,

- B will change his concepts so that they are nearly the same as A's, and B will therefore understand what A is saying, or,
- B will distort A's meanings to fit his own concepts behind the symbols, without changing his own concepts much, with the result that B will misunderstand A.

The constructivist perspective on learning, however, assumes that concepts are not taken directly from experience, that the learner does not passively absorb knowledge. Rather, the learner is an active participant in the construction of his own knowledge, because knowledge arises from the interaction of the learner's existing ideas and new ideas that are interpreted and understood in the light of the learner's own current knowledge. A person sees the world through the lens of his existing knowledge and therefore each individual sees the world differently.

The constructivist believes that we see what we understand. Conceptual knowledge can therefore not be transferred ready-made and intact from one person to another, each learner must necessarily construct his own conceptual knowledge. The new curriculum has set the stage for a move in South African pedagogy from a predominantly transmission mode of learning, where skilled educators view children as empty vessels easily filled with knowledge, to an understanding of learning that appreciates that learners are much more actively involved in constructing knowledge.

2.6 Van Hiele and the Development of Geometrical Thinking

Two mathematics educators in the Netherlands, Pierre van Hiele and Dina van Hiele-Geldorf developed a pedagogical theory to explain geometrical thinking of children. They identified relatively stable, qualitatively different levels of understanding through which an individual passes when learning geometry. The Van Hiele model is currently the best known theoretical account of learning geometry. This is a good example of what is called a ‘pragmatic theory’. Van Hiele (1986:56) claimed that children have to take a sequence of steps in a fixed order in their geometric learning about shape. Through their research they have identified five levels of understanding spatial concepts through which children move sequentially on their way to geometric thinking. The hierarchy for learning geometry described by the van Hiele parallels Piaget’s stages of cognitive development. One should note that the van Hiele model is based on instruction.

The van Hiele model supports Vygotsky’s notion of the “zone of proximal development” which is the “distance between the actual developmental level as determined by independent problem-solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers” (Vygotsky, 1978,85). Language at the visual level serves to make possible communication for the whole group about the structures that learners observe.

Van Hiele (1986:56) hypothesised five sequential levels of geometric reasoning. These are visualisation, analysis, informal deduction, formal deduction and rigour (Burger and Shaughnessy, 1986:31). The model suggests that, assisted by appropriate instructional experiences, the learner moves sequentially from the initial or basic level (visualisation), where space is simply observed and the properties of figures are not explicitly recognised, through the sequence listed above to the highest level (rigour), which is concerned with formal abstract aspects of deduction (Fuys and Liebov, 1997:250). At level one of the van Hiele (1986:57) hierarchies the analysis of geometric concepts begins (visualisation is at level zero).

For example, through observation and experimentation learners begin to discern the characteristics of figures. These emerging properties are then used to conceptualise classes of shapes. Learners at this level cannot yet explain relationships between properties, interrelationships between figures are still not seen and definitions are not yet understood. At the level of informal deduction (level two) learners are able to establish the interrelationship of properties both within figures (for example in a quadrilateral, opposite sides being parallel necessitates opposite angles being equal) and among figures (a square is a rectangle because it has all the properties of a rectangle). Thus, they can deduce properties of a figure and recognise classes of figures (van Hiele 1986:58).

Class inclusion is understood and definitions are meaningful. Informal arguments can be followed and given but the learner at this level does not comprehend the significance of deduction as a whole or of the role of axioms. Empirically obtained results are often used in conjunction with deduction techniques. Formal proofs can be followed, but learners do not see how the logical order can be altered nor do they know how to construct a proof starting from different or unfamiliar premises (van Hiele 1986:58).

At level three learners understand the significance of formal deduction as a way of establishing geometric theory within an axiomatic system. They are able to see the interrelationship and role of undefined terms, axioms, postulates, definitions, theorems and proof. Learners at this level can construct, not just memorise, proofs; they accept the possibility of developing a proof in more than one way. The interaction of necessary and sufficient conditions is understood; distinctions between a statement and its converse can be made. Level four (rigour) is the highest van Hiele (1986:58) level. At this level, learners can work in a variety of axiomatic systems, that is, non-Euclidean geometries can be studied and different systems can be compared.

Geometry is seen in the abstract. What is important in terms of pedagogy is that, as Wirszup (1976:78) suggests, people at different levels of mathematical understanding speak, use and understand terms differently and that educators often use terms that can only be understood by learners who have progressed to the third or fourth van Hiele level. Consequently, when trying to communicate with learners who function at lower levels, their intentions may be completely misunderstood. A major purpose for distinguishing learners' levels of understanding is to recognise obstacles that they may experience in the learning process and to allow educators to develop strategies which will enable children to progress in terms of conceptual development (Bishop, 1997:180).

Following Piaget, another significant contribution to understanding the development of geometric thinking was advanced by Pierre van Hiele (1986:68). The van Hiele model, on the other hand, suggests that learners advance through levels of thought in geometry. Van Hiele categorises these levels as visual, descriptive, abstract/relational, formal deduction and rigour (Table 1).

Stage and Name	Geometry Cognitive Developments
Level 1. (Visualisation) Corresponds to Sensori-motor on a Piagetian scale.	Learners recognise figures as total entities, but do not recognise properties of these figures.
Level 2. (Analysis) Corresponds to Concrete Operations on a Piagetian scale.	Learners analyse component parts of the figures, but interrelationships between figures and properties cannot be explained.
Level 3. (Informal Deduction) Corresponds to Concrete Operations on a Piagetian scale.	Learners can establish interrelationships of properties within figures and among figures. Informal proofs can be followed but learners do not see how the logical order could be altered nor do they see how to construct a proof starting from different or unfamiliar premises.
Level 4. (Deduction) Corresponds to Formal Operations on a Piagetian scale.	At this level the significance of deduction as a way of establishing geometric theory within an axiom system is understood. The interrelationship and role of undefined terms, axioms, definitions, theorems, and formal proof is seen. The possibility of developing a proof in more than one way is seen.
Level 5. (Rigour)	Learners at this level can compare different axiom systems. Geometry is seen in the abstract with a high degree of rigour, even without concrete examples.

Table 1: The van Hiele Model (1986:56)

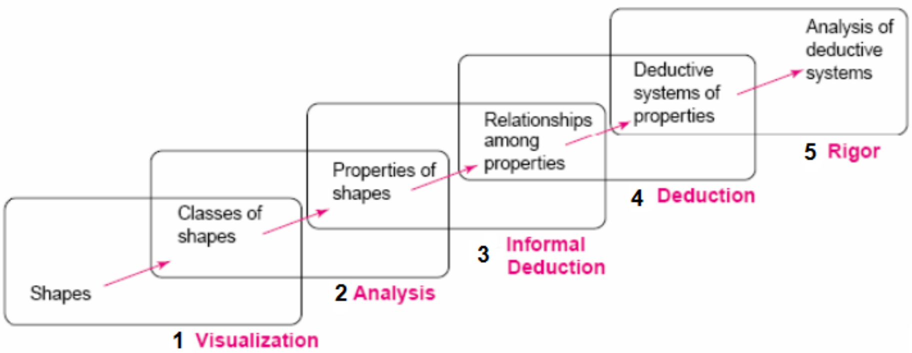


Figure 5: van Hiele's stages (van de Walle, 2004:57)

According to this model, progress from one of van Hiele's levels to the next is more dependent upon teaching method than on age (Figure 5). Given traditional teaching methods, research suggests that lower secondary learners perform at levels one or two with almost 40 percent of learners completing secondary school below level two. The explanation for this, according to the van Hiele model, is that educators are asked to teach a curriculum that is at a higher level than the learners.

According to the van Hiele model, it is not possible for learners to bypass a level. They cannot see what the teacher sees in a geometric situation and therefore do not gain from such teaching. While research is generally supportive of the van Hiele levels as useful in describing learners' geometric concept development, it remains uncertain how well the theory reflects children's mental representations of geometric concepts. Various problems have been identified with the specification of the levels. For example, that the visualising of the lowest level as "visual" when visualisation is demanded at all the levels and the fact that learners appear to show signs of thinking from more than one level in the same or different tasks, in different contexts. An integral component of the van Hiele model is a specified teaching approach involving four phases. There is little research on this aspect of the model and hence uncertainty about its success.

The dynamic representation and manipulation of geometrical objects allow a particular configuration of elements to be modified in order to span the whole range of possibilities. This feature could help the learner visualise the multiplicity of configurations included in a single geometrical figure and therefore overcome the difficulties related to the conflict of drawing (Laborde, 1993:50).

Clements and Battista (1992:420) suggest that there is another level of geometric thinking that develops before the van Hiele level of visualisation. At this level, children perceive geometric shapes, but attend to only one aspect of a shape's visual characteristics. For example, children may differentiate between figures that are curved and those that are angular, but not among figures in the same class. They postulate a pre-recognition level where children perceive geometric shapes by attending to only a part of the shape's characteristics and state that: "Children perceive geometric shapes, but perhaps because of a deficiency in perceptual activity, may attend to only a subset of a shape's characteristics".

Learners are unable to identify many common shapes. They may distinguish between figures that are curvilinear and those that are rectilinear but not among figures in the same class; that is, they may differentiate between a square and a circle, but not between a square and a triangle. Thus, learners may be unable to identify common shapes because they lack the ability to form requisite visual images (Clements and Battista, 1992: 429).

It is clear that both Piaget and van Hiele, in their developmental approaches to cognition, emphasise the active construction of knowledge and the importance of children building relationships among geometric concepts and processes. It is the construction and abstraction of these relationships that is important in the personal meaning-making for children in their mathematical thinking. What is important in terms of pedagogy is that, as Wirszup (1976:78) suggests, people at different levels of mathematical understanding speak, use and understand terms differently and educators often use terms that can only be understood by learners who have progressed to the third or fourth van Hiele level.

Consequently, when trying to communicate with learners who operate at lower levels, their intentions may be completely misunderstood. A major purpose for distinguishing learners' levels of understanding is to recognise obstacles that they may experience in the learning process and to allow educators to develop strategies which will enable children to progress in terms of conceptual development (Bishop, 1997:224).

2.6.1 Van Hiele levels and the Revised National Curriculum Statement

The intermediate phase assessment standards for geometry, as expressed in the South African Revised National Curriculum Statement (RNCS) documents, require that learners are able to name shapes, describe and/or classify shapes using properties and construct shapes correctly in order to attain learning outcome three; that is, the learner is able to describe and represent the characteristics and relationships between two-dimensional shapes and three-dimensional objects in a variety of orientations and positions (Department of Education, 2002a: 6).

At van Hiele level one (Fuys and Liebov, 1997:250), the learner identifies, names, compares and functions on geometric figures according to their appearance. Similarly, the RNCS (DoE, 2002b: 6) assessment standards, which are guided by van Heile levels, are characterised by the naming and visualising of shapes and objects in natural and cultural forms. As such, both van Hiele and the RNCS assessment standards characterise this level by recognition of the shape as a whole. Van Hiele level one is characterised by the analysis of figures in terms of their components and their relationships, a stage which allows learners to discover properties or rules of a class of shapes empirically.

The characteristics of the RNCS's assessment standards are the definition of shapes and objects in terms of properties such as their faces, vertices and edges. The characteristics of both the van Hiele level and the assessment standards are concurrent in that they define shapes and objects using their properties. At the informal deduction level (van Hiele level two) learners logically relate previously discovered properties or rules by giving or following informal arguments such as drawing, interpreting, reducing and locating positions.

This fits well with the RNCS assessment standards which state that learners must be able to provide informal arguments such as drawings, interpretations and the reducing and locating of positions. The first three van Hiele levels (levels zero to two) cover all the assessment standards of the intermediate phase as stated in the RNCS (Department of Education, 2002c: 8). Therefore, the exit level outcomes for learners in the intermediate phase of the South African curriculum can be related to the expectations of van Hiele level two.

2.7 Piaget and the Learning / Teaching of Mathematics

Jean Piaget was a Swiss psychologist famous for his studies of the intellectual growth of children and his influential theories of cognitive development. In studying children, Piaget (1970:13) found four stages of mental growth. These are a sensori-motor stage, from birth to age 2, when mental structures concentrate on concrete objects; a pre-operational stage, from age 2 to 7, when they learn symbols in language, fantasy, play and dreams; a concrete operational stage, from age 7 to 11, when they master classification, relationships, numbers and ways of reasoning about them; and a formal operational stage, from age 11, when they begin to master independent thought and other people's thinking.

Piaget (1970:15) studied the development of children's understanding, through observing them and talking and listening to them while they worked on exercises he had set. His view on how children's minds work and develop has been enormously influential, particularly in educational theory. His particular insight was the role of maturation (simply growing up) in children's increasing capacity to understand their world: they cannot undertake certain tasks until they are psychologically mature enough to do so.

The Piagetian (1970:23) work has two major themes in geometry. The first theme is that our mental representation of space is not a perceptual "reading off" of what is around us. Rather, we build up from our mental representation of our world through progressively re-organising our prior active manipulation of that environment. Secondly, the progressive organisation of geometric ideas follows a definite order and this order is more experiential. That is, initially topological relations such as connectedness, enclosure and continuity are constructed, followed by projective and Euclidean relations. The first of these Piagetian themes, concerning the process of the formation of spatial representations, remains reasonably well-supported by research. The second hypothesis has received, at best, mixed support. The available evidence suggests that all types of geometric ideas appear to develop over time, becoming increasingly integrated and synthesised.

Piaget (1970:13) believes that individuals work with independence and equality on each other's ideas, so when the learner is opposed to new knowledge and interacts with others he or she encounters something that contradicts his or her beliefs or current understanding. This is what Piaget calls "cognitive conflicts" (Mugny and Doise, 1978:181). This conflict results in a case of disequilibrium. Working co-operatively and activating meta-cognitive strategies such as planning, monitoring and evaluating are likely to engage learners to assimilate or accommodate their knowledge and therefore re-equilibrate their thinking. When learners employ their meta-cognitive strategies, they are more than likely to revise, evaluate and guide their ways of thinking to provide a better fit with reality.

Cognitive structure is the central idea of Piaget's (1970:13) theory. These structures are patterns of physical or mental action that underlie specific acts of intelligence and correspond to stages of child development. Piaget described two processes used by the individual in their attempt to adapt: assimilation and accommodation. Both processes are used throughout life as the person increasingly adapts to the environment in a more complex manner. Assimilation is the process of using or transforming the environment so that it can be set in pre-existing cognitive structures. Accommodation is the process of changing cognitive structures in order to accept something from the environment. Both processes are used simultaneously and alternately throughout life.

2.7.1 Stages of Cognitive Development

Piaget identified four stages in cognitive development (Piaget, 1980:59):

1. **Sensori-motor stage** (Infancy). In this period (which has six stages), intelligence is demonstrated through motor activity without the use of symbols. Knowledge of the world is limited (but developing) because it is based on physical interactions and experiences. Children acquire object permanence at about 7 months of age (memory). Physical development (mobility) allows the child to begin developing new intellectual abilities. Some symbolic (language) abilities are developed at the end of this stage (Table 2).
2. **Pre-operational stage** (Toddler and Early Childhood). In this period (which has two sub-stages), intelligence is demonstrated through the use of symbols, language use matures and memory and imagination are developed, but thinking is done in a non-logical, non-reversible manner. Egocentric thinking predominates.
3. **Concrete operational stage** (Elementary and early adolescence). In this stage (characterised by seven types of conservation: number, length, liquid, mass, weight, area, volume), intelligence is demonstrated through logical and systematic manipulation of symbols related to concrete objects. Operational thinking develops (mental actions that are reversible). Egocentric thought diminishes.
4. **Formal operational stage** (Adolescence and adulthood). In this stage, intelligence is demonstrated through the logical use of symbols related to abstract concepts. Early in the period there is a return to egocentric thought. Only 35 percent of high school graduates in industrialised countries obtain formal operations; many people do not think formally during adulthood.

<table>
<tr><td rowspan="4">Piaget's cognitive development stages</td><td>1 Sensori-motor (Birth – 2 years old)

Children are learning and thinking through their senses and the manipulation of objects in the world around them.</td></tr>
<tr><td>2 Pre-operational (2 – 7 year old)

In this stage children are using symbols to represent the previous stage's findings. Language, memory and the use of objects in make-believe play are acquired.</td></tr>
<tr><td>3 Concrete Operational (7 – 11 Year old)

In this stage reasoning is in the process of becoming logical. There is organisation of collected information, but not adult abstract reasoning is apparent.</td></tr>
<tr><td>4 Formal Operational (adolescence - adulthood)

Using logical thinking adolescents in this stage apply symbols to abstract concepts. Adolescents can now solve scientific problems with numerous potential conclusions.</td></tr>
</table>

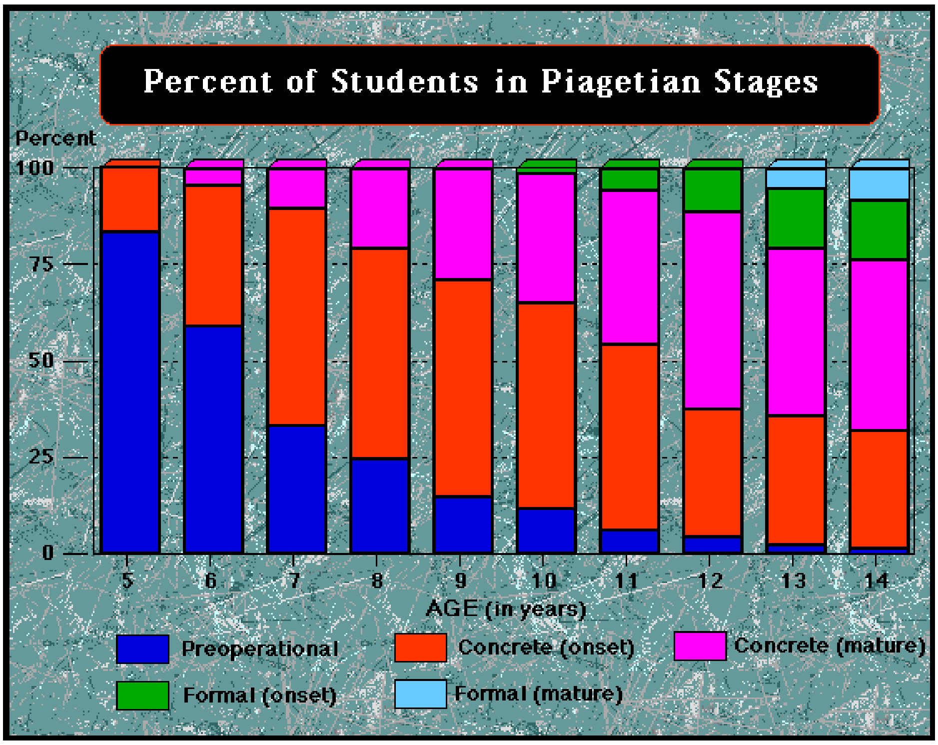
Table 2: Piaget's stages of cognitive development (Berk, 2000:56)

Piaget maintains that cognitive structures are not stable and they change through the processes of adaptation; that is, assimilation and accommodation. Piaget affirms that all children construct, or create logic and number concepts from within rather than learn them by internalisation from the environment (Piaget, 1970:27; Inhelder and Piaget, 1964:375; and Kamii , Lewis and Jones, 1993:200).

The application of Piaget's theory results in specific recommendations for a given stage of cognitive development. For example, with learners in the concrete operational stage, learning activities should involve problems of classification, ordering, location and conservation using concrete objects.

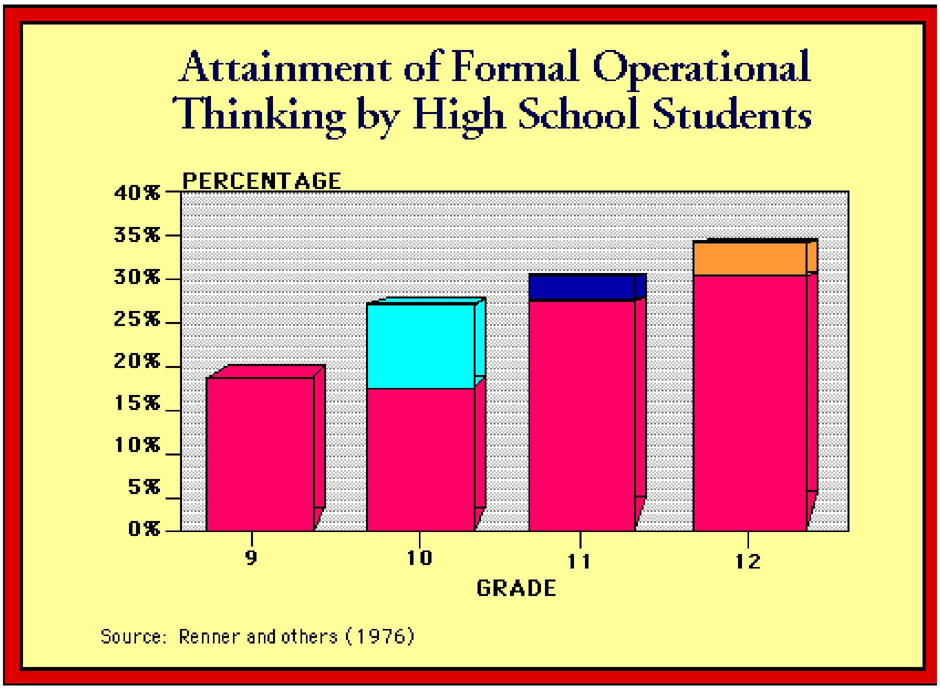
Educators should also try to provide a rich and stimulating environment with logical matters that depend on concrete objects and try to prepare learners for the next stage, namely, formal operational which involves abstract thinking. Discovery learning and supporting the developing interests of the child are two primary instructional techniques. Parents and educators need to challenge the child's abilities, but not present material or information that is too far beyond the child's level. Educators must also use a wide variety of concrete experiences to help the child learn (Graph1).

Piaget based his research methods primarily on case studies. He believed that biological development drives the movement from one cognitive stage to the next. Data from cross-sectional studies of children in a variety of western cultures seem to support this assertion for the stages of sensori-motor, pre-operational and concrete-operational stages (Renner, Stafford, Lawson, McKinnon and Kelog, 1976:123).



Graph 1: Percentage of students in Piagetian stages (Renner *et al*, 1976:32)

However, data from similar cross-sectional studies of adolescents do not support the assertion that all individuals will automatically move to the next cognitive stage as they biologically mature. Data from adolescent populations indicate only 30 to 35 percent of high school learners attain the cognitive development stage of formal operations. For formal operations, it appears that maturation establishes the basis, but a special environment is required for most adolescents and adults to attain this stage (Graph 2).



Graph 2: Attainment of formal operational thinking by high school students (Renner *et al*, 1976:33)

2.7.2 Piaget and Papert: Similar Goals, Different Means

Piaget and Papert (1999:105) are both constructivists in that they view children as the builders of their own cognitive tools, as well as of their external realities. For them, knowledge and the world are both constructed and constantly reconstructed through personal experience. Each gains existence and form through the construction of the other. Knowledge is not merely a commodity to be transmitted, encoded, retained and re-applied but a personal experience to be constructed. Similarly, the world is not just sitting out there waiting to be uncovered, but gets progressively shaped and transformed through the child's personal experience.

Piaget and Papert (1999:106) are also both developmentalists in that they share an incremental view of knowledge construction. The common objective is to highlight the processes by which people outgrow their current views of the world and construct deeper understandings about themselves and their environment. In their empirical investigations, Piaget and Papert both study the conditions under which learners are likely to maintain or change their theories of a given phenomenon through interacting with it during a significant period of time. Piaget and Papert (1999:106) define intelligence as adaptation**,** or the ability to maintain a balance between stability and change, closure and openness, continuity and diversity, or, in Piaget's words, “between assimilation and accommodation”. Piaget's interest was mainly in the construction of internal stability, whereas Papert is more interested in the dynamics of change.

Piaget’s (Papert, 1999:105) theory relates to how children become progressively detached from the world of concrete objects and local contingencies, gradually becoming able to mentally manipulate symbolic objects within a realm of hypothetical worlds. He studied children’s increasing ability to extract rules from empirical regularities and to build cognitive invariants. He emphasised the importance of such cognitive invariants as a means of interpreting and organising the world. One could say that Piaget’s interest was in the assimilation pole. His theory emphasises all those things needed to maintain the internal structure and organisation of the cognitive system. What Piaget describes particularly well is precisely this internal structure and organisation of knowledge at different levels of development.

The emphasis of Papert’s (1999:105) lies almost at the opposite pole. His contribution is to remind us that intelligence means being situated, connected and sensitive to variations in the environment. In contrast to Piaget, Papert draws our attention to the fact that “diving into” situations rather than looking at them from a distance, that connectedness rather than separation, are powerful means of gaining understanding. Papert’s (1999:105) research focuses on how knowledge is formed and transformed within specific contexts, shaped and expressed through different media and processed in different people’s minds. While Piaget liked to describe the genesis of internal mental stability in terms of successive plateaus of equilibrium, Papert is interested in the dynamics of change. He stresses the fragility of thought during transitional periods. He is concerned with how different people think once their convictions break down, once alternative views sink in, once adjusting, stretching and expanding their current view of the world become necessary. Papert (1999:105) always points toward this fragility, contextuality and flexibility of knowledge under construction.

Piaget’s “child” (Papert, 1999:105), often referred to as an epistemic subject, is a representative of the most common way of thinking at a given level of development. The “common way of thinking” that Piaget captures in his descriptions is that of a young scientist whose purpose is to impose stability and order over an ever-changing physical world.

Papert (1980:8) describes two major research themes from the early 1980s that are relevant to research in technology and education even today. “Children can learn to use computers in a masterful way and that learning to use computers can change the way they learn everything else” (Papert, 1980:83). Papert then explains a very important aspect of learning through technological means. Learning through technology is more than just fun; very powerful kinds of learning are taking place.

