THE DEVELOPMENT, USE AND EVALUATION OF SELF INSTRUCTION MATERIAL FOR THE NUMERICAL METHODS SECTION OF MATHEMATICS II AS TAUGHT TO TECHNIKON STUDENTS

by

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I declare that the dissertation represents my own work, both in conception and execution. Conclusions and comments given are my own and do not necessarily reflect the views of Technikon Natal.

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SUMMARY

The preparation of this dissertation, besides considering (in Chapter 1) some general and specific factors that prompted research into self-instruction in a component of mathematics as taught in technikons, involved the pursual of four overall aims.

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1. The **first aim** was the **identification of factors that affect the performance of students using self-instructional material** (Chapter 2). By studying literature on the preparation of self instructional texts by authorities in the field such as Derek Rowntree (1986), A.J.Romiszowski (1984) and others, the author identified factors that these authorities regarded as important in the development of self-instructional material. These factors were incorporated, where applicable, in the self-instructional material developed by the author.

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2. The **second aim** was the **development of self-instructional material in the Numerical Methods component of Mathematics II** <u>**as taught to Technikon students**</u> **in Engineering and Applied Science** (Chapter 3). The factors regarded by experts as crucial in the writing of effective self-instructional texts were incorporated by the author, as was the use of mathematical programming techniques that have proved

successful when used by Technikon students studying mathematics, such as those used in *Engineering Mathematics* by K.A.Stroud (1982). Textbooks including *Engineering Mathematics - a Programmed Approach* by C.W.Evans (1989) and especially *Numerical Methods for Engineers and Scientists* by A.C.Bajpai, I.M.Calus and J.A.Fairley (Bajpai et al 1977) were utilized in the subject-matter part of the preparation of the self-instructional text.

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3. The **third aim** was the **use and evaluation of a prepared self-instructional text** (Chapter 4). Such use and evaluation of the material developed by the writer constituted the research project which the dissertation reports. Evaluation of the text took the following form:

3.1 A post-test experiment, in which the experimental group was provided with the self-instructional text while a matched control group received conventional classroom instruction. The achievement of both groups of 31 students in the Numerical Methods section of a common Mathematics II examination paper formed the basis of the comparative statistical analyses.

3.2 Analysis of responses to questionnaires given to students subjected to the self-instructional approach, prior and subsequent to the self-instructional exercise, was also used to evaluate students' perception of this Short Lecture Self-Instructional (SLSI) approach ...in many cases this was their first exposure to self-instruction.

Data collected during the execution of the third aim has been presented using standard statistical methods, and the author (who during the last twenty years in mathematics education has lectured on statistical methods to numerous Technikon student groups including those doing post-diploma research methodology courses) kept to the maxim "Let the facts speak for themselves", in order to avoid bias.

Analysis of the results obtained, relating to the two points mentioned above, led to the following conclusions :

i) The majority of students who participated in the Short Lecture Self-Instruction (SLSI) project indicated in their responses to a pre- SLSI questionnaire that they were excited by the prospect of participating in the SLSI project.

ii) Responses by students in the post-SLSI session indicated that the majority of participating students had enjoyed this instructional method. They believed that the approach could be used successfully in certain mathematical topics, although opinions varied on what proportion of Mathematics II was suited to self-instruction (SI). Responses were virtually unanimous that the short introductory lecture preceding each lesson was extremely helpful in enabling them to understand the new concepts introduced in the lesson as well as enabling them to work through the follow-up self-instructional lessons unaided.

iii) The experiment itself yielded the following information: The mean examination percentage in the numerical methods question of a common examination question paper obtained by the experimental SLSI group was ≈ 5,5% higher than that obtained by the matched Conventional Classroom Approach (CCA) control group. Statistically, however, the evidence was not sufficient to indicate that the SLSI method was significantly better than the CCA.

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4. The **fourth aim**, important in terms of this study being one in the field of **post-school education**, was to develop **recommendations** (arising from the research conclusions and factors that led to the research) for the utilization of self-instructional material in mathematics at a post-school level.

Finally, nine respondents to a questionnaire sent to other South African technikon mathematics departments (Annexure G) indicated that besides being interested in the Short Lecture Self-Instructional approach, they also felt that there are various mathematical topics in the current technikon syllabi where this approach could be used.

OPSOMMING

Die voorbereiding van hierdie tesis behels die inagneming van algemene en spesifieke faktore (Hoofstuk 1) wat gelei het tot navorsing in self-onderrig in 'n komponent van technikon wiskunde II, asook 'n ondersoek van vier algemene doelwitte, naamlik :

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1. **Doelwit 1 :** Die **identifikasie van faktore** (Hoofstuk 2) **wat die prestasie vermoë van studente, wat self-onderrig materiaal gebruik, beïnvloed.** Faktore wat belangrik geag word deur deskundiges op hierdie gebied soos Derek Rowntree (1986), A.J. Romiszowski (1984) en andere word na verwys. Hierdie faktore word, waar van toepassing, deur die outeur in berekening gebring in die ontwikkeling van sy self-onderrig materiaal.

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2. **Doelwit 2 :** Die **ontwikkeling van self-onderrig materiaal in Numeriese Metode komponent van Wiskunde II <u>soos aangebied aan Technikon studente</u> in Ingeneurswese en Toegepaste Wetenskap** (Hoofstuk 3). Faktore wat belangrik geag is deur deskundiges op die gebied van effektiewe self-onderrig tekse, sowel as wiskundige programmerings tegnieke, word deur die outeur geimplementeer. Laasgenoemde tegnieke, soos byvoorbeeld die wat in *Engineering Mathematics* deur K.A.Stroud (1982) gebruik is, is in die verlede suksesvol deur Technikon studente

aangewend. Handboeke wat gebruik is in die voorbereiding van hierdie teks sluit in : *Engineering Mathematics - a Programmed Approach* deur C.W. Evans (1989) en veral *Numerical Methods for Engineers and Scientists* deur A.C. Bajpai, I.M. Calus and J.A. Fairley (Bajpai et al 1977).

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3. **Doelwit 3 :** Die **gebruik en ontleding** van 'n **voorbereide self-onderrig teks**. Hierdie gebruik en ontleding van materiaal is die basis waarop hierdie navorsings projek gebasseer is. Die evaluering het die volgende vorm aangeneem.

3.1 'n Na-toets eksperiment, waartydens die eksperimentele groep 'n self-onderrig module ontvang, terwyl die kontrole groep konvensionele klaskamer onderrig ontvang. Die resultate wat deur die twee groepe (31 studente elk) in die Numeriese Metodes afdeling van 'n algemene Wiskunde II vraestel behaal is, vorm die basis van die Statistiese vergelykingsanalise.

3.2 Ontleding van meningsvraelyste wat voor en na die self-onderrig oefening deur die studente ingevul is, is ook gebruik om hulle persepsie ten opsigte van self-onderrig te meet. In die meeste gevalle was dit hulle eerste blootstelling aan self-onderrig.

Data wat tydens die uitvoering van die derde doelwit ingesamel is, is deur middel van algemene statistiese metodes verwerk en opgesom. Die outeur (dosent vir die afgelope 20 jaar in statistiese metodes vir technikon studente, insluitend die wat na-diploma navorsing metodologie kursusse gedoen het) het hom by die goeie reël: "Let the facts speak for themselves", gehou ten einde partydigheid te voorkom.

Ontleding van resultate, voortspruitend uit doelwit 3, het tot die volgende gevolgtrekkings gelei:

i) Die meerderheid studente wat aan die self-onderrig program deelgeneem het, het in die eerste meningsvraelys aangedui dat hulle opgewonde was oor die vooruitsig om aan so 'n program deel te neem.

ii) Antwoorde deur studente, nadat hulle aan die self-onderrig program deelgeneem het, het aangedui dat hulle hierdie onderrig metode geniet het. Hulle was van mening dat hierdie benadering suksesvol in sekere wiskundige onderwerpe gebruik kan word, maar menings het verskil oor watter dele van Wiskunde II wel geskik sou wees vir self-onderrig. Hulle was eenparig dat die kort inleidende lesing wat elke les voorafgaan, hulle in staat gestel het om die nuwe begrippe te verstaan sowel as om die opvolg oefeninge sonder hulp deur te werk.

iii) Die eksperiment het die volgende inligting opgelewer: Die gemiddelde persentasie verkry in die Numeriese Metodes afdeling van 'n algemene eksamen vraestel deur die eksperimentele groep was ≈ 5,5% hoër as die van die kontrole groep. Statisties was daar nie voldoende bewyse dat die self-onderrig metode beter is as die gewone konvensionele klaskamer onderrig metode nie.

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4. **Doelwit 4 :** Aangesien hierdie 'n studie is in na-skoolse opvoeding, is dit belangrik om aanbevelings (voorspruitend uit die gevolgtrekking en faktore wat tot hirdie studie gelei het) te maak oor die gebruik van self-onderrig materiaal in wiskunde op tersiere vlak.

Ten slotte, nege respondente van 'n menings vraelys wat aan onder Suid-Afrikaanse Technikons (Bylae G) gestuur is, het aangedui dat hulle in die self-onderrig benadering belang stel. Volgens hulle is daar verskeie wiskundige onderwerpe in die huidige technikon sillabusse waar hierdie voorgestelde onderrig metode gebruik kan word.

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ABBREVIATIONS

ANC	African National Congress
ASP	Academic Support Programme
CAI	Computer Aided Instruction
CAL	Computer-Aided Learning
CCA	Conventional Classroom Approach
Eng.	Engineering
H.C.	Heavy Current
HSRC	Human Sciences Research Council
IMU	Individual Mathematics Instruction
IPI	Integrated Programmed Instruction
L.C.	Light Current
LFM	Learning For Mastery
MIME	Micros In Mathematics Education
OFS	Orange Free State
PI	Programmed Instruction
PSI	Personalized System of Instruction
SAQ	Self Assessment Question
SI	Self-Instruction
SLSI	Short Lecture Self-Instruction
TN	Technikon Natal
UOFS	University of the Orange Free State

CHAPTER 1

INTRODUCING THE PROBLEM

1.1 Introduction

The importance of this research project can be viewed in many ways. In this chapter two broad facets which prompted the project will be reviewed:
firstly, the need for different tertiary teaching strategies in South Africa in the light of increased student numbers, decreasing finance and limited infrastructure; secondly, the perception that mathematics is a difficult subject and hence the importance of trying a variety of approaches in teaching mathematics students at a post-school level.

From the outset, when the writer first became involved in this project, it was hypothesized that the problems caused by the environment in which the learning of mathematics takes place (particularly at technikons), and by the perceived difficulty of the subject matter, could partially be resolved, or alleviated, through the development and use of self-instructional material.

1.2 Education: a National Problem

With current political striving towards a new democratic South Africa, the need for a strong economy is emphasized daily in the national media by political and industrial leaders. By implication, there is the associated need for an education system that will provide the necessary manpower to manage and run the economy. Technikons are specifically concerned with career-orientated education, and with the expected increase in student numbers entering technikons, it becomes imperative that teaching strategies and the related factors that have a bearing on technikon tertiary education receive the attention of concerned educators.

In South Africa, technikons fulfil many of the functions of technological universities found in some other countries. According to Bester (1988;14) they are "technologically-oriented tertiary educational institutions ... involved in technology praxis". He goes on to define technology as "the result of a process of utilizing a resource-complex of means, skills and knowledge to extend human capability". In the light of this, technikons are specifically concerned with the generation and transfer of technology, including instructional technology. It seems reasonable to suggest that, through both teaching and research, technikons should (through their various disciplines) constantly pursue new and better means of imparting existing knowledge or of encouraging students to apply such knowledge. The present dissertation sets out to report on one small effort in that direction.

In order to highlight the problems and challenges facing tertiary education (and specifically technikon tertiary education), publications by various authorities have been studied and have yielded, inter alia, the following information:

Du Preez (1991) has drawn attention to some of the challenges facing post-school education, i.e. that there will be **more** students, a need for **more** career-education, with **less** money from the state, **less** academic space, and **less** academic staff.

The growth in student numbers (real and estimated) in technikons is borne out by the NATED 02-300 report which states that "..technikon education is at present growing at a very high rate of 17% per annum .." (NATED, 1991:12), as well as by Kirsten (1991) who provides statistics on the number (4113) of engineers and scientists receiving qualifications from universities in 1985, and the number (1967) of technologists, technicians and computer scientists (his terminology) receiving qualifications from technikons in 1987. These figures are then compared by Kirsten to the projected numbers that would be required to complete studies in the year 2020 if the balance between the two types of institutions is to be adjusted (according to Kirsten) to a ratio of 3,0 : 1,0 (technikon : university) rather than the then existing ratio (Kirsten, 1991 : 129) of 0,8 : 1,0 (technikon : university) . The projected figures for 2020 given by Kirsten are 7000 students in engineering and science graduating annually from universities, and about 20 000 in engineering and computer science from technikons. Obviously with the short-term situation in South Africa being very uncertain, medium to long-term predictions are extremely dangerous; however a pro-active approach necessitates that these sort of predictions are taken into account when formulating educational strategies.

The"**less**" referred to by Du Preez in monetary terms is even more of a problem when considering tertiary education because of the vast amounts of state revenue that will have to be channelled into primary and secondary education in order to redress the imbalances that exist between the education offered to the different racial groups within South Africa. Despite vociferous opposition from tertiary educationists, it is likely that financial provision for universities and technikons in South Africa will continue to drop (so-called "under-provision" has been the trend of at least the past five years), particularly under any new government.

The Times Higher Education Supplement of 20 March 1992 reported that South African universities could have their budgets cut by as much as 50% under an African National Congress government. In the same report the ANC's budget specialist, Viv McMenamin is quoted as saying the organisation was "exploring the options of emphasising primary education".

The government's Educational Renewal Strategy plans, unveiled in 1991, reveal that if the ideal of nine years' compulsory free schooling for all were to be introduced, there would be little money available for anything more than the absolute minimum in mainstream education.

For these reasons it is clear that, for example, technikons will have to become more economical, more self-sufficient and more productive.

The 1980 census indicated that 38% (NATED, 1991:35) of the adult population in South Africa had not passed standard three, indicating the real need for an upgrading of the primary phase of education for the third-world population of South Africa. Du Preez points out that South Africa is essentially a third-world country with the odd smattering of the first world (op. cit.,:9). The majority will need to get a share of government funds, that are unlikely to grow substantially until the economy grows substantially. Again this is a dilemma in that economic growth depends on highly-trained manpower which can "only" be provided by an educational system which has adequate funds coming directly or indirectly from a vibrant prosperous economy. NATED 02-300 (NATED, 1991:31) acknowledges that the "State has experienced increasing difficulties in financing the rapidly-expanding education system" and cites the fact that the economic growth has not kept pace with the growth in pupil and student numbers as one of the main sources of this financial problem. The manpower cost is already very high with 37% of employees in the public sector being in the education sector (NATED, 1991:23). With the "brain drain" and the shortage of high level manpower, the prospect of tertiary academic staff being attracted to industry is also a very real one.

What is apparent from the above is that nationally the education problem is such that innovative alternative educational approaches are imperative if the country is to survive, let alone prosper.

1.3 **Mathematics the Problem**

The vast majority of technikon students seem to experience difficulties with the study of mathematics, which is a compulsory component of all engineering and many other national diploma curricula.

secondary schools

The **high failure** rate in mathematics at various, tertiary institutions coupled with constraints such as limited finance, shortage of qualified teaching personnel, and increasing student numbers has led to various different teaching strategies being tried and adopted in the teaching of mathematics in such institutions. Mathematics, specifically Mathematics II, is regarded by many students at technikons to be the subject that holds them back.

The examination statistics for this subject at Technikon Natal for Engineering and Applied Science students for the two semesters in 1991 as well as the first semester in 1992 are shown in Table 1.1 below :

	MATHEMATICS II : Semesters 1 & 2 1991 and 1 1992 Number of students who :				
Instructional Programme	Registered (R)	Wrote the exam (W) *	Passed exam (P)	% Pass P/R x 100	% Pass P/W x 100
Civil Eng.	206	148	71	34,5	48,0
Surveying	38	31	15	39,5	48,4
Electrical L.C.	229	152	79	34,5	52,0
Electrical H.C.	57	41	25	43,9	61,0
Mechanical Eng.	147	120	65	44,2	54,2
Chemical Eng.	53	39	27	50,9	69,2
Extra Credit	38	24	13	34,2	54,2
TOTAL :	768	555	295	38,4	53,2

Table 1.1

Source : Examination control sheets for June 1991, November 1991 and June 1992, Technikon Natal.

* : A semester or course mark of at least 40% determined from tests and/or assignments is required in order to be admitted to the examination.

Besides reflecting the low pass rate, the statistics in Table 1.1 also show that the dropout rate for students who register for Mathematics II is very high. This is mainly due to students not obtaining the required course mark. The course-mark problem does not only apply to Mathematics II but also Mathematics I. With the re-curriculated Engineering courses that have been implemented from January 1992, an attempt has been made at Technikon Natal to obviate the problem of course-mark failure in Mathematics I by introducing a system of continuous assessment (see Appendix I).If this continuous assessment results in the pass rate for Mathematics I improving, it will probably also be introduced for Mathematics II.

These statistics further show that even without the more general problems facing education in the future, the methods used in the teaching of mathematics probably require urgent attention.

Similar problems exist in mathematics education elsewhere. Various institutions have tried teaching and instructional strategies to overcome such problems, and several of these are discussed below. At **Monash University, Victoria, Australia,** amongst the aims of introducing a Personalized System of Instruction (PSI) in engineering mathematics was the attempt to **increase the pass rate** and to ensure that students who passed **mastered each section.** (Cumming & McIntosh, 1983:203)

Guided reading as a teaching technique in engineering mathematics modules at the University of Bristol, England was introduced to **utilize staff and student time more efficiently and to develop**

learning skills that other methods did not. This approach, besides relieving students of the unproductive note-taking that is such an integral part of the conventional lecture system, also had the advantage that students could work at their own pace (within reason). Students intimated that this method was worth retaining (Clements & Wright, 1983:95-98).

The need for **improvement of the teaching of mathematics** to engineers and scientists was one of the factors that led to the MIME (Micros In Mathematics Education) project at the Loughborough University of Technology in England. This project, which also aimed at exploiting the use of the microcomputer in mathematics teaching, resulted in material being written in two types of units: as a teacher's aid and as a programmed text (Bajpai et el., 1984:781-810).

Evans (1989), a lecturer at what was then Portsmouth Polytechnic, favours a programmed approach for essentially two reasons : the **inability of students to cope with mathematics textbooks** - "They find them remote and the concepts difficult to handle" (Evans, 1989:xvii) - and the fact that **tertiary institutions are under pressure to increase productivity.**

After conducting research into mathematics teaching in Kenya, Eshiswani points out that the quality of **teaching in many African countries has deteriorated** due to factors including "use of inappropriate teaching methods" (Eshiswani, 1985:479).

Taking the selected examples given above into account, as well as the objective of this research project to develop self-instructional material in a mathematical topic, the following quotation attributed to Professor W. Mödinger seems apt:

> " Without mathematics there can be no science, without science there can be no technical and industrial development, and without that a country will remain in the group of underdeveloped countries"(Deston, 1991:27).

The two facets of the problem underlying the present research, i.e the national tertiary education problem and the subject (mathematics) problem described in this chapter, as well as an interest in the welfare of students struggling with engineering mathematics taught at Natal Technikon were amongst the factors that led to this research project. An outline of the scope of the study will now be given.

1.4 **Scope of the present study**

1.4.1 **Aim**

This research was mainly concerned with a study of the extrinsic factors that affect the <u>performance</u> of students using self instructional-material, and the development and evaluation of self-instructional material for the Numerical Methods component of Mathematics II as taught in National Diploma courses. Intrinsic factors (e.g. Students' own language ability) inevitably affect the situation also, but they have not formed part of this study. The factors specifically borne in mind by the author were those named in Chapter 2.2.

1.4.2 **Statement of intent**

The intent was to show that the performance of students (in the Numerical Methods component of Mathematics II) using well-designed self-instructional material can be as good as, or better than, that of students receiving "traditional" instruction.

1.4.3 **Limitations**

The study was limited to the Numerical Methods section of the new Mathematics II instructional offering (implemented during 1990) at Technikons. The syllabus of the instructional offering is taught at all technikons in South Africa.

The study was also limited to students in the Schools of Engineering and Applied Science at Technikon Natal, although contacts were made with other technikons.

1.4.4 **Conceptual clarifications and definitions**

Self instruction material

The self-instruction material developed and used was similar to texts used by K.A. Stroud in his *Engineering Mathematics* (1982), a format that had previously been found very useful for self instruction by Technikon students. Use was also be made of material derived from the author's own lecturing experience, and information obtained from literature, in the preparation of the self-instruction material.

Traditional Instruction Methods or Conventional Classroom Approach

These terms refer to conventional existing methods used by lecturers in Mathematics at Technikon Natal. As in most post-school institutions, these methods tend to concentrate on information transmission, lectures, taught tutorials and the use of textbooks or notes.

Note: The terms **tertiary** and **post-school** are used interchangeably in this dissertation.

Mathematics II (Subject code 160401922)

This subject is offered to Engineering and Applied Science students in their second semester (six months) at the Technikon. There are ≈ 16 weeks of instruction with four forty-minute lecture periods and one double tutorial period per week.

SYLLABUS

1. Differentiation
2. Partial differentiation
3. Series
4. Integration
5. Solutions of first order differential equations
6. Numerical methods
7. Statistics

Numerical Methods component of Mathematics II

The syllabus for this component covers the following:

Errors : Finite differences, difference tables, correcting &

extending difference tables, interpolation using the Newton Gregory forward difference formula, numerical differentiation (deduced from interpolation formulae), numerical integration (Trapezoidal and Simpson's rule), and numerical solution of linearly independent equations (Gauss-Seidel method).

1.4.5 **Assumptions**

It was assumed that the groups (control and experimental) used in this study were comparable because the Numerical Methods section of Mathematics II was covered over the same period by students using the self-instructional material and by those receiving "traditional" lectures.