Children are learning to speak mathematics and acquiring a new image of themselves as mathematicians (Papert, 1980:8). Papert proposes the use of computers in mathematics education as a means of overcoming cultural barriers. He postulates that children can learn to use computers in a masterful way and learning to use computers can change the way they learn everything else (Papert, 1980: 8). As opposed to the traditional learning environment where the educator instructs and the learner follows, in this learning environment the learner assumes the role of instructor and programmer. The learners' interaction with Logo via the computer is not a one-way process. The programme provides feedback in such a manner as to provide learners with a choice as to how they want to react to it. Logo is a computer programming language designed for learning.

It allows the learner access to creating screen effects and to the mathematical concepts that underlie it (Noss, 1988:251). There have been a number of longitudinal studies done which have sought to analyse the power of this environment from a mathematical perspective, and which have illustrated that children are able to explore and use a variety of mathematical ideas in a wide range of contexts (Papert, 1985:56; Hoyles, 1985:237). The body of research suggests that learners working with Logo, by creating and interacting with objects that are visible, quantifiable and adhere to conventional mathematics, build connections between spatial and algebraic thinking. Mathematics becomes more concrete to learners and algebraic formalisation is supported through Logo procedures.

Furthermore, learners are afforded opportunities to try out ideas and modify plans, which are essential elements of mathematical problem-solving. Learners can make and test conjectures, a vital component of mathematical reasoning (Jones, 2005:34). These studies have confirmed Papert's (1985:56) claim that by learning Logo the child is behaving as a mathematician. Papert proposes that active learning brought about through Logo has its roots based on the theories of Piaget. Papert (1985:56) cites Bruner's work as closer to Logo in that he postulates that learning is enactive, iconic and symbolic. Bruner's constructivist theory is a general framework for instruction based upon the study of cognition. Much of the theory is linked to child development research. Learners firstly physically manipulate the Turtle, then they direct the pictorial Turtle on the screen and thirdly they write procedures in Logo which is symbolic (Ernest, 1989:33). The turtle is an on-screen cursor, which can be given movement and drawing instructions, and is used to programmatically produce line graphics.

Papert (1985:145) explains that when we say we educate children, it sounds like something we do to them. That is not the way it happens. We do not educate them. We create contexts in which they will learn. The dynamic-geometry supported classroom offers a challenge regarding the creation of the contexts mentioned by Papert. Learners in such classes may spend much of their class time interacting with a computer programme, rather than communicating with an educator. The learners are expected to actively explore visual images and discuss, analyse and communicate their findings. Learners can use dynamic software to construct and manipulate their own diagrams, but pre-constructed sketches are also available for use in geometry tasks.

2.8 Mathematics Cognitive Development

Moursund (2005:25) developed a mathematics cognitive development model. It represents his current insights into a six-level, Piagetian-type model. (Table 3).

Stage and Name	**Mathematics Cognitive Developments**
Level 1. Piagetian and Mathematics sensori-motor. Birth to age 2.	Infants use sensory and motor capabilities to explore and gain understanding of their environments. There is some innate ability to deal with small quantities such as 1, 2 and 3. As infants gain crawling or walking mobility, they can display innate spatial sense. They can move to a target along a path requiring moving around obstacles and can find their way back to a parent after having taken a turn into a room where they can no longer see the parent.

Level 2. Piagetian and Mathematics pre-operational. Age 2 to 7.	Children begin to use symbols, such as speech. They respond to objects and events according to how they appear to be. The children are making rapid progress in receptive and generative oral language. They adapt the language environments they spend a lot of time in. They learn number words and name the number of objects in a collection and how to count them, with the answer being the last number used in this counting process. A majority of children discover or learn “counting on” and counting on from the larger quantity as a way to speed up counting of two or more sets of objects. Children gain increasing proficiency in such counting activities.
Level 3. Piagetian and Mathematics concrete-operations. Age 7 to 11.	Children begin to think logically. In this stage, which is characterised by seven types of conservation: number, length, liquid, mass, weight, area, volume, intelligence is demonstrated through logical and systematic manipulation of symbols related to concrete objects. Operational thinking develops. While concrete objects are an important aspect of learning during this stage, children also begin to learn from words, language, pictures, video and learning about objects that are not concretely available to them. The time span of concrete operations is approximately the time span of elementary school. The level of abstraction in the written and oral mathematics language quickly surpasses a learner’s previous mathematics experience. There is a substantial difference between developing general ideas and understanding of conservation of number, length, liquid, mass, weight, area, volume and learning the mathematics that corresponds to this stage.

Level 4. Piagetian and mathematics formal operations. After age 11.	At age 11 or 12, or so, thought begins to be systematic and abstract. Intelligence is demonstrated through the logical use of symbols related to abstract concepts, problem solving and gaining and using higher-order knowledge and skills. Mathematics maturity supports the understanding of and proficiency in mathematics at the level of a high school mathematics curriculum. Piagetian and mathematics formal operations includes being able to recognise mathematics aspects of problem situations in both mathematics and non-mathematics disciplines, convert these aspects into mathematics problems and solve the resulting mathematics problems if they are within the range of the mathematics that one has studied. Such transfer of learning is a core aspect of Level 4. Cognitive development can continue well into college and most learners never fully achieve Level 4 mathematics cognitive development.
Level 5. Abstract mathematical operations. Moving far beyond mathematics formal operations.	Mathematical content proficiency and maturity at the level of contemporary mathematics texts used at the upper division undergraduate level. Good ability to learn mathematics through some combination of reading required texts and other mathematics literature, listening to lectures, participating in class discussions, studying on one's own, studying in groups. Pose and solve problems at the level of one's mathematics reading skills and knowledge. Follow the logic and arguments in mathematical proofs. Fill in details of proofs when steps omitted in textbooks and other representations of such proofs.
Level 6. Mathematician.	A very high level of mathematical proficiency and maturity at this level is required. This includes speed, accuracy and understanding in reading, writing and oral communication of research-level mathematics.

Table 3: Mathematics Cognitive Development model by Moursund (2005:25)

2.9 Vygotsky and the Learning / Teaching of Mathematics

Vygotsky studied the role of social and cultural factors in the making of human consciousness. His work emphasises the socially transmitted knowledge of the teacher and the active engagement of the child in the learning process.

In their search for alternative models to explain learning, many researchers have turned their attention to Vygotsky's notion of mediation, where a more competent peer or adult is viewed as assisting performance, bridging the gap between what the child knows and can do and what the child needs to know. Vygotsky (1978:56) conceptualised this gap between unassisted and assisted performance as the zone of proximal development (ZPD), that 'space' where learning leads to development. For Vygotsky (1978:56), human consciousness is social. Essentially, Vygotsky enables us to conceptualise the prior existence of complex cognitive structures as existing in the child's culture, rather than in the individual child. Every experience, then, that the child has is mediated through cultural tools (Figure 6).

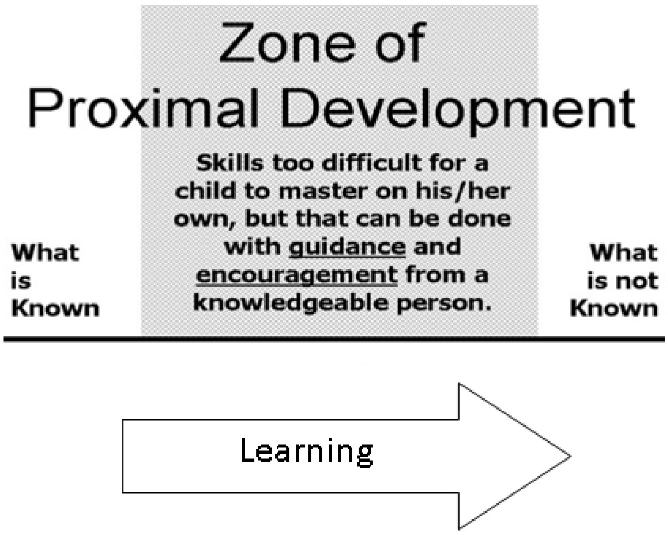


Figure 6: Zone of Proximal Development (Nardi, 1998:31)

Vygotsky (1978:56) suggests that learners can be guided by explanation, demonstration and can attain higher levels of thinking if they are guided by more capable and competent adults. This conception is better known as the Zone of Proximal Development (ZPD). The Zone of Proximal Development is the gap between what is known and what is not known, that is, generally higher levels of knowing. The ability to attain higher levels of knowing is often facilitated and, in fact, depends upon, interaction with other more advanced peers, who, for Vygotsky, are generally adults.

Through increased interaction and involvement, learners are able to extend themselves to higher levels of cognition. Vygotsky (1978:56) defines the Zone of Proximal Development as “the distance between the actual developmental level as determined by independent problem-solving and the level of potential development as determined through problem-solving under the guidance or in collaboration with more capable peers”. The ZPD is thus the difference between what learners can accomplish independently and what they can achieve in conjunction or in co-operation with another more competent person (Figure 7).

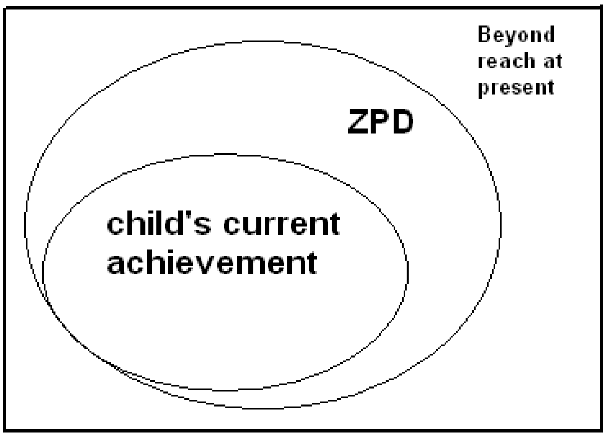


Figure 7: Child’s current achievement (Moore and Kearsley, 1996:89)

The socio-cultural development enriches the active learning processes and contributes to encouraging constructing knowledge. Moore and Kearsley (1996:89) have indicated that socio-cultural development is an area that is missing in most traditional or objectivist learning environments. Interactions between the learner and the content, between the learner and the instructor and between the learners themselves are necessary for learning and for the shared social construction of knowledge (American Psychological Association [APA], 1995; Moore and Kearsley, 1996:89).

Vygotsky (1978:56) proposes that knowledge is not constructed individually, but happens through the internalisation of social knowledge, which is embodied in social and cultural practices. Based on this cognitive theory, social constructivist theory acknowledges the active construction of knowledge formed by the learner on the basis of experiences and prior knowledge. It places major importance on the use of language and the role of interactions with others and their physical world as contributing factors to the construction of knowledge.

Vygotsky (1978:59) suggests that an active learner and an active social environment co-operate to produce developmental change. The learner actively explores and tries alternatives with the assistance of a more skilled partner, as in an instructor, or a more capable peer. The educator and the partner guide and structure the learners' activity, scaffolding their efforts to increase current skills and knowledge to a higher competency level. Scaffolding is the support during a teaching session, where a more skilled partner (adult or peer) adjusts the level of assistance given based on the level of performance indicated by the learner. A greater level of support is offered if the task is new and less is provided as competency grows (Berk and Winsler, 1995:25). The learner is able to move forward and continues to develop new capabilities.

Mastering space and shape concepts in geometry offers opportunities to practise logical reasoning and to acquire abilities in various reasoning patterns. Troutman and Lichtenberg (2003:407) argue that through the listing of properties and classifications learners begin to build concepts enabling them to develop the spatial sense to function in their environment. Reasoning skills are necessary to advance from a procedural to a conceptual approach.

2.10 Collaborative Learning

Collaborative learning has been practised in schools for many decades (Slavin, 1989: 232). There is much professional expertise and experience in developing strategies for collaborative learning. There is a robust research tradition addressing a myriad of issues to do with collaborative learning, from the pioneering work of Vygotsky and early researchers to studies investigating the links between new pedagogy and information technology. However; there are limitations in the research; only a few studies suggest that working in a small team achieves cognitive outcomes that cannot be matched or exceeded by the most capable group member (Schwartz, 2001:197). Nonetheless, collaborative learning has been shown to be a more effective.

2.11 Skemp's Theory of Understanding

The theory of the psychology of learning mathematics formulated by Skemp (1989:34) identifies the importance of learning mathematics with understanding, which he describes as reflective intelligence. Richard Skemp was the major pioneer in Mathematics Education who first integrated the disciplines of mathematics, education and psychology. He points out those individual schemas are very important in the formation of conceptual structures. If the early schemas were inappropriately made the learner may later have difficulties with assimilation of more complex ideas. In his argument, Skemp points out that schemas that are well formed are long term. These schemas take into account the bigger picture, not just the immediate task.

This leads to what he calls instrumental and relational understanding. He argues that logical understanding is the ability to connect mathematical symbolism and notation with relevant mathematical ideas and to combine these ideas into chains of logical reasoning (Skemp, 1989:45). Thus if learners have well formed schemas they are able to combine relevant mathematical ideas into logical reasoning, creating a network of ideas that could be called upon and used appropriately.

Focusing on creating a network of ideas, Hiebert and Carpenter (1992:69) suggest that when learning with understanding occurs, a mathematical idea, procedure, or fact that is understood thoroughly is linked to existing networks with stronger or more numerous connections where mental representations are enriched by being connected with a network of ideas (Hiebert and Carpenter, 1992:67). The growth of mathematical understandings involves learners in the construction and/or assimilation of new schemas into existing schemas in order to create a network of reference points, which they in turn use to connect and give meaning to new ideas. In order for learners to construct, link and assimilate their schemas they must be located in social settings where they can construct useful and powerful connections.

Mathematics is presented and communicated with symbols and words. The symbols system has been invented to model meaning precisely. Skemp (1989:30) argues that learners can, however, operate at the superficial level only; that is, they can use the symbols without entering into their meaning. Skemp also stresses the necessity for learners to delve into the semantics of mathematics if meaning and understanding are to be constructed. Communication seems to play a major role in influencing understanding in mathematics. This is supported by Vygotsky's (1978:29) theory which asserts that an individual's thinking is formed through internalised conversations devised from interactions with others.

Advocating the perspective that language is a powerful factor in the accommodation of individual and collective thinking, Vygotsky (1978:29) argues that an individual constructs his/her own understanding through the use of language and social interactions. He describes a child's cultural development as appearing on two planes: firstly, on the social plane between two people as an inter-psychological category and then on the psychological plane; that is, within the child as an intra-psychological category. What happens within the intra-psychological stage supports the learner to move into the inter-psychological stage where the learner accommodates the learning into existing schemas, creating a network of ideas (Hiebert and Carpenter, 1992:67). There is an emerging consensus recognising that schemas and networks relate to how learners construct and retain knowledge, that some are more coherent than others and that effective mathematics understanding depends on how educators recognise and accommodate this in their pedagogy. This is, however, not the whole story, as we now need to look at the role of meta-cognition in the construction of mathematical understanding.

2.12 Conceptual and Theoretical Location of Study

Learning is a process in which children must be actively involved using concrete material. Textbooks, by their nature, are iconic (two-dimensional) or symbolic (one-dimensional). They cannot provide the enactive (three-dimensional) experiences that learners need. Programmes need to provide activities that reflect how youngsters learn, including the use of three - dimensional materials.

There are three major implications of the theories proposed by the cognitive psychologists (Flavell, 1976:231):

- Knowledge is actively constructed or invented by learners, it either fits into what is known (an existing framework) or it requires alteration of what is known (a new framework);
- Learners construct understanding by manipulating materials and ideas physically and mentally and by reflecting on those manipulations, relationships and patterns to make generalisations and abstractions and to integrate those into existing cognitive structures;
- The social dialogue of ideas among learners and between learners and educators supports learning. When learners explain ideas, they clarify and solidify their own understanding. When they listen to others they evaluate, compare, learn and become aware of other ways of thinking. When educators listen to learners they hear how ideas are

understood, evaluate where learners are in the learning process and are better able to make decisions about next moves.

Currently, many educationists conceive of learners as architects building their own knowledge structures. The view of the learner has changed from that of a passive recipient of knowledge to that of an active constructor of knowledge. Current learning perspectives incorporate three important assumptions:

* Learning is a process of knowledge construction, not of knowledge recording or absorption;
* Learning is knowledge-dependent; people use current knowledge to construct new knowledge;
* The learner is aware of the processes of cognition and can control and regulate them; this self-awareness, or meta-cognition significantly influences the course of learning (Flavell, 1976:231). Rather than passively receiving and recording information, the learner actively interprets and imposes meaning through the lenses of his or her existing knowledge structures.

Piaget (1970:23) argued that the progressive organisation of geometric ideas in children follows a definite order and this order is more experiential (and possibly more mathematically logical) than it is a reflection of the historical development of geometry. For example, although topology is a recently developed area of mathematics, Piagetian research suggests that for learners, topological relations, such as connectedness, enclosure and continuity are formed first. After this come the ideas of rectilinearity (such as the outline of objects) associated with projective geometry. Finally, the child is ready to acquire Euclidean notions of angularity, parallelism and distance. At best, this suggested learning sequence has received mixed support from research. The available evidence indicates that all types of geometric ideas appear to develop over time, becoming increasingly integrated and synthesised as children progress.

In recent literature on the role of Information and Communications Technology in education, computers are seen as particular cognitive technologies. Media helps transcend the limitations of the mind (for example, attention to goals, short-term memory span) in thinking, learning and problem-solving activities (Pea, 1987:91). The view of the learner has changed from that of a passive recipient of knowledge to that of an active constructor of knowledge. Learners construct understanding by manipulating materials and ideas physically and mentally and by reflecting on the knowledge that they will create. The social dialogue of ideas among learners and between learners and educators supports learning. When learners explain ideas, they clarify and solidify their own understanding. Media can help transcend the limitations of the mind (for example attention to goals, short-term memory span) in thinking, learning and problem-solving activities (Pea, 1987: 91)

2.13 Categorisation of Errors

According to Donaldson (1963:183), there are three types of errors made by learners when they deal with problem-solving in mathematics. Although identified nearly fifty years ago, this classification is still valid, and since no newer literature seems to exist, it is used here to highlight these errors. The first is arbitrary error, meaning that the learner frequently applies concepts from real life experiences which fail to take into account the constraints of the problem. Here, the learners overlook a part of the available information, while working on the rest (Donaldson, 1963:201). The learner behaves randomly and fails to take account of the limitations laid down in what was given (Orton, 1983:4). The second type of error is executive error whereby the learner fails to perform the required manipulations. It refers to those involving the inability by the learner to carry out manipulations, though the learner may have understood the theory (Orton, 1983:6).

The third type of error is structural error which occurs when learners fail to understand the basic principle of a problem. It arises from a false expectation about the structure of a problem and a fundamental failure to understand the relationships concerned in the problem or to grasp some essential rule to solution (Donaldson, 1963:185). Structural errors happen when there is a lack of understanding of topics (Orton, 1983:7).

The critical goal in understanding the notion of errors is to support the learner in becoming an effective thinker. It could be argued that changes in the way teaching and learning are conceptualised and the understanding of errors made by learners would lead to a significant transformation of the education process.

2.14 Computers and Learning

Technology affects what learners learn and how learning is accomplished. Educators need to understand and be able to use technology in an ever-growing number of ways consistent, with how people use it outside the classroom (Robinson *et al*, 2000:123). Using computers can change teaching. A study of the effect of introducing computers into mathematics teaching involved 12 New Zealand educators who were involved in the action research. The resulting change in perspective, which for some educators took as long as a year to achieve was, characterised by:

- a lessening of control and greater use of guided discovery learning that made use of discussion and group work;
- a willingness to learn along with the learners;
- a desire to plan lessons involving the computer where its role is a tool for learning;
- an ability to make mathematics and its implications and not the computer, the focus of concern.

An architect will build a three-dimensional model to display the spatial relationships in an intended construction. We would find it difficult to move around our cities and towns without access to maps. The maps, of course, being two-dimensional, distort the three-dimensional world in which we live. Sometimes we draw pictures of real-life situations to make the spatial properties of the situation more obvious or clear; for example, to show that the roof of a house is rectangular. Many learners, of course, have difficulty 'seeing' spatial relationships in real-life situations; they cannot generalise or map their spatial knowledge over the real-life situation.

The nature and role of visualisation and imagery in the teaching and learning of mathematics is complex. Fischbein (1993:144) posits that "a visual image not only organises the data at hand in meaningful structures, but is also an important factor guiding the analytical development of a solution". Bishop (1997:186) concludes his review by saying that “there is value in emphasising visual representations in all aspects of the mathematics classroom”.

Keller *et al* (2002:1) studied the use of Java applets for visualisation of three-dimensional objects in middle and secondary education. In their study, they looked at the effects of use on both learners and educators. Based on the data they had collected they concluded that use of the applets "improve learners’ spatial visualisation skills, as indicated by improved ability to create isometric drawings, connect isometric drawings with other two-dimensional representations of three-dimensional objects; translate among these representations and enhance future educators’ pedagogical content knowledge, as indicated by growth in their own spatial visualisation skills and increased awareness of the teaching and learning issues related to isometric drawings".

The mathematics education literature lends support to learning geometry with pre-constructed shapes by young learners. Research about learning with the ‘‘Shape Maker,’’ a specially designed micro-world for learners at the pre-proof stage of geometrical thinking, is one such example (Battista, 2001:110). In this environment, each class of common triangles and quadrilaterals has a ‘‘shape maker,’’ a Geometer’s Sketchpad construction that can be dynamically transformed in various ways to produce different shapes within that class. Battista found that as learners manipulated and reflected on their manipulations with the ‘‘Shape Maker’’ they abstracted certain actions and integrated these abstractions into a mental model of the ‘‘shape maker’’ that constituted their construction of meaning for the device. Battista (2001:110) found that as the learners worked with the ‘‘Shape Maker’’ they moved from visual to property based thinking and, hence, to thinking that utilised inference to relate and organise both attributes and classes of shapes.

To overcome this problem, it is necessary to teach within what Vygotsky (1978:55) called the ‘‘zone of proximal development’’. When this type of teaching and learning is assisted by software, it is important to rethink the design features that encourage learners to develop better distinctions between geometric attributes and infer what has been preserved during the use of software.

2.15 Conclusion

If views on how learning takes place are examined, no single view of learning will be completely effective for all learners. Cognitive and constructivist learning theories, respectively, emphasise the role of meta-cognition or the self-monitoring of learning and thinking (Shephard, 2000:4) and the idea that knowledge is constructed through a process of creating personal meaning from new information and prior knowledge within realistic settings (McMillan and Schumacher, 2001:12). Educators assist learners to link new knowledge to existing knowledge and develop instructional techniques that would facilitate cognitive growth and change.

Key cognitive processes are examined in assessing a particular concept and, hence, instructional methods are designed to help learners develop these processes. Van de Walle (2004:36) views teaching in this regard as assisting learners to construct knowledge through problem-posing and engaging learners in mathematical discourse so that they may examine their new assumptions about mathematics. Contemporary views underline the idea that the cognitive pre-requisites for mastering mathematics involve more than traditional computation skills. Cognitive development, according to Troutman and Lichtenberg (2003:10), involves internal representations and external representations. They argue that conceptual learning occurs if children build internal representations composed of networks of concepts and relationships that mirror desired external representations.

Troutman and Lichtenberg (2003:407) warn against explaining, showing and telling, implying that this is procedural teaching. Mastering space and shape concepts in geometry offers opportunities to practise logical reasoning and to acquire abilities in various reasoning patterns. They argue that through the listing of properties and classifications, learners begin to build concepts enabling them to develop the spatial sense to function in their environment. Reasoning skills are necessary to advance from a procedural to a conceptual approach.

Cognitive development in the learning of geometry has been a major focus of research. Piaget argues that the development of learners' concept of space progresses through various stages of acquisition, representation and characterisation of spatial concepts. He considers this development as a maturation process (Geddes and Fortunato, 1993:200). The van Hiele model, on the other hand, suggests different levels of thinking, focusing on experience through different phases of learning (van Hiele,1986:25). These phases may be recursive and are not necessarily achieved in a linear Piagetian progression. Contemporary views (van de Walle, 2004:348) support the van Hiele levels of geometric thought which propose a five level progression towards the understanding of spatial ideas.

Some of the problems experienced in space and shape in geometry classes were:

respondents had difficulty in representing characteristics of and relationships between two-and three-dimensional objects; three-dimensional activities were specifically experienced as problematic; if geometric objects were placed in a variety of orientations and positions, respondents experienced problems in analysing and solving problems and problems related to viewing objects from different angles revealed difficulties.

Chapter Three will concentrate on the research design and methodology. The various processes involved in the design of the study will be addressed.

CHAPTER 3: RESEARCH DESIGN AND PROCESS

3.1 Introduction

This chapter highlights the research design and process of this study. There are three common research designs, which dominate the educational landscape. Reeves and Hedburg (2003:25) have identified them as the quantitative, qualitative and the eclectic-mixed mode pragmatic research model. Researchers showing preference to work in the quantitative paradigm present their results primarily in the language of numbers. In a quantitative paradigm, the purpose is “not to report data verbally, but to represent those data in commercial values” (Leedy and Omrod, 2001:143). Data collected in a quantitative paradigm is usually analysed using inferential and descriptive statistical techniques.

Qualitative research uses *inter alia* a case study design meaning that the data analysis focuses on one phenomenon, which the researcher selected to understand in depth regardless of the number of sites or participants for the study (McMillan and Schumacher, 2001:398). This study utilised both qualitative and quantitative aspects. The qualitative approach enables a better understanding of the experiences, opinions and perceptions of participants. The qualitative research approach elicits participant reports of meaning, experiences or perceptions. The quantitative approach uses certain statistical methods. These include frequency counts and the use of tables and graphs to illustrate averages, distribution of scores and difference and hypothesis testing.

Crotty (1998:6) argues that in developing a research design, the researcher should answer two basic questions: firstly, what methodologies and methods will be employed in the research and, secondly, how this choice and use of methods and methodologies are testified? The second question deals not only with the purpose of the research but also with the researcher’s understanding of theoretical perspective and about what human knowledge is and what is the epistemology. This study design was structured according to Crotty (1998:5) research processes. Thus, the two initial questions have expanded into four: what epistemology is embedded in the theoretical perspective? What theoretical perspective lies behind the methodology? What methodology controls our choice and use of methods? What methods are proposed to be used? These four elements are presented separately because each element is substantially different from the other.

3.2 Research Methods

Methods refer to the techniques or procedures used to gather and analyse data related to the research questions. The researcher used qualitative methods of collecting data (questionnaires given to the learners). In this qualitative approach based on constructivist perspectives, the researcher collected open-ended emerging data with the primary intent of developing themes from the data. In the model followed here the distinction between qualitative and quantitative research occurs at the level of methods.

According to Holloway and Wheeler (2002:3), qualitative research is a form of social inquiry that focuses on the way people interpret and make sense of their experiences and the world in which they live. Quantitative research aims at testing a hypothesis; the approach is context free and the research is often conducted. Data collection methods include questionnaires and interviews and the outcomes of the research have measurable results (Creswell, 2002:62; Holloway and Wheeler, 2002:16).

The methodology followed in this research can be described as a case study**.** Case study research is a methodology in which the investigator explores a bounded system within its real life context over a sustained period of time through data collection involving multiple sources of information in order to gain an in-depth exploration and analyse systematically the manifold phenomena that constitute this system. A single instrumental case study was conducted: the research was focused on an issue and selected one bounded case to illustrate this issue. Case study research provided a way to study and describe the behaviour of individual learners within their social context (the classroom) over a period of time, allowing different methods of data collection to be incorporated.

Constructivism helps a case study researcher to justify many narrative descriptions since the emphasis is on detailed description and interpretations of the people comprising the case. One of the characteristics of case studies and qualitative studies in general that is most criticised is the lack of external validity or generalisability because of the small number of research units. However, the aim of this study was not to generalise findings to schools but to obtain a deeper understanding of the implementation of Poly as experienced by the participants.

3.3 Sampling

The participants were selected through a process of purposive sampling. The sample was composed of elements which contained the most characteristic and representative attributes of the learner population (McMillan and Schumacher, 1997:25). Age was not a significant factor in the sampling process owing to the participants being in the same grade. It followed naturally that learners in grade seven would fall in the 11-12 year age group. However, the researcher had to ensure that there was fair representation as far as gender was concerned as studies have shown that there is a discrepancy in performance in mathematics between boys and girls (The Annual Report of Her Majesty's Chief Inspector of Schools, 2005:29). Samples of twenty learners each were selected for the experimental and control groups respectively. Two groups of grade seven learners were selected through purposeful sampling, with one group exposed to traditional teaching methods and the other using the computer. The learners were chosen randomly.

Learners in the control group were taught using the traditional method. The traditional instruction method in this study was lessons given by an educator, use of textbooks and other materials and a clear explanation of procedural knowledge and conceptual knowledge of two-and three- dimensional shapes to learners. The educator demonstrated two- dimensional and three-dimensional shapes using the chalkboard and the textbook. The learners did not have any tasks that made use of representations on computers (see Table 4).

Method	**Computer- Based Instruction (*n* = 20)**	**Traditional Instruction (*n* = 20)**
Instructional content	The instruction included lessons on two-and three- dimensional concepts.	The instruction included lessons on two-and three- dimensional concepts.
Forms of instruction	The learners worked in pairs on a computer.	The learners worked in groups without using any computers.
Learning environments	Learners received instruction using computer-assisted instructional software.	Learners used the traditional classroom.

Table 4 Computer-based Instruction vs Traditional Instruction

Both experimental and control participants were from disadvantaged socio-economic communities with minimal or no access to computers. However, for the purposes of this research both groups of participants had the same level of computer operating competence which was based on their exposure to computers during their schooling. Furthermore, the participants received negligible academic stimuli from their immediate families. The teaching methods employed at the school were strictly traditional and adhered to the Department of Education guidelines.

The research involved a seventh grade class in a public primary school in Verulam, KwaZulu-Natal. The school was selected because of its easy accessibility and acceptance by management to conduct the research. Forty learners were selected for this study (22 boys and 18 girls), aged from 11 to 13 years. The research followed various stages: sending the proposal of the research and getting permission from the KwaZulu-Natal Department of Education; getting permission from the principal of the school and the parents of the learners involved (Table 5).The study was designed such that it offered the least disruption to the normal academic programme.

RESEARCH STEPS	**METHOD**	**PARTICIPANTS**
Step 1	Questionnaire	Experimental and control group
Step 2	Class exercises / pre-test on statistics/ nature of errors	Experimental and control group
Step 3	Post-tests	Experimental and control group
Step 4	Computer activities	Experimental group
Step 5	Interview	Experimental group

Table 5: Methods of Data Collection

Step 1: The research questionnaire (Appendix F) was handed out to both the experimental group and the control group so as to garner responses from the participants prior to the study itself. The learners' understanding of mathematics, technology use and data handling strategies and skills are vital to this study.

Step 2: In introducing the aspect of two-and three-dimensional shapes, both groups of participants were involved so as to assess prior knowledge and teach the aspects that are relevant to the grade seven syllabi (Appendix D).

A "paper and pen" test was administered to serve as a pre-test to all grade seven learners before the sample selection to determine the baseline mathematical knowledge of the learners. Thereafter, the learners were randomly divided into two groups. Both the experimental and the control group consisted of twenty learners each. This was followed by an intervention programme conducted with the experimental group, where the software programme, Poly, was used. The post-test was administered to the groups at the conclusion of the contact session. Individual questionnaires were given. A test was designed to elicit the background knowledge of the learners. The questions were based on the curriculum.

Step 3: Tests (Appendix A) based on the lessons taught in step three were carried out to ascertain the learners' understanding of the topic.

Step 4: The experimental group was involved in the computer-based exercises (Appendix I).

Step 5: The researcher conducted the interview with the experimental group (Appendix E).

The experimental group learners were involved in classroom work and interactive group work without having any experience of Poly prior to their involvement in the project. Learners worked autonomously and engaged in collaborative learning and group work. It was an opportunity to exercise cognitive and social development, flexibility and organisation for the teacher, the ability to co-operate, discuss and negotiate with others. Learning in small groups can improve achievement and mental functions. In establishing pairs, learners chose their own partners and remained constant throughout the lessons.

Another consideration was the extent to which the experimental group were competent in using the computer. Both the experimental and the control group have had at least four years operating experience with computers; therefore, it was inferred that the progress made by the experimental group regarding the relationship with the Poly software was due to the influence of the software and not their competence in operating the computer. During the lessons, the experimental group not only experienced a presentation of Poly as a demonstration tool but also interacted with it. A screenshot of one of the tasks that the learners were exposed to is provided below. One can observe that the programme allowed the learners to manipulate the different orientations of the shape presented (see figure 8).

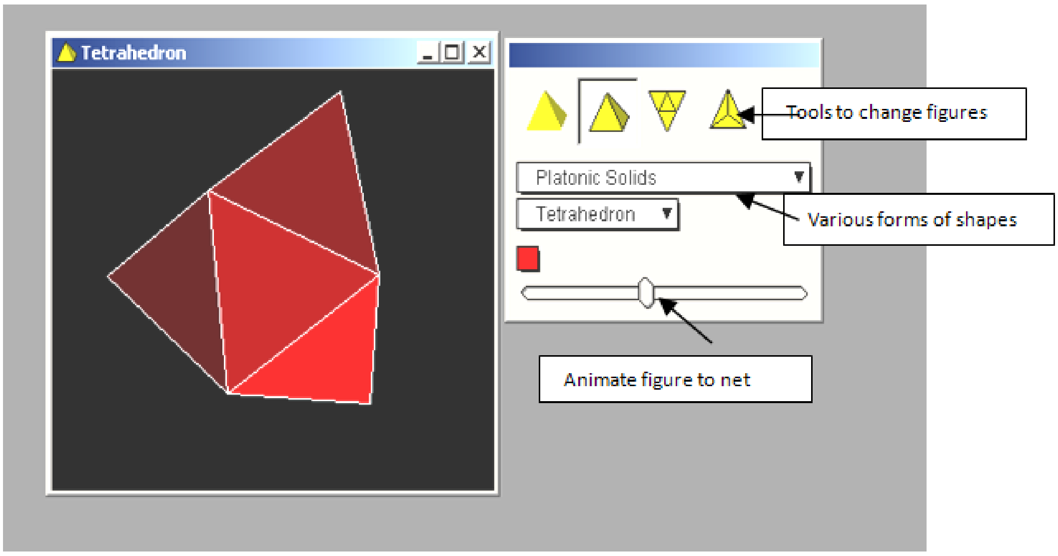


Figure 8: Screenshot from a Poly window with the various nets of solids unfolding

Both the co-operative and the individualised learning sessions included three parts: teacher introduction to the whole class (about 10 minutes), co-operative or individualised work (about 20-25 minutes), and teacher review with the whole class (about 5-10 minutes). This was done over a period of 4 weeks.

- Introduction: the key concept was introduced to the learners. At the end of this phase, the teacher set up the problem.
- Practical session: learners engaged in activities on the computer in order to explore the different orientation of the nets of the solids.
- Plenary: this is where the new material was discussed; learners discussed what they had discovered and the teacher tried to reinforce and extend learners' learning; the learners' results of their experiments with Poly were the basis for discussions in class; the teacher's intervention at this stage aimed to help learners understand what they had discovered in order to make richer meanings.

3.4 Research Instruments

For the purpose of this study, three research instruments were used for collecting data. These included questionnaire, an interview and activities on nets of solids using the software Poly. The instruments used for the research consisted of a test which was administered to the learners as pre-and post-test to the two participating groups. It was also administered as a delayed post-test to the experimental group. The test consisted of two types of questions. The first type of 25 questions dealt with matching two-dimensional plane figures. The second task required learners to study nets of given platonic solids and to indicate which would fold into three-dimensional solids.

At the commencement of the research project, the pre-test was administered to both participating groups in March 2009. Then an intervention programme was conducted with the experimental group. At the conclusion of the contact session, the post-test was administered to the two groups whilst the experimental group also wrote a delayed post-test.

3.4.1 Questionnaire, Measuring Instrument and Interviewing Schedule

The researcher used the questionnaire as it had less bias possibilities. For testing purposes, a paper and pencil test of two-dimensional geometry questions and nets of solids were designed by the researcher. Individual interviewing was carried out through self-completion questionnaires. This was because the researcher was less likely to influence the responses of the subjects, for example, by asking questions in a particular tone which may lead the respondent towards a particular line of thought. The questionnaires therefore ensured anonymity, were cost effective and permitted data collection from a large sample (Gay, 1992:27). In spite of the advantages of the questionnaire as outlined above, the researcher was aware of its inherent limitations which included the respondents being unable to clarify issues when completing the questionnaire. This was because the researcher could not “face-to-face” probe the respondent to clarify issues. Pre-test intervention and post-test designs are uniquely appropriate for investigating the effects of educational innovations and are commonly used in educational research (Dugard and Todman, 1995:15).

With regard to the questionnaire design, instructions to the respondents were simple and concise. This was aimed at facilitating the completion of the questionnaires. The researcher made every attempt to ensure that the questions were simple and straight forward so as to avoid ambiguity.

The questionnaire was administered to the experimental and the control group and was divided into four categories. Section A was designed to elicit biographic information from the respondents. These included questions on age and gender. Section B included both open and closed-questions that looked at the concepts of two-and three-dimensional shapes. Questions were also structured to elicit responses about learners' prior knowledge and if and how it influenced their ability to recall definitions of shapes. The open-ended questions were designed for the respondents to express their ideas and feelings about specific issues. It was the researcher's intention for these questions to provide "rich information" to enhance the findings of this study. Section C questions comprised closed-questions about specific aspects related to learners' perceptions about using computers. Section D category consisted of two questions which were aimed at obtaining a list of additional mathematical concepts that learners may have learned during their activities.

Questionnaires were given to the learners because a questionnaire is more reliable than other methods of collecting data. A questionnaire is anonymous and it encourages honesty. In order to ensure the validity and reliability of the questionnaire, a pilot study of the questionnaire was undertaken. The aim of the questionnaire pilot study was to simulate the real thing as closely as possible by using a similar population and setting up the same conditions for administration and response in order to examine how long it takes to answer the questionnaire and if the questions are clear or need further explanations. The pilot study of the questionnaire helped to make the language and syntax of the questions less complex in order for the questionnaire to be more clear, comprehensible, reliable and valid. The questionnaires were given to the learners one day after the completion of the lessons; they were brief, easy to understand and reasonably quick to complete. They included a small introduction followed by open-ended questions.

3.4.2 Interviews

The use of the interviews in research shows a move away from seeing participants as being manipulated and data as somehow external to individuals and towards regarding knowledge as generated between humans, often through conversations. Influenced by constructivism, knowledge is seen as constructed between humans and interviews can serve as a significant tool in this process of construction. An interview can be regarded as a change of views between two or more people on a topic, enabling verbal, non-verbal, spoken and heard channels to be used (Yin, 2003:132).

An interview can be defined as a conversation between two people, which begins with the interviewer, with the purpose of collecting data relevant to their research and focuses on content which is determined by the researcher’s goals of the research. However, an interview cannot be considered an ordinary conversation since it has a specific purpose; it is based on questions asked by the interviewer and the responses have to be as explicit as possible.

Semi-structured interviews were used in this research since they were flexible, allowing new questions to be brought up during the interview and that helped the researcher gather data sought. During the interviews, the researcher’s aim was to establish whether the learners’ had achieved the objectives and if the use of Poly had facilitated learning. The researcher wanted to gather information about positive and negative aspects concerning the use of the software which was used during the lessons; how the lessons conducted differed from the traditional mathematical lesson; whether Poly contributed in achieving the objectives of the lessons and to what extent these objectives would be achieved if the researcher had not used it.

Bearing in mind these points, researcher had some probing questions that would get the interviewees to expand their responses. Open-ended questions were used during the interview since the researcher did not want the interviewees to be constrained in their answers. Open-ended questions are flexible, show the limits of the interviewee's knowledge, encourage co-operation between the interviewer and the interviewee, allow the interviewer to have a more informed account of what the interviewee supports and can produce unexpected answers (Yin, 2003: 135).

The interviews were recorded on tape as recording is convenient and also assists in reducing distraction to both the interviewer and the participants; furthermore, the interviewer does not have to impede the flow of discussion to make copious notes. In addition, "Interviews recorded on tape may be replayed as often as necessary for complete and objective analysis at a later time" (Best and Kahn, 1989:202). In other words, reliability checks can be facilitated.

The researcher was aware that the presence of a tape recorder could cause some uneasiness amongst certain participants. However the participants were guaranteed anonymity and did not see this as a threat. Although the interviews were audio recorded, brief notes were also taken on paper during the interviews by the researcher to record non- verbal cues. Data consisted of voice-recorded individual interviews conducted with the learners and questionnaires completed by each learner separately. The voice data from the interviews were transcribed manually into a written form while learners' responses to the questions were documented more systematically.

After each question, space was provided for learners to give detailed responses. These questions were used in order to collect in-depth data about learners' experiences and beliefs while avoiding the limitations of pre-set categories. Learners were afforded an opportunity to decide what they wanted to report without much prompting. Thus, the researcher had the chance to explore and generate items that could not be predicted otherwise, as open-ended questions can catch the authenticity, richness and depth of response that would not be captured with other forms of questions. Nevertheless, open-ended questions might lead to irrelevant, superficial and redundant information and require more time for the responder to complete the questionnaire and more data handling for the researcher. By pilot testing the questionnaire, the disadvantages were limited (Best and Kahn, 1989:204).

3.4.3 Activities on nets of solids

The learners in the experimental group and control group had to complete a worksheet.

The activities comprised the following sections:

- Matching two-dimensional shapes
- Selecting the correct nets of the cube
- Choosing the correct nets of the tetrahedral
- Selecting the correct nets of the octahedral

3.5 Pilot Study

Robson (2002:130) refers to a pilot study as a small-scale version of the real research. It is a trial of the proposed research to check its feasibility. Yin (2003:96) views pilot tests as helping the researchers to improve their data collection plans regarding the content of the data and the procedures to be followed. To determine the suitability of the items, a pilot test was held in which the lessons designed were taught informally. The participants were learners from another grade who did not participate in the research, had never used Poly before and were selected in a purposeful way. In the pilot test, which lasted three days, these learners were taught the lessons as they were designed. As a result of the pilot test, a number of changes to the research design were made.

3.6 Analysing the data

The techniques employed to analyse the results were statistical computations involving Microsoft Office Excel, Wessa and Statistical Package for Social Science programmes. Analysis of the data was carried out predominantly through content analysis of the responses from the open-ended questions in the questionnaires and answers in the activities. Closed-questions in the questionnaires were analysed through frequency counts and the use of tables and graphs to illustrate averages, distribution of scores and difference.

Reliability and validity of an instrument are not independent from one another. If the instrument is unreliable, it cannot be valid because it cannot validly measure an attribute or construct if it is inconsistent and inaccurate - consistency refers to reliability. However, an instrument can be reliable without being valid (Polit and Beck, 2008:458). Reliability refers to the consistency or dependability with which an instrument measures the attribute it is designed to measure. Validity refers to the degree to which an instrument measures what it is supposed to measure. In this study, it implies that the questionnaire should contain items which test quality assurance practices and activities to ensure quality in an educational institution.

The two main threats to the validity of observation and interview study according to Airasian and Gay (2003:213) were observer bias and the observer effect. Observer bias refers to the invalid information that results from the perspective a researcher brings to a study. It occurs when a researcher consciously or unconsciously interprets data based on attitudes or beliefs held prior to the research. The observer effect occurs when a researcher’s presence leads participants to behave atypically. Airasian and Gay (2002:224) suggest that the way to handle observer bias and observer effect is to make the researcher aware of it so that they can be as inconspicuous as possible.

According to Airasian and Gay (2002:224), an interview is a purposive interaction between two or more persons, with the one (the researcher) trying to obtain information from the other (participant). Interviews permit a researcher to obtain information that cannot be obtained from observations. Qualitative interviews are free flowing and open-ended, with the interviewer probing to clarify and extend the participant's comments. Airasian and Gay (2002:224) strongly encourage transcribing, as transcripts are the interviewer's field notes and become the data the researcher will analyse. According to Cohen *et al,* (2000:305), unstructured observations are used when the researcher is less clear on what he or she is seeking.

The purpose of analysing qualitative data was to determine the categories, relationships and assumptions that inform the respondents' view of the topic studied. The following analysis was a descriptive narrative with issues raised throughout. The analysis began with the transcription of the interviews, observations and questionnaires. The research involved an experimental and control group with pre-and post-tests being administered to ascertain whether a shift in learners understanding had occurred. There were two groups of 20 learners each in the experimental and control groups. The experimental group spent equal amounts of time working with the manipulation of the platonic solid. Learners from a grade seven class were selected. The names of the learners were assigned numbers and the computer randomly chose these learners. The research method includes the collection of qualitative data by means of learners' questionnaire to establish their perceptions of the classroom environment during the intervention. A variety of other data collection methods were introduced, including observations and interviews with selected learners.

3.7 Ethical Issues

Many learners come from less affluent or less richly resourced backgrounds; all learners who enter school do not have the same computer skills. Thus, some learners with higher digital skills used the software with more expertise than others. In order to limit this disparity, all learners who participated in the research had a training lesson in Poly. As stated before, Poly is freely-available, open-source software. However, the most important ethical issue in this research concerns confidentiality and anonymity (See appendix B). Participants were assured about the confidentiality of the data and protection of identities. Although there was a form of intrusion into the teaching, the researcher had sought consent from the Department of Education, the Principal and from the learners' parents. These people were provided with adequate information (a written statement that outlined the nature, purpose and procedure of the research, what data would be gathered and how it would be used) about the research.

3.8 Conclusion

This chapter concludes an investigation into the research design and process of the effectiveness of computer-aided teaching on the quality of learning geometry in mathematics by grade seven learners. A variety of aspects of the research design have been underlined. The research approach and sampling were highlighted. Discussion of the empirical investigation dealing with classroom observation, questionnaires and interviews was done. The procedure, data collection, presentation and analysis as well as the ethical issues and limitations were set out.

In the next chapter, discussions of the findings will be conducted. The results and analysis of the findings will be examined in detail.

CHAPTER FOUR: PRESENTATION AND DISCUSSION OF FINDINGS

4.1 Introduction

The purpose of this chapter is to present and discuss the findings emerging from the data collected in this study. As indicated in Chapter Three, the questionnaires were administered to the experimental group and the control group. Also, as already indicated, the questionnaires comprised a series of closed-and open-ended questions. The closed-questions were analysed through frequency counts and percentages. The percentages were rounded-off to one decimal place and, where necessary, graphic representation is given. The open-ended questions in the questionnaires and the interviews were addressed through elementary content analysis. Learners' efforts in the questionnaires were assigned one mark for the correct response and no marks if the response was incorrect. The presentation of findings from such analysis is supplemented with direct quotation. The presentation of the findings is discussed in the context of the relevant literature, as outlined in Chapter Two.

The findings are categorised as follows:

4.2 Experimental Group Results

4.2.1 Experimental group profile

This was done to elicit the gender composition and ages of the participants.

4.2.2 Pre-Tests: Analysis of Errors in Pre-concepts

This was important in ascertaining the errors made by the learners of the pre-knowledge concepts.

4.2.3 Qualitative Analysis of Two-and Three-Dimensions of Cubes, Tetrahedra and Octahedra

Probing what caused the learners' misconceptions of two-and three-dimensions was undertaken.

4.2.4 Learners' Perception of the Concepts of Two-and Three-Dimensional Shapes

How did the learners understand two-and three-dimensional shapes?

4.2.5 Comparison of the Pre-and Post-Test of Nets of Cube

Did the computer intervention make an impact on the understanding of the nets of the cube?

4.2.6 Comparison of the Pre-and Post-Test of Octahehra.

Was there an improvement in the results with the use of Poly?

4.2.7 Qualitative Analysis of the Four categories for Pre-and Post-Test

Which category improved the most with the use of the computer?

4.2.8 Correlation: Analysis for Pre-and Post-Test.

This was carried out to determine the measure of relationship between the pre-and the post-test.

4.2.9 Measures of Central Tendency.

The measure of central tendency was done to find a single value that attempted to describe the set of data by identifying the central position within that set of data.

4.2.10 Testing of Normality of Pre-and Post-Test using histogram.

The histogram was done to identify whether the results followed a normal bell curve.

4.2.11 Hypothesis Testing (Mann-Whitney Test) of Pre-and Post-Test.

The Mann-Whitney Test was used to test whether two independent samples of observations were drawn from the same distribution.

4.3 Control Group Results

4.3.1 Control Group Profile

This was done to elicit the gender composition and ages of the participants.

4.3.2 Pre-Test: Analysis of Errors of Pre-concepts

This was important in ascertaining the errors made by the learners of the pre-knowledge concepts.

4.3.3 Post-Test: Qualitative Analysis of Two-and Three-Dimension of Cube, Tetrahedra and Octahedra

Probing what caused the learners not to understand the underlying concepts of two-and three-dimensions was undertaken.

4.3.4 Comparison of the Pre-and Post-Test of Nets of Cube

Did the computer intervention make an impact on the understanding of the nets of the cube?

4.3.5 Comparison of the Pre-and Post-Test of Nets of Octahedra

Was there an improvement in the results with the use of Poly?

4.3.6 Qualitative Analysis of the Four Categories of Tasks for the Pre-and Post-Test.

Which category improved the most with the use of the computer?

4.3.7 Measures of Central Tendency

The measure of central tendency was done to find a single value that attempted to describe the set of data by identifying the central position within that set of data.

4.3.8 Testing of normality of Pre-and Post-Test Using Histogram

The histogram was done to identify whether the results followed a normal bell curve.

4.3.9 Hypothesis Testing (Mann-Whitney Test) of Pre-and Post-Test

The Mann-Whitney Test was used to test whether two independent samples of observations were drawn from the same distribution.

4.4 Comparison of the Control and Experimental groups

4.4.1 Comparison of the Post-Test of Two Dimensional Shapes

This was done to compare the post-test of the experimental and the control groups.

4.4.2 Comparison of the Post-Test of Cube

The experimental and the control group’s post-test of the cube were examined to find which group improved their scores.

4.4.3 Comparison of the Post-Test of Tetrahedron

The experimental and the control group’s post-test of the terahedra were examined to find which group improved their scores.

4.4.4 Comparison of the Pre-and Post-Test of nets Octahedra

The experimental and the control group's pre-and post-test of the octahedra were examined to find which group improved their scores.

4.5 Qualitative and Quantitative Analysis of the Interview

This was done to verify the results of the questionnaire.