Mathematics I results have been used for matching, and the sampling done by convenience.

It was assumed that students who had reached second level would be self motivated and thus adjust to the self-instructional technique.

1.4.6 **The importance of the study**

Several factors encountered in Mathematics education at Technikon level motivated the study :

1) Mathematics is generally perceived to be a problem subject for Engineering students at the Technikon.
2) Mathematics II specifically is regarded as a difficult instructional offering by students, and has a high failure rate.
3) This research examined an alternative lecturing and teaching

approach, that could result in improving the situation.

4) Similar self-instructional material can possibly be developed and used in bridging the gap between secondary and tertiary education, assisting students who struggle or miss certain topics, or those who are allowed to rewrite without attending classes.

5) Self-instruction material could also help by reducing contact time, thus freeing staff to pursue research.

1.4.7 **Information used** includes the following:

(1) Secondary data obtained from literature on self-instructional methods and designs, were used to identify the factors that affect the performance of students;

(2) Relevant texts on Numerical Methods, as well as the information obtained from literature were used in the development of the self-instructional material;

(3) Marks achieved by experimental and control groups in the applicable section of common examinations, as well as data obtained from student responses to pre-SLSI and post-SLSI questionnaires, were used to evaluate the effectiveness of the material.

(4) An overview of the whole situation provided the basis for recommendations made.

1.4.8 Methodology and Use of the Data

The methods used in the research project have included literature and questionnaire surveys, a controlled experiment, and critical analysis. The self-instructional material was issued to students in the "experimental" group in the form of a booklet. Normal class groupings were used with participating students being selected in terms of convenience.

1.4.9 Conclusions and general recommendations

Data obtained from questionnaires and the experiment have been presented statistically in chapter 4 along with the corresponding conclusions. Chapter 5 contains general recommendations and suggestions as to where self-instruction can possibly be used in mathematics teaching.

CHAPTER 2

A SURVEY OF AVAILABLE LITERATURE ON THE USE OF SELF-INSTRUCTION, WITH PARTICULAR REFERENCE TO POST-SCHOOL MATHEMATICS

2.1 **Introduction**

In this chapter :

(i) the factors that, according to the literature, should be taken into account in the preparation of SI materials, or which are considered to be important for the success of this medium of instruction will be examined;

(ii) different forms of SI will be reviewed by referring to projects that are running, and have been run, at various educational institutions.

Literature sources were obtained by various means including a SABINET search conducted by the library at Technikon Natal, a HSRC search conducted by that organization's Databases and Publications Section, and by means of following up on references in books and journal articles relating to the research topic.

Self-instruction obviously refers to a method whereby the **learner learns on his own**, normally **using specially prepared teaching materials.** As a student progresses through school and further formal education, the emphasis should ideally shift from **teacher-centred instruction** to **learner-centred individualized progress.**

The importance of active participation by the learner is supported by Grayson (1977:99) who states, "It is what the learner does that determines what he learns, not what the teacher does".
Huber (1989), in an article entitled *Teaching and Learning - students and university teachers*, points out, that in the field of education, more attention is currently being directed to promoting independent learning. Self-instruction can by implication be incorporated with other more conventional forms of instruction and need not necessarily be an alternative to them.

Authorities, such as Rowntree (1986) and Romiszowski (1984), refer to the variety of names given to what is essentially self-instruction. These include the following:
Computer-aided instruction ; distance education ; resource-based learning ; correspondence education ; packaged learning ; flexistudy ; individualized learning ; directed private study; guided reading; programmed learning etc.
Self-instruction as applied in programmed learning came into prominence with a resurgence of interest in the work of B.F.Skinner (Margulies, 1962 : 15) during the period from the mid-1950s to the late 1960s. Skinner based his theory on the psychological theory of operant conditioning, according to which behaviour is only learned if it is immediately reinforced. Hence the learning exercise was to be such that correct answers to exercises, assignments or questions should be followed by a positive reward (praise for example), while incorrect responses should not be rewarded. Skinner favoured a

system which would result in a minimum of incorrect responses since this would minimize the danger of the learner developing a negative attitude towards the learning activity. Eraut (1989), in describing these early developments of programmed learning, points out that the carefully designed control needed to have a learning system satisfying Skinner's theories resulted in him inventing his first teaching-machine. A main feature of teaching-machines, as well as programmed texts developed in the 1950s and 1960s, was that the instructional material was presented in frames one step at a time. Normally each instructional frame required some sort of response from the learner and the next frame provided the correct answer. Features of this **linear programmed** approach ("linear" since all the users followed the same "linear" sequence) were the presentation of subject matter in a logical sequence of small steps with as few errors as possible, and the immediate presentation of the correct response once the learner had responded to a step. Another feature was that learners were able to work at their own pace.

The self-instructional material developed in the present research project is essentially linear, but with the difference that although it is hoped that student answers to assignments are correct, incorrect responses will occur and do result in further learning in that the correct solutions with applicable explanations are given (see Annexure F for student comments).
The extension that followed from the linear programming approach was the development of branching programmes in which the learner

was required to answer a multiple-choice-type question on completion of the instructional frame. Depending on the response, the programme would branch to the applicable sequence of frames, which could involve further explanation, revision or the next stage of the instruction.

Problems experienced in the nineteen sixties with the use of programmed learning included matching students to the right programmes, managing classes where students did not proceed at the same pace, and the fact that the electronic and computer age were in their infancy, which meant that the writing of programmes, as well as the presentation of the data, was not as easy as it is these days. Computer Aided Instruction (CAI), with the current advances in technology and specifically electronic media, can easily use linear and branching techniques.

Eraut (1989:412) infers that subsequent to the pioneering decade (1954-1964) of linear programming, the approach has become more pragmatic. Task Analysis and the statement of learning objectives came to the fore. In the Task Analysis stage of programme preparation, Eraut (1989:413) mentions that four ideas (**mastery performer**, **hierarchy**, **matrix** and **classification by knowledge type**) have led to further developments. These four ideas are briefly described below.

"Mastery performer" - here the learner would be able to reach the behaviour objectives which are stated in performance terms.

"Hierarchy" - the assumption here is that the material to be learnt can be sequenced into small steps or stages where the previous stage is often a prerequisite for the next stage.

"Matrix" - the principles, terms and examples of the subject content are listed in a two-dimensional-matrix form and then linked within by colour coding , e.g. an element of the matrix that is shaded in a particular colour indicates the relationship between corresponding components of the matrix (Table 2.1 illustrates this idea).

Terms, principles and examples :	A	B	C	D	E
A					
B					
C					
D					░░░
E					

Source : Author.

Table 2.1

The shading of the element of the matrix corresponding to components D (an example) and E (a principle) using the colour ░░░ could mean that D is an example of principle E.

"Classification by knowledge type" - this idea has essentially two bases, namely Bloom's *Taxonomy of Educational Objectives*, or an extension of it (Bloom et al., 1971); and *typologies of learning psychologists*, which attempt to differentiate between categories such as discriminations, chains, concepts and principles (Gagné, 1977).

The post-sixties period also saw the analysis and evaluation of the material presented in programmed texts as well as the examination of factors which hinder the use of such material. The move has been towards instructional systems where a variety of methods are used (such as the present project which uses a mix of short lecture and self-instruction). Some of the selected self-instructional projects mentioned in 2.3 below reflect a systems approach.

Before referring to recent literature dealing with factors that need to be taken into account when preparing self-instructional material, it is perhaps apt to mention that although a flood of inferior programmed-learning material hit the market in the fifties and early sixties, the lessons learned, teaching approaches developed, and the principles (such as focusing on the individual) are worth considering when preparing instructional material these days. Programmed instruction could certainly form a useful component of Computer-Aided Learning (CAL) and an integrated instructional system.

2.2 Factors to be taken into account when preparing SI material

Sources, including Osborn (1981), Jones and Lewis (Units 5 & 10, 1981), Holmberg (1981), Rowntree (1986), Williams (1988), Jackson & Prosser (1989), Marquis (1989), Hanson (1990) and Grobler (1991) studied by the writer, identify particular factors which need consideration.

These factors will be discussed under the following sub-headings : prospective students, content, objectives, medium of instruction, sequence, style of presentation, assessment and evaluation.

2.2.1 Prospective students

A variety of factors, including demographic, motivational and learning factors (including subject background), need to be known about the prospective users of self-instructional material. These general factors are interrelated and should be considered collectively when deciding on the type and structure of self-instructional material that is to be used. **Demographic** factors include not only numbers of students, and their ages and sex, but also their personal circumstances such as whether they are employed (a factor that is bound to affect motivation), what other subjects they are studying (how much time they have available), and their study facilities (areas where they can work). **Motivational** factors can best be considered by examining why they are studying the particular subject, why they would choose self-instruction, as well as what they can expect to achieve and how they can benefit from this method of instruction. When considering the motivation of students studying mathematics, consideration must be given to anxiety in the subject. Williams (1988) expresses the opinion that understanding and reasoning rather than memorizing would reduce mathematics-anxiety. The preparation of self-instructional material will obviously depend on the type of learners who will be using the material, and **learning** factors to be considered include general factors such as intelligence, prior-level of education, their experience with different instructional methods (especially self-instruction), as well as subject-related factors relating to the knowledge, skills and opinions the learners hold. Grobler (1991) identified two further factors that influence the effectiveness of SI, viz. **level of development** and **personal learning styles.**

2.2.2 **Content**

In order to decide on the content to be incorporated into the self-instructional material, a variety of techniques are useful. The first stage would seem to be to determine the basic concepts required for mastering the subject. **Concept analysis,** according to Rowntree (1986), involves isolating the main ideas or concepts, and then defining each concept. This examination can be done by considering examples, counter examples, borderline examples and invented examples of the concept. In order to ensure that all the main ideas are included, the use of an appropriate diagram could be useful. Diagrams such as **spray diagrams** (where the main idea is stated at the centre of the diagram with lines spraying out to other aspects), **concept maps** (a structured diagram that links concepts and aspects, showing cause and effect) and **matrices** (normally a two dimensional matrix which relates to subject matter that can fall into a variety of categories) are useful for this purpose. In Mathematics and Science subjects, an **algorithm** which joins the concepts by arrows that explicitly show the connections between them is particularly useful. Rowntree (1986:58) adds the following advice: "However you decide on the possible content of your course, **don't overload it.**"

2.2.3 **Objectives**

Once the **aims** - i.e. a general statement or statements of what one hopes the course will achieve - has been formulated, **objectives** which state what the learners should be able to do as a result of having worked through the lesson should be stated. These objectives in mathematics at post-school technikon level should presumably include both the subject matter or knowledge type as well as the ability to use such knowledge or skill. The statement of clear objectives before each lesson serves to communicate the intentions of the developer of the self-instructional material, not only to himself but also to others that are interested in such material (colleagues and students). They also assist in the further development of such material by enabling the developer to distinguish between necessary and unnecessary content, decide on the sequencing of such material, help with the decision on the most appropriate teaching media and learning activities, and help with the formulation of suitable ways of assessing and evaluating whether the objectives have been achieved.

2.2.4 **Medium of instruction**

The choice of the teaching media to be used should be considered in the early stages of the course-planning. Rowntree (1986) regards the question of which media are accessible to both developer and learners as a crucial one. The main medium remains **print** which may or may not be supported by other media, including **audio** tapes or records. Language laboratories and mathematics school revision courses are examples of these media, the latter being used in conjunction with printed material. The use of radio as an instruction medium is effective in distance education. (Australia is a leader in the use of radio for school education). **Audio-visual** video is becoming one of the most popular media which can be used on site as well as on television (at present local programmes in English, Mathematics, Biology and Science are presented on television for matriculation level). **Practical work** can be done by providing "kits" or enabling learners to use laboratory facilities while **personal interaction** between learner(s) and tutor remains one of the common media. Finally **computers** as medium of instruction are being utilized more and more. With the advances in technology, computers can combine most of the media mentioned above.

In choosing the media for self-instruction, consideration must be given to the functions that the media will perform. These functions - including getting the learners' interest, recalling previous learning, stimulating new learning, explaining, provoking, encouraging, getting active response, giving feedback and assessment - are usually performed by the lecturer in face-to-face meetings with students. Furthermore, cost factors (money, time and flexibility) have to be taken into account.

Where feasible, a combination of media that will offer the learner sufficient variety of stimulus and activity will probably be the most effective.

2.2.5 Sequence

A variety of techniques can be used to assist in the preparation of lesson material in a sequence that will ensure that concepts covered early in the teaching will not interfere with those in later learning. In factual subjects, topic-by-topic and/or chronological-sequencing could be used. The present project, being concerned primarily with mathematics, made use of instruction-sequence methods such as **structural logic** (the sequence is dictated by the logical structure of the subject, in which prior understanding of certain concepts is a prerequisite for understanding later concepts) and **problem-centred** sequence (a problem is presented and then the solution is developed with the necessary concepts being introduced where necessary); these seemed to be the most applicable. Experience has also shown that reinforcement of concepts is crucial to basic understanding in mathematics and therefore **spiral** sequencing (the same concept is presented over and over as the learners work through the lessons, each time in more detail requiring greater understanding) is also important.

2.2.6 Style of presentation

The effectiveness of the SI will to a large extent depend on the style of presentation. The material-writer will have to put in writing **everything** he would want to say to the learners if they were in a conventional teaching situation. The text will have to assist the learners find their way around the subject using instructions that, according to Osborn (1981:38), should be "clear, unambiguous, and easy to follow; brevity is a virtue". In essence, the instructions should inform them what they need to know before tackling a lesson, clearly indicate what they should be able to do on completion of the lesson, advise them how to tackle the work, explain the subject matter in such a way that they can use and relate to existing knowledge, encourage them, engage them to actively do exercises, give them feedback so that they can see their progress and determine the amount they have learned.

In order to achieve these aims, the material-writer will have to consider the amount of material the student can study in a lesson. In presenting the material a variety of factors should be taken into account, amongst which are : the use of existing materials as a part of the lesson, teaching through activities that will serve to keep learners more alert and involved and hence **motivate** them, use of stimuli (graphs, figures, cartoons, tables, pictures) that will to some extent provide variety and interest, and the physical format (e.g. highlighting, changing fonts, or enclosing material in boxes). Variety can also be introduced in the touch given to the lesson by the writer in the form of good-humour , respect and dramatization. Rowntree (1986:156), states "One of the best safeguards against monotony is to **READ IT ALOUD**" and then adds that, if the material sounds dull, the necessary revision should take place.

Another factor that especially applies to scientific text is the careful use of specialist vocabulary, (i.e. careful explanation of new terms that are essential, and gradual introduction of new terms).

2.2.7 **Assessment**

Assessment is a way of "obtaining and interpreting information about the knowledge, skills and attitudes of another person"(Jones & Lewis quoting Rowntree, 1980:unit 5 p.9). Rowntree (1986), in examining the purposes for assessment, emphasizes **formative assessment** (which has as object the improvement of future learning) and **summative assessment** (which reports on what the student has already learned, in terms of a mark or grade). When writing self-instructional texts, questions such as **"when", "what"** and **"how"** to assess should be borne in mind.

These considerations will depend on whether the self-instructional material forms a major or minor part of a course. **'When'** could be on a continuous basis and/or an end-of-course examination (preferably a combination of both). One of the advantages of self-instructional material written in a programmed-learning type format is that learners are continually assessing themselves by answering and then checking their answers to the self assessment questions or exercises.

'What' refers mainly to the content and should be linked to the lesson and course objectives. **'How'** the assessment is carried out is clearly linked to **what** the objectives are. Assessment could take the form of so-called objective tests (multiple-choice, true or false) and/or own-answer, so-called "subjective tests". The objective tests have the advantage that marking can be done by computer or people not necessarily familiar with the subject. However such objective tests are unlikely to be appropriate for testing subject objectives such as being able to recall, explain, invent, argue a case, etc. Assessment in mathematics can make use of both types of tests because students are required to discriminate and recognize as well as integrate and solve problems.

2.2.8 **Evaluation of SI material**

Evaluation should be a continuous process, the results of which can be utilized to develop and improve the instructional method and material. Evaluation of Self-instructional material, according to Jones & Lewis (1980, Unit 10), besides determining the effectiveness of communicating ideas and intentions, should also indicate if the students have learned the required facts and concepts. Areas where the material can be improved and information about future use should also be obtained. Evaluation, besides using measurements of student achievement which can be subjected to statistical evaluation, should also take into account what Gillham (Lewis & Jones, Unit 10, 1980:9) refers to as "soft illuminative evaluation". By this is meant the areas such as opinions, feelings, perceptions and circumstances that could be helpful in explaining why the facts are as they are, and therefore, in conjunction with the more direct assessment grades, provide a broader view on the effectiveness of the instructional method and material. Chapter 4 of this dissertation includes "soft evaluation" data obtained from questionnaires answered by students both prior to using the self-instructional material and subsequent to its use. Gillham (Lewis & Jones, Unit 10, 1980:18-21) discuss myths that often put educators off using evaluation which is an extremely important part of the development of effective instructional methods and material. These myths are that evaluation has to be **large-scale, sophisticated, involve an outside agent** and is a **threat**, merely an **academic exercise** and is **difficult**. Gilham discounts each of these myths and shows that evaluation can be an effective simple procedure that can be usefully employed by educators. With regard to the development of self-instructional material Rowntree (1986), suggests that the following procedure be followed : critical analysis of the first draft by the developer lead to the second draft that is given to colleagues and other experts for their comments. Consideration of these comments should lead to a third draft that is tried out in a tutorial situation by individual students whose suggestions lead to further improvement. A field trial can now be run and, based on analysis and discussion, the developed product can be produced. Further revisions could take place subsequently, based on feedback from users.

The self-instructional material that was developed for this investigation (Chapter 3) was evaluated in a similar manner.

2.3 An examination of some selected SI Projects

Self-instruction projects have been carried out in a variety of different ways at various educational institutions. Some of these will now be considered, the information being drawn from published reports on selected projects.

2.3.1 The Keller Plan

(Romiszowski, 1984 ; Cumming et. al., 1983 ; Kulik, 1989)

This method of instruction was originally devised in 1963 by Fred Keller and associates to meet the needs of a new psychology programme at the University of Brazilia.

The main features of the Keller plan included the **use of individual study units**, often specifically written for the course, sometimes presented in programmed instructional form allowing students to progress at their own rates; **self-tests** which, once attempted by the student, are then discussed with a monitor who assists the course instructor (a system which according to Kulik (1989) distinguishes this personalized system of instruction from other individualized approaches); **study guides** containing detailed objectives, reading references and practical assignments; and **individual and/or group practical work and discussions** controlled by the guide notes. The **instructor manages** the system with monitors who help with assessment and tutoring.

On reaching the required degree of proficiency or mastery, students earn the right to attend a number of lectures given by a lecturer. A high proficiency level (as high as 100%) was normally required in each study unit test before the students could move to the next unit.

Besides being utilized in the teaching of psychology at the universities of Colombia and Brazil, the Keller plan was used in the teaching of third-year engineering mathematics-courses at the

Monash University, Victoria. At Monash, this Personalized System of Instruction (PSI) (or Keller plan) was organized so that the same number of teaching hours were used as in the case of conventional lecture methods. Cumming and McIntosh (1983), who introduced this method, felt that they could preserve the basic principles of PSI without increasing the number of teaching hours for which PSI courses were notorious. They also felt that the initial extra effort and cost would be offset against the long term saving of teacher time.

In the Keller research project the following specific PSI principles were included :

- **Self-pacing** .. students worked through the units with the aid of the study guides at their own pace.
- **Frequent testing** .. when students had completed a study unit they did the applicable test which was marked immediately giving them **immediate feedback** about their progress.
- **Mastery of material** ..80-90 % mastery was required before students progressed to the next section of work, and **frequent individual discussions** between lecturer and student took place as detailed below.

The students had few formal lectures and learnt from texts or notes using detailed study guides.

The time commitment of the lecturers was kept within reasonable bounds because suitable texts were available so that besides limited additional notes, only the study guides had to be prepared. Students who did not achieve the required proficiency (80%) at the first attempt could take similar tests on the same material, without penalty, until the required standard was obtained. In order to obtain a grade higher than a pass, students could sit for one or both of two optional tests on the same material, but at a higher standard. In such a case, only one attempt was permitted. This course had no final examination.

Questionnaires completed by students during the first three years of this project indicated that about 75 % preferred this method and thought that it should be continued.
The lecturers felt that the method had been successful in that certain PSI aims had been achieved, including an increased pass-rate with students having attained reasonable mastery of each section; students having rapid feedback on their progress; increased individual contact with lecturers; greater flexibility in organizing study plans; students had learned the art of reading textbooks selectively; the method prevented mindless transcription of notes and avoided the distractions of large classes. Furthermore the pressures of an examination were also avoided.
It was further felt that this method, if introduced at first and subsequent levels, would break down the initial barriers between student and lecturer and would enable individual students to become aware of their particular problems sooner. The method was seen as particularly suitable for technique-oriented material.

More generally Kulik et al. (1979), in summarizing the results of 75 studies that compared the effectiveness of PSI and conventional classes, reported that, in 57 out of 61 of these studies (in which final examination averages were compared), the PSI scores were higher.

2.3.2 **Project at the University of the Orange Free State (Grobler, 1991)**

The purpose of this study was similar to that of the present research project, i.e. to find out whether students embarking on a self-study module could do as well as those attending classes in the usual way.

Factors that led to Grobler's research included continuous attention given to tertiary teaching methods and limitations and disadvantages of conventional lectures (such as the passive nature of learners, emphasis on syllabi rather than on student, overemphasis on the importance of the lecturer as an authority in the subject, and student boredom).

Advantages, according to Grobler, of students using self-instruction included the fact that individual differences between students could be taken into account; learners were actively involved; clearly- stated objectives implied better motivation; it was easier to provide regular feedback; and the emphasis shifted from the syllabus to the student.

Disadvantages mentioned by Grobler that could arise were that interest could wane with time and students could become bored using this instructional method; there could be a lack of personal contact between student and lecturer, and students could put off doing the work until it was too late.