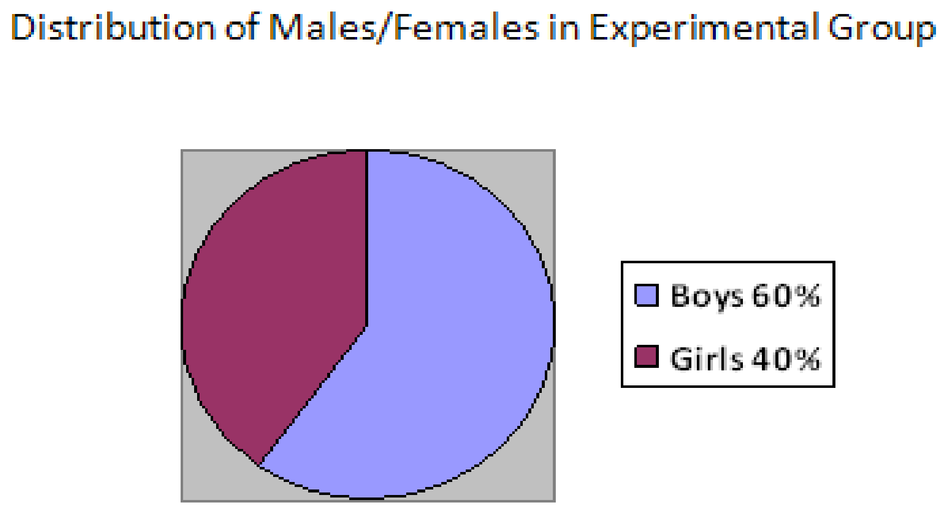
4.6 Conclusion

The detailed discussion of the above follows:

4.2 Experimental Group Results

4.2.1 Experimental Group Profile

Section A provided biographical details regarding the learners. This was done to ascertain the Piaget (1970:70) level of the learners. The following graph (See Graph 3) indicates the gender of the respondents.



Graph 3*:* Distribution of males and females in the experimental group

A total of 12 males and 8 females responded to the questionnaires. In other words, 60 percent of the respondents were male and 40 percent were female. The distribution was in line with the gender distribution of learners in this grade in the sample school. The distribution might imply that findings in this study may be transferable to other primary schools of similar contexts.

Furthermore, the Annual Report of Her Majesty's Chief Inspector of Schools (2005:21) suggests that we ought to be concerned about the more limited progress of girls in mathematics, relative to boys. Therefore, it is important to note the proportion of males to females in a study (See Table 6).

Age (in years)	**12**	**13**	**Total**
Number of Learners	12 (60 percent)	8 (40 percent)	20

Table 6: Age distribution of the experimental group learners.

It was gleaned from table 2 that the entire sample in the experimental group fell within Piaget's formal operations (12 and above) where thinking involves abstractions (Saler and Edgington, 2006:129). According to Piaget, children can contemplate and understand abstract problems in this stage (Moursund, 2006:56). However, data from this study has indicated otherwise and will be discussed in greater detail.

4.2.2 Pre-Tests: Analysis of Errors in Pre-concepts

Pre-tests were given to the learners to determine the errors that they made in terms of three categories. The following concepts were tested; the properties of squares, the properties of cube, two-dimensional shapes, sorting out various three-dimensional shapes and identifying nets of a cube. This was done to ascertain the extent to which the learners understood two-and three-dimensional shapes (Refer to table 7).

The table provides a summary of the different type of errors that were made by the experimental group.

Type	Items	Error or misconception
Arbitrary error	What is meant by parallel lines? What is meant by perpendicular lines?	The learners confused attributes of shapes with their properties.
Structural error	How many edges can you see? How many vertices can you see? Does it have a width?	The learner was unable to define , count and identify edges, faces and vertices.
Executive error	Does it have a height? Does it have a length? Imagine unfolding a cube so that all of its faces are laid out as a set of squares attached at their edges. The resulting diagram is called a net for a cube. Which of the following will make a cube?	Unable to identify different two-dimensional nets that can be folded into a three-dimensional cube. Learners confused the names of components of three- dimensional shapes. They looked at the shape and found that nets do not fold properly because they did not take into account the matching congruent edges. Think the way a shape is oriented is part of what defines it.

Table 7: Exemplars of errors

4.2.3 Qualitative Analysis of Two-and Three-Dimensions of Cubes, Tetrahedra and Octahedra

CLASSIFICATION OF ITEMS	**ARBITRARY ERRORS**	**STRUCTURAL ERRORS**	**EXECUTIVE ERRORS**
Properties of squares	30 percent	40 percent	20 percent
Properties of Cube	30 percent	30 percent	30 percent
2-Dimensional Shapes	10 percent	0 percent	0 percent
3-Dimensional Shapes	10 percent	10 percent	10 percent
Nets of a Cube	20 percent	50 percent	30 percent

Table 8: Classification of errors for the experimental group

Table 8 displays the three types of errors that occurred in the post-test of the experimental group. It is evident that the learners are able to identify two- and three dimensional shapes. This concurs with van Hiele level 1. The structural error of the nets of the cube is high because learners find it difficult to disembed the squares in different orientations.

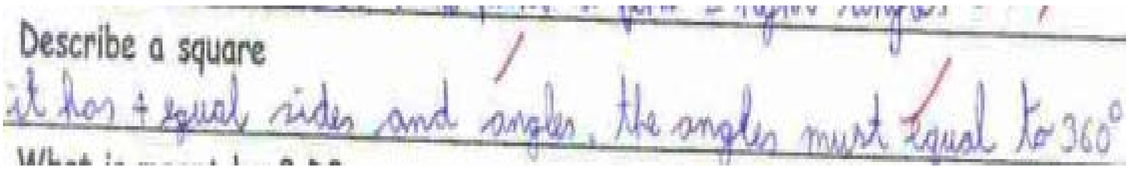
4.2.4 Learners' Perception of the Concepts of Two-and Three-Dimensional Shapes

This section was concerned with the learners' perceptions of the concepts of two-and three-dimensional shapes, nets of solid shapes and learners' prior knowledge and if and how it influenced their ability to visualise three-dimensional shapes.

<u>Qualitative and Quantitative Analysis of Question on Properties of Squares (Appendix G)</u>

The square was selected as it represented the core understanding of a cube. This was based on the concept of plane geometric shapes. The idea of determining what the learners recall about properties of polygons was paramount to their understanding of two-and three-dimensional shapes. 40 percent of the experimental group failed to make relationships with the equal sides and that each angle was a right angle. It was observed from table 5 that these errors were primarily structural errors. Thirty percent of the learners from the experimental group made arbitrary errors; they failed to state the correct characteristic of the squares, thus impeding their understanding of three-dimensional shapes.

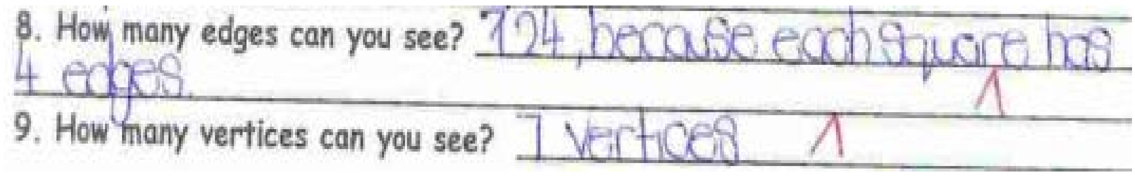
A response from a learner is shown below to indicate his understanding of a square. This is van Hiele level two. Learners needed to have an in depth knowledge of a square in grade seven, that is, in the 12 year range.

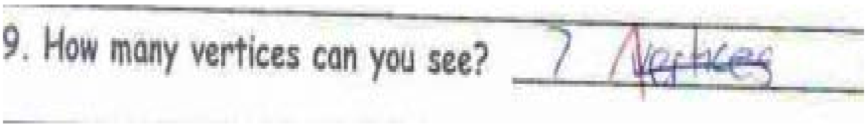

Describe a square
it has 4 equal sides and angles, the angles must equal to 360°

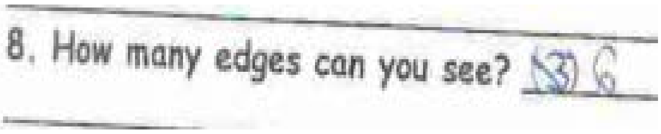
The required frames were sketchy and vague. Clearly, learners needed help with understanding the properties of a square.

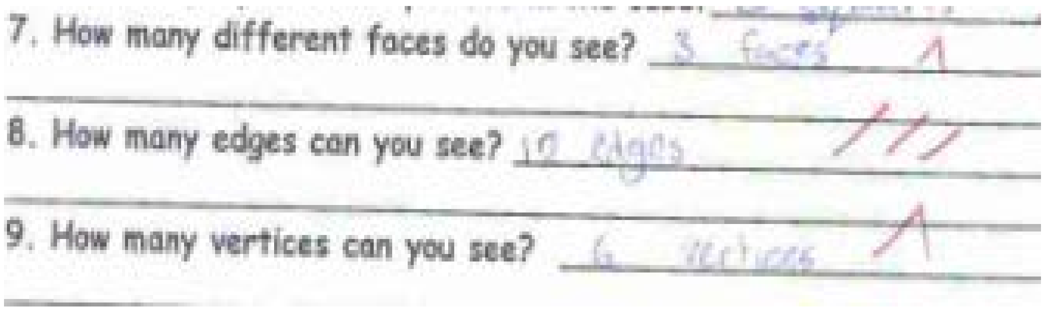
Qualitative Analysis of Properties of a Cube (Appendix G)

This was based on the concept of a regular platonic solid with six equal square faces. It was apparent that the learners did not grasp this meaning. Clearly, learners were unable to retrieve the frame faces, edges, vertices, length, height and width.


8. How many edges can you see? 7 24, because each square has 4 edges.
9. How many vertices can you see? 7 vertices

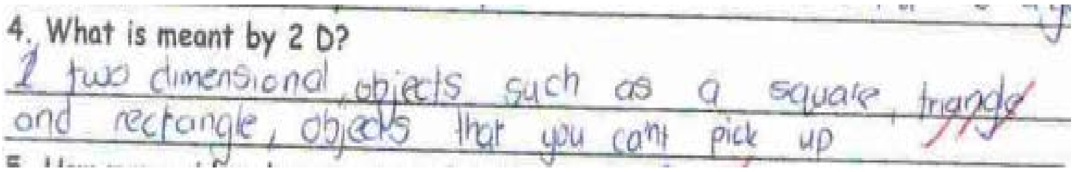

9. How many vertices can you see? 7 vertices


8. How many edges can you see? (3) 6


7. How many different faces do you see? 3 faces
8. How many edges can you see? 12 edges
9. How many vertices can you see? 6 vertices

Qualitative and Quantitative Analysis of Two-dimensional Shapes (Appendix G)

This item was concerned with identifying two-dimensional shapes. This was done to see whether learners had a background knowledge of two-dimensional shapes. Ten percent of the experimental group made arbitrary mistakes. An interesting observation was that these learners had a good understanding of two-dimensional shapes. A response from one of the learners is given below.



<u>Qualitative and Quantitative Analysis of Three-dimensional shapes. (Appendix G)</u>

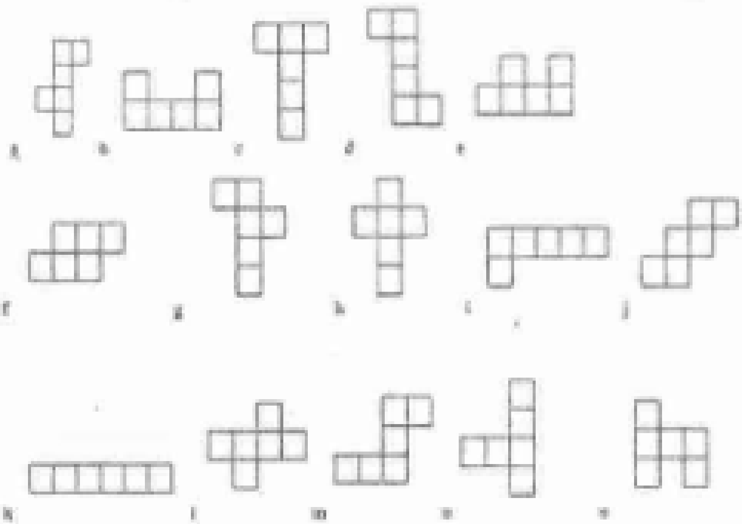
Ten percent of the experimental group made structural errors. It was evident that these learners did not understand the definition of three-dimensional shapes. This can be attributed to the fact that a single answer response was needed for this task and it became a problem to classify a wrong answer.

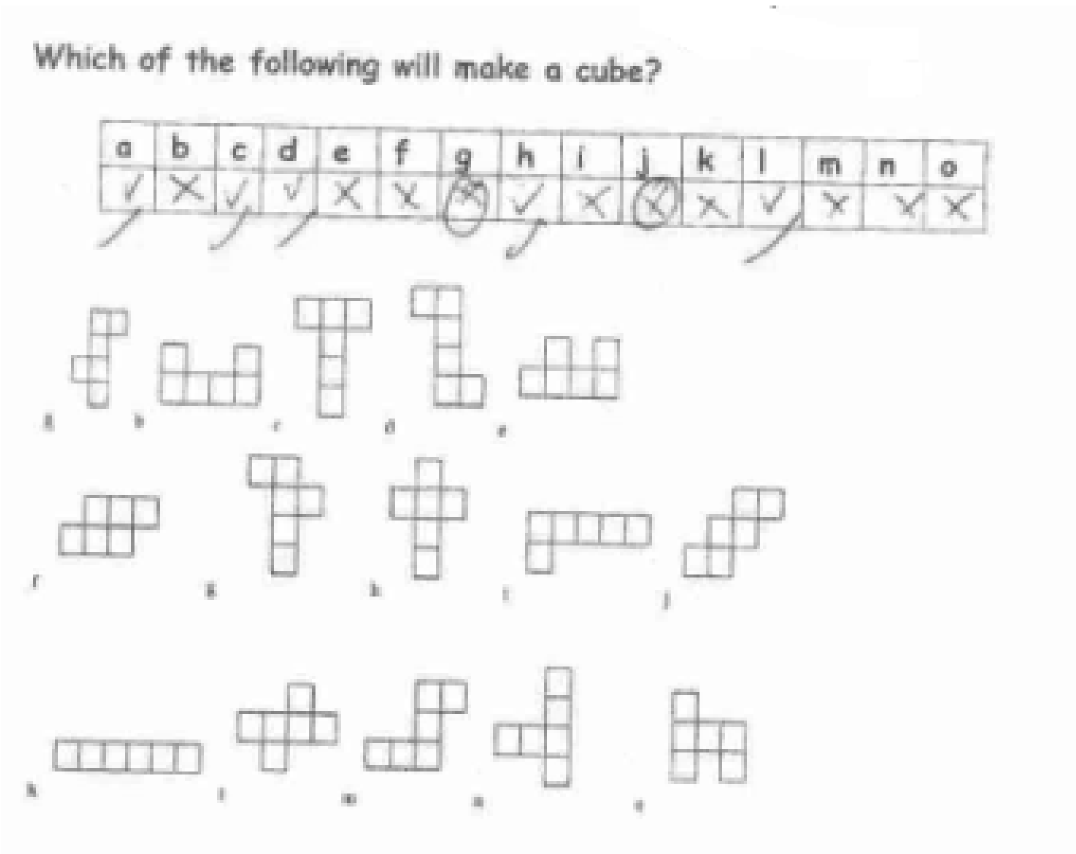
<u>Qualitative and Quantitative Analysis of Nets of a Cube. (Appendix G)</u>

This was done to assess the learners' ability regarding the different orientation of nets of a cube. Fifty percent of the experimental group recorded structural errors. Learners have lost track of the unfolding of the cube which was done in their mind. Davis (1998:29) refers to this as a control error. The learner has memorised a rule that he/she has been following or they behaved in a certain way because they know from experience that this is an effective or appropriate way to tackle the problem. Thirty percent of the experimental group made executive errors. They were unable to use rotations and flips to compare various nets. Because the orientation of the shape changed, they were unable to form a square. Evidence of these errors can be observed from the responses of learners which are given below.

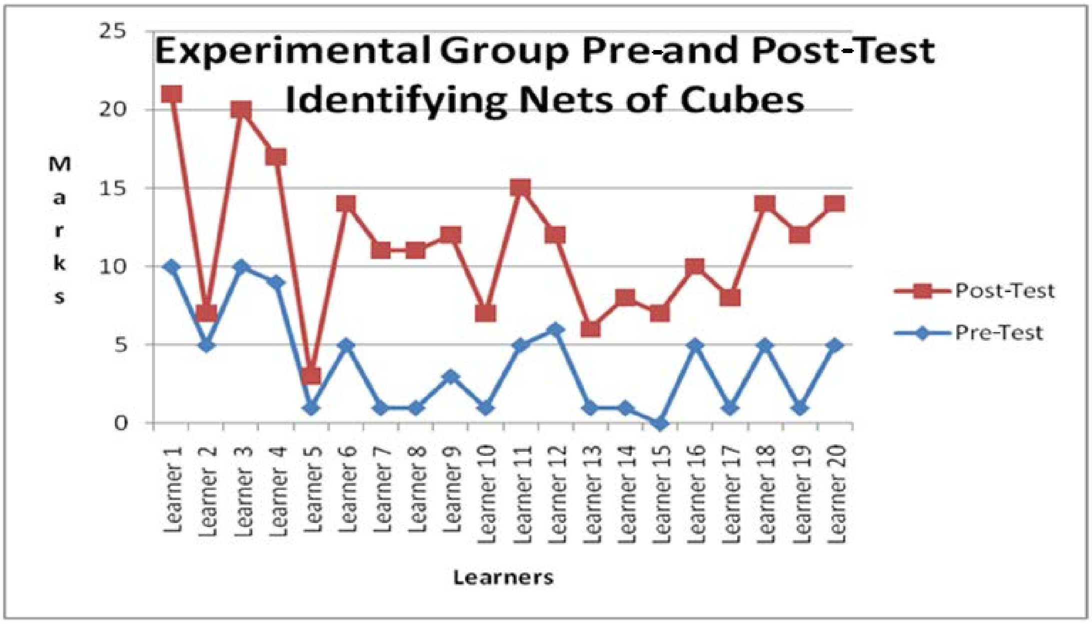
Which of the following will make a cube?

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
✗	✗	✓	✗	✓	✓	✗	✓	✓	✗	✓	✓	✗	✓	✗





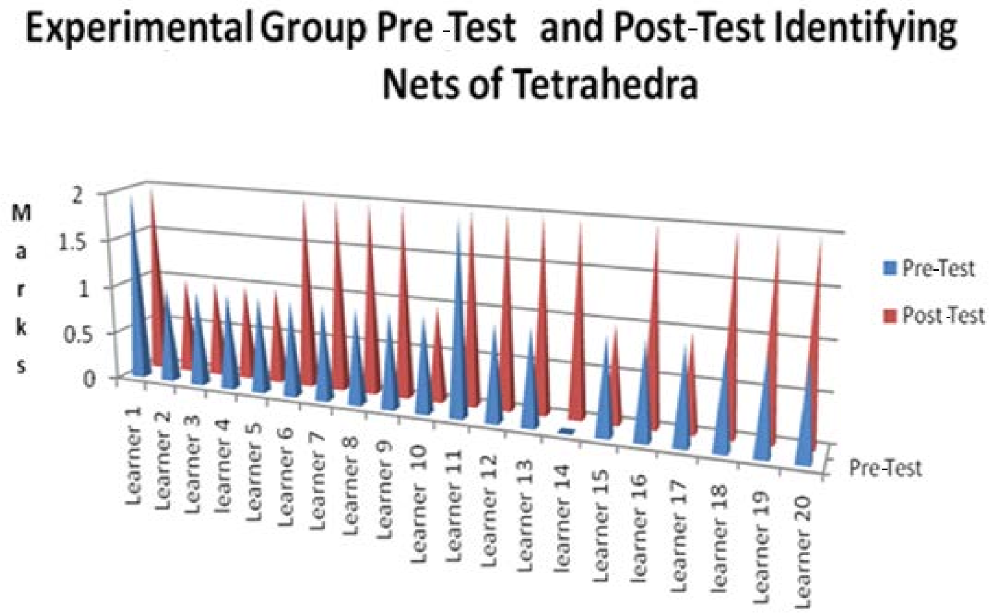
4.2.5 Comparison of the Pre-and Post-Test of Nets of the Cube



Graph 4: Experimental Group Pre-and Post-Test Result for Identifying Nets of Cubes

It is evident that there is a vast improvement in the learners understanding of finding the nets (Graph 4). Thus, one can attribute this improvement to the intervention of the computer programme.

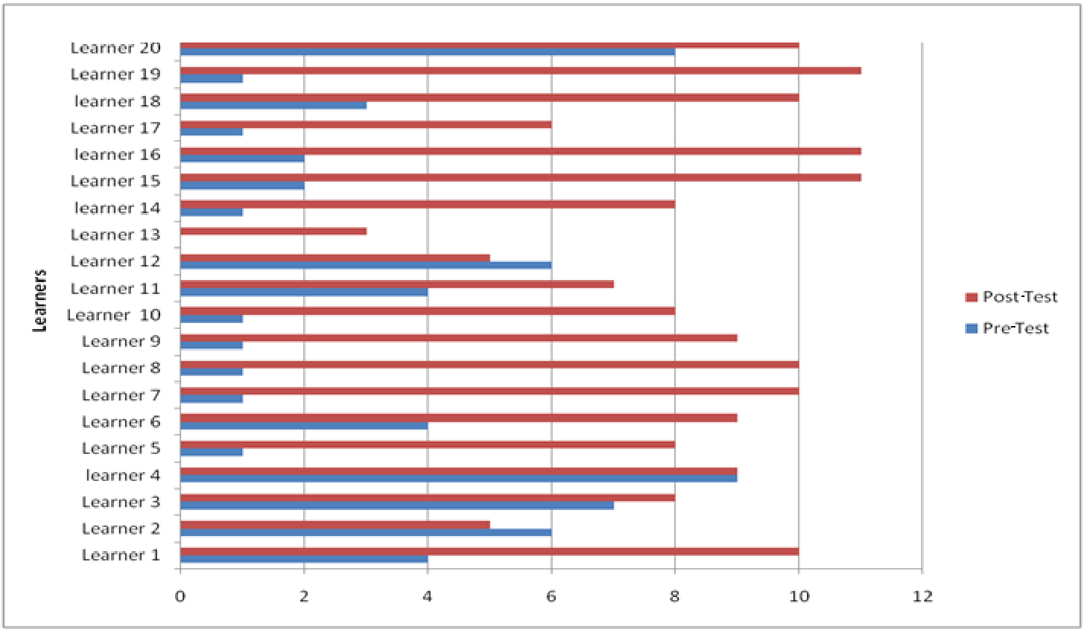
4.2.6 Comparison of the Pre-and Post-Test of Tetrahehra



Graph 5: Experimental Group Pre-and Post-Test Result for Identifying Nets of Tetrahedra

The scores of the post-test are higher than the scores of the pre-test. This can be attributed to the intervention of the computer programme (Graph 5).

4.2.7 Comparison of the Pre-and Post-Test of Octahehra

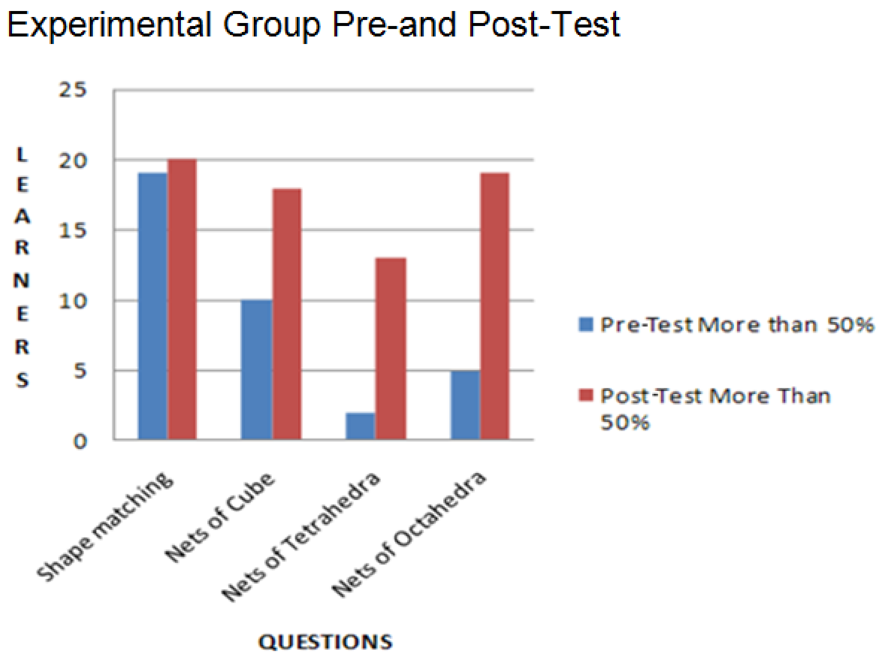


Graph 6: Experimental Group Pre-and Post-Test Result for Identifying Nets of Octahedra

Once again, the difference in the scores can be attributed to the intervention of the computer programme (Graph 6).

4.2.8 Qualitative Analysis of the Four Categories for Pre-and Post-Test

The analysis of the results for the experimental group pre-and post-test indicates the extent to which the computer made a significant impact. Graph 7 depicts the improvement made.



Graph 7: Experimental Group Pre-and Post-Test

It is evident that there was a drastic improvement in the learners' conceptual understanding of nets of solid shapes in both the experimental and the control groups. However, there was a marked change in the experimental group. Poly has the potential to present the same thing in different ways and generally it has incredible potential, reconstructing, expanding, shrinking and flipping. Using this software, one has the chance to flip the shape and see through it and learners can see that the same shape will move up, down, left and right.

A lot of emphasis is given to 'understanding': "Understanding means having a great deal of pre-requisite knowledge at one's fingertips; it means having multiple perspectives on the objects involved; it means having multiple representations for them, and co-ordinated means of moving among perspectives and representations. And it means having all this knowledge organised in ways that derive power from redundancy" (Schoenfeld 1990: 4).

Thus, understanding is a complex process in which didactic, cognitive, epistemological and mathematical aspects intertwine in determining the processes leading to understanding (Dreyfus, 1993:231). The dynamic environment fosters the interaction between construction and proof, between doing on the computer and justifying the conjectures with mathematical arguments, making learners more systematic in their conjectures (Laborde, 2000:156). It provides a bridge to understanding through exploration leading to the acceptance of the idea by providing a dynamic representation of the mathematics involved (Hoyles and Sutherland, 1989:4).Evidence from researchers shows that ICT can support mathematical learning and teaching, encourage mathematical thinking, enhance learner's learning (Smith, 1998:326), and raise pupil attainment while the knowledge remains long-term in learners' minds (Holzl, 2001:64).

4.2.9 Correlation: Analysis of Pre-and Post-Test

The correlation of the experimental pre-and post-test is 0,11 and the correlation of the control pre-and post-test is 0,41. This indicates that the pre-knowledge maybe not be a significant influence in the present study concept. The pre-knowledge concept of the grade seven curriculum should be revisited. The correlation of the experimental pre-test and the control pre- test is 0,28 which indicates a weak correlation and can be assumed to be independent samples. In addition, the correlation of the experimental and control post-test is -0,07. They are weakly correlated, indicating that the two groups are independent populations. Tables 12 and 13 shows the calculation for the T-Test and the correlation.

4.2.10 Measures of Central Tendency - Mean and Standard Deviation

The relationship between the pre-and post-test scores of the experimental groups with respect to correlation and hypothesis testing was undertaken. The mean and standard deviations of pre-and post-test scores were calculated and presented in Table 9. As seen in the table, the experimental group had a higher mean than the control group on the pre-and post-test.

	Experimental	
	Pre-Test	Post-Test
Mean	31,05	42,25
Std dev.	6,3	4,76

Table 9: Mean and Standard Deviation of the Experimental Group

A large standard deviation indicates that the data is far away from the mean. A small standard deviation shows that the data is clustered close to the mean. It is evident from table 9 that the experimental group post-test had the least standard deviation and substantiates the positive effect of technology on learning geometry.

Before	41	36	34	21	43	26	35	28	24	30	39	28	34	38
After	48	31	46	41	43	36	45	47	45	45	46	40	44	38
Before	27	22	28	33	28	26								
After	35	42	44	43	37	49								
Before														
After														

Claimed Difference in Means 0

TEST FOR MEANS

T-Statistic 1.5000504 **The P-Value** 0.075265

Conclusion for Means

Suggestive evidence against the null hypothesis

Claimed Difference in Variances 0

TEST FOR VARIANCES

T-Statistic 1.2053599 **The P-Value** 0.122085

Conclusion for Variances

Little or no real evidences against the null hypothesis

General Statistics

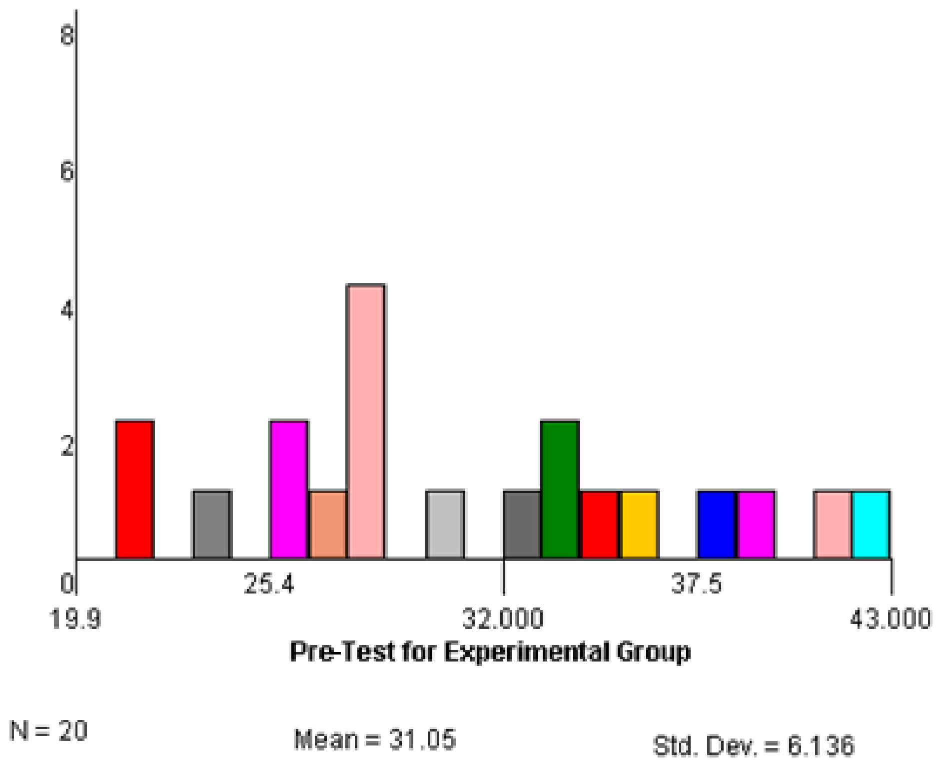
Mean(X)	31.05	**Mean(Y)**	42.25
Variance(X)	39.6289474	**Variance(Y)**	22.6184211

Correlation (X, Y) 0.108554

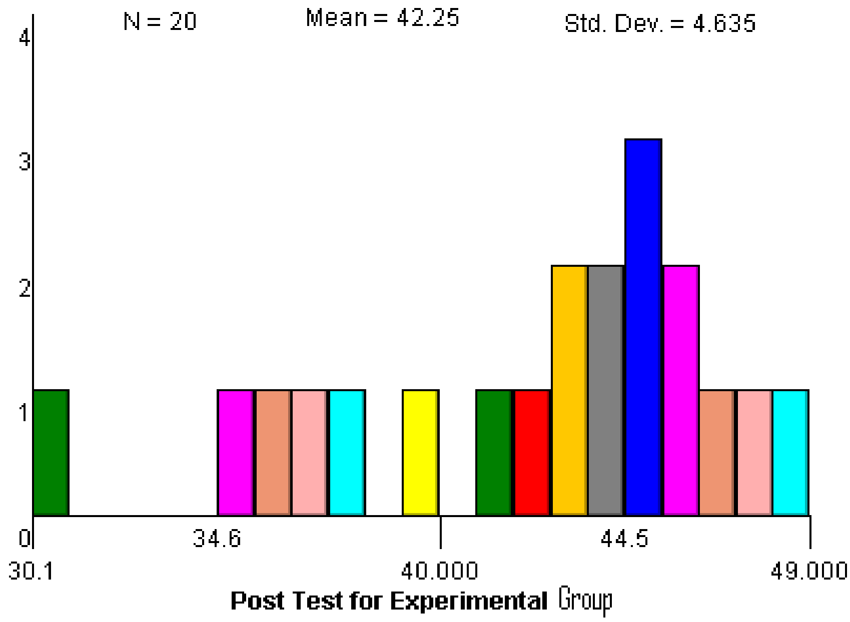
Table 10: Experimental Group T- Statistic, Variance and Correlation

The result obtained suggests that we need to ignore the null hypothesis that both the means of the pre-and post-test are equal (Table 10). The P-value of 0,122085, indicates that there was no significant difference in the performance of the two groups in the pre-test. This implies that the experimental and the control groups could be considered comparable concerning their level of mathematical knowledge and skills.

4.2.11 Testing of Normality of Pre-and Post-Test Using Histogram



Graph 8: Histogram of the pre-test of the experimental group



Graph 9: Histogram of the post-test of the experimental group

A comparison of the above histogram (Graph 8 and 9) indicates that the standard deviation of the post-test (4,635) of the experimental group is smaller than the standard deviation of the pre-test (6,136). It affirms that technology could improve geometrical understanding.

4.2.12 Hypothesis Testing (Mann-Whitney Test) of Pre-and Post-Test

It was decided to perform a Mann-Whitney test because the data did not follow a normal distribution (See Table 11). The null hypothesis indicates that the mean score of two groups are equal. H_0: $\mu_1 = \mu_2$

H_0 = the null hypothesis μ_1 = the mean of population 1, and μ_2 = the mean of population 2. That is, H_1: $\mu_l > \mu_2$ μ = the learners' mean of the experimental group. The effective null hypothesis is H_0: $\mu_l = \mu_2$

Mann-Whitney U Test for the Pre- and Post-Test of the Experimental Group

Pre-Test		Post-Test	
Scores (A)	Ranks (A)	Scores (B)	Ranks (B)
41,0	25,5	48,0	39,0
36,0	18,5	31,0	12,0
34,0	14,5	46,0	36,5
21,0	1,0	41,0	25,5
43,0	29,0	43,0	29,0
26,0	4,5	36,0	18,5
35,0	16,5	45,0	34,0
28,0	8,5	47,0	38,0
24,0	3,0	45,0	34,0
30,0	11,0	45,0	34,0
39,0	23,0	46,0	36,5
28,0	8,5	40,0	24,0
34,0	14,5	44,0	31,5
38,0	21,5	38,0	21,5
27,0	6,0	35,0	16,5
22,0	2,0	42,0	27,0
28,0	8,5	44,0	31,5
33,0	13,0	43,0	29,0
28,0	8,5	37,0	20,0
Scores (A)	Ranks (A)	Scores (B)	Ranks (B)
26	4,5	49,0	40,0
ΣA=621,0	ΣRA=242,0	ΣB=845,0	ΣRB=578,0

Table 11: Mann-Whitney U Test for the Pre-and Post-Test of the Experimental Group

Calculation

$$U^1 = N^A N^B + \frac{N^A (N^A + 1)}{2} - \sum R^A$$

$$U^2 = N^A N^B - U^1$$

Smallest value of U^1 and U^2 = U (the value on which significance is tested)

U_1= 20 X 20 + ((20 X (20+1)) ÷ 2) – 242 = 368

U_2= (20 X 20) – 368 = 32

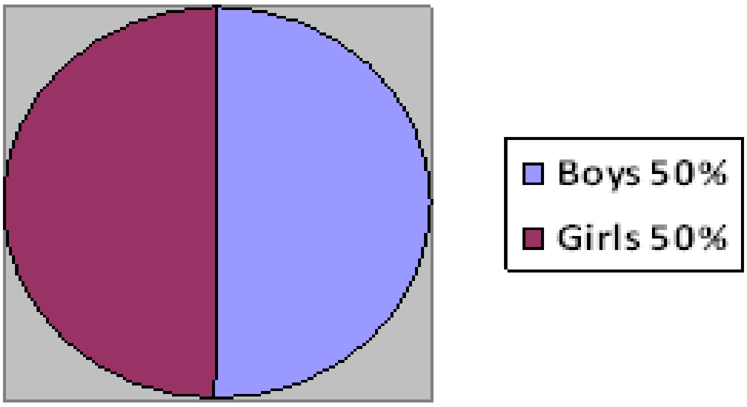
Critical Values 127 (significant (U=32, N_a=20, N_b=20, p=0,05 (one-tailed); p=0,025 (two-tailed))

4.3 Control Group Results

4.3.1 Profile of the Control Group Learners

Question 1: Comparison of the number of male and female learners.

Graph 10 indicates the gender of the respondents.



Graph 10: Comparison of number of male and female learners in the control group

A total of 10 males and 10 females responded to the questionnaires (Graph 10). In other words, 50 percent of the respondents were male and 50 percent were female. The distribution was in line with the gender distribution of learners in this grade in the case study. The distribution might imply that findings in this study may be transferable to other primary schools of similar contexts to that of the sample school.

Age distribution of the learners

Age (in years)	12	13	Total
Number of Learners	13 (65 percent)	7 (35 percent)	20

Table 12: Age distribution of the learners.

Cognitive development in the learning of geometry has been a major focus of research. Piaget argues that the development of learners' concept of space progresses through various stages of acquisition, representation and characterisation of spatial concepts (Piaget, 1967:37). Piaget considers this development a maturation process (Geddes and Fortunato, 1993: 200). It can be gleaned from table 12 that the entire sample in the control group fell within Piaget’s formal operations (12 and above) where thinking involves abstractions. (Saler and Edgington, 2006:161). According to Piaget, children can contemplate and understand abstract problems at this stage (Moursund, 2006:60). However, data from this study has indicated otherwise and will be discussed in greater detail.

4.3.2 Pre-Test: Analysis of Errors of Pre-concepts

This section is concerned with the learners’ perceptions of the concepts of two-and three-dimensional shapes, nets of solid shapes and learners’ prior knowledge and if and how it influences their ability to visualise three-dimensional shapes (Table 13). The following table classifies errors in a conventional post-test control group:

CLASSIFICATION OF ITEMS	ARBITRARY ERRORS	STRUCTURAL ERRORS	EXECUTIVE ERRORS
Properties of squares	20 percent	50 percent	10 percent
Properties of Cube	30 percent	50 percent	60 percent
2-Dimensional Shapes	0 percent	20 percent	0 percent
3-Dimensional Shapes	20 percent	30 percent	30 percent
Nets of a Cube	20 percent	80 percent	40 percent

Table 13 Classification of errors for the Control group

4.3.3 Post-Test: Qualitative Analysis of Two-and Three-Dimension of Cube, Tetrahedra and Octahedra

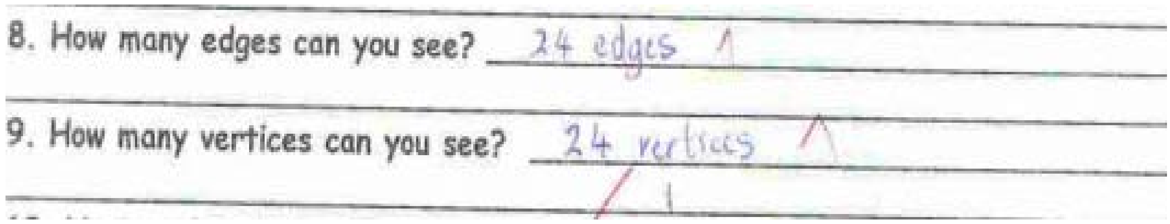
<u>Qualitative and Quantitative Analysis of Properties of Squares (Appendix G)</u>

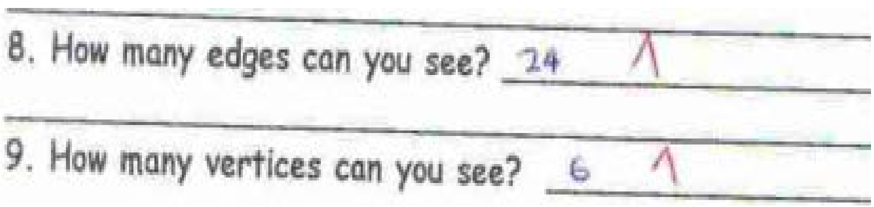
A larger percentage of the control group, 50 percent, displayed structural errors. Vague answers like “a square has 4 sides and 4 angles”, “a shape that has 4 right angles” and “the sides form 2 right angles” were characteristic of the incorrect responses.


3. Describe a square
It has 4 equal sides and 4 angles

<u>Qualitative Analysis of Properties of a Cube (Appendix G)</u>

This was based on the concept of a regular platonic solid with six equal square faces. It was apparent that both groups did not grasp this meaning. A fairly large amount of structural and executive errors were recorded in the control group. Clearly, learners were unable to retrieve the frame ‘faces, edges and vertices’, ‘length, height and width’. Learners’ conceptual understanding of the characteristics of the cube was poor. Evidence of the lack of knowledge could be assigned to van Hiele level one. Learners were asked about the visual aspect of a cube. Once again, it is evident that learners do not understand these concepts.


8. How many edges can you see? 24 edges
9. How many vertices can you see? 24 vertices


8. How many edges can you see? 24
9. How many vertices can you see? 6

Qualitative and Quantitative Analysis of Two-dimensional Shapes (Appendix G)

This item was concerned with two-dimensional shapes. It required learners to identify two-dimensional shapes. Twenty percent of the control group made structural errors. A greater percentage of learners from the control group were able to retrieve the frame required for the solution of this task. An interesting observation was that both groups had a good understanding of two-dimensional shapes. A response from one of the learners is given below.

Qualitative and Quantitative Analysis of Three-dimensional shapes (Appendix G)

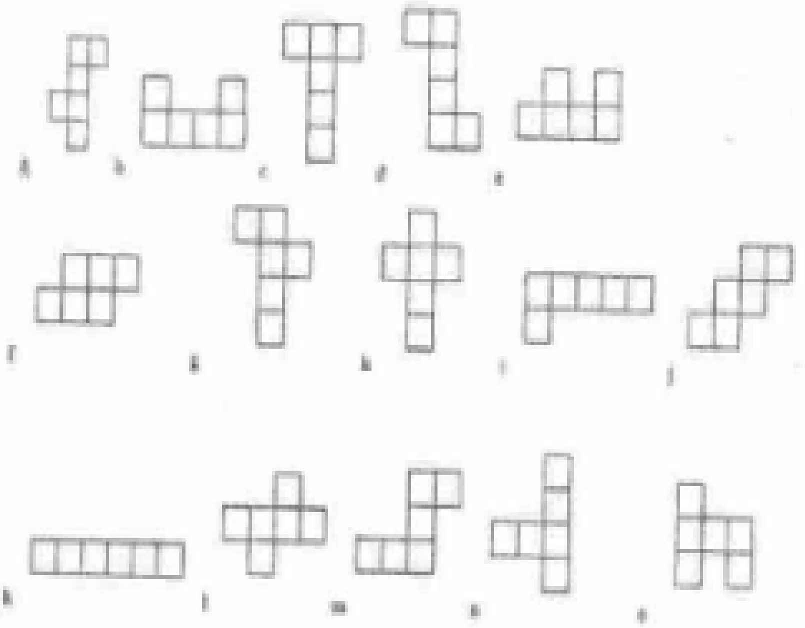
Thirty percent of the control group made structural errors. It is evident that these learners did not understand the definition of three-dimensional shapes. This can be attributed to the fact that a single answer response was needed for this task and it became a problem to classify a wrong answer.

Qualitative and Quantitative Analysis of Nets of a cube (Appendix G)

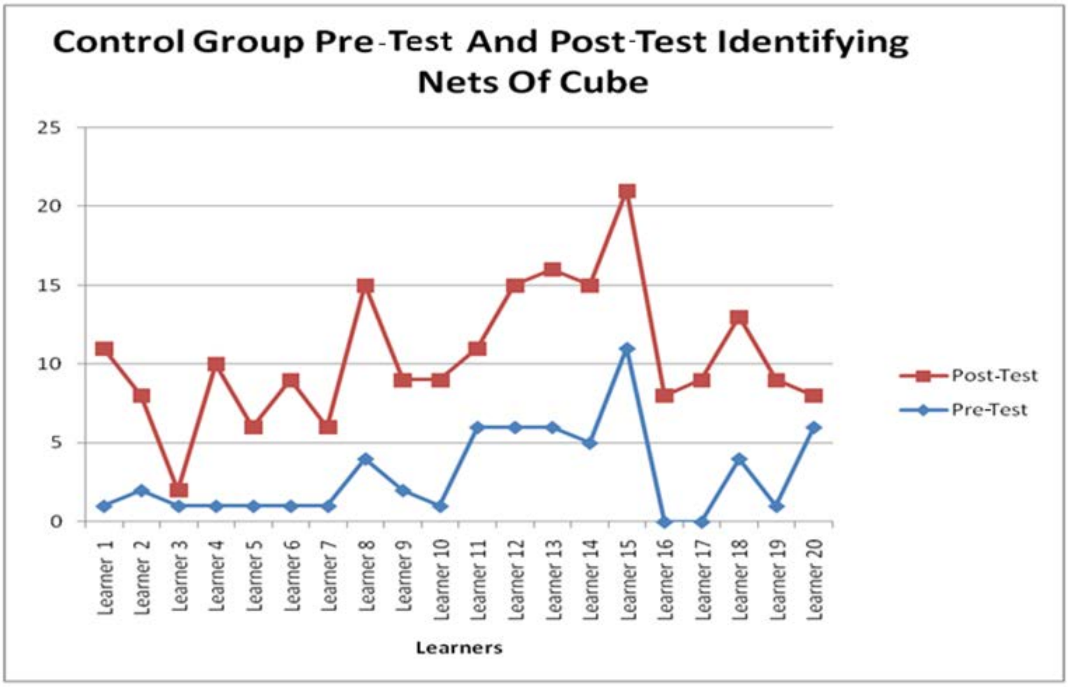
Eighty percent of the control group recorded structural errors. Learners had lost track of the unfolding of the cube which was done in their mind. Davis (1990:29) refers to this as a control error. The learner had memorised a rule that he/she has been following or they behave in a certain way because they know from experience that this is an effective or appropriate way to tackle the problem. Forty percent of the control group made executive errors. They were unable to use rotations and flips to compare various nets. These nets could not make a square because the orientation of the shape changed.

Which of the following will make a cube? 15

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
X	X	✓	X	X	X	✓	✓	X	?	X	✓	X	X	X



4.3.4 Comparison of the Pre-and Post-Test of Nets of Cube



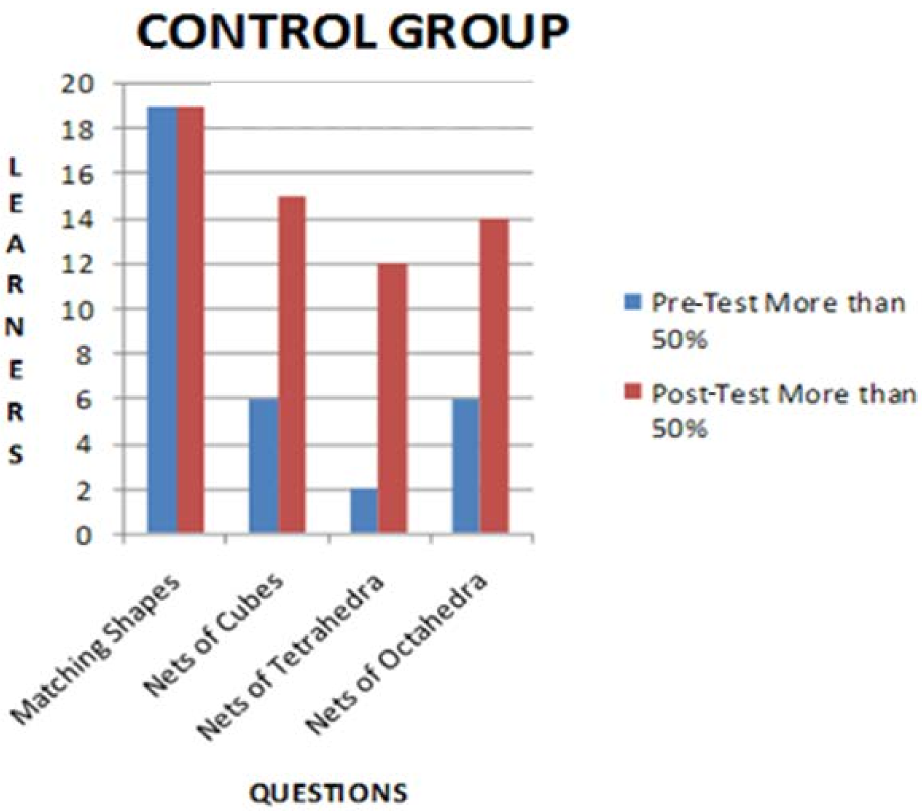
Graph 11: Control Group Pre-and Post-Test Result for Identifying Nets of Cubes

4.3.5 Comparison of the Pre-and Post-Test of Nets of Octahedra

In the pre-test 50 percent of the learners gave the correct solution. From the responses it can be inferred that the learners were unable to visualise how the faces of the cube will fold was limited. However, after the intervention of Poly with the experimental group, there was an increase in the identifying of the nets of the cube by learners to 90 percent (Graph 11). Poly provided an active way to manipulate visual mathematical objects ensuring a clearer understanding of concepts. Sliding makes these environments much more powerful than traditional paper and pencil learning; animating capability allows learners to experience the direct manipulation of geometrical objects created on the screen. Learners have the opportunity to see the dynamic representations of geometric configurations that cannot be easily illustrated without the use of technology. This can be further supported by the van Hielie model where, for a learner to function adequately at level 1, the learner should have grasped the basics of the previous level adequately. This type of pre-requisite building blocks is analogous to the Piagetian concepts of “assimilation and accommodation”. Thus, like the van Hiele model which emphasises an orderly growth path, Piaget’s theory also premises that all “intellectual behaviour has its beginnings in early infancy, and mature reasoning skills emerge through subsequent phases of conceptual development” (Helms and Turner, 1981:51).

4.3.6 Qualitative Analysis of the Four Categories of Tasks for Pre-and Post-Test

Graph 12 is an analysis of the results for the control group pre-and post-test. It is evident there is a significant improvement in the post-test and this can be attributed to how the teacher conducted the lesson.



Graph 12: Control Group Pre-and Post-Test

4.3.7 Measures of Central Tendency - Mean and Standard Deviation

The relationship between the pre-and post-test scores of the control group with respect to correlation and hypothesis testing was undertaken. The mean and standard deviations of pre-and post-test scores were calculated and presented in Table 14. As seen in the table, the mean of the post-test was higher than the pre-test and the standard deviation of the pre-test was higher than the post-test.

	Control	
	Pre-Test	Post-Test
Mean	29,7	38,45
Std dev.	7,23	6,43

Table 14: Control Group Mean and Standard Deviation

A large standard deviation indicates that the data is far away from the mean (Table 15). A small standard deviation shows that the data is clustered close to the mean. It is evident that traditional teaching did not make a significant impact on the learning of geometry.