Grobler added that the purpose of a self-instructional module was not necessarily to leave students entirely to their own devices, but to attempt to give them the opportunity to utilize their own academic potential.

The experiment involved a group of 30 volunteers , while the control group consisted of 30 matched students, both groups being from a first year psychology class.

The experimental group were given a self-instruction workbook on a section of the course and did not attend the normal lectures. The experimental group completed the required section in the same time as the other students and wrote the same class test. The comparison of class-test results is shown in Table 2.2 below:

	Marks out of 35	%
Experimental group	26,28	75,08
Control group	25,36	72,45
Total group	23,56	67,31

Table 2.2

(Grobler, 1991:104)

Statistical analysis of these results led to the conclusion that there was no significant difference in achievement between the two groups.

A questionnaire and discussions with participating students showed that there were no negative feelings with regard to their experiences using the SI module.

Implications resulting from this research project included lecturers being able to reduce contact time although the preparation of an effective self-instruction module implies thorough planning, hard work and continuous evaluation of the finished product. Self-instruction material could be effectively prepared with the minimum additional educational media. However, students need to be introduced to the SI approach gradually since many prefer contact with a lecturer. Furthermore the development stage and maturity of students as well as their personal learning style must also be taken into account.

2.3.3 The Kent Mathematics Project (Romiszowski, 1984)

This was a school-level self-instructional course where students selected their own path through the material. Their choice was restricted only by restraints on material and the sequencing of certain topics. The course was organized into tasks consisting of written and taped programmes, worksheets, exercises and post-programme exercises.

A sheet of general instructions was supplied as well as a card on which their work was recorded. Tasks were shown on a master matrix which students consulted when they had completed a task in order to select their next task. At the completion of each task, the student's record of work was written up and checked by the teacher, who would update a progress chart. Each task had a test associated with it and each task was only approved by the teacher when the student had completed it to his satisfaction.

2.3.4 Individualized Mathematics Instruction (IMU) project in Sweden (Romiszowski, 1984)

This project began on a pilot basis in 1964 and was extended to many secondary schools between 1968 and 1971.

Initially the aims included the construction and testing of self-instructional study-material in mathematics; testing of suitable teaching methods for the use of this material; trying out different ways of grouping pupils and making use of teachers; as well as measuring, using the materials developed, the effects of individualized instruction.

In principle this project was based on the premise that there would be no grade differentiation. A common curriculum was the starting point for all pupils, and the subject material was structured in units according to degree of difficulty. Each unit comprised four components (A,B,C and D). The basic course consisted of components A,B and C. Component A was common to all students while B and C were divided into levels of difficulty. Each basic unit was followed by the completion of a diagnostic test.

The material was individualized in the rate of progress through the work and the degree of penetration. Normally the intention was that students should complete three modules per year. Pupils within one grade could reach different points (or levels) within the material. It was possible to change level both within and between modules. The D component was not part of the basic course and comprised revision and tasks of a more independent form.

2.3.5 **Microcomputers In Mathematics Education (MIME) (Bajpai et.al., 1984)**

Professor Bajpai in the Department of Engineering Mathematics at Loughborough University of Technology , being concerned with the teaching of mathematics to engineers and scientists, was primarily responsible for this project. The idea was to develop programmed texts that could be used on microcomputers alongside text material. Units were written both as teaching aids and in the form of self-instructional programmes.

Factors emerging from this project included student reaction being generally favourable to the units but the development time for each unit was very much more than initially anticipated and modifications to improve material took almost double the time spent on the initial development of programmes. A team approach was preferred since it was felt that this would lead to the generation of ideas and techniques. The interactive nature of programmes using a computer was also seen as an advantage that could make learning "more fun".

Based on the favourable findings, the MIME team considered writing a series of units to enhance the teaching of various topics in mathematics.

2.3.6 **A Kenyan project (Eshiswani, 1985)**

This research project in mathematics (specifically involving a statistics module), in Kenyan high schools was conducted to compare the effectiveness of three teaching methods, viz. **P**rogrammed **I**nstruction (PI), **C**onventional **C**lassroom **A**pproach (CCA), and **I**ntegrated **P**rogrammed **I**nstruction (IPI).

Problems that led to the research included the shortage of mathematics teachers, a large number of failures in this subject, and the disruption caused by the frequent changes in teachers.

The effectiveness of the three methods was measured in terms of achievement and retention rates.

The sample used for the research was taken from four high schools (two for boys and two for girls) in Nairobi. Within each of the schools three second-form classes were randomly assigned to each of the three methods. Prior to the experiment, all subjects underwent several mathematics pretests measuring reasoning ability, knowledge of probability, attitude towards mathematics, comprehension of mathematical vocabulary, and comprehension of scientific terms.

The first (PI) group learned their probability through a programmed -text edited by Eshiswani; the second (CCA) group was taught by a

mathematics teacher (Eshiswani taught the girls and a trained assistant the boys) using a regular school mathematics text; and the third (IPI) group used programmed materials 75% of the time and were taught conventionally the rest of the time.
The teaching/instruction period was just over two weeks. Halfway through this period, and at the end of the instruction, the subjects were given achievement tests, while six weeks after instruction, a retention test was given. In the intervening period, the pupils continued with their normal mathematics programme.

Null hypotheses established were that there was no difference between the means of the three groups with respect to : achievement test scores, time needed to complete the unit on probability, and retention test scores.

Results from this research were :
Hypothesis (1) : PI and IPI groups obtained higher mean scores than the CCA group.
Hypothesis (2) : The IPI group retained more than the other two groups.
Hypothesis (3) : Programmed instruction resulted in a saving of nearly 50% of instruction-time; integrated programmed instruction resulted in a 20% saving of instruction-time.

2.3.7 A Guided Reading Project at Bristol University, England (Clements et.al., 1983)

This, another example of a self-instructional project, seems of particular relevance for tertiary education.

This project was the result of the (now familiar) search for efficient, effective teaching methods that would better utilize staff and student time as well as develop important learning skills which, according to Clements et. al. (1983, p. 95), "are difficult to foster by other means".

The project involved providing the students with a set of notes prepared by the lecturer, which, together with a series of lectures and tutorials, guided them through their reading and studies.

Advantages claimed included the development of student confidence in reading mathematical textbooks; fostering the ability to learn mathematics independently; and reduction of time-consuming note taking associated with more traditional lecturing methods. Students were able to study material more flexibly and were more motivated to discuss the work amongst themselves.

This method of instruction was originally used with fifteen students in a course on basic vector algebra and probability theory. The notes detailed the exact extent of the material to be studied, commented on the ideas, recommended exercises for the students to attempt, and gave some additional worked examples. The notes were divided into units of approximately one to two weeks' work. Each unit was associated with a lecture-period in which the lecturer spend time elaborating where necessary and helping students with specific difficulties. The authors admitted that an experimental method would have been the most objective way of assessing the effectiveness of the project, but due to financial and time factors, they assessed the effectiveness on the subjective responses of students and their personal judgement.

The conclusions reached showed that almost all the students felt that the guided reading course should be retained and that they had enjoyed studying this way, with the best features being freedom of pace and the ability to work ahead. The guided reading method enhanced their note-taking ability. To discourage procrastination it was important that assessment work be demanded.

Finally the courses used for this project were chosen because of readily-available, suitable texts and as such the preparation of the notes was not unduly difficult and the implementation did not give rise to practical problems.

2.3.8 General Comment on these SI projects

A comparative examination of the self-instructional projects named brings to light certain common principles. Firstly, there is a need for a structuring of learning materials into a logical sequence of **short units** (lessons, frames) which are either written specifically for the particular syllabus topic or could be referenced with the aid of a detailed study guide. To a varying degree the **students worked** through the self-instructional material at their **own pace,** and **mastery** of the material (evaluated by regular short tests which provided **immediate feedback** on progress) was in most cases a pre-condition for moving to the next unit. The advantages and disadvantages of each individual project have been mentioned in the brief descriptions given above.

It is important when considering the use of self-instructional methods to highlight some of the findings. Students generally enjoyed studying on their own and the research conducted in the projects indicated, almost without exception, that their performance was comparable or better than that obtained following more traditional methods. There were, however, indications that students could become bored with Self-instruction and that procrastination and lack of motivation could be problematic. Advantages such as flexibility, avoiding time-consuming note-taking associated with traditional lectures, working at their own pace, and learning independently are undeniable. Judging from these studies and comments from students who have used the self-instructional material in the present study (Annexure F), it may be concluded that an instructional system that uses a well-balanced combination of methods would probably be the most effective for student learning.

CHAPTER 3

SELF-INSTRUCTION MATERIAL ON NUMERICAL METHODS

Introduction

This chapter, after brief comment, contains the self-instructional material that was developed and used by the author in the research project on which this dissertation is based. Factors derived from the preceding study of similar projects (reported on in Chapter 2) have been borne in mind. With specific reference to the writer's material, the following comments relating to the medium used, the style of presentation, and the development of the material, are relevant :

Medium

The medium of instruction used in the self-instructional part of this project was **print**. A variety of factors led to the decision to use this medium. The **target population** were on-site engineering or applied science technikon students who had an extremely full, weekly lecture timetable. Many of these students did not have easy **access** to audio-visual or computer facilities, and even if they had used on-site computer facilities, the **limited access time and number** of terminals would have been unduly inconvenient and would most probably have resulted in the required studying not being done. The **cost** (in time and money) of producing a self-instructional text was also substantially less than would have been the case for other media. The self-instructional text was a single authorship venture and this meant that the time taken had to be reasonably short. Consideration as to the medium that would be the most effective, taking the above limitations into account, led the author to decide on print because past experience had been that students had successfully used programmed-texts (such as *Engineering mathematics for Engineers and Scientists* - Stroud, 1982) successfully, albeit as a supplementary text rather than a self study-text.

Style of presentation

Factors that led to the "**conversational style**" of presentation used in the self-instructional text developed by the author included those mentioned in Chapter 2. Specifically the idea that the text replaces the teacher in a one-to-one tutorial type situation as well as the point that many students were not familiar with this approach and hence would probably be intimidated by the usual "cold" factual mathematics textbook approach, led to the "open and friendly" style of presentation. Students' comments (Appendix F) indicate that this approach was easy to follow and obviously user-friendly. Many students also stated that they had enjoyed the SI work, thus indicating that such instruction could be intrinsically motivating, involving "the performance of an activity for no reward except the direct enjoyment of the activity itself" (Tang, 1989:220).

The material which was to be learned by the students was divided into short lessons, an approach adopted by proponents of both Bloom's Learning for Mastery (LFM) and Keller's Personalized System of Instruction (PSI) (Kulik et al., 1990).

Development of material through student involvement :
Reference to Appendix F and to the statistics provided in chapter 5 will show that student involvement (by means of responses to questionnaires and personal comments) was an important part of the development of the SI text. After the first draft was prepared in the first semester of 1990, it was distributed to students who covered the same numerical methods work in Applied Mathematics T3. They received normal traditional lectures on the topic and the SI booklet was used as a supplementary text on which they were asked to comment.

Taking the student comments into consideration, the text was revised and used more independently during both semesters in 1991. Once again, comments and suggestions from students and colleagues who had received copies of the text, were taken into account for the revision of the SI text at the end of 1991. The SI text that was used in the post-test experiment (reported in Chapter 4), conducted during the first semester of 1992, is what appears in this dissertation.

It is perhaps important to point out that this project involved self-instruction techniques only for the <u>numerical methods</u> section of the Mathematics II syllabus. South African technikons use nationally similar syllabuses in all diploma programmes, and the full syllabus for Mathematics II appears on page 11.

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On the following 55 pages, the full text of the self-instruction material is presented.

TECHNIKON NATAL

SELF-INSTRUCTION IN NUMERICAL METHODS

Prepared by :-

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INTRODUCTION AND STUDENT GUIDE <u>1</u>

A variety of self-instruction texts exist for numerous topics in mathematics. This series of lessons has specifically been written for the Numerical Methods section of Mathematics 2 given to Technikon Engineering and Applied Science students.

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Self-instruction using the lessons provided should proceed
*as follows: *
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1. **Attend** the introduction to each lesson (± 10 minutes)
2. **Work** through the lesson with the aid of a note pad, making sure you **<u>work through</u>** the worked examples AND **<u>do</u>** the follow-up tutorials (which are included in the lessons). Full solutions are given to each tutorial example (cover these up while you are doing the tutorial).

Note : Tutorial answers follow the following pattern:

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(place your solution mask immediately below such a pattern)

3. **Inform** your lecturer as soon as you have finished each lesson. Also mention any problems or difficulties you encountered while working through the lesson.
4. **Spend as much time as possible** on the further exercises (It is suggested that you do past exam-papers)
5. Move to next lesson.

A bookmark (cardboard) to be used as a solution mask is provided.

Symbols:
- L : Lesson
- W : Worked example
- T : Tutorial
- F : Further examples (provided separately)
- A : Answers
- S : Summary

PLEASE try to spend the following lecture periods on the lessons : (<u>NOTE :</u> If a lesson is not completed in lecture or tutorial time, you should finish the work in your <u>OWN</u> time).

LESSON :	LECTURE/TUTORIAL TIME	Suggested "own" time
1	1 period	30 minutes
2	1 period	60 minutes
3	1 period	60 minutes
4	1 period	60 minutes
5	1 period	60 minutes
6	2 periods	60 minutes
7	1 period	60 minutes
8	1 period	60 minutes

[LECTURE TIME $1\frac{1}{2}$ weeks]

SECTION: ERRORS AND ACCURACY

LESSON 1

After **working** through this lesson you should be able to:

1. **round** numbers correctly to a given number of decimal places or significant figures (whichever is asked for)
2. determine the **range** of a rounded number
3. **determine** the **maximum absolute error** of a given rounded number

General:
Errors can occur in a variety of ways when we are dealing with numbers. For example numbers can be recorded incorrectly, or errors can result from rounding or truncating numbers. We are going to examine errors that arise when numbers have been rounded.

The rounding procedure we will use will be that numbers will be rounded to the nearest, UNLESS the digits following the last digit required in the rounded number is exactly 5 (or 50 or 500 .. or 50000..) in which case the preceding digit remains even if it is even or is rounded up to even if it is odd.
(W: indicates a worked example - **work through the examples carefully**)

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W1: Round each of the following numbers to the indicated degree:

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a) 17,3251 to 2D (2 decimal places) or 4 significant figures. (To determine the number of significant figures in a number, start, from the left hand side, counting from the first non-zero digit i.e. the first **significant** digit).

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A 17,33 (since following digits "51" is greater than "50")

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b) 0,015499 to 3D or 2 significant figures

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A 0,015 (since following digits "499" is less than "500"

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c) 201,45 to 1D or 4 significant figures

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A 201,4 (following digit exactly "5" leaves preceding digit even)

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d) 9201,35 to 1D or 5 significant figures

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A 9201,4 (following digit exactly a "5" ups preceding odd digit to even).

(T indicates tutorial : Please cover the given ANSWER and do the solution yourself before checking your solution against the given solution. If your solution is not correct, look at the given solution and then cover the solution again and answer the question again)

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T1: Round each of the following numbers to the indicated degree:

a) 3,314 ; 0,0155 ; 121,3349 ; 14,615 .... to 2D

b) 0,1346 ; 19,83 ; 121,5 ; 982,500001 ... to 3 significant figures

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A a) 3,31 ; 0,02 ; 121,33 ; 14,62

A b) 0,135 ; 19,8 ; 122 ; 983

Easy isn't it?

****

If we are given a rounded (approximate) number, for example 13,4, we know that it comes from the range 13,35 to 13,45. <u>That is</u>, a number falling between 13,35 and 13,45 will be rounded to 13,4 correct to 1 D. If the exact value was "say" 13,42 then the error in rounding will be 13,42 - 13,4 = 0,02. In other words the error is defined as : the **exact value** - **approximate** (rounded or given) **value**. In symbols we have

ϵ (error) = x (exact) - X (approximate).

$$\boxed{\epsilon = x - X \qquad (1)}$$

Normally we don't know the exact value. If we are given 13,4 (rounded value) we can estimate the maximum error by using the extreme values of the range. In this case the max. error $\epsilon = 13,35 - 13,4 = -0,05$ or $\epsilon = 13,45 - 13,4 = 0,05$.

The maximum absolute error $|\epsilon|$ will be $|-0,05|$ which equals 0,05. Note $|x| = x$ if x is positive and $|x| = -x$ if x is negative. Eg: $|17| = 17$ and $|-17| = 17$.

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W2: Determine: i) Range
ii) Maximum error
iii) Maximum absolute error
of each of the following rounded numbers:

a) 3,314 b) 0,0155 c) 121,5 d) 13

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A a(i) Range: 3,3135 to 3,3145

(ii) $\epsilon = 3,3135 - 3,314 = -0,0005$ or
$\epsilon = 3,3145 - 3,314 = 0,0005$

(iii) $|\epsilon| = 0,0005$

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b(i) Range : 0,01545 to 0,01555

(ii) $\epsilon = 0,01545 - 0,0155 = -0,00005$ or
$0,01555 - 0,0155 = 0,00005$

(iii) $|\epsilon| = 0,00005$

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c(i) Range : 121,45 to 121,55
(ii) ϵ = 121,45 - 121,5 = -0,05 or
121,55 - 121,5 = 0,05
(iii) $|\epsilon|$= 0,05
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d(i) Range : 12,5 to 13,5
(ii) ϵ = 12,5 - 13 = -0,5 or
13,5 - 13 = 0,5
(iii) $|\epsilon|$= 0,5

Notice that the maximum error can be positive or negative and has one significant figure (5) in each case.

Using Rule (1), i.e. $\epsilon = x - X$ (or in words : error equals exact minus the given approximate value), it follows that $x = X + \epsilon$.
ie: we can express the exact value x in terms of the given value X and the error ϵ (in the **given** value).

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T2: Determine the :
i) maximum absolute error and
ii) range of each of the following numbers
(assume numbers are rounded unless otherwise specified)
of :
a) 22,3 b) 120 (exact) c) 0,012
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A a) i) $|\epsilon|$ = 0,05 ii) range from 22,25 to 22,35
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b) i) ϵ = 0 (since no error in an exact number)
ii) 120 only
****
c) i) $|\epsilon|$ = 0,0005 (so easy - a "5" in the next position)
ii) 0,0115 to 0,0125
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S: The maximum error in a given rounded number is a number containing a 5 in the next significant position. The absolute value "ignores" the sign. The range is from value minus maximum error **to** value plus maximum error.
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After working through this lesson you should be able to:

1) **estimate** the **maximum error** in an expression containing **addition** and **subtraction**.

2) **estimate** the **range** of the expression, and

3) **check** the **range** by **using** the **"Bush" method.**

Recall $\epsilon = x - X$ (error = exact - given approximate) =>

$$\boxed{x = X + \epsilon \ldots\ldots\ldots\ldots\ldots\ldots (2)}$$

(exact = approximate + error)
Note : " => " means implies.

Error in addition of numbers $(X_1, X_2 \ldots\ldots X_n)$

Error = exact value - approximate value

ie: Error in sum
$\epsilon = (x_1 + x_2 + x_3 + \ldots + x_n) - (X_1 + X_2 + X_3 + \ldots X_n)$

Using (2) this implies $\epsilon = (\{X_1 + \epsilon_1\} + \{X_2 + \epsilon_2\} + \{X_3 + \epsilon_3\} + \ldots + \{X_n + \epsilon_n\}) - (X_1 + X_2 + X_3 + \ldots\ldots + X_n)$ =>
$\epsilon = \epsilon_1 + \epsilon_2 + \epsilon_3 + \ldots + \epsilon_n$

Now the individual errors $\epsilon_1, \epsilon_2 \ldots \epsilon_n$ can be positive or negative. If we want the maximum error we assume the signs of the individual errors are such that the sum of individual errors reinforce each other, i.e. they are positive, or we simply use the absolute values, which leads to the maximum error in the addition being the sum of the absolute errors.

$$\boxed{\text{ie: max error } \epsilon = |\epsilon_1| + |\epsilon_2| + |\epsilon_3| + \ldots + |\epsilon_n| = \Sigma|\epsilon_i| \ldots (3)}$$

W1: Determine the maximum error in 13,21 + 17,1 + 12
****
A: max $\epsilon = \Sigma|\epsilon_i| = 0{,}005 + 0{,}05 + 0{,}5 = 0{,}555$

Notice the value of 13,21 + 17,1 + 12 = 42,31 ; so the range of 13,21 + 17,1 + 12 will be from 42,31 - 0,555 to 42,31 + 0,555 or 41,755 to 42,865 i.e. from (value - error) to (value + error).

We can check the range by determining the minimum and maximum possible values of the expression. I call this the "BUSH" method.

In our case 13,21 + 17,1 + 12 lies in the range (13,205 + 17,05 + 11,5) to (13,215 + 17,15 + 12,5) i.e. from 41,755 to 42,865 (Spot on ... see bottom of page 5).

From the range obtained using the "BUSH" method:

We can now go one step back and find the maximum error:

$$\boxed{\left[\frac{(X+\epsilon)-(X-\epsilon)}{2} = \frac{2\epsilon}{2} = \epsilon\right]}$$

i.e. error equals half the (upper minus lower value).
therefore error in our case using "BUSH" range will be :

$$\boxed{\epsilon = \frac{42,865 - 41,755}{2} = 0,555}$$

NOTE : You should use the "BUSH" method to check your answers.