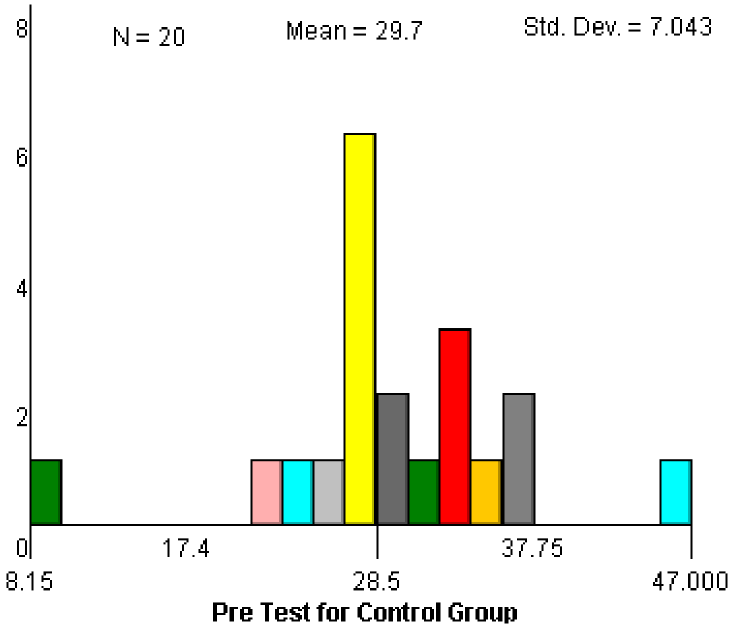
Before	28	28	30	27	10	28	29	22	35	33	37	32	47	24
After	33	43	41	33	21	44	44	41	38	42	42	33	39	39
Before	26	33	37	27	33	28								
After	45	44	44	43	29	31								
Before														
After														

Claimed Difference in Means	0		
TEST FOR MEANS			
T-Statistic	1.1738276	**The P-Value**	0.12773
Conclusion for Means			
Little or no real evidences against the null hypothesis			
Claimed Difference in Variances	0		
TEST FOR VARIANCES			
T-Statistic	0.505273	**The P-Value**	0.309995
Conclusion for Variances			
Little or no real evidences against the null hypothesis			
General Statistics			
Mean(X)	29.7	**Mean(Y)**	38.45
Variance(X)	52.2210526	**Variance(Y)**	41.3131579
Correlation (X, Y)	0.4087197		

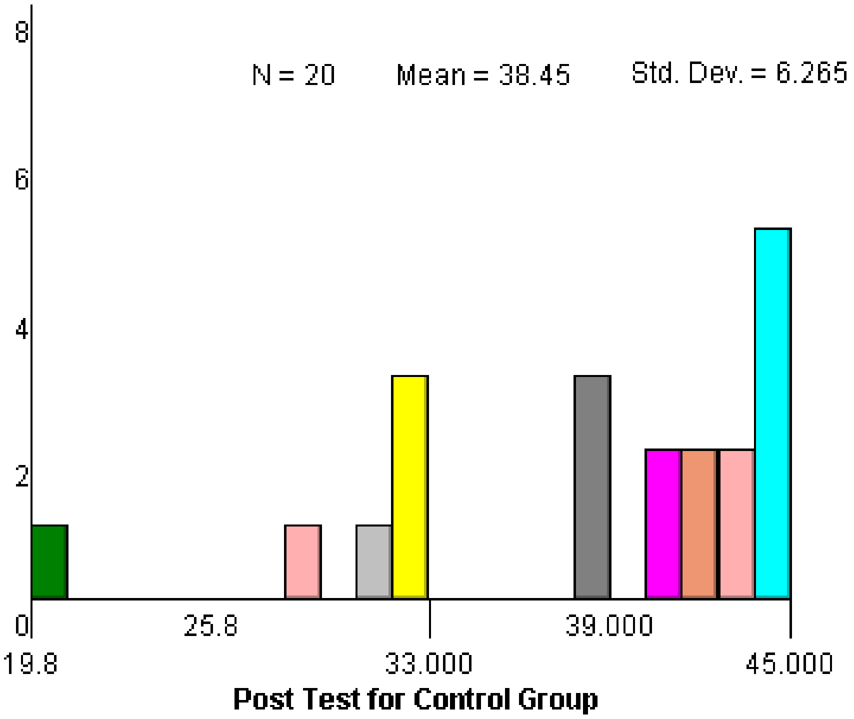
Table15: Control Group T- Statistic, Variance and Correlation

The result obtained suggests that we must take heed of the null hypothesis that both the means of the pre-and post-test are equal.

4.3.8 Testing of Normality of Pre-and Post-Test Using Histogram



Graph 13: Histogram of the pre-test of the control group



Graph 14: Histogram of the post-test of the control group

A comparison of the above histogram (Graphs 13 and 14) indicates that the standard deviation of the post-test (6,265) of the control group is smaller than the standard deviation of the pre-test (7,043). The difference is negligible.

4.3.9 Hypothesis Testing (Mann-Whitney Test) of Pre-and Post-Test

<u>Mann-Whitney U Test for the Pre-and Post-Test of the Control Group</u>

Pre-Test		Post- Test	
Scores (A)	Ranks (A)	Scores (B)	Ranks (B)
28,0	9,5	33,0	19,5
28,0	9,5	43,0	33,5
30,0	14,0	41,0	29,5
27,0	6,5	33,0	19,5
10,0	1,0	21,0	2,0
28,0	9,5	44,0	36,5
29,0	12,5	44,0	36,5
22,0	3,0	41,0	29,5
35,0	23,0	38,0	26,0
33,0	19,5	42,0	31,5
37,0	24,5	42,0	31,5
32,0	16,0	33,0	19,5
47,0	40,0	39,0	27,5
24,0	4,0	39,0	27,5
26,0	5,0	45,0	39,0
33,0	19,5	44,0	36,5
37,0	24,5	44,0	36,5
27,0	6,5	43,0	33,5
33,0	19,5	29,0	12,5
Scores (A)	Ranks (A)	Scores (B)	Ranks (B)
28,0	9,5	31,0	15,0
ΣA=594,0	ΣRA=277,0	ΣB=769,0	ΣRB=543,0

Table 16***:*** Mann-Whitney U Test for the Pre-and Post-Test of the Control Group

Calculation

$$U^1 = N^A N^B + \frac{N^A (N^A + 1)}{2} - \sum R^A$$

$$U^2 = N^A N^B - U^1$$

Smallest value of U^1 and U^2 = U (the value on which significance is tested)

U_1= 20 X 20 + ((20 X (20+1)) ÷ 2) - 277 = 333

U_2=(20 X 20) - 333 = 67

Critical Values

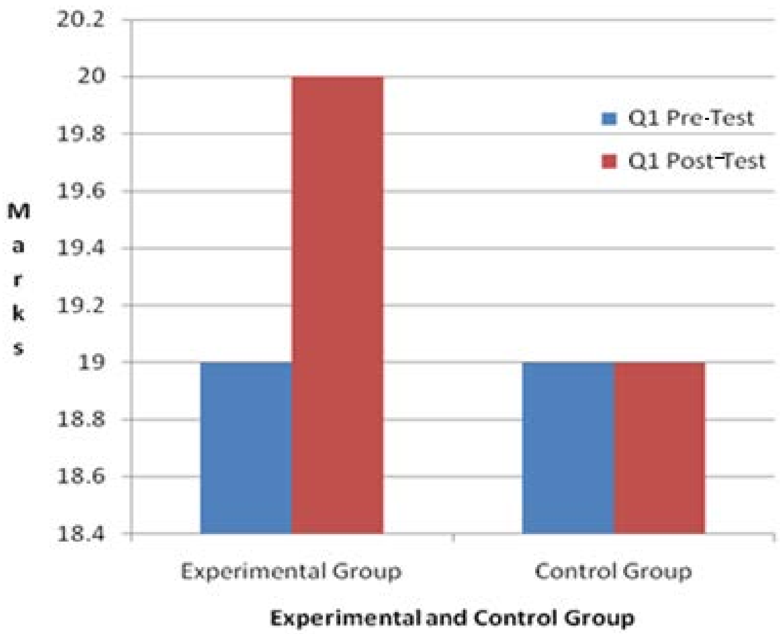
Critical Value: 127 [significant (U=67, N_a=20, N_b=20, p=0,05 (one-tailed); p=0,025 (two-tailed)].

Both the Mann-Whitney tests results indicated that although there was no intervention in the control group, the two pair sample (Pre-and Post-Test) scores between the pre- and post test were significantly different.

4.4 Comparison of the Control and Experimental group

4.4.1 Comparison of the Post-Test of Two Dimensional Shapes

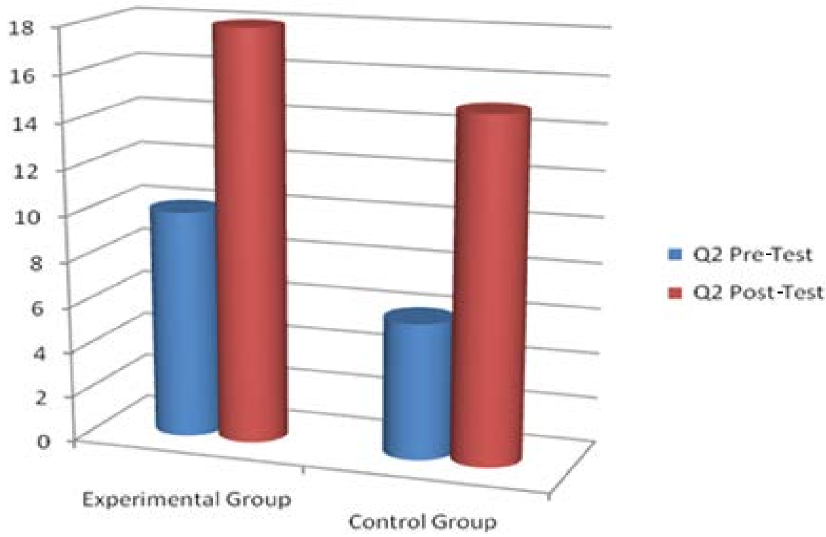
Graph 15 is an analysis of the results for the pre-and post-test of the experimental and control group for question 1. Clearly, it can be observed that there was a significant improvement in the experimental group.



Graph 15: Experimental and Control Group Result for Question 1

Question 1 required the learners to match two-dimensional shapes that were presented in different orientations. Learners in both the experimental and the control group fared exceptionally well. Thus, they were functioning at van Hiele level 1. The pre-knowledge framework to explore solid shape is good. The mean attained for the pre-test of the experimental group was 92 percent and that for the post-test was 98 percent. This result reveals that the learners have a fairly good understanding of plane figure. They were able to find the matching shapes and this concurs with van Hiele level 1.

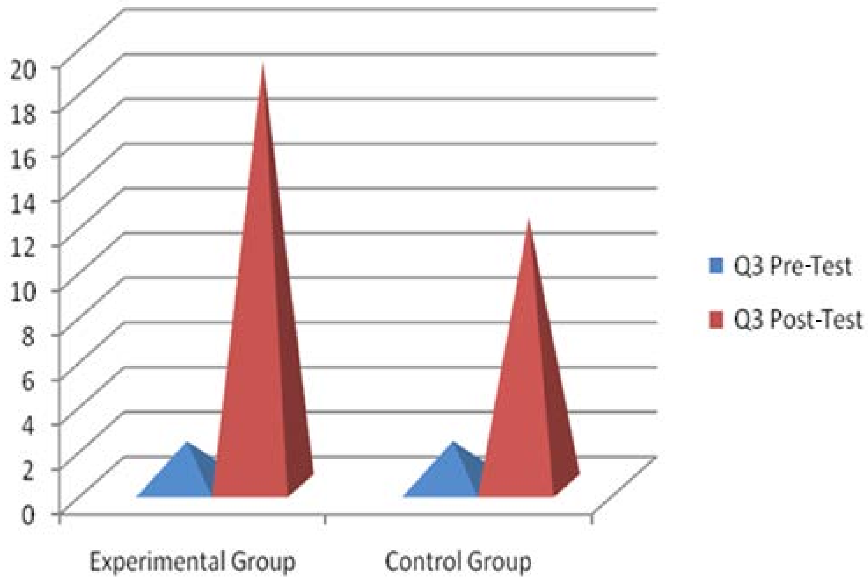
4.4.2 Comparison of the Post-Test of Cube



Graph 16: Experimental and Control Group Result for Question 2

Question 2 required learners to identify which nets of the cubes will fold into a solid cube. The experimental group was able to perform better than the control group (Graph 16). Learners had to have the pre-requisite knowledge of a face, edge and vertices. The learners were required to differentiate between plane figures and solid shapes. Their reasoning ability had to come to the fore as some of the shapes provided less faces. Constructivists believe that learning involves the generation of knowledge and learning strategies. According to this view, learning in schools has to emphasise the use of intentional processes that learners can use to construct meaning from information, experiences and their own thoughts and beliefs. Successful learners are active, goal-directed, self-regulating and assume personal responsibility for contributing to their own learning. So the learning of complex subject matter is most effective when it is an intentional process of constructing meaning from information and experience. Glasersfeld (1995:14) argues that: “from the constructivist perspective, learning is not a stimulus-response phenomenon; it requires self-regulation and the building of conceptual structures through reflection and abstraction”.

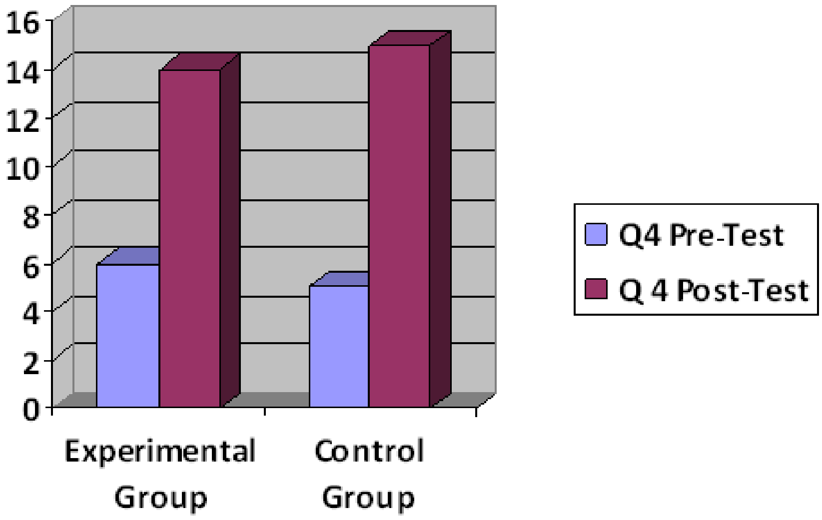
4.4.3 Comparison of the Pre-and Post-Test of Tetrahedra



Graph 17: Experimental and Control Group Pre-and Post-Test

In the pre-test, only 10 percent of the experimental and control group learners attained more than 50 percent whilst in the experimental post-test 95 percent scored more than 50 percent. It can therefore be inferred that in the control group geometrical ideas were difficult for many learners. Noss (1987:343) states that many misconceptions appear to be related to a failure of learners to view concepts in a dynamic rather than in a static form. Poly can present information in different ways or by enabling changes to be shown dynamically (Ainsworth *et al*, 1997:95), widening the range of possible activities and providing deeper reflection, exploration and heuristics than in a paper and pencil approach (Straesser, 2001:321). This dynamic environment enables learners to explore the problem and make mathematical conjectures (Graph 17).

4.4.4 Comparison of the Pre-and Post-Test of Octahedra



Graph 18: Experimental and Control Group Pre-and Post-Test Result for Question 4

In the pre-test, 25 percent of the experimental and the control group learners managed to identify the nets of the octahedron whilst in the experimental group, post-test 65 percent of the learners could identify the nets of the octahedron (Graph 18). This result can be attributed to the poor understanding of the physical shapes, that is, van Hiele level 2 and suggests that learners have limited experience with visualisation.

4.5 Qualitative and Quantitative Analysis of the Interview Questions of the Experimental Group

The demographic data set out in Table 17 shows the gender and age distribution of the participants in the experimental group.

Age	12	13	Total
Male	10	2	12
Female	6	2	8
Total	16	4	20

Table 17: Gender and age Distribution-Interview of the experimental group

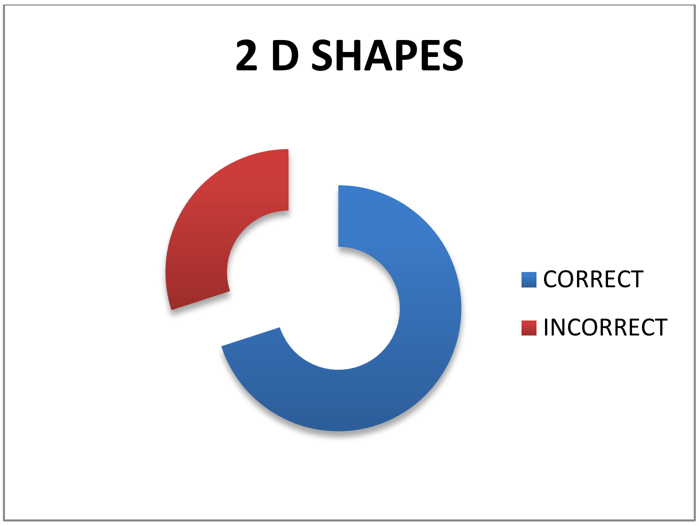
Two-and three-dimensional concepts

Question1. What did the learner understand by the concept ‘two-dimensional shape’?

From the responses, 70 percent of the learners of the experimental group were able to provide an accurate description of two-dimensional shapes. Some of their responses are depicted below:

- They are flat shape or object
- The measurement shown are length and breadth
- They have two - dimensions
- The shapes all have an inside region and an outer boundary. It has area but no thickness
- We can draw these shapes on paper and they are often represented in books

However 30 percent of the respondents provided answers that were not related to two-dimensional shapes (Graph 18). Their responses were that they were geometric shapes with sides and angles. Since the computer screen is inherently two-dimensional, it might be a more appropriate medium for studying some aspects of two-dimensional geometry. Additionally, the dynamic, clean, flexible, re-playable and controllable nature of computer environments may make them more advantageous over their concrete counterparts (Clements, 1999:28). Given some experience with computers, such transformations as rotating, flipping and dragging a two-dimensional shape.



Graph 19: Learners' understanding of two-dimensional shapes.

Question 2. What did the learner understand by the concept 'three-dimensional shape'?

Satisfactory descriptions of the concept of area were provided by 60 percent of the respondents. Some of their responses are depicted below:

- It has height, length and breadth
- It is not a flat shape
- It has three-dimension
- It has a top, side and front view
- One can pick it up, like books, pens, desks, etc
- The shapes all have an inside and an outer surface.
- Circle in two-dimension is extended to include a sphere in three-dimensional space
- A three-dimensional shape has space and has volume
- It is a solid shape and something that is hard or impossible to see through
- Three-dimensional figures can stand up-they have height

As per interviews conducted with the experimental group, 35 percent of the learners described three-dimensional shapes as being a flat object.

Computer-based learning environments commonly comprise symbolic as well as static and dynamic pictorial representations, frequently combined with the possibility of modifying them interactively. While multiple, dynamic and interactive external representations have the potential to improve learning in various ways, they also place specific demands on learners, such as the need to process and relate different representations, to control and evaluate interactions with these representations and to construct coherent mental representations. Computer software for the teaching of three-dimensional geometry should allow learners to see a solid represented in several possible ways on the screen and to transform it, helping them to acquire and develop abilities of different orientations.

Question 3. Was the learner able to explain the term 'nets of a shape'?
Fifty percent of the respondents from the experimental group indicated that they had prior knowledge of nets of a shape. This response was of significance as one of the theories discussed in the literature review espoused that pre-knowledge frames played an important role in shaping learners' abilities to deal with more abstract problems (Davis, 1990:39). Nets of a shape occur when the shapes are cut out as a plane figure.

Question 4. Was the learner able to explain two-dimensional shapes?
Only 60 percent of the learners from the experimental group answered correctly. Visualisation here refers to visual perception and visual imagery. Visualisation is an essential part of mathematics learning and Poly can play a powerful role not only in stimulating and shaping learners' visual images but in providing access to new forms of representation as well as to multiple and linked representations.

Question 5. If yes, please provide a brief explanation of what you understood a two-dimensional shape to be. Some of the responses from the experimental group were:

- Two-dimensional shapes were flat figures
- These shapes can be drawn on a page as a flat object

- Two-dimensions are having only the dimensions of length and breadth, like a square

Thus, experimental group learners have a solid pre-knowledge of two-dimensional shapes.

Question 6. Did the learner know what a three-dimensional shape was before these lessons?

Only 50 percent of the learners answered correctly. There is clear evidence that geometrical ideas are difficult for many learners; many misconceptions appear to be related to a failure to view concepts in a dynamic rather than in a static form (Noss, 1987:9). Poly can present information in different ways by showing it in different forms or by enabling changes to be shown dynamically (Ainsworth *et al*, 1997:93). Straesser (2001:321) asserts that Poly widens the range of possible activities and provides deeper reflection, exploration and heuristics than in a paper and pencil approach. This dynamic environment enables learners to explore the problem and make mathematical conjectures.

Observing changes can develop learners' understanding of mathematical relationships and convince the learner of the truth of the conjectured attribute, thus developing conceptual understanding (Jonassen, 2000:65). Learners who see shapes build them and carry out transformations of them on a computer screen, take these images and construct their own knowledge of shapes developing their understanding of mathematical relationships (Fey, 1989:142). Enabling the learner to see something happen dynamically can support the formation of mental images which will, in turn, assist in the process of understanding.

Question 7. The learners of the experimental group had to provide a brief explanation of what they understood a three-dimensional shape to be. Some of the responses were:

- A polyhedron is often defined as a geometrical solid with flat faces and straight edges
- These are solid shapes
- Three-dimensional shapes have length, breadth and depth, like a cube

Question 8. If the learners answered YES to questions 3, did this knowledge assist them in finding a method of finding nets of the solid shapes?

Yes □ No □

Learners were able to visualise the hidden details of the shape. The knowledge of faces, vertices and edges assisted them in identifying the solid shapes.

Question 9. If the learners answered yes to question 7, they had to explain how they describe a three-dimensional shape.

It was better seeing the shapes breaking up and unfolding rather than seeing a still shape on a piece of paper. The runner was moved and the shape opened and flipped. The net of the shape could then be observed. Learners saw a shape from different perspectives, they saw it reconstructing and constructing in front of their own eyes, enlarging, shrinking while the relationship remained the same.

Question 10. If the learners answered NO to question 7, they had to explain how they found the nets of three - dimensional shapes.

The learner moves from low level (according to van Hiele’s model) of recognition and description skills to higher order skills of classification and discrimination of two-dimensional objects. In skills such as construction of models, different views of objects are used to be developed in the learner.

Section C: Freedom of choosing own method of solution

The learner was allowed to try his own methods of finding answers.

The importance of technology in mathematics and mathematics education is examined by de Villiers' (2004:705). He argues that the computer, with the dynamic mathematics programmes, is an immensely powerful tool that could be used to improve the epistemological perspective. He convincingly shows that the opportunity to do investigations, explorations and the formulating of conjectures is much wider. De Villiers adds: "… the main advantage of computer exploration of topics … is that it provides powerful visual images and intuitions that can contribute to a person's growing mathematical understanding …" (de Villiers, 2004: 5). When conjecturing, verification, global refutation, heuristic refutation and understanding are active, the participant's intuition is activated.

The learners from the experimental group felt comfortable using methods that are inherent to their knowledge for solving the problem. In terms of the interface of the software (as illustrated in Figure 9), this is designed to be intuitive and to provide an open and generative environment that enables learners to learn through making and designing personally meaningful artefacts. The interface also employs rich semiotic resources that enable multiple perspectives and representations for mathematical meaning-making; for example, learners can represent a solid in three-dimensional or its correspondence in two-dimensional perspectives.

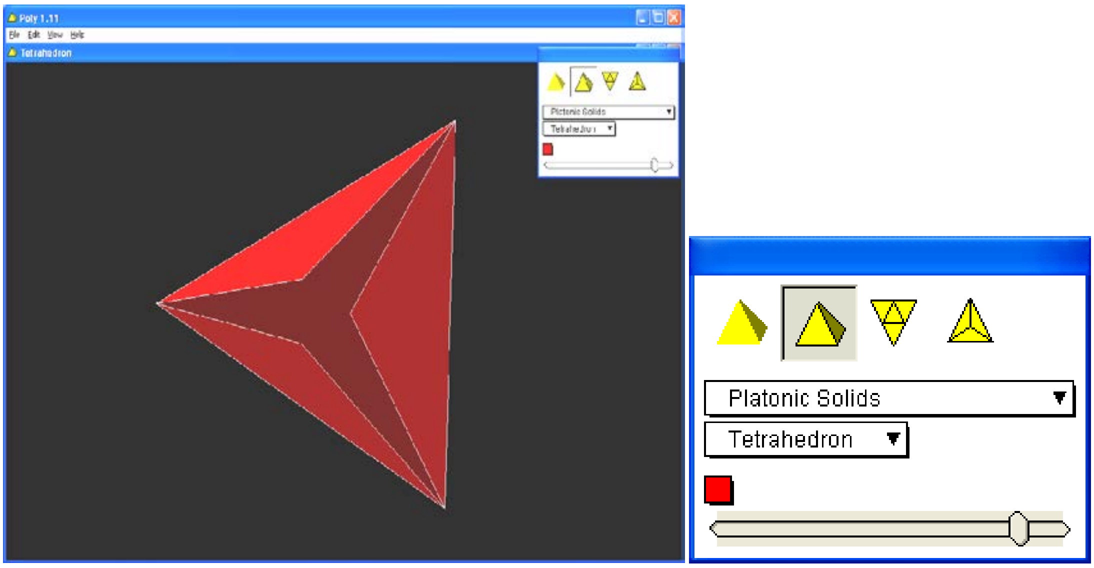


Figure 9: Net of a tetrahedron unfolding

A mental image is any kind of cognitive representation of a mathematical concept by means of spatial elements. Hence, the design of Poly aims to make it straightforward for learners to construct different solids and perceive them in a concrete or pictorial form. The repetition of this process is known to help learners formulate a “picture in their mind’s eyes” (Presmeg, 2006:101). Poly was designed to enable learners to see solids in many positions on the screen and consequently gain a rich experience that allows them to form richer mental images than from textbooks or other static resources. In this context, the Poly environment is designed to be a rich environment for manipulating and transforming representations of solids (Figure 10).

The learners from the experimental group were not fearful of giving incorrect responses because the feedback from the computer made them feel at ease. Observation allows learners to see and understand the third dimension, choosing perspectives and displaying visual feedback. The Poly software was designed so that learners can rotate a geometric object and thus gain a holistic view of the object. Features designed into the software include easy variation of the speed and the direction of the rotation of any object, directly controlled by the user of the software. In addition, the software is designed such that the drawing style of any object can be in a ‘solid colour’ view or in a ‘transparent line’ view, as illustrated in Figure 10, and learners can select, label and colour the edges and faces of the objects.

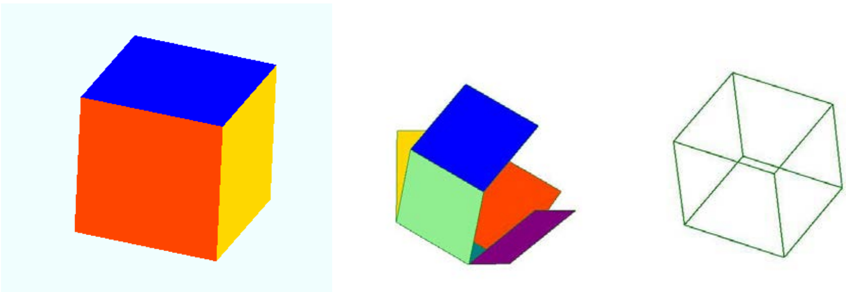


Figure 10: A Solid and Transparent View of a Cube

The dragging capability of the software enabled learners to rotate, move and resize three-dimension objects in much the same way as the commonly available two-dimension dynamic geometry software environments. The design approach for Poly focuses on enabling rotation to be executed in all directions through the provision of an on-screen rotation cursor that can also be used to determine the speed of the rotation. The design was made such that learners are able to resize all the dimensions of the object.

Giving the computer instructions made the learners from the experimental group feel in control.
As Papert (1990:42) notes,

> "*technology is not the cause of anything either positive or negative, but rather should be thought of as a tool that society can use to shape the environment."*

However, it can be seen that the real challenge is how to best utilise these technologies to achieve our educational objectives. In other words, how can we use technology to serve as a catalyst for positive change and as an accompaniment to the teaching and learning situation? In this case, technology should be used for "enrichment and improvement of the conditions in which human beings learn and teach" (and not as an end in itself) (Carnegie Commission, 1972:89). When used in this sense, research has shown that the benefits of using technology such as computers and graphing calculators are immeasurable and incomparable to traditional teaching approaches.

Learners were able to work at their own pace because the software was available to them whenever required. This was unlike a traditional classroom lesson where, once the educator finished the lesson the learner relied on the text for clarity or the educator when s/he was available to explain the concepts again. Thus, if the learner was unable to grasp the concept at any stage s/he could stop the programme and replay from the point of misunderstanding. Being able to work with other learners by sharing ideas on screen improved their understanding of the lessons.

Vygotsky supports the idea that individual cognitive development results from social interaction in the world and that speech, social interaction and co-operative activity are all important aspects of this social world (Vygotsky, 1962:64). The computer, as used in Poly is interactive and this interaction is social since it promotes co-operative learner work and learner interaction with their peers. Micro-worlds have been defined in various ways (Papert, 1980:25; Thomas *et al*, 1996:38; Hoyles and Sutherland, 1985:34); the term is used here to refer to an environment, based in a computer or another medium, in which the central objects and relations of a domain are represented into a concrete or semi-concrete form that is accessible to new learners. Semi-concrete means that the computer environment simulates to some degree actions or objects that exist in the physical world or that could be sketched or constructed using pencil and paper. Typically, a micro-world connects a symbolic representation of the objects and operations with a dynamic visual or graphical representation. These multiple, linked representations presented the learner with the opportunity to predict and act on mathematical objects in one modality and receive feedback in a different form.

Learners from the experimental group were able to share their ideas with the entire class and that made them feel that their ideas were important. Co-operative learning is the instructional use of small groups so that learners work together to maximise their own and each other's learning. Learners perceive that they can reach their learning goals if and only if the other learners in the learning group also reach their goals (Johnson and Johnson, 1987:142). They are not only responsible for learning the material that is presented, but also for ensuring everyone in the group knows the material as well (Sivin-Kachela, 1996:98). So learners need not only interact with materials or with the educator, but they also need to interact with each other to achieve their learning goals.

Johnson and Johnson (1987:88) identify three basic types of learning that goes on in any classroom: competitive learning where learners compete to see who is the best; individual learning where learners work individualistically toward a goal without paying attention to other learners and co-operative learning where learners work co-operatively with a vested interest in one another’s learning as well as their own. Of the three patterns, competition is presently the most dominant; here the learners view the school as a competitive enterprise where one tries to do better than others.

Co-operation among learners who celebrate one another’s successes, encourage each other to do homework and learn to work together regardless of ethnic backgrounds or whether they are male or female, bright or struggling, disabled or not, is still rare. Even though these three patterns are not equally effective in helping learners learn, it is important that learners learn to interact effectively in each of these ways. Learners will face situations in which all three patterns are operating and they will need to be able to be effective in each. They should also be able to select the appropriate pattern suited to the situation. However, in the ideal classroom, all three patterns are used. This does not mean that they should be used equally, but the co-operative pattern should dominate the classroom, being used 60 to 70 percent of the time. The individualistic pattern may be used 20 percent of the time and a competitive pattern may be used 10 to 20 percent of the time (Johnson and Johnson, 1999:88).

Consistent with this literature, Papert (1980:10) noted that using the computer has an intrinsic influence on the performance of the understanding of three- dimensional shapes. The learners in the experimental group, who were taught with Poly, demonstrated better performance in three - dimensional concepts than did the control group learners, who received traditional instruction. There was a positive effect on learner's geometric reasoning regarding two-and three-dimensional geometric shapes. The positive trend was obvious and the experimental group gained relatively more from the intervention than did the control group.

4.6 Conclusion

The aim of this study was to use technology to enhance the geometry learning experience. Thus, this chapter presented and discussed the findings derived from the questionnaires, interviews and activities on two-and three-dimensional activities. The findings revealed that learners using the computer programme Poly displayed better results than the learners who were exposed to traditional teaching methods only. The next chapter will summarise the main findings and the information gathered in this chapter will be used to develop recommendations for future research.

CHAPTER FIVE CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

The aim of this study is to investigate the potential and the implications of the implementation of Poly for teaching geometry in the primary school. Within this framework, the study set out to explore how classroom dynamics are shaped, the potential of Poly for mathematics teaching and learning; learners' learning attitudes and the feasibility of its use in the primary school classroom. From the analysis of learner's interviews and learners' questionnaires; it was found that the introduction of Poly influences the educational practice in three-dimensions, namely, classroom practice, cognitive development and learning attitudes. This assumption did not precede the research but was generated from the data gathered. This chapter provides a summary of the study (5.2), presents the main findings of the questionnaires and interviews (5.3), outlines the main conclusions emanating from the findings (5.4), and puts forward recommendations with regard to related issues for future research (5.5).

5.2 Summary of the study

The focus of this study is to enhance the understanding of three-dimensional geometry learning experience of primary school learners through the utilisation of the use of software. With regard to the literature review, a conceptual framework for this study was highlighted and selected relevant theories were discussed. This study centred on a qualitative research design with quantitative elements, in which a case study methodology was employed.

Through a process of purposive sampling, a primary school in Verulam, KwaZulu-Natal was selected and forty grade seven learners who were deployed in an experimental group and a control group in equal proportion, respectively, were also selected. The experimental group performed a series of activities on finding the nets of solid shapes using Poly whilst the control group undertook a series of activities on finding the nets of solid shapes using traditional learning methods. The research instruments included questionnaires and an interview. The interview was conducted with learners in the experimental and control group. Data analysis included frequency counts and content analysis.

5.3 Main findings of the study

Emanating from the study are the main findings which are summarised according to the research questions.

5.3.1 Research question: Can computer-generated instruction enhance the conceptual understanding of primary school learners?

Consistent with the literature, it appears that using the computer has a positive effect on the geometric reasoning of two-and three-dimensional geometric shapes. The positive trend is obvious and the experimental group gained relatively more from the intervention than did the control group. The findings that emerged from this study demonstrated that classroom management breaks from routine while using Poly. It facilitated the classroom activity and enhanced its productivity and quality.

Poly can be used to facilitate the display of information, increase access to pattern generation, assist in the execution of tasks and provide learners the opportunity to work at their own pace. At the same time, the educator's role remains central and essential to the classroom practice. The educator's role is based on identifying appropriate learning outcomes, choosing appropriate software activities, structuring and sequencing the learning process.

Emanating from the study are the main findings which are summarised according to the research questions. The way in which Poly was used by learners and educators made a difference. Computers should be used to help learners develop a better mathematical understanding. It appeared that while there are certainly no guaranteed results, Poly has the potential to improve learners' educational experiences. It can enable the effective application of constructive, cognitive and collaborative models of learning.

Poly is not just a mathematical tool but also a tool for thinking and helping to enhance learning. It can serve as a vehicle for helping learners to foster fundamental geometrical concepts. The assessment of the use of computers in mathematics by the learners of the experimental group revealed that the application of computers brought about an increased interest in mathematics and introduced more variety to their studies, making them more enjoyable and interesting. This can be observed from the histogram.

Research question: To what extent are educators willing to adopt a modified teaching strategy in pursuit of improved learner achievement?
These lessons showed that the use of Poly in an open setting has a mediating role for learners’ reinforcement of mathematical ideas and construction of knowledge. It contributed to learners’ use of their mathematical knowledge and stimulated them into making their thinking visible. Their attention was more focused on overarching ideas; they did not waste time on secondary tasks.

Mathematics with Poly seemed to have a more flexible structure. It widened the range of possible activities, provided an access route to deeper reflection and exploration than in paper and pencil practice. Understanding of mathematical concepts and relationships can also be achieved through the visual manipulation of objects provided by Poly. Although obstacles to understanding mathematical concepts did not disappear, the achievement of the learners in these lessons should not be underestimated. The findings of the research also revealed that the use of Poly can provide rich mathematical environments where learners are engaged in classroom activity. It appeared to have the potential to facilitate peer interaction, as well as to focus that interaction on learning.

Learners engaged in collaborative activity during the use of Poly and this collaboration was equal among peers. They saw themselves as learners, developed confidence to try things out in an experimental manner and were motivated to seek justifications for their conjectures. Poly motivated the learners; it is motivational because it enabled learners to make improvements to the quality of their work. Learners were more deeply involved with the activities which became more attractive and enjoyable; thus they learned more from the activities in a fixed period of time.

This case study was an honest account of a real lesson with ordinary learners. The classroom consisted of learners with mixed abilities and different learning styles. Learners worked collaboratively throughout the activities. After the experience with Poly, the researcher believed that although such results are not consistent and generalisable, the effort needed to incorporate it in the classroom is worthwhile. Such software, which encourages educators to examine the learning process, allows learners to assume personal responsibility and provides options in flexible and open-ended environments which cannot be easily rejected. Poly is not a panacea but the researcher personally believes that it can be a catalyst in mathematics learning and teaching. A further incentive is that this software is free and has no cost implications.

5.4 Limitations

The basic restriction of the research was that it was a single instrumental case study performed by one researcher over a small period of time. The study occurred in a particular time and under particular circumstances. The results could have been more reliable and reliant across the learner-population if more than one researcher participated, if Poly was implemented for a longer period of time and in different age groups and if more than one case was selected. Although these findings cannot be generalised to the overall population, because of the small number of research units, they can generalise theoretical propositions and this was the aim of this study.

More time was needed to have adequate training and familiarisation with the software. Throughout the research, and especially during the analysis, the researcher tried to be as objective as possible in order to present a real account of the research and not an evaluation of the effectiveness of the intervention and, finally to conclude un-biased findings.

This research had offered an insight into the ways a learning environment in a primary school classroom in Verulam was shaped with the implementation of Poly. However, there was a need for great caution concerning the distribution of the sources used. Copying the sources and using them in a classroom cannot guarantee the same results. Factors such as competence in using Poly and the adoption of a constructivist approach to learning played a vital role in improving the understanding of two-and three-dimensional shapes.

It is a challenge for educators to choose and to design appropriate technology-based activities for their classes. The knowledge required to teach with technology goes beyond the specific content of the subject at basic school level. It demands an integration of the subject and the specific mathematical knowledge of the instrumental dimension of the used technology. Moreover, either with or without the technology, the right teaching approach to mathematical knowledge in the learning environment requires the teacher to understand the context. The concept of pedagogical content knowledge turns out to be a framework to understand the intertwining of the key ingredients of the formation of educators prepared to use the technology as an asset in planning the lessons.

5.5 Conclusions Emanating from the Findings

With the introduction of ICT to mathematics education, one question to consider is whether mathematics education changes when ICT is introduced. The ICT-integrated environment and the paper-and-pencil environment suggests that the latter is relatively passive in supporting learning. Current studies have found that there are changes in terms of active engagement with the implementation of ICT into mathematics education as ICT holds higher efficiency in mathematics manipulation and communication as well as interactivity between educators, learners and mathematics. The conclusions are derived from the findings of this study and will be discussed according to themes.

5.5.1 Stages of Cognitive Development

Contemporary learning theories that reflect a social-constructivist view of learning and learning practices develop learners' capacity to self-regulate their own learning and provides the intellectual infrastructure for teaching and learning in many emerging ICT-integrated classrooms. The van Hiele levels were used to determine the finding of nets of solid for the pre- and post-tests. From the assessments, pre-test results showed that almost all learners were functioning at level 1, the visualisation level. Post-test results showed a growth in understanding for the majority of learners, moving towards level 2, where they could state and work with nets of solids rather than solely on what three-dimensional looked like. Interviews with the learners indicated that the use of the Poly had a pronounced effect on understanding the nets of solids. The findings suggest that technology could be used to broaden and deepen learners’ perceptions of two-and three-dimensional shapes.

5.5.2 Pre-mathematical Frames

The findings in this study revealed that pre-mathematical frames play an integral role in determining learners’ ability to solve more complex and abstract mathematical problems, as stated previously. Of the experimental group, in which 90 percent of learners were able to find the nets of the solid shapes during their activities, 80 percent of learners had pre-knowledge of two-dimensional and three-dimensional shapes whilst 90 percent had pre-knowledge of nets of solid shapes. This may be attributed to the benefits of learning with Poly.

5.5.3 Deep and Surface Learning

All twenty (100 percent) respondents in the experimental group indicated that they had begun to understand associated additional mathematical concepts during the course of the activities. Using computers in mathematics can change the way learners learn everything else. Furthermore, this may imply that the use of Poly through the medium of computers enables learners to think on a deeper level by relating other concepts to their activities. Therefore, Poly influenced the way learners thought and what they thought about. Computer-guided learning facilitated the learning of new concepts which influenced the understanding of deep structures. Research studies espouse that deep structures of the brain are inborn. One implication of this is that learning new languages does not change the way you think or what you think about, as this would be a surface structure, but learning new concepts would do this.

It was evident from this study that learners made more errors in the traditional mode of teaching. The question on properties of squares was answered equally well by both the control group and the experimental group. More errors were made by the control group than the experimental group when answering the question on cubes. This indicates that the learners were experiencing problems with three-dimensional shapes.

The experimental group recorded 30 percent of surface errors whilst the control group recorded 50 percent. However, 60 percent of executive errors were recorded by the control group as compared to 30 percent of the experimental group. This indicated that the computer has an advantage over the traditional teaching approach. The control group’s surface error was 80 percent as compared to the experimental group of 50 percent. This indicated that learners have a poor knowledge of nets of solid shapes in both the experimental and the control group.

Perceptual difficulties are frequently associated with learning disabilities. These learners may have difficulties integrating the components or parts of a spatial stimulus to form the whole, difficulties discriminating between the main visual information and irrelevant background information. In structural errors, learners failed to appreciate the relationship involved in the problem or to grasp some principle essential to the solution. Lack of earlier sensori-motor experiences, such as building, matching, shape manipulation, can lead to difficulties learning visually or tactually; some children prefer to learn using their auditory senses.

As discussed previously, there was a marked difference in the experimental and control groups scores. Thus, with the use of technology, the learner would be able to revisit gaps in his or her knowledge. It appeared that understanding two-and three-dimensional shapes with manipulatives, both computer and concrete, had a positive effect on learners' geometric reasoning and spatial tasks. The experimental group gained relatively more from the intervention than did the control group.

It has been argued that dynamic geometry software develops higher order thinking skills such as synthesising, analysing, conjecturing, experimenting, generalising and reasoning. It was noted that dynamic geometry software develops both deductive and inductive reasoning. The integration of technology in mathematics teaching not only develops mathematical thinking but could also enhance general thinking abilities, such as qualitative-analytic, spatial-imaginal and causal experimental thinking and promote higher-order thinking skills by developing appropriate teaching scenarios.

5.5.4 Result of Histogram and Mann- Whitney Test

The Mann-Whitney Test of the experimental group between the pre-and post-test showed that there was a significant difference in the scores. The result was also significant of the control group pre-and post-test. The results of the histogram for the experimental post-test group exhibited better scores in the

experimental group than the control group. Furthermore, the result of the histogram for the experimental group was also skewed to the right, indicating that the experimental group scored better than the control group. The above observation can be attributed to the intervention of the use of the computer.
The hypothesis, if computer-assisted teaching and learning took place, then the learners would be able to improve their understanding of two-and three-dimensional geometrical concepts, can be accepted.

5.6 Recommendations for future research

The results of this study showed that there is good potential in using Poly in teaching primary school mathematics but further research investigating mathematical ideas developed by learners through the use of Poly is necessary. Moreover, research on educators' professional development in order to use Poly effectively in the classroom is necessary. If Poly is going to become an established part of the curriculum, it is important to continue to address the following issues: the general impact of Poly activity on mathematics learning; the variety of learner approaches to Poly; the implications of discussion and collaboration between learners while using Poly, gender differences while using Poly; the use and developing understanding of mathematical ideas and the role of the educator in Poly environments.

Finally, longitudinal studies need to be carried out to examine the long-term effects of the use of Poly in learners' mathematical attainment and achievement. Integrating new technology into everyday teaching and learning of mathematics has proven to be a slow process that involves multiple challenges for educators and learners. Educators need adequate training and experience using software; they also need time to accept and adapt to the changes necessary to effectively integrate technology into their classrooms, including changes in teaching methods, learning situations, and also mathematical concepts and contents taught.

5.7 The Impact of Computer-Assisted Instruction

At the end of this study, it was found that the computer-supported teaching which was applied to the experimental group in the teaching of geometry was more effective than the traditional teaching. However, the pre-knowledge framework for both the experimental and the control group adheres to the theory of Piaget. The Mann- Whitney tests indicate that there are significant differences in both the pre-and post-tests for the control and experimental groups. It seemed that a blended approach is more meaningful as there is a difference in both scores. The experimental group learners found the opportunity of participating in the computer-supported teaching activities actively, with the help of the dynamic geometry programme Poly.

In addition to this, the computer-supported teaching environment in the experiment group was supported with group study. In these computer-supported teaching environments, the opportunity of discussing their opinions, discussing the results they found with their friends and constructing their knowledge, were given to the learners. In the conclusion of this study, the dynamic geometry programme Poly can be seen as an important reason for increasing experimental group learners' geometry achievement since this dynamic geometry programme gives learners the opportunity of moving the geometric figures which were given to them, constructing new figures, making observations and constructing their own knowledge. Besides, this software made geometry more enjoyable for learners, providing visuals of geometric figures.

5.8 Conclusion

Today, many educators and learners have access to computers and, although appropriate software is available both in schools and at home, technology is rarely integrated into everyday teaching. Being aware of the vital role educators play in a technology-supported mathematics classroom, professional development opportunities need to be adapted in order to better prepare educators for this new challenge of effectively integrating technology into their teaching practice.

Analysis of the data collected through the questionnaires, activities on finding nets of solids and focus group interviews displayed a significant disparity in the learning experiences of the control group, who attempted activities on finding nets of solids using traditional classroom teaching and learning methods, and the experimental group, who conducted their activities on finding nets of solids using software. The research study conducted in the context of this dissertation represents a first step towards the goal of providing more successful introductory materials for professional development with dynamic mathematics software by identifying impediments educators face when being introduced to this new technological tool.

For different reasons, like its open source nature and versatility, the dynamic mathematics software Poly was selected from the pool of available educational mathematics software in order to evaluate a series of introductory technology workshops and assess the usability of the software itself. New instructional materials can be developed or existing instructional materials for professional development can be modified with the goal of making the introduction of dynamic geometry software easier for mathematics educators.

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APPENDIX A

QUESTIONNAIRE

SECTION A

BIOGRAPHICAL INFORMATION

Place a tick in the appropriate block.

1. Gender   Male □ Female □

2. Age   11□ 12 □ 13 □ 14 □

SECTION B

This section comprises questions about 2-dimensional and 3-dimensional shapes that require you to place a tick in the appropriate block. Some questions may require explanations.

1. What do you understand by the concept '2-dimensional shape'?

____

____

2. What do you understand by the concept '3-dimensional shape'?

____

____

3. What do you understand by the term 'nets of a shape'?

____

____

4. Did you know what 2-dimensional shape was before these lessons? Yes □ No □

5. If yes, please provide a brief explanation of what you understood a 2-dimensional shape to be.

____

____

____

6. Did you know what a 3-dimensional shape was before these lessons? Yes □ No □

7. If yes, please provide a brief explanation of what you understood a 3-dimensional shape' to be.

____

____

____

8. If you answered YES to questions 3 and 5, did this knowledge assist you in finding a method of finding nets of the solid shapes? Yes □ No □

9. If you answered Yes to question 7, please explain how you arrived at your answer.

____

____

____

10. If you answered NO to question 7, please explain how you found the nets of 3 dimensional shapes

____

____

____

SECTION C

Tick the statements that you agree with.

1. I was allowed to try my own methods to find answers. □

2. I felt comfortable trying my own method. □

3. I did not fear getting answers wrong because the feedback from the computer made me feel at ease. □

4. Giving the computer instructions made me feel in control. □

5. I was able to work at my own pace. □

6. Being able to work with other learners by sharing ideas on their/my screen improved my understanding of the lessons. □

7. Being able to share my ideas with the entire class made me feel that my ideas were important. □

SECTION D

1. Did you learn or gain a better understanding of any other mathematical concepts from this activity? Yes □ No □

2. If YES, list these concepts and explain how.

____

____

APPENDIX B – Parent Consent Form

Lotusville Primary School
58 Trevennen Road
Verulam
Tele/Fax : 032 5332607

Dear Parent

Re: Informed Consent

I am currently pursuing a Master of Technology degree at the Durban University of Technology under the supervision of Proffessor R. Naidoo (Tel: 2042371). My research is on "*The effectiveness of computer-aided teaching on the quality of learning geometric concepts by grade seven learners at a selected primary school in KwaZulu-Natal."* The primary aim of this study is to improve the quality of your child's learning experience through the use of the resources available in the school's computer centre in conjunction with the web-based facilities made available by the Durban University of Technology. Permission has been obtained from the KwaZulu-Natal Education Department to conduct this study as part of the normal school program.

All lessons will be conducted in school during normal school hours. Furthermore the content of the lessons form part of the Grade 7 syllabus. In addition to being involved in the lessons your child will be required to answer a short questionnaire and participate in a brief interview based on his/her perceptions of the learning model.

Please note that participation in this study is voluntary. If you decide to grant permission for your child to participate you are free to withdraw your consent and discontinue participation at any time.

Further information regarding this study can be obtained from P.Yegambaram (0832965632).

Thanking You
P.Yegambaram

DECLARATION BY PARENT

I, ……………………………………………………… (full names of parent), hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to my child,
………………………………………………………………, participating in the research project.
I understand that I am at liberty to withdraw my child from the project at any time, should I so desire.

…………………………………………… ………………
SIGNATURE OF PARENT DATE

<u>APPENDIX C Letter from Department Of Education</u>

kzn education
Department:
Education
KWAZULU-NATAL

MR P YEGAMBARAM
43 MARINE AVENUE
SEATIDES
TONGAAT
4399

Enquiries: Sibusiso Alwar
Date: 07/01/2010
Reference: 0006/2010

RESEARCH PROPOSAL: THE EFFECTIVENESS OF COMPUTER AIDED TEACHING ON THE QUALITY OF LEARNING GEOMETRIC CONCEPTS AT A SELECTED PRIMARY SCHOOL IN KWAZULU NATAL

Your application to conduct the above-mentioned research in schools in the attached list has been approved subject to the following conditions:

1. Principals, educators and learners are under no obligation to assist you in your investigation.
2. Principals, educators, learners and schools should not be identifiable in any way from the results of the investigation.
3. You make all the arrangements concerning your investigation.
4. Educator programmes are not to be interrupted.
5. The investigation is to be conducted from 07 January 2010 to 07 January 2011.
6. Should you wish to extend the period of your survey at the school(s) please contact Mr Sibusiso Alwar at the contact numbers above.
7. A photocopy of this letter is submitted to the principal of the school where the intended research is to be conducted.
8. Your research will be limited to the schools submitted.
9. A brief summary of the content, findings and recommendations is provided to the Director: Resource Planning.

...dedicated to service and performance
beyond the call of duty.

KWAZULU-NATAL DEPARTMENT OF EDUCATION
POSTAL: Private Bag X9137, Pietermaritzburg, 3200, KwaZulu-Natal, Republic of South Africa
PHYSICAL: Office G25, 188 Pietermaritz Street, Metropolitan Building, PIETERMARITZBURG 3201
TEL: Tel: +27 33 341 8610/8611 | Fax: +27 33 341 8612 | E-mail:

kzn education

Department:
Education
KWAZULU-NATAL

10. The Department receives a copy of the completed report/dissertation/thesis addressed to:

The Director: Resource Planning
Private Bag X9137
Pietermaritzburg
3200

We wish you success in your research.