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T1. Determine the maximum error and range of each of the following expressions. Check the range using the "BUSH" method.

a) 12,31 + 0,1 + 1,0013
b) 112,1 + 1,43 + 7 (where 7 is exact)

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A a) $\epsilon = \Sigma|\epsilon_i|$ = 0,005 + 0,05 + 0,00005 = 0,05505
Now: 12,31 + 0,1 + 1,0013 = 13,4113

Therefore Range from 13,4113 - 0,05505 to 13,4113 + 0,05505
i.e. from 13,35625 to 13,46635

"BUSH" check: range from 12,305 + 0,05 + 1,00125 to 12,315 + 0,15 + 1,00135
i.e. 13,35625 to 13,46635

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b) $\epsilon = \Sigma|\epsilon_i|$ = 0,05 + 0,005 + 0 = 0,055
Now 112,1 + 1,43 + 7 = 120,53
Therefore Range from 120,53 - 0,055 to 120,53 + 0,055
i.e. 120,475 to 120,585
"BUSH" check: range from 112,05 + 1,425 + 7 to 112,15 + 1,435 + 7 i.e.120,475 to 120,585

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It is important to note that although the answers to the above problems are given to a large number of significant figures, it is normally only necessary to give an answer to the same number of significant figures as the number containing the fewest significant figures when dealing with multiplication, division or powers; **OR** as the number containing the fewest significant figures after the decimal comma when dealing with addition and subtraction.

Error in subtracting numbers ($X_1 - X_2$)

Error = actual - approximate (or given value) => error in difference
$\epsilon = (x_1 - x_2) - (X_1 - X_2)$

Now replacing x_1 by $X_1 + \epsilon_1$ and x_2 by $X_2 + \epsilon_2$ gives us
$\epsilon = [(X_1 + \epsilon_1) - (X_2 + \epsilon_2)] - (X_1 - X_2)]$
$= X_1 + \epsilon_1 - X_2 - \epsilon_2 - X_1 + X_2 = \epsilon_1 - \epsilon_2$

But these errors (ϵ_1 and ϵ_2) could be positive or negative, hence if we want the maximum error in the difference we choose their signs in such a way so that the errors reinforce each other. For example, if X_1 and X_2 are positive we take ϵ_1 as positive and ϵ_2 as negative then $\epsilon = \epsilon_1 - \epsilon_2$ will be a maximum. Or more simply, rather than a (positive ϵ_1) - (negative ϵ_2) we use

$$\boxed{\epsilon = |\epsilon_1| + |\epsilon_2|}$$

So for subtraction the maximum error "ϵ" also equals the sum of absolute ϵ's

i.e. $\boxed{\text{maximum } \epsilon = |\epsilon_1| + |\epsilon_2| \ldots\ldots (4)}$

W2: Find the maximum error and range of:
a) 123,14 - 17,0 (= 106,14)
b) 13,1 + 19,32 - 10,3 (= 22,12)

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A: a) maximum $\epsilon = |\epsilon_1| + |\epsilon_2| = 0{,}005 + 0{,}05 = 0{,}055$
Range from 106,14 - 0,055 to 106,14 + 0,055 i.e. 106,085 to 106,195
"BUSH" method: 123,135 - 17,05 to 123,145 - 16,95 i.e. 106,085 to 106,195

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b) maximum $\epsilon = 0{,}05 + 0{,}005 + 0{,}05 = 0{,}105$
Range from 22,12 - 0,105 to 22,12 + 0,105 i.e. 22,015 to 22,225
"BUSH" method: 13,05 + 19,315 - 10,35 to 13,15 + 19,325 - 10,25 i.e. 22,015 to 22,225

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T2: Find the maximum error in:

a) 1,012 + 7,41 - 3,21
b) 18 (exact) + 101 - 72,1

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A: a) $\epsilon = 0,0005 + 0,005 + 0,005 = 0,0105$
b) $\epsilon = 0 + 0,5 + 0,05 = 0,55$

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S: When you add and/or subtract numbers, the maximum error will be equal to the sum of the absolute errors.

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After working through this lesson you should be able to :
1) **Estimate** the **maximum error** in an expression containing multiplication or division (i.e. a product or quotient)

2) **Determine** the **range** of the expression.

3) **Check** the **range** using the "BUSH" method.

Error in a product $(X_1.X_2)$

Using error ϵ = exact - approximate
i.e. $\epsilon = x_1x_2 - X_1X_2$ and again replacing x_1 by $X_1 + \epsilon_1$ and x_2 by $X_2 + \epsilon_2$, gives us :

$$\epsilon = (X_1 + \epsilon_1) \ . \ (X_2 + \epsilon_2) - X_1X_2$$

$$\Rightarrow \epsilon = X_1X_2 + \epsilon_1X_2 + \epsilon_2X_1 + \epsilon_1\epsilon_2 - X_1X_2 = \epsilon_1X_2 + \epsilon_2X_2 + \epsilon_1\epsilon_2$$

Now if ϵ_1 and ϵ_2 are relatively small (in comparison with X_1 and X_2 respectively) then $\epsilon_1\epsilon_2$ will be insignificant and hence if we ignore $\epsilon_1\epsilon_2$, we get : the error in the product to be $\epsilon \approx \epsilon_1X_2 + \epsilon_2X_1$. Again if we choose the signs of the errors so that the terms reinforce each other, we get the maximum error in the product

$$\boxed{\epsilon \approx |\epsilon_1 \ X_2| \ + \ |\epsilon_2 \ X_1| \ldots\ldots (5)}$$

In words, maximum error is approximately equal to absolute value of ({error in X_1} times X_2) plus absolute value of ({error in X_2} times X_1).
W1: Find the maximum error and the range of 12,06 x 3,1. Check the range using the "Bush" method.
****
A: Error in product $\epsilon \approx |\epsilon_1 \ X_2| + |\epsilon_2 \ X_1|$ = 0,005 x 3,1 + 0,05 x 12,06 = 0,6185
Now 12,06 x 3,1 =37,386
=> Range is from 37,386 - 0,6185 to 37,386 + 0,6185
i.e. 36,7675 to 38,0045
"Bush" check: range from 12,055 x 3,05 to 12,065 x 3,15
i.e. 36.76775 to 38,00475
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T1: Find the maximum error and range of the following products. Check your range using the "BUSH" method.
a) 2,31 x 10,6 b) 117,2 x 32,01 c) 2 x 3,63 (where 2 is exact).
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A a) $\epsilon \approx$ 0,005 x 10,6 + 0,05 x 2,31 = 0,1685
Now 2,31 x 10,6 = 24,486 therefore Range is from 24,486 - 0,1685 to 24,486 to 0,1685 i.e. 24,3175 to 24,6545
"BUSH" method: Range is from 2,305 x 10,55 (min) to 2,315 x 10,65 (max) i.e. 24,31775 to 24,65475. ## The difference between the two ranges is due to us using a formula that ignores a relatively small term $(\epsilon_1 \ \epsilon_2)$.##
If we consider that each number in this example was

given to 3 significant figures, then rounded to 3 significant figures, the range will be:
Using the (truncated) formula: 24,3 to 24,7
Using the BUSH method: 24,3 to 24,7

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b) max $\epsilon \approx$ 0,05 x 32,01 + 0,005 x 117,2 = 2,1865
Now 117,2 x 32,01 = 3751,572. Therefore Range is from 3751,572 - 2,1865 to 3751,572 + 2,1865 i.e. 3749,3855 to 3753,7585
"BUSH" method: 117,15 x 32,005 to 117,25 x 32,015 i.e. 3749,3858 to 3753,7588.
Again to 4 significant figures this gives the same result as the formula.

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c) max ϵ = 0 x 3,63 + (0,005 x 2) = 0,01
Range from 7,26 - 0,01 to 7,26 + 0,01 i.e. 7,25 to 7,27
"BUSH" method: 2 x 3,625 to 2 x 3,635 i.e. 7,25 to 7,27

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Error in Quotient

$$\left(\frac{X_1}{X_2}\right)$$

As before: Error = exact value - approximate value. i.e.:

$$\epsilon = \frac{x_1}{x_2} - \frac{X_1}{X_2} = \frac{(X_1 + \epsilon_1)}{(X_2 + \epsilon_2)} - \frac{X_1}{X_2}$$

$$= \frac{X_2(X_1 + \epsilon_1) - X_1(X_2 + \epsilon_2)}{X_2(X_2 + \epsilon_2)} = \frac{X_1X_2 + \epsilon_1X_2 - X_1X_2 - X_1\epsilon_2}{X_2(X_2 + \epsilon_2)}$$

$$= \frac{\epsilon_1X_2 - X_1\epsilon_2}{X_2(X_2 + \epsilon_2)}$$

if we replace the denominator:

$$X_2(X_2 + \epsilon_2) \quad by \quad X_2^2(1 + \frac{\epsilon_2}{X_2})$$

[multiplying out will show the two terms are equal]
we get:

$$\epsilon = \frac{\epsilon_1X_2 - X_1\epsilon_2}{X_2^2\ (1 + \frac{\epsilon_2}{X_2})}$$

Now if ϵ_2 is relatively small compared to X_2 then

$$\frac{\epsilon_2}{X_2}$$

is (relatively), even smaller compared to 1, and ignoring

$$\frac{\epsilon_2}{X_2}$$

in the above error formula, we get error in a quotient:

$$\epsilon \approx \frac{\epsilon_1 X_2 - X_1\epsilon_2}{X_2^2}$$

Choosing the signs so that errors reinforce each other gives us the maximum error

$$\epsilon \approx \frac{|\epsilon_1 X_2| + |X_1\epsilon_2|}{X_2^2}$$

which can be rewritten as: Error in quotient

$$\left(\frac{X_1}{X_2}\right)$$

is:

$$\epsilon \approx \left(\left|\frac{\epsilon_1}{X_1}\right| + \left|\frac{\epsilon_2}{X_2}\right|\right) \cdot \left|\frac{X_1}{X_2}\right| \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

In words, maximum error in quotient is approximately equal to {absolute value of (error in numerator divided by numerator) + absolute value of (error in denominator divided by denominator)} **all** multiplied by absolute (numerator divided by denominator).
Although the above theory is messy the problems are easy.
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W2: Determine the maximum error and range of

$$\frac{12{,}31}{4{,}8}$$

(Check range using "BUSH" method).

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A:

$$\epsilon \approx \left(\frac{0,005}{12,31} + \frac{0,05}{4,8} \right) \cdot \left(\left| \frac{12,31}{4,8} \right| \right) = 0,027756$$

Now :

$$\frac{12,31}{4,8} = 2,5645833$$

Therefore Range is from 2,5645833 - 0,027756 to 2,5645833 + 0,027756
i.e.: 2,5368273 to 2,5923393

or to 2 significant figures (since 4,8 is only to 2 significant figures) the range is from 2,5 to 2,6.

"BUSH" check: Range from:

$$\frac{12,305}{4,85} \quad to \quad \frac{12,315}{4,75}$$

i.e: 2,5371136 to 2,5926316 (2,5 to 2,6 to 2 significant figures)

Notice for division

$$\text{min} = \frac{\text{min}}{\text{max}} \quad \text{and} \quad \text{max} = \frac{\text{max}}{\text{min}}$$

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T2: Determine the maximum error and the range of each of the following : a) 0,315 ÷ 2,1 b) 17,83 ÷ 0,18
Check the range using the "Bush" method.

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A:

$$a) \quad \epsilon \approx \left[\left| \frac{\epsilon_1}{X_1} \right| + \left| \frac{\epsilon_2}{X_2} \right| \right] \cdot \left| \frac{X_1}{X_2} \right|$$

$$= \left[\frac{0,0005}{0,315} + \frac{0,05}{2,1} \right] x \; \frac{0,315}{2,1} = 0,0038095$$

$$Now\ \frac{0,315}{2,1} = 0,15$$

Therefore Range is from 0,15 - 0,0038095 to 0,15 + 0,0038095 i.e. 0,1461904 to 0,1538095

"BUSH" check: Range from

$$\frac{0,3145}{2,15}\ to\ \frac{0,3155}{2,05}$$

i.e. 0,146279 to 0,1539024

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$$b)\quad \epsilon \approx \left[\left|\frac{\epsilon_1}{x_1}\right| + \left|\frac{\epsilon_2}{x_2}\right|\right] \cdot \left|\frac{x_1}{x_2}\right|$$

$$= \left[\frac{0,005}{17,83} + \frac{0,005}{0,18}\right] x\ \frac{17,83}{0,18} = 2,779321$$

$$Now\ \frac{17,83}{0,18} = 99,055556$$

Therefore Range is from 99,055556 - 2,779321 to 99,055556 + 2,779321 i.e. 96,276235 to 101,83488

"BUSH" check: Range from

$$\frac{17,825}{0,185}\ to\ \frac{17,835}{0,175}$$

i.e. 96,351351 to 101,91429

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S: The maximum error in the product of two numbers is approximately equal to the absolute value of (first factor times error in second factor) plus the absolute value of (second factor times error in first factor). The maximum error in a quotient is approximately equal to (absolute value of {error in numerator over numerator} plus absolute value of {error in denominator over denominator}) all multiplied by absolute value of {numerator over denominator}

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After working through this lesson you should be able to :
1) **Determine** the **maximum error** in a **root** or **power.**

2) **Determine** the **range** of a root or power.

3) **Check** your **range** using the "BUSH" method.

and also be able to :
4) **Determine the maximum error in an expression containing a combination of operations** dealt with before (i.e. addition, subtraction, multiplication, division, roots and powers).

5) **Determine the range of such an expression.**

6) **Check your range using the "BUSH" method.**

Errors in powers (X^n where n can be an integer or rational number) Using Error = exact - approximate, gives:

$$\epsilon_{power} = x^n - X^n = (X + \epsilon)^n - X^n$$

where ϵ is error in X.

Now using binomial theorem

$$(X + \epsilon)^n = X^n + n X^{n-1}\epsilon + \frac{n(n-1)}{1x2} X^{n-2}\epsilon^2 + \ldots$$

If we ignore the relatively small terms containing ϵ^2 ; ϵ^3 and higher powers of ϵ we get:

$$(X + \epsilon)^n \approx X^n + nX^{n-1}\epsilon \ldots\ldots$$

which gives us:

$$\epsilon_{power} \approx (X^n + n X^{n-1}\epsilon ..) - X^n = nX^{n-1}\epsilon$$

$$i.e. \qquad \epsilon_{power} \approx nX^{n-1}\epsilon \ldots\ldots\ldots\ldots (7)$$

where in $nX^{n-1}\epsilon$, ϵ is the error in X.
Again the maximum error will be obtained if we use absolute values.
W1: Using the formula, find the maximum error and range of $13,21^{1/2}$
(= 3,6345564)
****

A: **15**

$$\epsilon_{power} \approx nX^{n-1}\epsilon = \frac{1}{2}.13,21^{\frac{1}{2}-1}.0,005 = \frac{1}{2}.13,21^{-\frac{1}{2}}.0,005 = 0,0006878$$

Therefore, the range is from 3,6345564 - 0,0006878 to 3,6345564 + 0,0006878 i.e.: 3,6338685 to 3,6352442
Or to 4 significant figures 3,634 to 3,635

"BUSH" check: Range is from $\sqrt{13,205}$ to $\sqrt{13,215}$
i.e.: 3,6338685 to 3,6352441
or: 3,634 to 3,635 to 4 significant figures.
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T1: Using the formula, determine the maximum error and range of:

a) $32,3^{\frac{1}{4}}$

b) $2,105^5$

As before, check the range using "BUSH" method.
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A:

a) $\epsilon_{power} \approx n\, X^{n-1}\, \epsilon = \frac{1}{4}\, x\, 32,3^{-\frac{3}{4}}\, x\, 0,05 = 0,0009225$

Now

$32,3^{\frac{1}{4}} = 2,3839692$

Therefore the Range is from 2,3839692 - 0,0009225 to 2,3839692 + 0,0009225 i.e. 2,3830466 to 2,3848917
"BUSH" range is from

$32,25^{\frac{1}{4}}$ *to* $32,35^{\frac{1}{4}}$

i.e. 2,383046 to 2,3848912 (Not bad!!)
****

b) $\epsilon_{power} \approx n\, .\, X^{n-1}\, .\, \epsilon = 5\, x\, 2,104^4\, x\, 0,0005 = 0,0490849$

Now $2,105^5$ = 41,329533
Therefore range is from 41,329533 - 0,0490849 to 41,329533 + 0,0490849 i.e. 41,280448 to 41,378618
"BUSH" range: $2,1045^5$ to $2,1055^5$ i.e. 41,280472 to 41,378642

NB :
Errors in an expression containing a combination of the given operations

The following worked examples illustrate how we use the formulae we have developed to determine the maximum error in such expressions. The range is then easy to work out. As before, the "BUSH" method is used to check our range. Take careful note of the setting out of the answers.
If you set your solution out neatly and carefully you won't make careless mistakes.
****
W2. In each of the following expressions determine: (i) the maximum error; (ii) the range. Check the range using the "BUSH" method.

a) $(1,512 + 2,418)^2$ where 2 is exact
****
A: i) Error in 1,512 + 2,418 = 0,0005 + 0,0005 = 0,001 $(\Sigma|\epsilon|)$
Error in $(1,512 + 2,418)^2 \approx 2 \,.\, (1,512 + 2,418) \times 0,001$ = 0,00786 $(n\, X^{n-1}\, \epsilon)$

ii) Now $(1,512 + 2,418)^2$ = 15,4449. Range is from 15,449 - 0,00786 to 15,4449 + 0,00786
i.e.: 15,43704 to 15,45276 [15,44 to 15,45]

"BUSH" check for range: $(1,5115 + 2,4175)^2$ to $(1,5125 + 2,4185)^2$
i.e.: 15,437041 to 15,457761 (good enough)
****
b)

$$A = \left(rs + \frac{p}{q}\right)^{\frac{1}{2}}$$

where p = 20,56 ; q = 10,34 ; r = 2,06 and s = 1,84
****
A: Error in rs (i.e.: 2,06 . 1,84) ≈ 0,005 x 1,84 + 0,005 x 2,06 = 0,0195

$$(|\epsilon_1 X_2| + |\epsilon_2 X_1|)$$

Error in

$$\frac{p}{q} \text{ i.e. } \frac{20,56}{10,34} \approx \left(\frac{0,005}{20,56} + \frac{0,005}{10,34}\right) x \frac{20,56}{10,34} = 0,001445$$

$$\left(\left|\frac{\epsilon_1}{X_1}\right| + \left|\frac{\epsilon_2}{X_2}\right|\right) x \left|\frac{X_1}{X_2}\right|$$

Error in

$$rs + \frac{p}{q} = 0,0195 + 0,001445 = 0,020945 \qquad (\Sigma \mid \epsilon \mid)$$

Error in

$$\left(rs + \frac{p}{q}\right)^{\frac{1}{2}} \approx \frac{1}{2}\left(2,06 \;.\; 1,84 + \frac{20,56}{10,34}\right)^{-\frac{1}{2}} .\; 0,020945 = 0,0043564$$

using

$$(\mid n\, X^{n-1}\, \epsilon \mid)$$

Now

$$\left(rs + \frac{p}{q}\right)^{\frac{1}{2}} = 2,4039123$$

Therefore, the range is from 2,4039123 - 0,0043564 to 2,4039123 + 0,0043564
i.e.: 2,3995559 to 2,4082687 [2,40 to 2,41]

BUSH check:

$$\left(2,055 \times 1,835 + \frac{20,555}{10,345}\right)^{\frac{1}{2}} \quad to \quad \left(2,065 \times 1,845 + \frac{20,565}{10,335}\right)^{\frac{1}{2}}$$

i.e.: 2,3995573 to 2,4082702 (2,40 to 2,41)

(Notice: When using the "BUSH" method, we use extreme values of numbers that will give us a minimum and a maximum respectively.

****

T2: Determine (i) maximum error and (ii) Range of each of the following (as before check your answer using the "BUSH" method).

a) $\dfrac{3,9214 \times 8,147^2}{12,981}$

b) $\sqrt{1,25^2 + 2,7 \times 3,416}$

c) $(1,25\ x\ 3,82) \div 0,5$

d) $\left(\dfrac{12,1^2 - 10,31}{\sqrt{17,11}}\right)^{\frac{1}{3}}$

****
****

A: a) i) Error in $8,147^2 \approx 2 \text{ x } 8,147 \text{ x } 0,0005 = 0,008147$
Error in $3,9214 \text{ x } 8,147^2 \approx 0,00005 \text{ x } 8,147^2 + 0,008147 \text{ x } 3,9214 = 0,0352663$
Error in $12,981 = 0,0005$

Error in:

$$\frac{3,9214 x 8,147^2}{12,981} \approx \left(\frac{0,0352663}{3,921 x 8,147^2} + \frac{0,0005}{12,981}\right) x \frac{3,9214 x 8,147^2}{12,981}$$

$= 0,0034893$

ii) Range $20,050649 - 0,0034893$ to $20,050649 + 0,0034893$
i.e.: $20,04716$ to $20,054138$

"BUSH" check:

$$\frac{3,92135\ x\ 8,1465^2}{12,9815} \quad to \quad \frac{3,92145\ x\ 8,1475^2}{12,9805}$$

i.e.: $20,04716$ to $20,054138$

****

b) i) Error in $1,25^2 \approx 2 \text{ x } 1,25 \text{ x } 0,005 = 0,0125$
Error in $2,7 \text{ x } 3,416 \approx 0,05 \text{ x } 3,416 + 0,0005 \text{ x } 2,7 = 0,17215$
Error in $1,25^2 + 2,7 \text{ x } 3,416 \approx 0,0125 + 0,17215 = 0,18465$
Error in:

$$\sqrt{1,25^2 + 2,7\ x\ 3,416} \approx \frac{1}{2}\ (10,7857)^{-\frac{1}{2}}\ x\ 0,18465 = 0,0281122$$

ii) Range is from $3,2841589 - 0,0281122$ to $3,2841589 + 0,0281122$
i.e.: $3,2560467$ to $3,3122711$

"BUSH" check: Range from

$$\sqrt{1,245^2 + 2,65\ x\ 3,4155} \quad to \quad \sqrt{1,255^2 + 2,75\ x\ 3,4165}$$

i.e.: $3,255933$ to $3,3121594$

****

c) i) Error in 1,25 X 3,82 $\approx$ 0,005 x 3,82 + 0,005 x 1,25 = 0,02535
Error in 0,5 $\approx$ 0,05
Error in:

$$\frac{1,25 \times 3,82}{0,5} \approx \left(\frac{0,02535}{1,25 \times 3,82} + \frac{0,05}{0,5} \times \frac{1,25 \times 3,82}{0,5}\right) = 0,9603089$$

ii) Range 9,55 - 0,9603089 to 9,55 + 0,9603089
i.e. from 8,5896911 to 10,510309
"BUSH" check:

$$\frac{1,245 \times 3,815}{0,55} \quad to \quad \frac{1,255 \times 3,825}{0,45}$$

i.e.: 8,6357727 to 10,6675
(slightly out - due mainly to relatively large error in denominator).
****
d) i) Error in $12,1^2 \approx 2 \times 12,1 \times 0,05 = 1,21$
Error in $12,1^2 - 10,31 = 1,21 + 0,005 = 1,215$
Error in

$$\sqrt{17,11} \quad or \quad 17,11^{\frac{1}{2}} \approx \frac{1}{2} \times 17,11^{-\frac{1}{2}} \times 0,005 = 0,0006043$$

Error in

$$\frac{12,1^2 - 10,31}{\sqrt{17,11}} \approx \left[\frac{1,215}{12,1^2 - 10,31} + \frac{0,0006043}{\sqrt{17,11}}\right] \times \frac{12,1^2 - 10,31}{\sqrt{17,11}}$$
$$= 0,2985388$$

Error in

$$\left(\frac{12,1^2 - 10,31}{\sqrt{17,11}}\right)^{\frac{1}{3}} \approx \frac{1}{3}(32,90282)^{-\frac{2}{3}} \times 0,2985388 = 0,0096914$$

ii) Range: 3,2043827 - 0,0096914 to 3,1946913 + 0,0096914
i.e.: 3,1946913 to 3,2140741
"BUSH" check:

$$\left(\frac{12,05^2 - 10,315}{\sqrt{17,115}}\right)^{\frac{1}{3}} \quad to \quad \left(\frac{12,15^2 - 10,305}{\sqrt{17,105}}\right)^{\frac{1}{3}}$$

$$32,604929^{\frac{1}{3}} \quad to \quad 33,202008^{\frac{1}{3}}$$

or 3,1946826 to 3,2140656

Now work through problems from past exam papers or 'further' problems.

****

S: Maximum error in X^n is approximately equal to :
the absolute value of $(nX^{n-1}\epsilon)$ **where ϵ is the error in X.**
For expressions containing a combination of addition, subtraction, multiplication, division and roots or powers you use the rules as shown in the worked examples.

****

****

****

SECTION : FINITE DIFFERENCES, NUMERICAL INTERPOLATION, NUMERICAL DIFFERENTIATION and NUMERICAL INTEGRATION

LESSON 5 :

After working through this lesson you should be able to:

1. **Form**,
2. **Correct**, and
3. **Extend** a **finite difference table**

Consider the following set of x and f(x) values:

x	-4	-2	0	2	4	6	8
f(x)	-55	-3	1	5	57	205	497

If we arrange these values into columns as shown below, and then form differences (see 1st difference column below) between successive f(x) (function values), we get:

x	f(x)	1st Differences
-4	-55	
		52
-2	-3	
		4
0	1	
		4
2	5	
		52
4	$\boxed{57}$	
		$\boxed{148}$
6	$\boxed{205}$	
		292
8	497	

NOTE: First Differences are formed by subtracting a function value from the following function value. For example:

In this table there is a finite increment (h) between the x values. We have h = 2 or the step size h = 2 (Since x values go up in steps of 2, i.e. -4; -2; 0; 2; 4 etc).

If we form second differences by finding the differences between first differences, and so on the table becomes:

Note: as before

x	f(x)	1st Diff	2nd diff	3rd diff	4th diff
-4	-55				
		52			
-2	-3		-48		
		4		48	
0	1		0		0
		4		48	
2	5		48		0
		52		48	
4	57		$\boxed{96}$		0
		148		$\boxed{48}$	
6	205		$\boxed{144}$		
		292			
8	497				

In this finite difference table we can see that the 3rd differences are constant, and fourth and higher order differences are zero.
As it happens, the x and f(x) values given above come from $f(x) = x^3 - 2x + 1$ for $x = -4(2)8$ i.e. x going from -4 to 8 in steps of 2.

It is interesting to see that if we differentiate f(x) successively, we get: $f(x) = x^3 - 2x + 1$; $f'(x) = 3x^2 - 2$ $f''x) = 6x$ and $f'''(x) = 6$ {a constant}. Higher order derivatives are zero. We also have 3rd finite differences being constant and higher order finite differences being zero.
In general, if we form a finite (finite => constant step size h) difference table for a <u>polynomial function of degree n,</u> the <u>n^{th} differences</u> will be <u>constant</u> and <u>higher order differences will be zero</u>.

<u>Note :</u> In forming the difference table, it is convenient to leave a blank line between successive function values ... this will enable you to easily fit first differences on the line between the function value lines, etc.
****

T1:(Please attempt tutorials before looking at the solution)

a) Form a finite difference table up to the stage where differences are zero's for the table of x and f(x) values given below:

x	-2	-1	0	1	2	3	4	5
f(x)	13	-1	-1	1	17	83	259	629

b) What is the finite or constant step size h

c) What is the degree of the polynomial function from which the set of table values were obtained.

****
****

A a)

x	f(x)	1st Diff	2nd Diff	3rd Diff	4th Diff	5th Diff
-2	13					
		-14				
-1	-1		14			
		0		-12		
0	-1		2		24	
		2		12		0
1	1		14		24	
		16		36		0
2	17		50		24	
		66		60		0
3	83		110		24	
		176		84		
4	259		194			
		370				
5	629					

b) $h = 1$ $(x = -2\ (1)\ 5)$

c) degree = 4 (since fourth finite differences are constant) The table, in fact, comes from $f(x) = x^4 + x - 1$.

Let's now see the effect an error in a single function value will have on a finite difference table.

For example, the difference table formed for $f(x) = 2x^3 - x^2 + 3$ for $x = -4\ (1)\ 6$ is:

x	f(x)	1st Diff			
-4	-141				
		81			
-3	- 60		2nd Diff		
			-38	3rd Diff	
		43			
-2	- 17		-26	12	4th Diff
					0
		17		12	
-1	0		-14		0
		3		12	
0	3		-2		0
		1		12	
1	4		10		0
		11		12	
2	15		22		0
		33		12	
3	48		34		0
		67		12	
4	115		46		0
		113		12	
5	228		58		
		171			
6	399				

If $f(2)$, that is the function value when $x = 2$, was recorded as 16 instead of 15 the table will be:

x	f(x)				
-4	-141				
		81			
-3	-60		-38		
		43		12	
-2	-17		-26		0
		17		12	
-1	0		-14		0
		3		12	
0	3		-2		1
		1		13	
1	4		11		-4
		12		$\boxed{9}$	
2	16		20		$\boxed{6}$
		32		$\boxed{15}$	
3	48		35		-4
		67		11	
4	115		46		1
		113		12	
5	228		58		
		171			
6	399				

See how the error has "fanned" out.

If we only had the above table, we could see by **"fanning" back** which function value is incorrect. It is quite simple to correct such an error. Recall, in a difference table,

```
                 ┌──┐    ****                            ***   ║  ║   ┌──┐
the entries      │  │    *  *   as before satisfied    *  *  = ║__║ - │  │
                         ****                            ***
                 ║  ║
                 ║__║
                                               ║  ║     ***
                 ┌──┐   ║  ║     ****          ║__║  =  *  *  +  ┌──┐
which implies    │  │ = ║__║  -  *  *   or              ***       │  │
                                 ****
```

Now consider the following example:

W

The function values in the following table contains one error. Correct the error and correct the difference table. **(NOTE : the x and f(x) values include values for x from -13 to 20 with step size h = 3)**

x	-13	-10	-7	-4	-1	2	5	8	11	14	17	20
f(x)	-2559	-1224	-465	-120	-27	-42	51	360	1065	2328	4311	7176

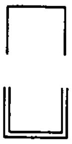
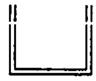
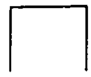
A
First we form the "finite" difference table (try it yourself).

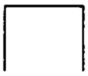
x	f(x)	1st Diff	2nd Diff	3rd Diff	4th Diff
-13	-2559				
		1335			
-10	-1224		-576		
		759		162	
-7	-465		-414		0
		345		162	
-4	-120		-252		-18
		93		144	
-1	-27		-108		72
		-15		216	
2	-42		108		-108
		93		108	
5	51		216		72
		309		180	
8	360		396		-18
		705		162	
11	1065		558		0
		1263		162	
14	2328		720		0
		1983		162	
17	4311		882		
		2865			
20	7176				

Examination of the differences indicates that 3rd differences should all be 162 (constant).

So we correct this column of differences, (merely draw a line through the incorrect values and write the correct value above the deleted value). This results in the following table:

x	f(x)				
-13	-2559	1st Diff			
		1335	2nd Diff		
-10	-1224		-576	3rd Diff	
		759		162	4th Diff
-7	-465		-414		0
		345		162	
-4	-120		-252		-18
		93		~~144~~ 162	
-1	-27		-108		72
		-15		~~216~~ 162	
2	-42		108		-108
		93		~~108~~ 162	
5	51		216		72
		309		~~180~~ 162	
8	360		396		-18
		705		162	
11	1065		558		0
		1263		162	
14	2328		720		0
		1983		162	
17	4311		882		
		2865			
20	7176				

By using  ****
* *
**** satisfying ****
* *
**** =  - 

or  =  - ****
* *
****

we can correct above the lower diagonal above to give:

x	f(x)	1st Diff	2nd Diff	3rd Diff	4th Diff
-13	-2559				
		1335			
-10	-1224		-576		
		759		162	
-7	-465		-414		0
		345		162	
-4	-120		-252		~~-18~~ 0
		93		~~144~~ 162	
-1	-27		-108		~~72~~ 0
		-15		~~216~~ 162	
2	~~-42~~ -24		108		~~-108~~ 0
		~~93~~ 75		~~108~~ 162	
5	51		~~216~~ 234		~~72~~ 0
		309		~~180~~ 162	
8	360		396		~~-18~~ 0
		705		162	
11	1065		558		0
		1263		162	
14	2328		720		0
		1983		162	
17	4311		882		
		2865			
20	7176				

NOTE: 396 - 162 gives the 234 then 309 - 234 gives the 75 and 51 - 75 corrects the error in original table of values. (i.e. -42 becomes -24). Fourth differences also corrected.

and then reform the incorrect portion of the table giving:

x	f(x)	1st Diff	2nd Diff	3rd Diff	4th Diff
-13	-2559				
		1335			
-10	-1224		-576		
		759		162	
-7	-465		-414		0
		345		162	
-4	-120		-252		~~-18~~ 0
		93		~~144~~ 162	
-1	-27		~~-108~~ -90		~~72~~ 0
		~~-15~~ 3		~~216~~ 162	
2	~~-42~~ -24		~~108~~ 72		~~-108~~ 0
		~~93~~ 75		~~108~~ 162	
5	51		~~216~~ 234		~~72~~ 0
		309		~~180~~ 162	
8	360		396		~~-18~~ 0
		705		162	
11	1065		558		0
		1263		162	
14	2328		720		0
		1983		162	
17	4311		882		
		2865			
20	7176				

We could use a similar procedure to extend a finite difference table (up or down). For example if asked to find f(23) in the above corrected table, we extend table (down) as follows:

x	f(x)	1st Diff	2nd Diff	3rd Diff	4th Diff
-13	-2559				
		1335			
-10	-1224		-576		
		759		162	
-7	-465		-414		0
		345		162	
-4	-120		-252		0
		93		162	
-1	-27		- 90		0
		3		162	
2	-24		72		0
		75		162	
5	51		234		0
		309		162	
8	360		396		0
		705		162	
11	1065		558		0
		1263		162	
14	2328		720		0
		1983		162	
17	4311		$\boxed{882}$	*****	0
		2865		*162*	
20	7176		$\boxed{1044}$	*****	
		3909			
23	11085				

obtained using $\begin{smallmatrix}\ast\ast\ast\ast\\ \ast\quad\ast\\ \ast\ast\ast\ast\end{smallmatrix}$ = $\sqcup$ - $\sqcap$ or $\sqcup$ = $\begin{smallmatrix}\ast\ast\ast\ast\\ \ast\quad\ast\\ \ast\ast\ast\ast\end{smallmatrix}$ + $\sqcap$

starting with 0 + 162 = 162, then 162 + 882 = 1044, 1044 + 2865 = 3909 etc Hence f(23) = 11085.

****

T2:

a) Consider the data

x	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7
f(x)	2,0	2,2001	2,4016	2,6081	2,8256	3,0625	3,3296	3,7401

i) Form and correct the finite difference table

ii) Find f(0,8) by extending the table

iii) What is the step size h in this problem?

b) The following table contains an error:

x	0	1	2	3	4	5	6
f(x)	10	19	50	121	250	455	745

30

Find and correct the error if f(x) is satisfied by a cubic polynomial.

****
****

A2:

a)

x	f(x)					
0,0	2,0					
		0,2001				
0,1	2,2001		0,0014			
		0,2015		0,0036		
0,2	2,4016		0,005		0,0024	
		0,2065		0,006		0
0,3	2,6081		0,011		0,0024	
		0,2175		0,0084		0
0,4	2,8256		0,0194		0,0024	
		0,2369		0,0108		0,1000
0,5	3,0625		0,0302		~~0,1024~~ 0,0024	
		0,2671		~~0,1132~~ 0,0132		
0,6	3,3296		~~0,1434~~ 0,0434		0,0024	
		~~0,4105~~ 0,3105		0,0156		
1) 0,7	~~3,7401~~ 3,6401		0,059			
		0,3695				

2) 0,8 4,0096

3) h = 0,1 (x goes from 0 to 0,7 in steps of 0,1)

A2:

b)

x	f(x)			
0	10			
		9		
1	19		22	
		31		18
2	50		40	
		71		18
3	121		58	
		129		18
4	250		76	
		205		~~9~~ 18
5	455		~~85~~ 94	
		~~290~~ 299		
6	~~745~~ 754			

therefore f(6) = 754 instead of 745

****

NOTE: When the error is near the bottom or near the top of the f(x) values, the "fan" is not as obvious as the case when the error is near the centre of the f(x) values.

LESSON 6:

After this lesson you should be able to:

1. **interpolate** (numerically) using the Newton Gregory (NG) forward difference interpolation formula.

2. **differentiate** (numerically) using a formula derived from the Newton Gregory forward difference interpolation formula.

Consider the following difference table:

x	f(x)				
-6	-203				
		148			
-4	-55		-96		
		52		48	
-2	-3		-48		0
		4		48	
0	1		0		0
		4		48	
2	5		48		0
		52		48	
4	57		96		0
		148		48	
6	205		144		
		292			
8	497				

If we label one of the given x values in the above table x_o and label the corresponding function value $f(x_o)$ or more simply, f_o, and then label the x and f(x) values below $(x_o;f_o)$ as $(x_1;f_1)$, $(x_2;f_2)$, $(x_3;f_3)$ ... and the $(x;f(x))$ values above $(x_o;f_o)$ as $(x_{-1};f_{-1})$, $(x_{-2};f_{-2})$, $(x_{-3};f_{-3})$ ... the table using 2 as x_o becomes:

x	f(x)
x_{-4}	f_{-4}
x_{-3}	f_{-3}
x_{-2}	f_{-2}
x_{-1}	f_{-1}
x_0	f_0
x_1	f_1
x_2	f_2
x_3	f_3

Note: If the step size between successive x values is h (h = 2 in our example) then

$$x_p = x_0 + ph \qquad \textit{an important equation}$$

e.g.: $x_3 = x_0 + 3 \,.\, h = 2 + 3 \,.\, 2 = 8$ etc...

The table below shows the forward difference notation for a finite difference table:

x	f(x)	1^{st} diff	2^{nd} diff	3^{rd} diff	
x_{-2}	f_{-2}				
		Δf_{-2}			
x_{-1}	f_{-1}		$\Delta^2 f_{-2}$		
		Δf_{-1}		$\Delta^3 f_{-2}$	
x_0	f_0		$\Delta^2 f_{-1}$		etc
		Δf_0		$\Delta^3 f_{-1}$	
x_1	f_1		$\Delta^2 f_0$		
		Δf_1		$\Delta^3 f_0$	
x_2	f_2		$\Delta^2 f_1$		$\Delta^4 f_0$
		Δf_2		$\Delta^3 f_1$	
x_3	f_3		$\Delta^2 f_2$		
		Δf_3			
x_4	f_4				

Note: All differences on the downward sloping diagonal from left to right have the same subscript (see above the dotted line).

In general using the forward difference notation, the r^{th} difference is given by:

$$\Delta^r f_n = \Delta^{r-1} f_{n+1} - \Delta^{r-1} f_n$$

$$eg \quad \Delta^4 f_0 = \Delta^3 f_1 - \Delta^3 f_0$$

If we examine a difference table, we notice that any function value $f(x_p)$ or simply f_p can be expressed in terms of a chosen function value in the table (say f_0) and corresponding forward differences:

$$\Delta f_0 \; ; \; \Delta^2 f_0 \; ; \; \Delta^3 f_0 \;\; etc$$

Please note $\Delta^3 f_0$ means a 3rd difference NOT $(\Delta f_0)^3$

For example, given:

x	f(x)				
x_{-1}	f_{-1}				
		Δf_{-1}			
x_0	f_0		$\Delta^2 f_{-1}$		
		Δf_0		$\Delta^3 f_{-1}$	
x_1	f_1		$\Delta^2 f_0$		$\Delta^4 f_{-1}$
		Δf_1		$\Delta^3 f_0$	
x_2	f_2		$\Delta^2 f_1$		$\Delta^4 f_0$
		Δf_2		$\Delta^3 f_1$	
x_3	f_3		$\Delta^2 f_2$		
		Δf_3			
x_4	f_4				

$f_4 = f_3 + \Delta f_3$ .... comes from $(f_4 - f_3 = \Delta f_3)$

$\Rightarrow f_4 = f_2 + \Delta f_2 + \Delta f_2 + \Delta^2 f_2$ (since $f_3 = f_2 + \Delta f_2$ and $\Delta f_3 = \Delta f_2 + \Delta^2 f_2$)

$\Rightarrow f_4 = f_1 + \Delta f_1 + \Delta f_1 + \Delta^2 f_1 + \Delta^2 f_1 + \Delta^3 f_1$ (similar reason)

$\Rightarrow f_4 = f_0 + \Delta f_0 + 2(\Delta f_0 + \Delta^2 f_0) + 2(\Delta^2 f_0 + \Delta^3 f_0) + \Delta^3 f_0 + \Delta^4 f_0$

i.e. f_4 expressed in terms of (i.t.o.) f_0 ; Δf_0 ; $\Delta^2 f_0$ etc

Generally we can express f_p i.t.o. f_0 ; Δf_0 ; $\Delta^2 f_0$ ... and p where

$$p = \frac{x_p - x_0}{h}$$

{comes from $x_p = x_0 + ph$}

The Newton Gregory (NG) forward difference interpolation formula gives us the relationship as follows:

$$f_p = f_0 + p \cdot \Delta f_0 + \frac{p(p-1)}{1 \times 2} \Delta^2 f_0 + \frac{p(p-1)(p-2)}{1 \times 2 \times 3} \Delta^3 f_0 + \ldots$$

OR

$$f_p = f_0 + \binom{p}{1} \Delta f_0 + \binom{p}{2} \Delta^2 f_0 + \binom{p}{3} \Delta^3 f_0 + \binom{p}{4} \Delta^4 f_0 + \ldots$$

$$\text{where} \quad p = \frac{x_p - x_o}{h}$$

and

$$\binom{p}{r} = \frac{p!}{(p-r)!\,r!} = \frac{p(p-1)(p-2)\ldots(p-r+1)}{1 \times 2 \times 3 \times \ldots \times r}$$

Note: In

$$\binom{p}{r} = \frac{p(p-1)(p-2)\ \ldots(p-r+1)}{1 \times 2 \times 3 \ \ldots \ \times r}$$

both numerator and denominator have r factors.

If certain differences are zero, we merely use the formula up to and including the non-zero differences. For example, if 3rd and higher differences are zero then the NG forward formula will be:

$$x_p = x_0 + p\Delta f_0 + \frac{p(p-1)}{1 \times 2} \Delta^2 f_0$$

(+ zero terms).

Suppose we are asked to find f(2,5), given the difference table at the beginning of this lesson, i.e. the function value corresponding to x = 2,5. We select one of the given x values as x_0 (say $x_0 = 0$) and take x_p as the given x value, i.e. $x_p = 2,5$. Since h = 2 we get

$$p = \frac{x_p - x_0}{h} = \frac{2,5 - 0}{2} = 1,25$$

x	f(x)				
-6	-203				
		148			
-4	-55		-96		
		52		48	
-2	-3		-48		0
		4		48	
0	1		0		0
		4		48	
2	5		48		0
		52		48	
4	57		96		0
		148		48	
6	205		144		
		292			
8	497				

Now using NG forward

$$f_p = f_0 + p\,\Delta f_0 + \frac{p(p-1)}{1 \times 2}\Delta^2 f_0 + \frac{p(p-1)(p-2)}{1 \times 2 \times 3}\Delta^3 f_0$$

+ zero terms

we get

$$f(2,5) = 1 + 1,25 \times 4 + \frac{1,25 \times 0,25}{1 \times 2} \times 48 + \frac{1,25 \times 0,25 \times -0,25}{1 \times 2 \times 3} \times 48$$

= 12,875

Note: x_p merely means a variable x which can be expressed in terms of p and the constants x_0 and h. i.e. $x_p = x_0 + ph$.

****

T Using -4 as x_0 in the finite difference table given above, determine f(-3,2).