Kind regards

[redacted]

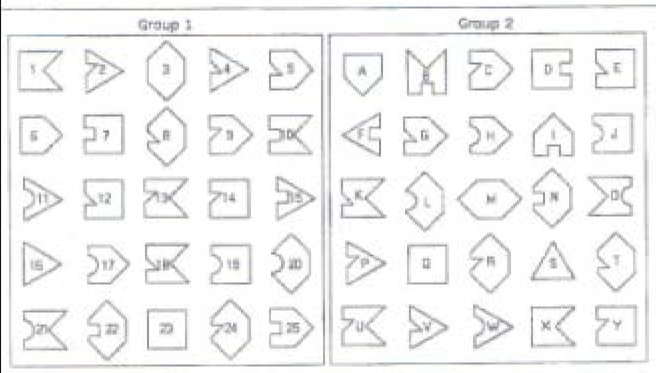
R. Cassius Lubisi (PhD)
Superintendent-General

...dedicated to service and performance
beyond the call of duty.

KWAZULU-NATAL DEPARTMENT OF EDUCATION
POSTAL: Private Bag X9137, Pietermaritzburg, 3200, KwaZulu-Natal, Republic of South Africa
PHYSICAL: Office G25, 188 Pietermaritz Street, Metropolitan Building, PIETERMARITZBURG 3201
TEL: Tel: +27 33 341 8610/8611 | Fax: +27 33 341 8612 | E-mail:

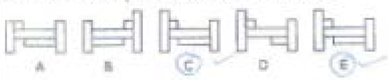
2D and 3Dimensional Questions for learners

1. In this example, you are asked to look at two groups of simple, flat objects and find pairs tha are exactly
the same size and shape. Each group has 25 small drawings of these 2-dimensional objects. Which shape in
Group 2 corresponds to the shape in Group 1?

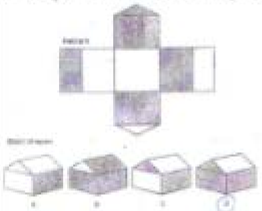


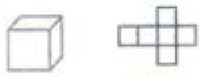
1	X	14	Y
2	P	15	F
3	M	16	S
4	V	17	H
5	G	18	K
6	A	19	J
7	D	20	L
8	T	21	O
9	C	22	N
10	B	23	Q
11	W	24	R
12	E	25	i
13	U		

2. Which two pictures are identical?



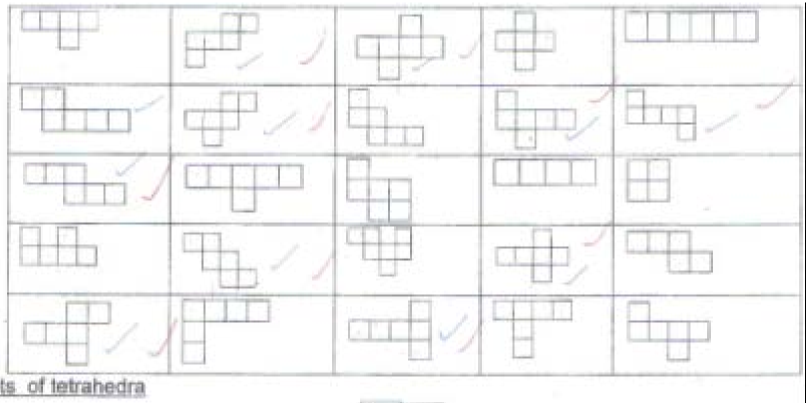
3. Which of the solid shapes shown could be made from the pattern?





cube net of a cube

4. If you take apart the cube while keeping each square attached to the entire side of at least one other square and then lay the resulting figure flat, you will have a **net** for the cube consisting of the six square faces of the cube. Below is one example of a net for the cube. Study the following shapes and tick the one that will make a cube.

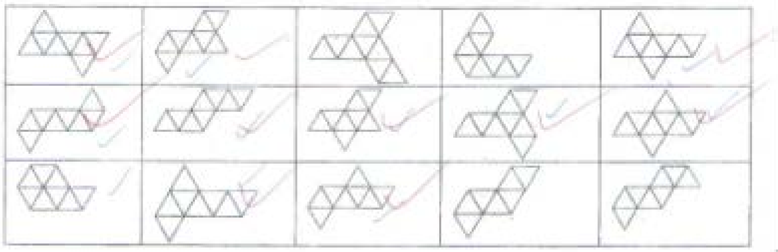


Nets of tetrahedra

ee if you can work out which nets will make a tetrahedron.

Nets of octahedron

f course, for an octahedron, you must have eight triangles in the net. There are also four iangles round a point. See if you can find all the nets for an below.



APPENDIX E
Response From One Participant

Age 12 Male / Female? Male

These are 2 dimensional shapes. Tell me how you matched group 1 with group 2. How do you know which ones from group 1 to match with group 2?

First you think carefully and look at group 1 shapes and to find the same shapes as that one you have the shape can be upside down or sideways to confuse you.

What were you thinking?

What was challenging during this exercise?

To find the shapes because some were upside down, some turned around, some sideways and it was challenging and confusing

2. Nets of a cube

Cut the edges of the box so that you can open it up and lie it flat.
Which of the following nets will fold into a cube?

How do you know?

I knew and saw that the one that they are showing has 6 sides and think and estimate which one goes and which does not.

What were you thinking?

You need to think as I was which one goes there and which one goes there and ask yourself if this is a cube?

What was challenging during this exercise?

You have to think and estimate where does each one go and this was challenging.

3. Nets of tetrahedra

Which of these nets will make a tetrahedron?
How do you know? I estimated which is which.

What were you thinking? I was thinking of the original shape and take a shape from the other shape and see

What was challenging during this exercise?

4. Nets of octahedron

How do you know?

What were you thinking?

What was challenging during this exercise?

Can you draw some 2 d shapes?
Can you draw some 3 d shapes cube/cyclinder/cone
Did you visualize?

How did the computer assist you to understand nets of solid shapes?

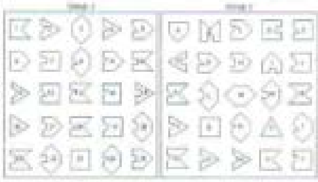
Thank you for your participation in this interview.

APPENDIX F QUESTIONNAIRE – Response from a learner

GRADE 7: INTERVIEW: 2 and 3 d Shapes

Name : ________________ Age 12 Male or Female? Female

These are 2 dimensional shapes.Tell me how you matched group 1 with group 2.



How do you know which ones from group 1 to match with group 2 ? It's the same shape but just placed differently in the other group

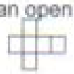
What were you thinking? ~~It was quite confusing~~. I looked at the shape and studied it because it was placed differently.

What was challenging during this exercise? When the shapes were in different ~~die~~ directions and some looked the same.

2. Nets of a cube

Cut the edges of the box so that you can open it up and lie it flat.

The flat box looks like this figure



Which of the following nets will fold into a cube

How do you know? Some had six blocks which was correct and you have to think how it will fold in a cube

What were you thinking? ~~It's quite easy~~. How the net will fold in a cube.

What was challenging during this exercise? They were confusing

3. Nets of tetrahedra

Which of these nets will make a tetrahedron.

How do you know?

It had 4 triangles and it look like the net given.

What were you thinking? It's not difficult to choose because there's little shape nets to choose from.

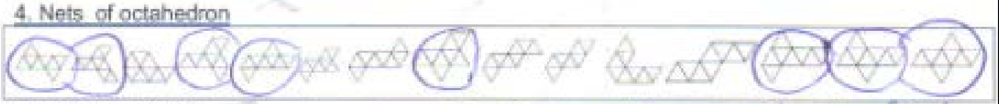
What was challenging during this exercise? Nothing

tyger yegambaram Page 1

~~And some had 3 or 5 tri~~

I also counted the amount of triangle

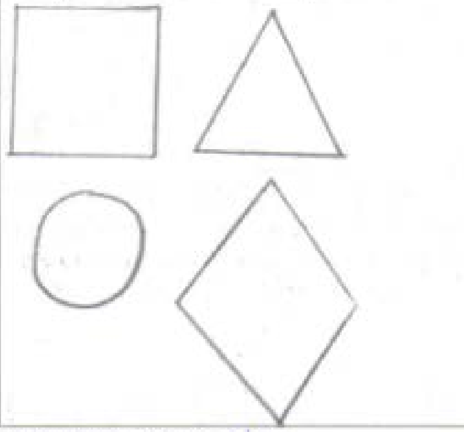
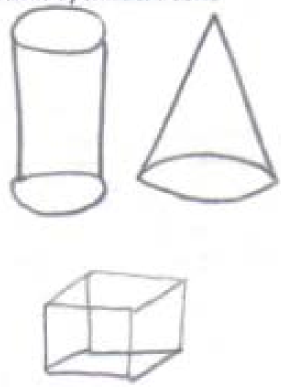
4. Nets of octahedron



How do you know? It has 8 triangles and it folds into a octahedron.

What were you thinking? Quite difficult because it was hard to think how it folded.

What was challenging during this exercise? ~~They all looked~~ Some were ~~It was~~ confusing because it had 8 triangles.

Can you draw some 2 d shapes?	Can you draw some 3 d shapes cube/cyclinder/cone
	

Did you visualize? Yes

How did the computer assist you to understand nets of solid shapes? It ~~show~~ showed how the net of a shape opens and closes which makes it easier to understand. It also made it ~~simpiler~~ easy to understand and also taught some methods.

Thank you for your participation in this interview.

APPENDIX G – Learner Interview - answer memorandum

Grade 7 Interview Age: ___________

1. What is meant by parallel lines? Lines that are equidistant apart

2. What is meant by perpendicular lines? Lines that form 90^0 angle

3. Describe a square A. 4 sided figure with all sides equal and all angles 90^0

4. What is meant by 2 D? A plane figure which has length and breadth

5. How many sides does a square have? 4

6. How many squares can you see in the cube? 6

7. How many different faces do you see? 6

8. How many edges can you see? 12

9. How many vertices can you see? 8

10. Name a few shapes like a cube that you find in your daily life. Dice,box

11. Does this figure have a height? Yes

12. What is a 3 D shape? It has height, length and breadth

Does it have a height? Yes

Does it have a length? Yes

Does it have a width? Yes

13. Is this cube a 3 D figure/Why? Yes – It has height, length and breadth

14. Can 7 squares make a cube? No

15. Can 5 squares make a cube? No

16. Will only squares make a cube? Explain. Yes– because all the faces must be the same

17. Does the size of the square need to be the same for it to form a cube? Explain

Yes- otherwise it will form a cuboid

Nets of Cube

Which of the following will make a cube?

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
✓	x	✓	x	✓	x	✓	✓	x	✓	x	✓	x	x	x

Is it a 2 dimensional or 3 dimensional shape? van Hiele's stage Level 1- Visualisation

Explain your answer. van Hiele's stage Level 2 - Analysis

How many faces: ___ van Hiele's stage Level 1 Visualisation

How many edges: _____ van Hiele's stage Level 1 Visualisation

How many vertices: _______ van Hiele's stage Level 1 Visualisation

What is the shape of each face? van Hiele's stage Level 1 Visualisation

Will a different cube (bigger or smaller) have a different number of faces, edges, or vertices? van Hiele's stage Level 2- Analysis

What are some of the common characteristics of the nets that you created? van Hiele's stage Level 1 Visualisation

How many squares does each net have? van Hiele's stage Level 1 Visualisation

Which arrangements of squares will not form a cube? van Hiele's stage Level 2 - Analysis

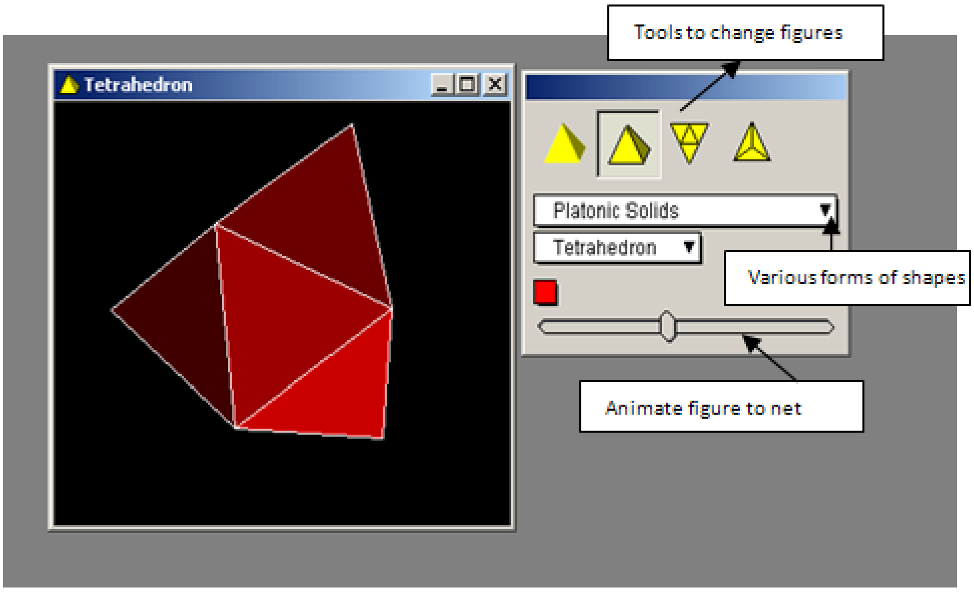
Nets of Cube

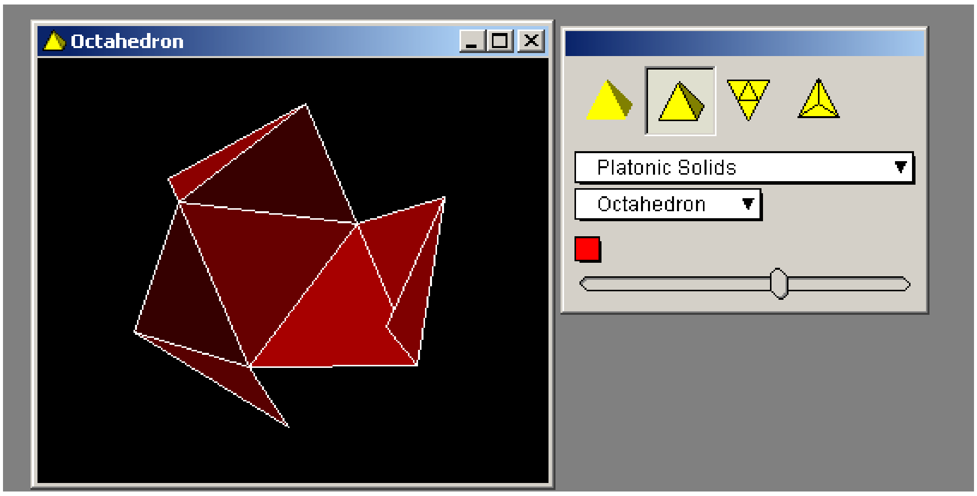
Which of the following are nets for a cube? van Hiele's stage Level 1 Visualisation

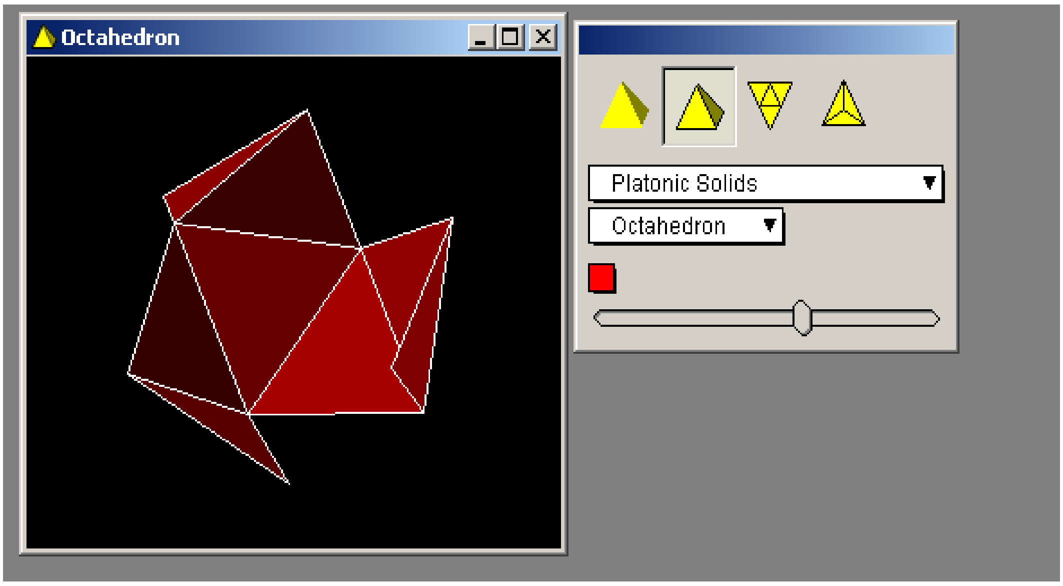
Explain how you decided. van Hiele's stage Level 2 - Analysis

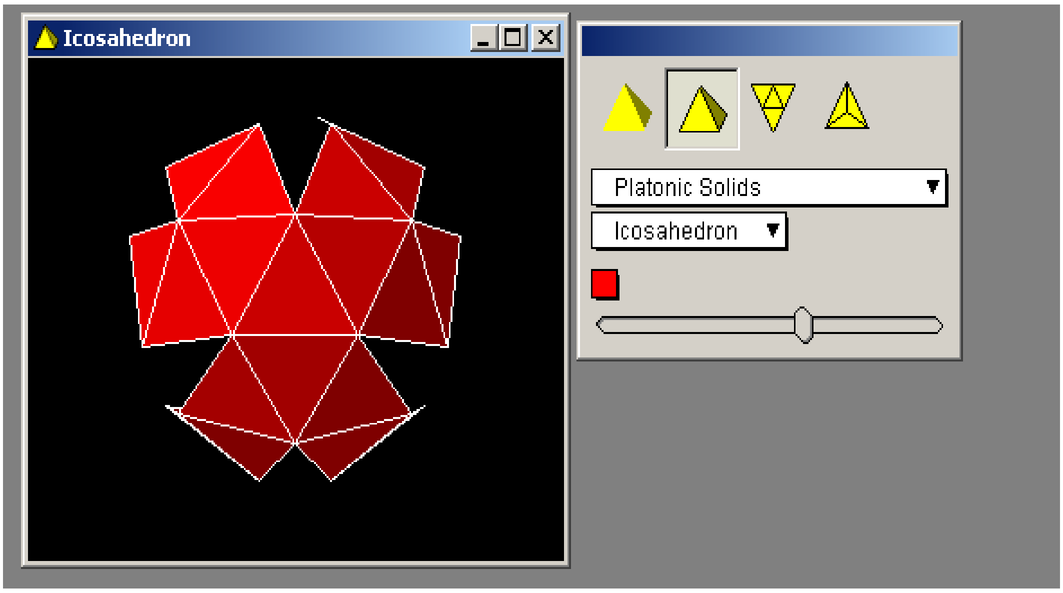
What will happen if I increased the number of squares? van Hiele's stage Level 2 - Analysis

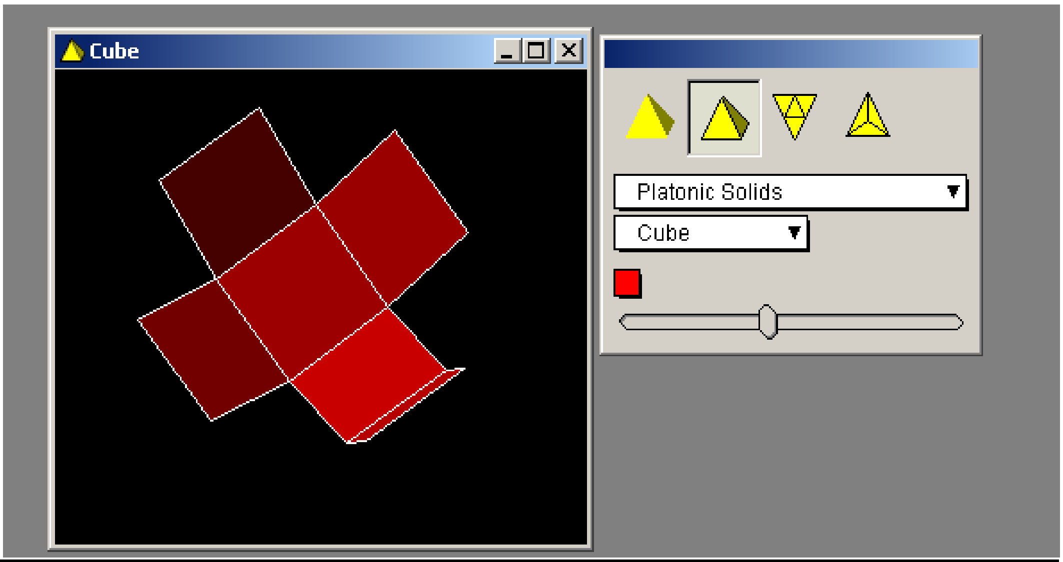
APPENDIX H : Screenshots from a Poly window



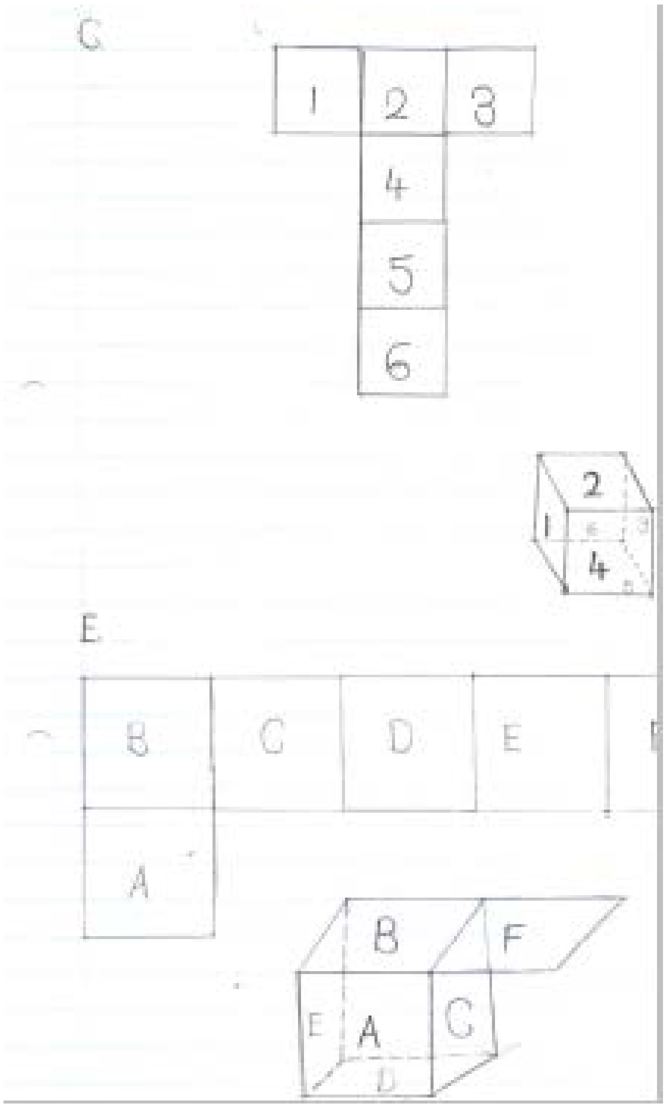



Octahedron
Platonic Solids
Octahedron


Icosahedron
Platonic Solids
Icosahedron

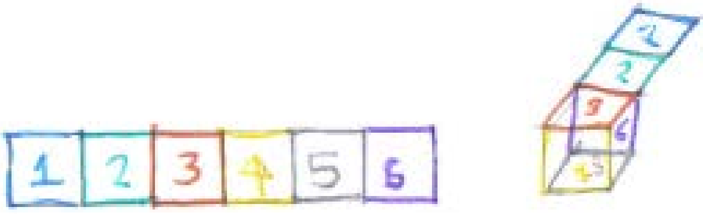

Cube
Platonic Solids
Cube

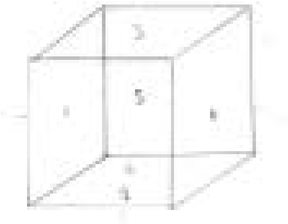
APPENDIX I – Sample of learners sketch of a cube



h)

k)



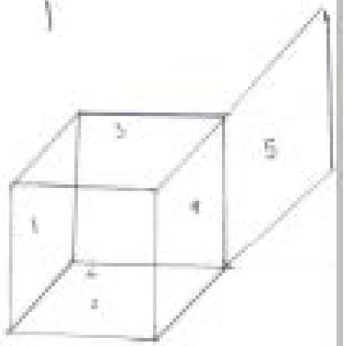


1 - left side
2 - front side

3 - top

- right side
- back
- base

j



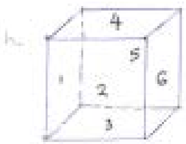
- left side
- front side

- top

- right cide

-

base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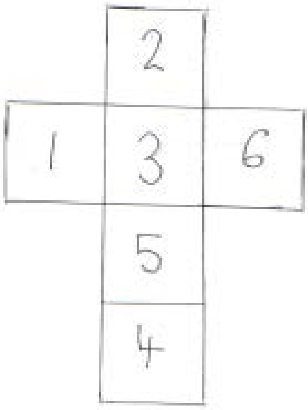
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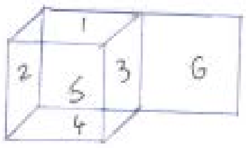
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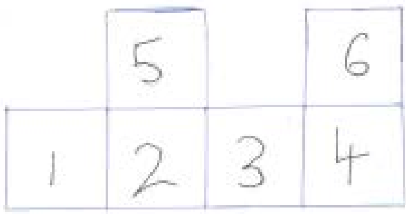
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