****

****

A $x_0 = -4$

Now

$$p = \frac{x_p - x_0}{h} = \frac{-3,2 - -4}{2} = \frac{0,8}{2} = 0,4$$

Therefore, using NG forward i.e.

$$f(x_p) \quad or \quad f_p = f_0 + p\Delta f_0 + \frac{p(p-1)}{1x2}\Delta^2 f_0 + \frac{p(p-1)(p-2)}{1x2x3}\Delta^3 f_0$$

we get :

$$f(-3,2) = -55 + 0,4x52 + \frac{0,4(-0,6)}{1x2}x{-48} + \frac{0,4(-0,6)x(-1,6)}{1x2x3}x48$$

= -25,368

Recap: $x_p = x_0 + ph$ or

$$p = \frac{x_p - x_0}{h}$$

and

$$f_p = f_0 + p\Delta f_0 + \frac{p(p-1)}{1 \ x \ 2}\Delta^2 f_0 + \frac{p(p-1)(p-2)}{1 \ x \ 2 \ x \ 3}\Delta^3 f_0 + \ldots$$

This formula gives $f(x_p)$ i.t.o. constants f_0 ; Δf_0 ; $\Delta^2 f_0$ .. and p.

If we want to find

$$\frac{df(x_p)}{dx} \quad i.e. \quad f'(x_p)$$

we use the function of a function rule.

i.e.

$$\frac{df(x_p)}{dx} = \frac{df}{dp} \ x \ \frac{dp}{dx} = \frac{dp}{dx} \ x \ \frac{df}{dp} \ \text{where}$$

$$\frac{dp}{dx} = \frac{1}{h} \ \text{since} \ p = \frac{x_p - x_0}{h} = \frac{1}{h}x_p - \frac{x_0}{h} \qquad (x_0 \ ; \ h \ \text{constants})$$

$$\Rightarrow \frac{dp}{dx} = \frac{1}{h} - 0 = \frac{1}{h}$$

$$f'_p \text{ i.e. } f'(x_p) = \frac{1}{h} x \frac{d\left(f_0 + p\Delta f_0 + \frac{p(p-1)}{1x2}\Delta^2 f_0 + \frac{p(p-1)(p-2)}{1x2x3}\Delta^3 f_0 \ldots\right)}{dp}$$

$$= \frac{1}{h} x \frac{d\left(f_0 + \Delta f_0 p + \frac{\Delta^2 f_0}{1x2}(p^2 - p) + \frac{\Delta^3 f_0}{1x2x3}(p^3 - 3p^2 + 2p) + \right)}{dp}$$

$$= \frac{1}{h}\left(\Delta f_0 + \frac{\Delta^2 f_0}{1 \ x \ 2}(2p - 1) + \frac{\Delta^3 f_0}{1 \ x \ 2 \ x \ 3}(3p^2 - 6p + 2) + \ldots\right)$$

Now we can do numerical differentiation using this formula. If 3^{rd} differences are constant => 4th and higher differences are zero.

NOTE: If 4^{th} differences are constant => 5^{th} and higher differences are zero, we would have to go one term further in NG and the resultant formula for differentiation.
In other words, the formula obviously does not include zero terms.
W: Using the table below, determine:

a) $f'(2)$ ....using $x_0 = 2$
b) f' (6,5) ......using $x_0 = 0$

x	f(x)				
-6	-203				
		148			
-4	-55		-96		
		52		48	
-2	-3		-48		0
		4		48	
0	1		0		0
		4		48	
2	5		48		0
		52		48	
4	57		96		0
		148		48	
6	205		144		
		292			
8	497				

A: Using $x_0 = 2$ =>

$$a) \quad p = \frac{x_p - x_0}{h} = \frac{2 - 2}{2} = 0$$

Now since

$$f'(x) = \frac{1}{h}\left(\Delta f_o + \frac{\Delta^2 f_o}{1x2}(2p-1) + \frac{\Delta^3 f_o}{1x2x3}(3p^2-6p+2)\right) + zero\ terms$$

$$\therefore f'(2) = \frac{1}{2}\left(52 + \frac{96}{1x2}(-1) + \frac{48}{1x2x3}(2)\right) = 10$$

b) Using $x_o = 0$

$$Now\ p = \frac{x_p - x_o}{h} = \frac{6,5 - 0}{2} = 3,25$$

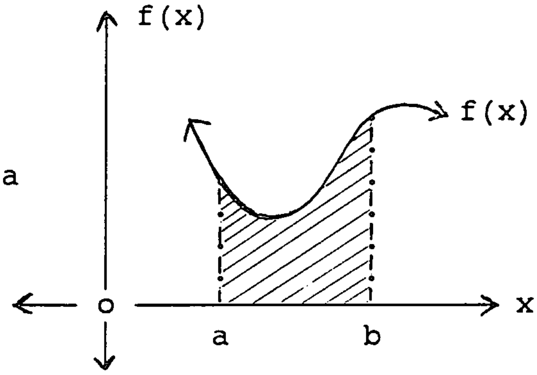
Now using the formula for f'(x) given above, it follows that:

$$f'(6,5) = \frac{1}{2}\left(4 + \frac{48}{1x2}(2x3,25 - 1) + \frac{48}{1x2x3}\left(3x3,25^2 - 6x3,25 + 2\right)\right) = 124,75$$

After working through this lesson you should be able to perform numerical integration using:
a) Trapezoidal rule or;
b) Simpson's rule.

Recall (from Mathematics 1) $\int_a^b f(x)\,dx$ = area between the graph of f(x), the x-axis and the ordinates x = a and x = b (Area above the x-axis will be positive and the area below the x-axis will be negative).
i.e.:

$$\int_a^b f(x)\,dx = \text{shaded area}$$



Now in numerical integration we reverse the above. That is, we use the area as an estimate of the definite integral. Or more specifically, we estimate the value of:

$$\boxed{\int_a^b f(x)\,dx}$$

by estimating the area between f(x), the x-axis and the ordinates x = a and x = b.

There are numerous ways of estimating the required area. We will examine two such methods, namely the Trapezoidal Rule and Simpson's rule.

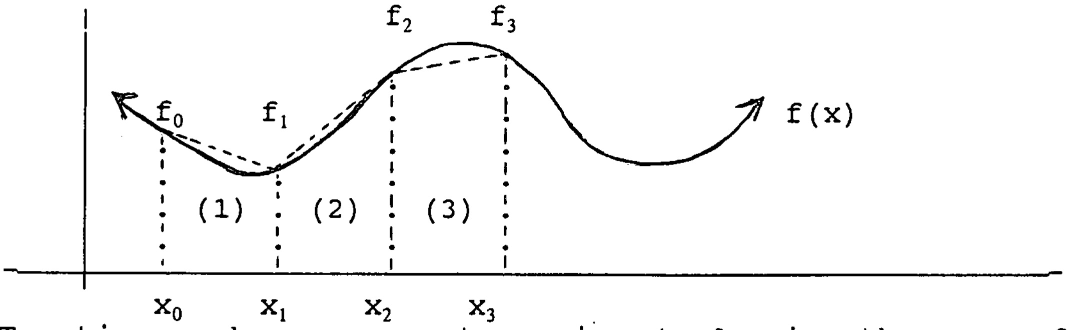
1. **Trapezoidal rule :**
Suppose we are given a set of (x; f(x)) values where, for example, x varies from x_0 to x_3 in steps of h, i.e.

x	x_0	x_1	x_2	x_3 ....
f(x)	f_0	f_1	f_2	f_3

If we now want to find

$$\int_{x_0}^{x_3} f(x)\ dx$$

we estimate the area under the curve between x_o and x_3.
For example:



Treating each area as a trapezium (and using the area of a trapezium as half the sum of parallel sides times the perpendicular distance between parallel sides), we get:

Area under curve between x_o and $x_3 \approx$ area of trapezium (1) + area of trapezium (2) + area of trapezium (3)

$$\approx \frac{f_o + f_1}{2} \times h + \frac{f_1 + f_2}{2} \times h + \frac{f_2 + f_3}{2} \times h$$

$$= \frac{h}{2} (f_o + f_1 + f_1 + f_2 + f_2 + f_3)$$

$$= \frac{h}{2} (f_o + f_3 + 2\ [f_1 + f_2])$$

Generally, if we were using areas of trapeziums to estimate the area under f(x) between x_o and x_N we would get

$$Area \approx \frac{h}{2} (f_o + f_N + 2\Sigma\ f_{remaining})$$

In words, the Trapezium rule states:

Area is approximately equal to half the step size (h) multiplied by (first plus last function values added to twice the sum of the remaining function values).

So $\int_{x_o}^{x_N} f(x)\ dx \approx \frac{h}{2}\ (f_o + f_N + 2\ \Sigma\ f_{remaining})$

W1: Find

$\int_0^8 f(x)\ dx\ given$

x	0	2	4	6	8
f(x)	1	5	57	205	497

****
A1: We have h = 2

$\int_0^8 f(x)\ dx$

[using trapezoidal rule .. see top of this page ..]

$\approx \frac{2}{2}\ (1 + 497 + 2\ (5 + 57 + 205)) = 1032$

****
The actual f(x) = $x^3 - 2x + 1$

$\therefore \int_0^8 (x^3-2x+1)\ dx = {}_0^8[\frac{1}{4}x^4-x^2+x] = \left(\frac{8^4}{4}-8^2+8\right)-(0) = 968$

(If we decrease the step size, the approximation improves).
****
W2: For example, if we use h = 1 in the example above we get the table :

x	0	1	2	3	4	5	6	7	8
f(x)	1	0	5	22	57	116	205	330	497

****
A2:

$\int_0^8 f(x)\ dx \approx Area\ using\ trapezoidal\ rule$

$\approx \frac{1}{2}\ (1 + 497 + 2\ (0 + 5 + 22 + 57 + 116 + 205 + 330)) = 984$

which is a better approximation

Using h = 0,5 gives area ≈ 972 (still a better approximation)
Try it yourself by forming the table of values etc.
****

T1: Given:

x	0	3	6	9	12	15	18
f(x)	20	22	19	21	25	27	35

determine: $\int_0^{18} f(x)\,dx$

A1: $\int_0^{18} dx$ *using the Trapezoidal Rule* $\approx \frac{h}{2}\left[f_0+f_0+2(f_1+f_2+f_3+f_4+f_5)\right]$

$= \frac{3}{2}[20+35+2(22+19+21+25+27)] = 424{,}5$

T2 Determine $\int_2^9 (x^3+7x)\,dx$ *numerically*

[Hint: form your own table using h = 1 {a smaller h value will improve the approximation of the definite integral).
****
****

A2:

x	2	3	4	5	6	7	8	9
f(x)	22	48	92	160	258	392	568	792

Now using the Trapezoidal Rule

$$\int_2^9 (x^3+7x)\,dx \approx \frac{h}{2}[f_0 + f_7 + 2\,\Sigma\, f_{remaining}]$$

$$= \frac{1}{2}\,[22+792+2(48+92+160+258+392+568)] = 1925$$

Using integration, we get:

$$\int_2^9 (x^3+7x)\,dx = {}_2^9\left[\frac{x^4}{4}+\frac{7x^2}{2}\right] = \left(\frac{9^4}{4}+\frac{7x9^2}{2}\right) - \left(\frac{2^4}{4}+\frac{7x2^2}{2}\right) = 1905{,}75$$

Simpson's Rule:

If instead of considering each strip area as a trapezium, we determine the respective areas by assuming the ordinates are joined by segments of a parabola, we obtain a better approximation of the definite integral. In order to do this, it is necessary to divide the area under the curve into an <u>even</u> number of strips (i.e. we have an <u>odd</u> number of points).

If we have:

x :	x_o	x_1	x_2	x_3	...	x_N
f(x):	f_o	f_1	f_2	f_3	...	f_N

where N is even, i.e. an odd number of points,

then Simpson's rule states:

$$\int_{x_o}^{x_N} f(x)\ dx \approx \frac{h}{3}\ (f_o + f_N + 2\ \Sigma\ f_{even} + 4\Sigma\ f_{odd})$$

where Σ f $_{even}$ is the sum of the remaining f values with even subscripts and $\Sigma\ f_{odd}$ is the sum of remaining f value with odd subscripts.

W3: Use Simpson's rule to find:

$$\int_0^8 f(x)\ dx$$

given

x	0	1	2	3	4	5	6	7	8
f(x)	1	0	5	22	57	116	205	330	497

(Recall exact answer = 968) while Trapezoidal rule gave 1032 (h = 2) and 984 (h = 1)).

A3: Using Simpson's Rule we get:

$$\int_0^8 f(x)\,dx \approx \frac{1}{3}(1+497+2(5+57+205)+4(0+22+116+330))=968$$

NB: Use Simpson's Rule, unless otherwise specified, when you are given an odd number of points.

If we are asked to find

$$\int_a^b f(x)\ dx$$

where f(x) is not easily integrated, we can get an approximate numerical solution by forming our own table of x and f(x) values. We normally choose step size h so that we obtain a reasonable approximation. A smaller step size will normally give a better approximation.

W4: Numerically, determine:

$$\int_0^3 x^2\sqrt{4x^3-x}\ dx \text{ using } h = 0,5$$

Now, working correct to 4 significant figures, gives:
****
A4:

x	0	0,5	1	1,5	2,0	2,5	3,0
f(x)	0 f_0	0 f_1	1,732 f_2	7,794 f_3	21,91 f_4	48,41 f_5	92,22 f_6

Using Simpson's Rule

$$i.e. \quad \int_{x_0}^{x_6} f(x)\ dx \approx \frac{h}{3}[f_0 + f_6 + 2(f_2 + f_4) + 4(f_1 + f_3 + f_5)]$$

$$\Rightarrow \int_0^3 x^2\sqrt{4x^3-x}\ dx$$

$$= \frac{0,5}{3}(0+92,22+2(1,732+21,91)+4(0+7,794+48,41))=60,72$$

****

T3: a) If:

x	0	3	6	9	12	15	18
f(x)	20	22	19	21	25	27	35
	f_0	f_1	f_2	f_3	f_4	f_5	f_6

determine, using Simpson's Rule:

$$\int_0^{18} f(x)\,dx$$

****

****

A3: a)

$$\int_0^{18} f(x)\,dx \approx \left[f_0+f_6+2(f_2+f_4)+4(f_1+f_3+f_5)\right]$$

$$= \frac{3}{3}[20 + 35 + 2(19 + 25) + 4(22 + 21 + 27)]$$

$$=423$$

b): Determine:

$$\int_2^{8} (x^3+7x)\,dx$$

using Simpson's Rule and the table formed in T2:

A3: b)

$$\int_2^{8} (x^3+7x)\,dx$$

$$\approx \frac{h}{3}\left[f_0+f_6+2(f_2+f_4)+4(f_1+f_3+f_5)\right]$$

$$= \frac{1}{3}[22 + 568 + 2(92 + 258) + 4(48 + 160 + 392)]$$

$$= 1230$$

****

T4: Given

x	2	-1	10	6	3,5	-7
f(x)	-17	-8	-393	-141	-48,5	-206

it can be shown that $f(x) = -4x^2 + 1x - 3$.

a) Now using h = 0,5 determine, numerically:

$$\int_0^3 f(x)\,dx$$

b) Check your answer by finding the definite integral.

****

****

A4:

a) Table:

x	0	0,5	1,0	1,5	2	2,5	3,0
f(x)	-3	-3,5	-6	-10,5	-17	-25,5	-36
	f_0	f_1	f_2	f_3	f_4	f_5	f_6

Using Simpson's Rule we get

$$\int_0^3 f(x)\,dx \approx \frac{h}{3}[f_0+f_6+2(f_2+f_4)+4(f_1+f_3+f_5)]$$

$$= \frac{0,5}{3}[(-3)+(-36)+2((-6)+(-17))+4(-3,5)+(-10,5)+(-25,5)] = -40,5$$

****

b)

$$\int_0^3 (-4x^2+1x-3)\,dx = {}^3_0[-\frac{4}{3}x^3+\frac{1}{2}x^2-3x] = (4-\frac{4}{3}x3^3+\frac{1}{2}x3^2-3x3)-(0) = -40,5$$

****

<u>**S:**</u> Numerical integration can be done by estimating the area under a curve. We have considered two methods :

i) Trapezoidal Rule and

ii) Simpson's Rule (used when we have an **<u>odd</u>** number of points).

SECTION : NUMERICAL SOLUTION of n LINEARLY INDEPENDENT EQUATIONS in n UNKNOWNS.

LESSON 8 :

After completing this lesson you should be able to **solve** a system of **independent linear equations numerically** using the Gauss-Seidel iteration method.

Briefly the iteration technique will involve:

1. **Guessing a solution** to the problem. (If possible an "accurate" guess).
2. Then using your guess as a starting point, **obtain a (hopefully) better approximation by a suitable numerical technique.**
3. **Proceed in a similar manner**, that is use the "better" approximation in step 2 to obtain a still better approximation, and continue obtaining successive approximations **until the desired accuracy is obtained.**

Note: If differences between successive approximations gets smaller and smaller - the iteration procedure is said to converge. If not you may have to adjust your initial guess or numerical technique to obtain a solution.

Consider the following set of linear equations (3 linearly independent equations in 3 unknowns - independent in the sense that one equation cannot be expressed as a linear combination of the other equations .. i.e. one equation is not a linear combination of the other).

$$\begin{aligned} 3x + y - z &= 6 \\ x - 4y + 2z &= 1 \\ x + y + 3z &= 9 \end{aligned}$$

In using the Gauss-Seidel method it is useful to write the equations in standard form, with the absolute coefficients of the first unknown (x in the given case) as large as possible in the first equation, the absolute coefficient of the second unknown (y in our case) as large as possible in the second equation etc. In other words interchange the equations to obtain the (absolute) diagonal elements as large as possible (partial pivoting). The set of equations given above satisfies this criteria.
Now we rewrite each of the equations, as follows:
Using first equation:

$$x = \frac{6 - y + z}{3} \quad \ldots (1)$$

Using second equation:

$$y = \frac{x + 2z - 1}{4} \quad \ldots (2)$$

and using third equation:

$$z = \frac{9 - x - y}{3} \quad \ldots (3)$$

The equations (1), (2) and (3) will be used in the iterative procedure.

We follow the iteration steps outlined before:

1) Guess a solution: Say x = 0 ; y = 0 and z = 0
2) Now in Gauss-Seidel we use this guess to obtain a "hopefully" better approximation, by substituting into equation (1). i.e.:

$$x = \frac{6 - y + z}{3} = \frac{6 - 0 + 0}{3} = 2$$

Now
x = 2 ; y = 0 ; z = 0

3) Using this approximation we now use equation (2) to give us:

$$y = \frac{x + 2z - 1}{4} = \frac{2 + 0 - 1}{4} = \frac{1}{4}$$

and therefore we now have x = 2 ; y = $\frac{1}{4}$; z = 0

4) Using this approximation and equation (3) we get:

$$z = \frac{9 - x - y}{3} = \frac{9 - 2 - \frac{1}{4}}{3} = 2\,\frac{1}{4}$$

and therefore x = 2 ; y = $\frac{1}{4}$; z = 2,25 at the end of the first run.
We now repeat this procedure (for 2nd run) i.e.:
using equation (1) and our approximate solution at the end of the first run, gives :

$$x = \frac{6 - y + z}{3} = \frac{6 - 0,25 + 2,25}{3} = 2,666667$$

Then using equation (2) with the most recent x & y values gives:

$$y = \frac{x + 2z - 1}{4} = \frac{2,666667 + 2 \times 2,25 - 1}{4} = 1,541667$$

(Notice in Gauss-Seidel we use the latest set of x ; y & z values).

Then using equation (3), we get :

$$z = \frac{9 - x - y}{3} = \frac{9 - 2,666667 - 1,541667}{3} = 1,597222$$

i.e. at the end of the second run x = 2,6666667 ;
y = 1,541667 and z = 1,597222

T: Work through the above three steps again, in order to determine the approximate solution at the end of the third run :

****
****

A:

(1) $$x = \frac{6 - y + z}{3} = \frac{6 - 1,541667 + 1,597222}{3} = 2,018518$$

(2) $$y = \frac{x + 2z - 1}{4} = \frac{2,018518 + 2 \times 1,597222 - 1}{4} = 1,053241$$

(3) $$z = \frac{9 - x - y}{3} = \frac{9,2,018518 - 1,053241}{3} = 1,97608$$

Giving x = 2,018518 ; y = 1,053241 ; z = 1,97608

If we proceed we get:
after (4) runs x = 2,307613 ; y = 1,314943 ; z = 1,792481
after (5) runs x = 2,159179 ; y = 1,18605 ; z = 1,884978
after (6) runs x = 2,232964 ; y = 1,250705 ; z = 1,838777
. . . .
. . . .
. . . .
after (19) runs x = 2,208331 ; y = 1,229164 ; z = 1,854169
after (20) runs x = 2,208335 ; y = 1,229168 ; z = 1,854166

So if we had to solve to 4 significant figures we would have x = 2,208 ; y = 1,229 and z = 1,854.

It is easy to write a simple BASIC programme to do the iterations for us: (Note: **if you wish, you** can make the programme more sophisticated).

```
10    REM: Gauss-Seidel iteration
20    LET  x = 0
30    LET  y = 0
40    LET  z = 0
50    FOR  I = 1 to 30
60    LET x = (6 - y + z) / 3
70    LET y = (x + 2 * z - 1) / 4
80    LET z = (9 - x - y) / 3
90    PRINT "AFTER "I" runs : x = "x" y = "y" AND z = "z
100   NEXT I
110   END
```

(It is obvious that the use of such an elementary programme will make life "easy" for us. We merely edit lines 60, 70 and 80 to suit our system of equations.)

Finally, check the solution by substituting back into the original system, in our case:

$3x + y - z = 3 \times 2{,}208 + 1{,}229 - 1{,}854 = 5{,}999$ (close to 6)
$x - 4y + 2z = 2{,}208 - 4 \times 1{,}229 + 2 \times 1{,}854 = 1$ (equal to 1)
$x + y + 3z = 2{,}208 + 1{,}229 + 3 \times 1{,}854 = 8{,}999$ (close to 9)

****

T: Solve each of the following systems of equations correct to 2 D (i.e. to 2 decimal places) using the Gauss-Seidel method:
PLEASE use your programme (see the programme above) to do the iteration)

a)
$$1x - 2y + 3z = 3{,}5$$
$$4x + 2z = 9$$
$$5x - y + z = -2$$

b)
$$8{,}321x + 4y - 2z = 14$$
$$3{,}4x - 1y - 16z = -13$$
$$2x + 17y + 5z = 23$$

****
****

A: a) Rewritten system:

$$\begin{aligned} 5x - 1y + 1z &= -2 \\ 1x - 2y + 3z &= 3,5 \\ 4x + 0y + 2z &= 9 \end{aligned}$$

which gives :

$$x = \frac{y-z-2}{5} \quad \ldots\ldots \quad (1)$$

$$y = \frac{x+3z-3,5}{2} \quad \ldots\ldots \quad (2)$$

$$z = \frac{9-4x}{2} \quad \ldots\ldots\ldots \quad (3)$$

Starting with x = y = z = 0: using (1) gives:

$$x = \frac{-2}{5} = -0,4$$

Then using (2) gives:

$$y = \frac{-0,4+3(0)-3,5}{2} = -1,95$$

and then using (3) gives:

$$z = \frac{9-4(-0,4)}{2} = 5,3$$

ie. after the first run x = -0,4 ; y = -1,95 ; and z = 5,3.

Repeating the above procedure for the second run, gives:

$$(using\ (1)\ ) \Rightarrow x = \frac{-1,95-5,3-2}{5} = -1,85$$

$$(using\ (2)\ ) \Rightarrow y = \frac{-1,85+3(5,3)-3,5}{2} = 5,275$$

and

$$(using\ (3)\ )\ \Rightarrow\ z = \frac{9-4(-1,85)}{2} = 8,2$$

ie. after the second run x = -1,85 ; y = 5,275 ; and z = 8,2 etc.

After the third run x = -0,984999 ; y = 10,0575 ; z = 6,47
After the fourth run x = 0,3174998 ; y = 8,11375 ; z = 3,865
. . . .
After 29 runs x = -0,2722646 ; y = 5,687296 ; z = 5,044529
After 30 runs x = -0,2714466 ; y = 5,681071 ; z = 5,042894

Hence to 2D : x = -0,27 ; x = -0,27 ; y = 5,68 and z = 5,04

Substituting these values into the right hand side of the rewritten system gives:

5(-0,27) - (5,68) + (5,04) = -1,99 (close to -2)
(-0,27) - 2(5,68) + 3(5,04) = 3,49 (close to 3,5)
4(-0,27) + 2(5,04) = 9 (equals 9)

****

b) Rewritten system

$8,321x + 4y - 2z = 14$
$2x + 17y + 5z = 23$
$3,4x - 1y - 16z = -13$

Therefore

$$x = \frac{2x-4y+14}{8,321} \ \ldots\ldots\ (1)$$

$$y = \frac{23-2x-5z}{17} \ \ldots\ldots\ (2)$$

$$z = \frac{3,4x-1y+13}{16} \ \ldots\ldots\ (3)$$

gives us:

x = 1,68249 ; y = 1,155001 ; z = 1,097842 after 1 run
x = 1,39114 ; y = 0,8663831 ; z = 1,053968 after 2 runs
x = 1,519337 ; y = 0,8642049 ; z = 1,081346 after 3 runs
. . . .
x = 1,532963 ; y = 0,8535022 ; z = 1,084911 after 9 runs
x = 1,532966 ; y = 0,8535008 ; z = 1,084912 after 10 runs

Therefore to 2D x = 1,53 ; y = 0,85 and z = 1,08

Substituting into RHS of rewritten system gives:

8,321(1,53) + 4(0,85) - 2(1,08) = 13,97 (compared to 14)
2(1,53) + 17(0,85) + 5(1,08) = 22,91 (compared to 23)
3,4(1,53) - (0,85) - 16(1,08) = -12,93 (compared to -13)
****

S : Gauss Seidel is a numerical method that enables us to solve a system of n linearly independent equations in n unknowns.

CHAPTER 4

THE EXPERIMENT : PROCEDURE, DATA, ANALYSIS and CONCLUSIONS

4.1 Preliminaries

After consulting various texts on, and examples of, Self-instructional material, an initial draft SI text (available from the author : see Appendix H) was prepared for the numerical methods section of Mathematics II (second semester level) given to Technikon Engineering and Applied Science students. This draft was given for comment to one group of Applied Mathematics T3 students during the second semester of 1990. (These students had covered the same numerical methods work in this subject). The numerical methods section for them had been covered using conventional lecture methods. Following the students' comments (cited in Appendix H), the first draft was revised and subsequently given to Engineering Mathematics II students during both semesters in 1991. Colleagues in the Mathematics department, as well as the students, were asked for their comments. Comments from these sources were noted in draft and the final (current) text was prepared for use in the experiment conducted during the first semester of 1992.

4.2 General Procedure

One class group (Civil Engineering and Surveying) of Mathematics II students formed the **S**hort **L**ecture **S**elf **I**nstruction (SLSI) trial group.

During the first lesson the purpose of the exercise was explained to them, and they were given a brief introduction to what they were expected to do. Their role would be to attend the short lecture (≈ 10 to 15 minutes) preceding each lesson, then work through the lesson doing the necessary tutorial examples during their own time (which preferably would include the remainder of the 45 minute period). They were also asked to fill in the first of two questionnaires (see Appendix A). They were issued with the SI text, and the first short lecture (≈10 minutes) was given. The students then used the rest of the 45 minute period to work through lesson 1.

During the rest of the course, the short lecture was given during the first 10 to 15 minutes of the students' set lecture period and they then worked through the relevant SI lesson. At the completion of the section, they were asked to fill in the second questionnaire (see Appendix B).

Students from the SLSI trial group who wrote the Mathematics II examination formed the experimental group. The control group was selected at random from other Engineering students writing the relevant examination paper, who were lectured by conventional means by a variety of lecturers, and who had obtained matching marks for Mathematics I.

Data obtained by using the pertinent responses to the questionnaires by the trial group, and the examination performance by the experimental and control groups, were used for the following Statistical presentation and analysis.

In presenting the data, numerous tables, figures/graphs and statistics are shown. The following quotation supports the author's presentation of the data in a variety of forms : "Many researchers, however, fail to exploit the data fully. One cannot turn over the facts too often; look at them from too many angles; chart, graph and arrange them in too many ways, inspect them from too many vantage points." (Leedy, 1980, p.231)

4.3 **Pre-SLSI Questionnaire Statistics** (Student profile)

The questionnaire that was given is shown in Appendix A.

Briefly, the questionnaire was used to obtain certain information about the students, prior to the use of the self-instructional material. Details such as name, age, home language, type of accommodation, sex, and employment status were obtained. Also, section A of the questionnaire included a rating on a scale of 0 to 5 by students on how they enjoyed mathematics (0 indicating they did not enjoy mathematics at all and 5 indicating that they enjoyed it very much). Section B enquired into the students' Standard Ten and Technikon Mathematics I information. Section C included two questions which determined whether they were familiar with self-instruction texts as well as a rating of their attitude to the proposed SLSI teaching approach. This was also on a scale of 0 (indicating they were not looking forward to working on their own) to 5 (indicating that they were very excited at the prospect of doing the work).

This questionnaire thus provided a **profile** of the students who participated in the development and use of the SI material.

In order to clarify the attributes obtained, a table of values is given, followed by a visual statistical diagram (where applicable) and appropriate comment.

Total number of responses to the questionnaire : 134 (126 males, 8 females).

Comment : Homogeneous group (predominantly male students).

4.3.1 **Age distribution** of respondents :

Age of students	18	19	20	21	22	23	24	25	26	27	28	32
Number of students	3	27	29	25	22	9	9	5	1	1	2	1

Table 4.1

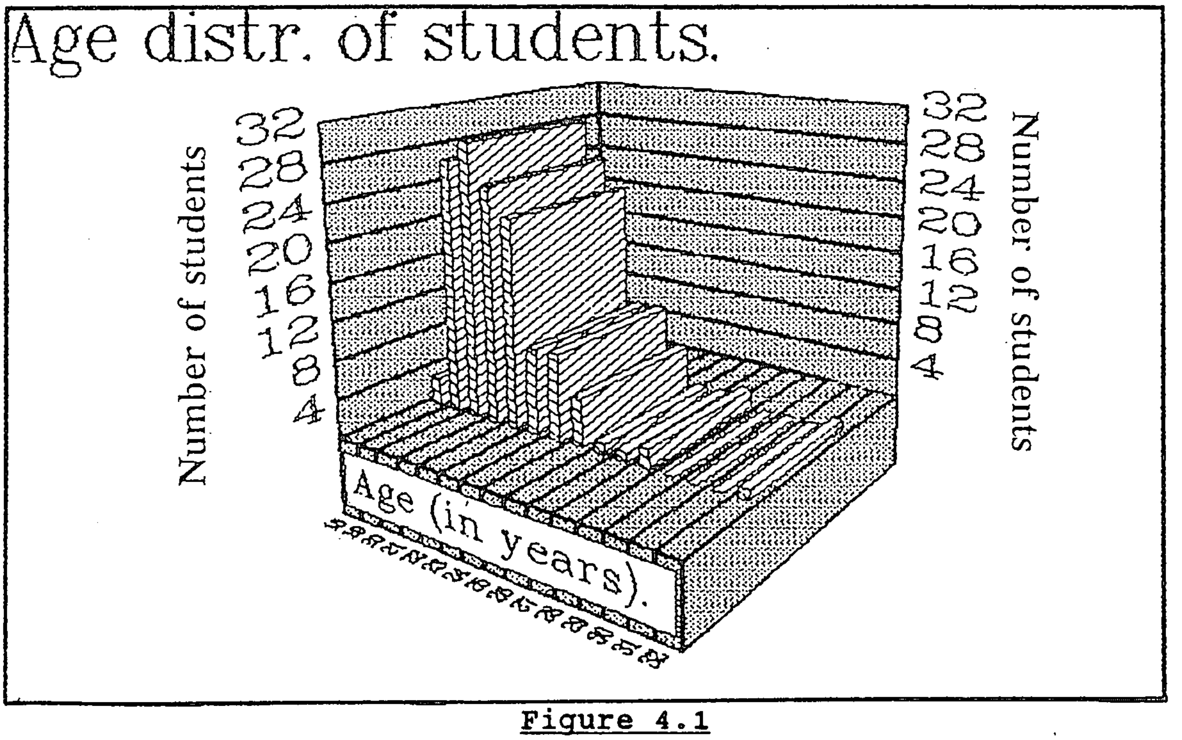


Figure 4.1

Statistics are : Arithmetic Mean $\overline{X}$ = 21,23 years
Standard Deviation s = 2,22 years.

Comment : Homogeneous group with an insignificant right hand side tail.

4.3.2 **Language Distribution :**

Home language	Number of students
English	119
Afrikaans	7
Tswana	2
German	2
Portuguese	2
French	1
Greek	1
TOTAL	134

Table 4.2

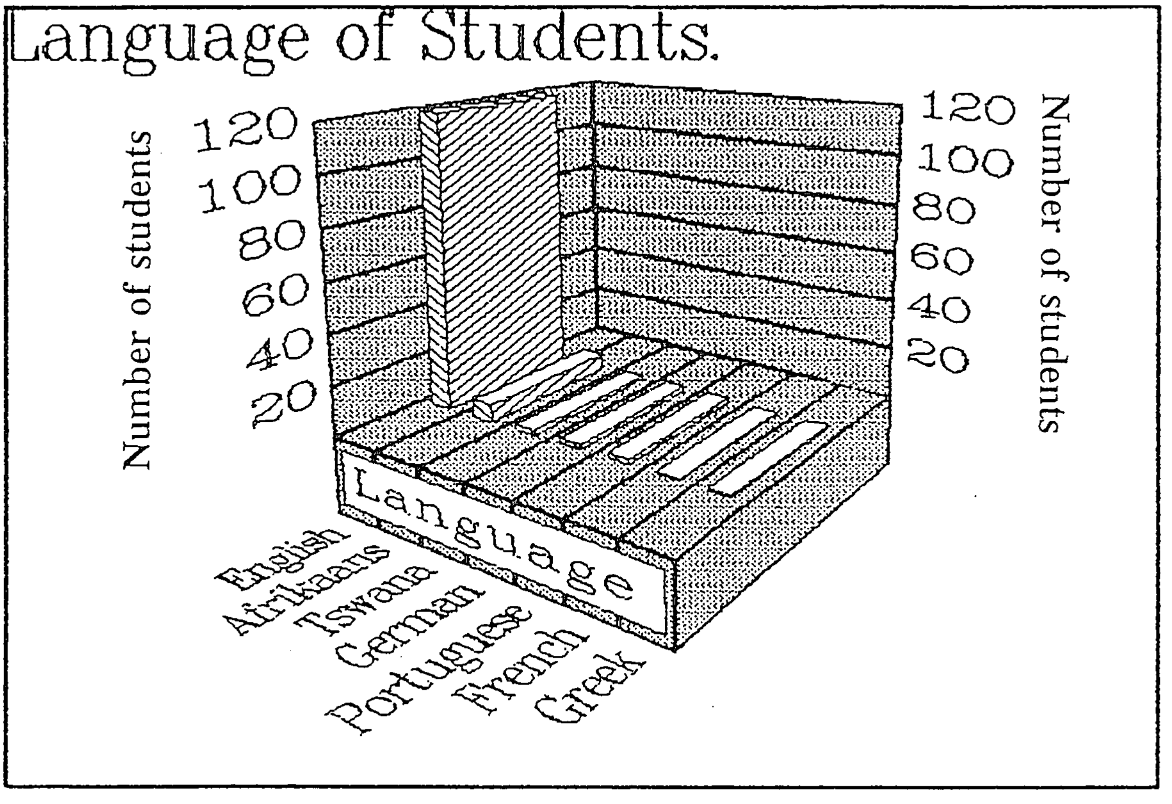


Figure 4.2

Comment : Homogeneous group (predominantly English speaking students).

4.3.3 **Accommodation Distribution :**

Type	Number of students
Home	80
Hostel	16
Private boarding	38
TOTAL	134

Table 4.3

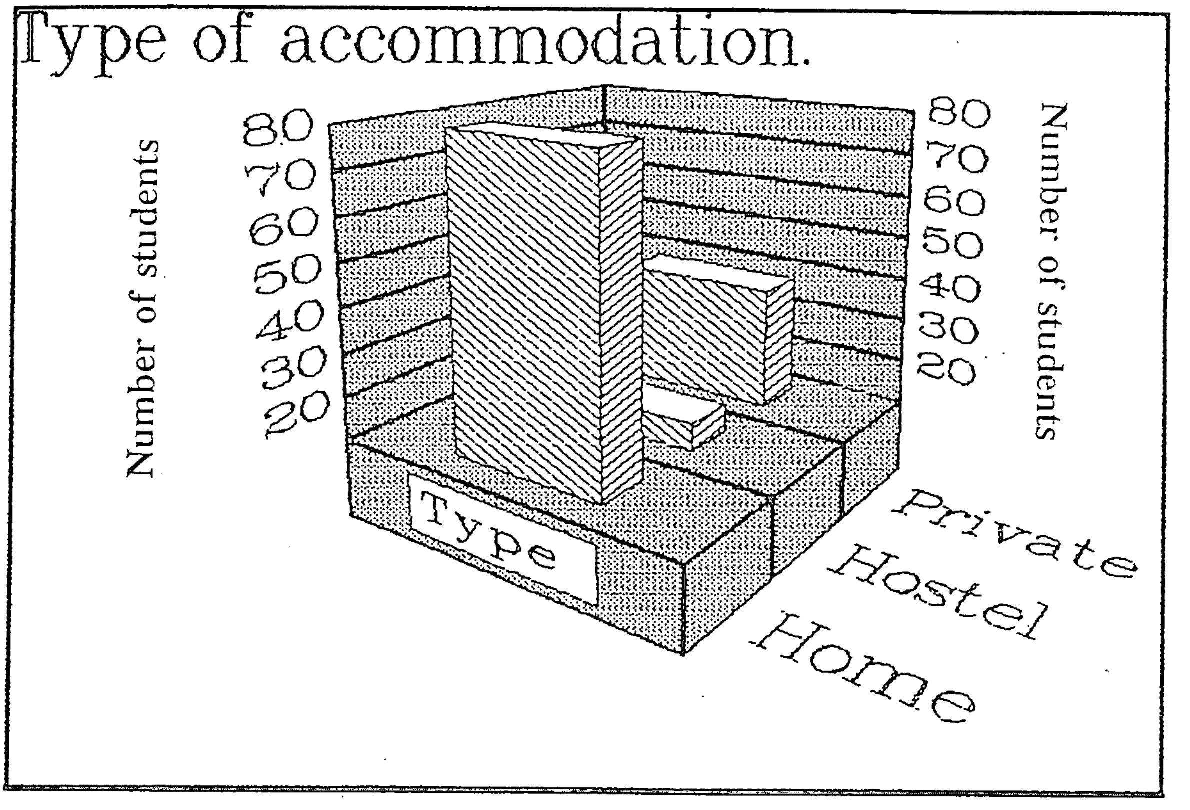


Figure 4.3

Comment : Mixed group.

4.3.4 **Employment Status** Distribution :

Employed	93
Not employed	41
TOTAL	134

Table 4.4

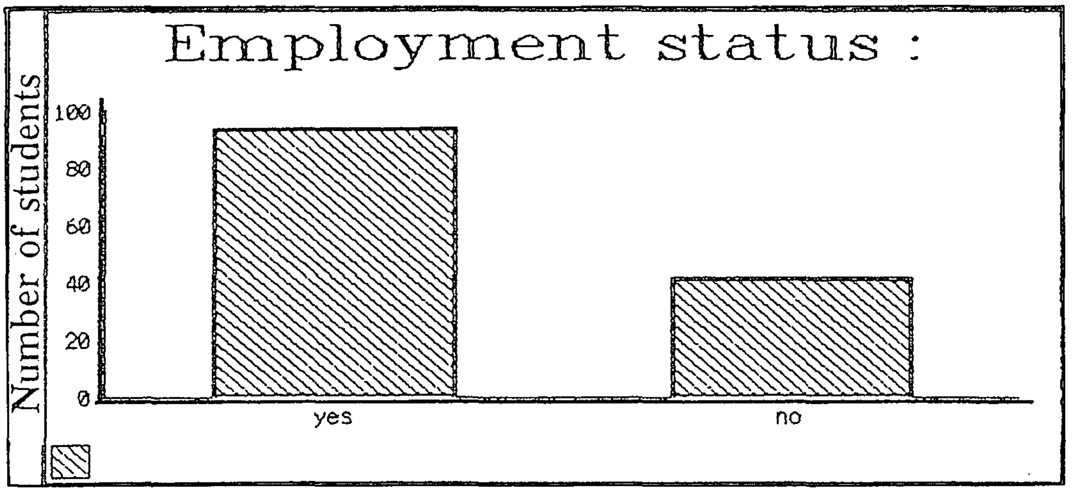


Figure 4.4

Comment : The high percentage of employed students would tend to support an initial assumption that the students would be self motivated.

4.3.5 **Enjoyment of Mathematics :**

Enjoyment rating (0.."not at all" to 5 "very much")

Rating (enjoy Maths?)	0	1	2	3	4	5
Number of responses	3	8	22	58	35	7

Table 4.5

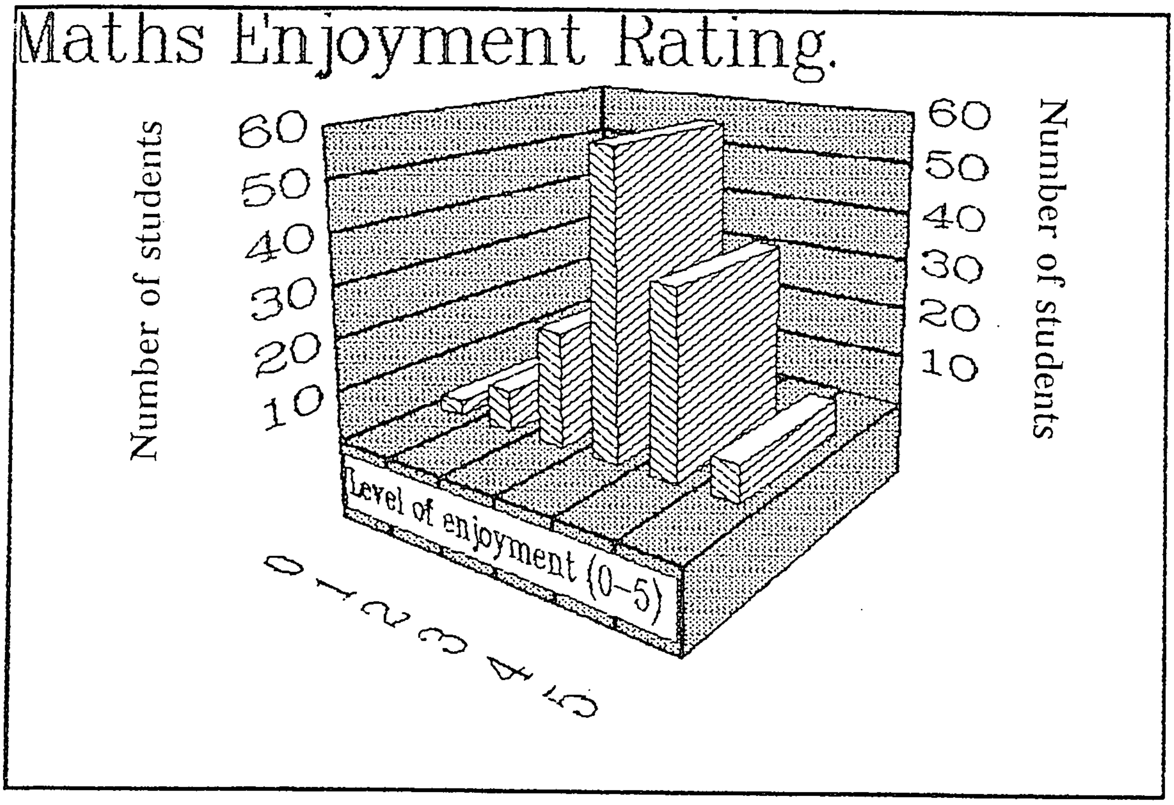


Figure 4.5

Number of responses : 133

Arithmetic Mean $\overline{X}$ = 3,02
Standard deviation s = 1,04

Comment : Despite poor performance in Mathematics (high failure rate, as indicated in Tables 4.6 and 1.1 on pages 110 and 6 respectively), these statistics (the majority responding with a rating of 3 or more) indicate that these students have a relatively positive attitude towards the subject.

4.3.6 Distribution showing **Standard Ten Mathematics symbols:**

Symbol obtained :		A	B	C	D	E	Other	Total
Grade	Higher	0	1	18	28	12	1	60
	Standard	12	16	25	15	5	1	74
							TOTAL	134

Table 4.6

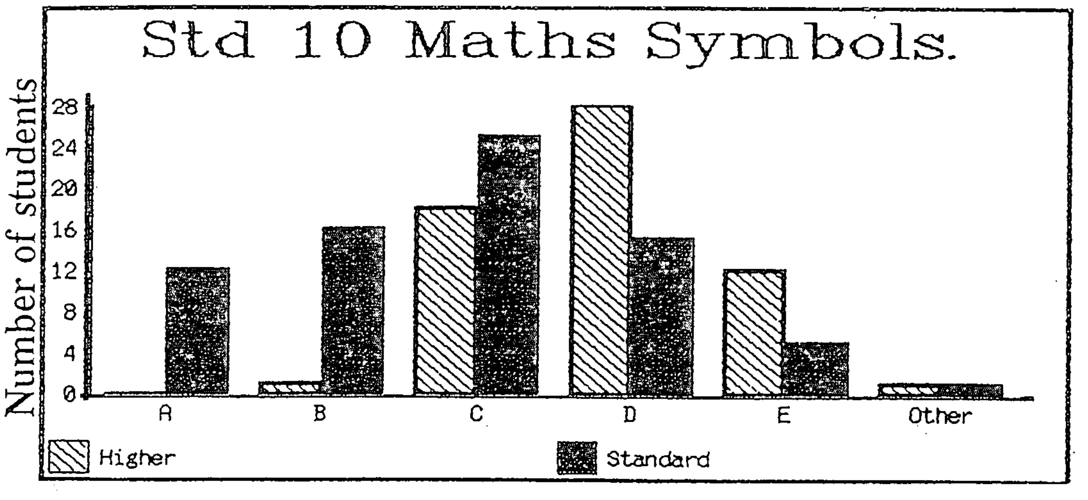


Figure 4.6

Comment: With virtually no A and/or B Higher-grade mathematics students, it is clear that the mathematical ability of these students is not ideal. Engineering and technology at tertiary level requires students with the ability to understand and apply difficult mathematical concepts.

4.3.7 Distribution of **Mathematics I** percentage **results :**

Frequency Distribution of Mathematics I marks.

Mathematics I %	Frequency
50 to 59	62
60 to 69	32
70 to 79	15
80 to 89	10
90 to 99	4
TOTAL	123

Table 4.7

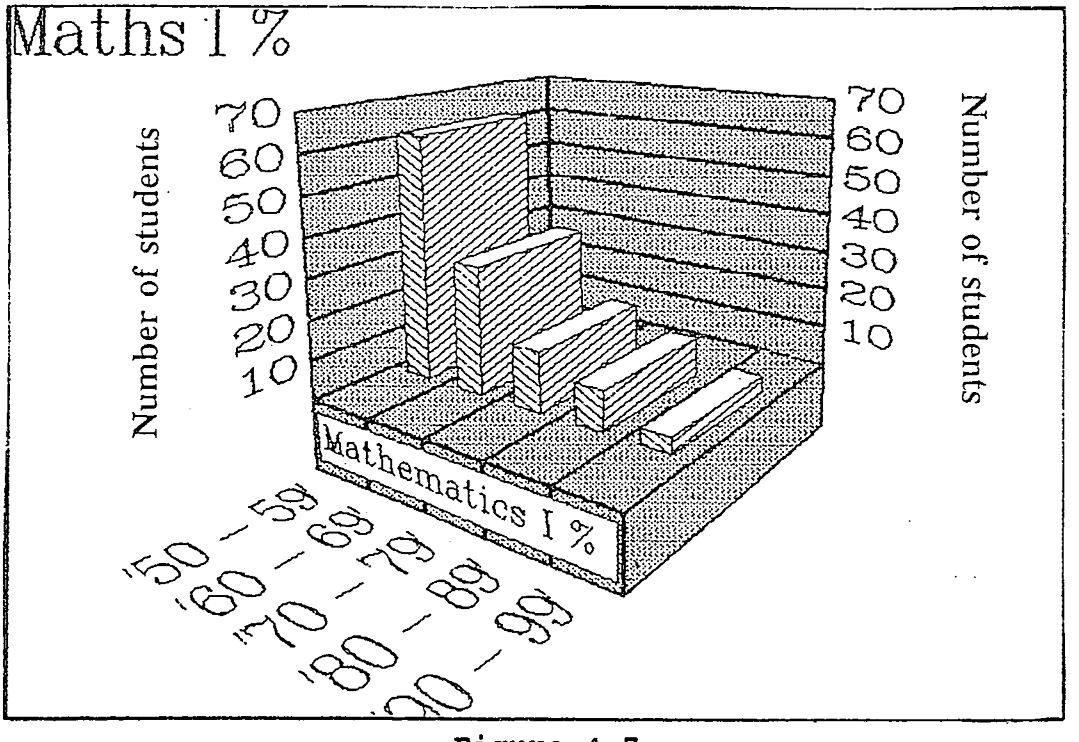


Figure 4.7

Arithmetic Mean (for the grouped data) $\overline{X}$ = 63,28 %
Standard Deviation s = 11,13 %
Note : Frequency distribution appears skewed. This is due to the fact that Table 4.7 only shows percentages for students who passed Mathematics I. If marks for students who had failed the subject were shown, the other tail would be evident.
Marks of students who obtain exemption from Mathematics I are not shown in Table 4.7.

4.3.8 Distribution of students with prior **knowledge of SI texts :**

Familiarity with self-instructional (SI) texts.

Familiar with SI texts		30 (22,9%)
Not familiar with SI texts		57 (43,5%)
Not sure		44 (33,6%)
	TOTAL	131

Table 4.8

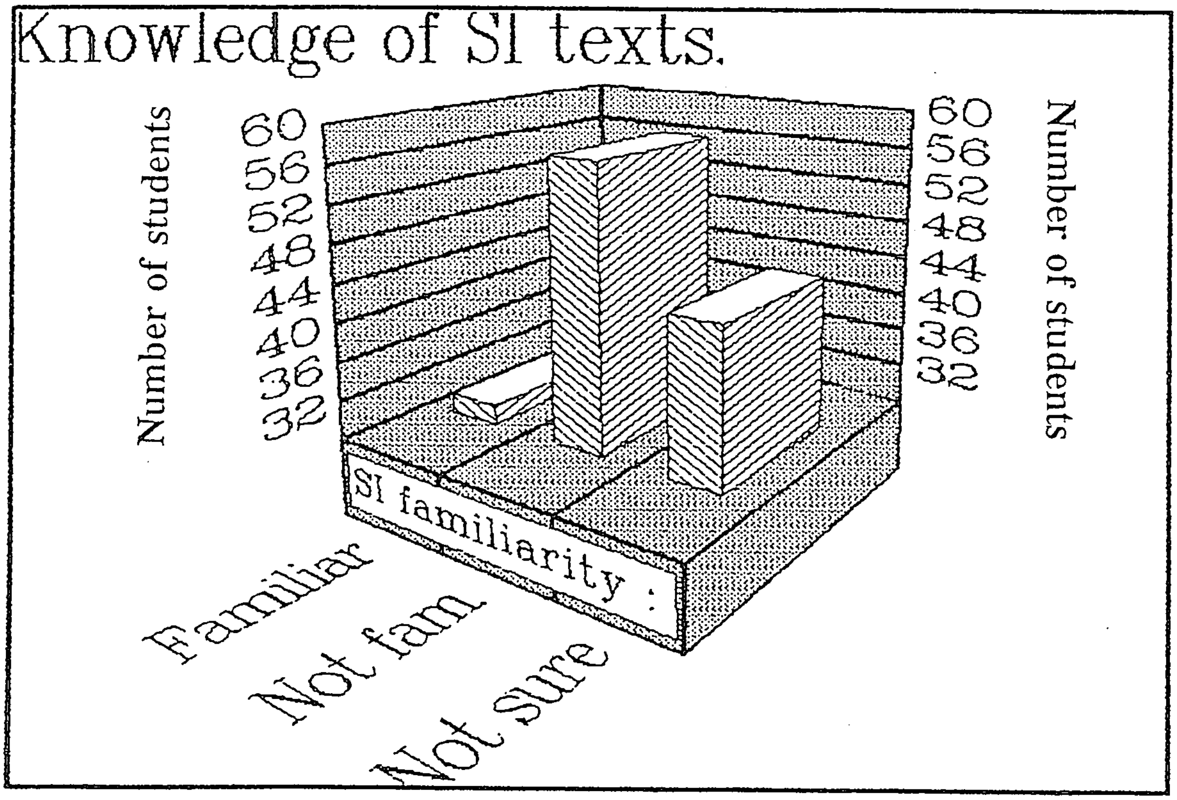


Figure 4.8

Comment: A low percentage (≈23%) of students indicated that they were familiar with SI texts.

4.3.9 Prior **attitude to SLSI approach :**

Attitude towards the short lecture self-instructional (SLSI) approach prior to its commencement. (Scale of 0 .. "not looking forward to working on own" to 5 .. "very excited and looking forward to doing the work".)

Attitude rating	0	1	2	3	4	5
Frequency	2	6	10	60	46	8
	Number of responses					132

Table 4.9

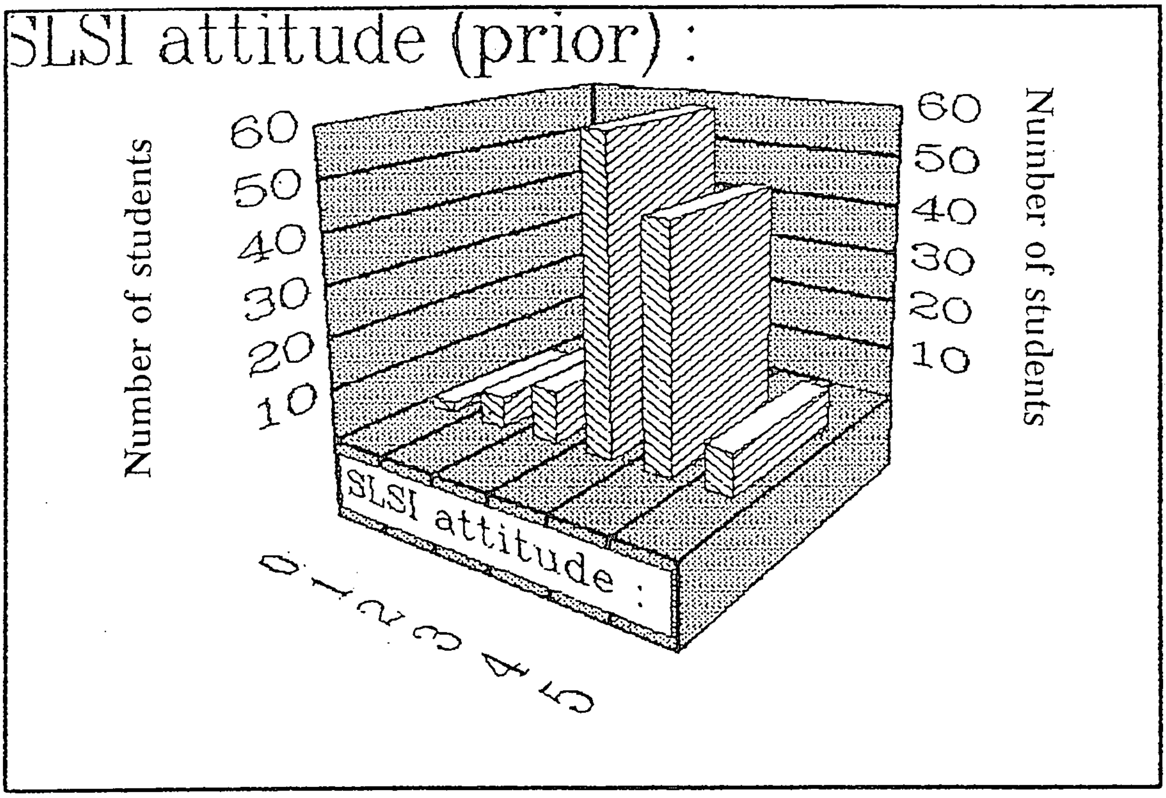


Figure 4.9

Arithmetic Mean Rating $\bar{X}$ = 3,26
Median M_e = 3,31 Mode M_o = 3,28
Note : In calculating the M_e and M_o, data have been treated as continuous, e.g. a rating of 3 has been assumed to fall between 2,5 & 3,5 with a corresponding class frequency of 60 in this case; the relevant grouped-data formulae have been used (see Appendix D).

Comment: These results clearly indicate that these students were keen to be subjected to alternative methods of instruction and were prepared to work on their own.

4.4 Post SI Questionnaire Statistics

The questionnaire that was given is shown in Appendix B.

The purpose of this questionnaire was to determine the attitude of students towards the SLSI method after having attended the short lectures and having used the self-instructional text in numerical methods; a further objective was to obtain their views on whether or not this method could be used successfully in Mathematics, and if so, to what extent. Students were also asked to make suggestions as to how the SI material could be improved.

Appendix F gives individual free responses by students to questions B4 and B6.

77 Students filled in this questionnaire.

Tables and diagrams are continued overleaf.

4.4.1 Distribution showing **enjoyment of SLSI** approach :

Response to B1 : "Did you enjoy doing this work (please rate on scale 5 (very much) to 0 (not at all)."

Rating	0	1	2	3	4	5
Frequency	0	0	5	26	35	10
				Total		76

Table 4.10

Mean $\bar{X} = 3,66$
Standard deviation s = 0,79

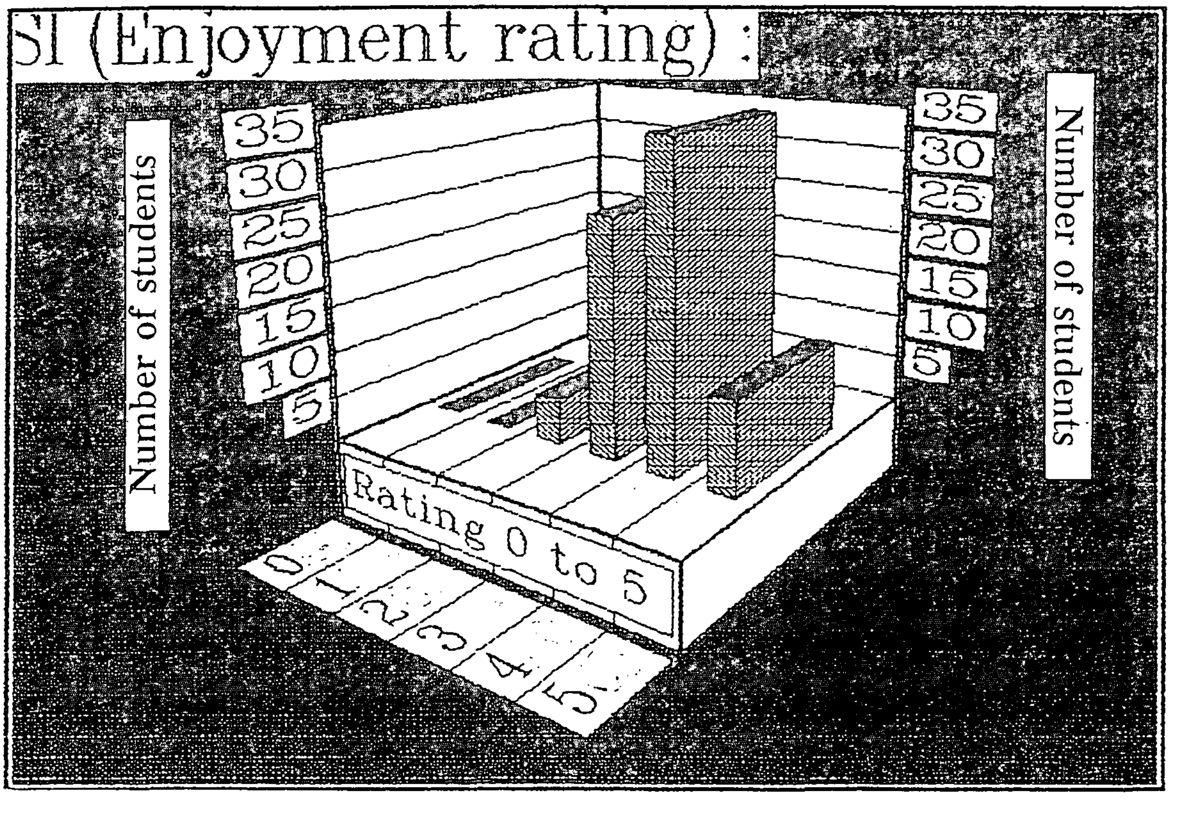


Figure 4.10

Comment: Except for a very small proportion (≈7%) who gave a rating less than 3, the vast majority of students indicated that they had enjoyed the SLSI approach.
The high percentage (≈60%) who gave ratings of 4 or 5 clearly is an indication that the method could be used more extensively in mathematics with comparable students.

4.4.2 **General attitude to SLSI approach :**

Question	Responses YES	Responses NO
B2:Do you think that the short introductory lectures helped you to understand the new concepts ?	77	0
B3:Do you think that you will do better in the Numerical Methods section of the exam paper than in the other sections ?	73	4
B4:Do you prefer this approach to the normal lecture, tutorial system.	64	13
B5:Do you think that similar self-instructional texts would be helpful in other sections of Mathematics II	56	21

Table 4.11

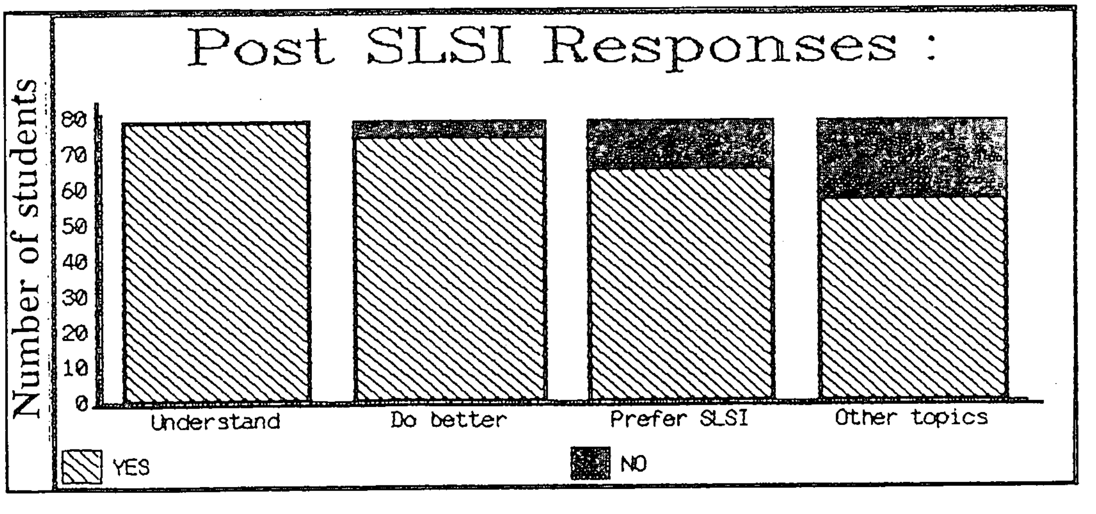


Figure 4.11

Comment: The responses of students to the questions shown in Table 4.11, and presented graphically in Figure 4.11 above, form the basis of whether or not the students perceived the SLSI method as given to them in this investigative study as worthwhile. The responses indicated, by and large, that the method was workable.

The component-bar-chart clearly shows that students felt extremely positive on most of the issues raised. They were unanimous that the short lecture had been useful, and the majority also indicated that they felt they would do better in the numerical methods section of the examination than in the other sections. (The mean percent obtained in the numerical methods question by the experimental group was 53,87%, while the mean percent for the other sections in the examination was 31,35%; for the control group, the comparable percentages were 48,19% and 42,32% respectively).

It is also important to note that ≈83% of the respondents indicated that they preferred the SLSI approach to the traditional lecture/tutorial approach, again an indication that the tried-and-trusted traditional conventional methods should be revised. To the question of whether they felt that similar texts would be helpful in other sections of Mathematics II, ≈27% indicated that they did not think so, while ≈73% felt this would be helpful. Further local work in this area is necessary to see whether the approach could be used successfully in other mathematical topics. Experience at other institutions (Chapter 2) has indicated that self-instruction has been used effectively in advanced mathematics.

Comment on the written responses (Annexure F, Part A) to extensions to question B4 i.e. "Please (in a few lines), indicate your reason for your answer".

The student responses to this question were constructive and on the whole very positive with regard to the use of the SLSI method. Virtually every student made a useful comment and these constitute part A of Annexure F.

Comment on the written responses (Annexure F, Part B) to extensions to question B6 i.e. "Please comment on the Self-Instructional text you have worked through the past two weeks, and make suggestions as to how you think the lessons can be improved ".

As in the previous case, the responses were constructive and generally positive. The majority of the students thought that the SI text was excellent and mentioned numerous advantages. The occasional negative comment was also received, especially from students who participated in the pilot sessions. These comments were taken into account in the preparation of subsequent texts. In certain aspects there were conflicting comments by different students, for example on the amount of tutorial type examples that should be included in the SI text : in some cases students felt that the text was "just right" while others felt that more examples could have been included. The author felt that additional examples would result in the students spending too much time on the section and could result in loss of interest, and referred the students who wanted more problems to past examination papers.

4.4.3 Distribution of students' **familiarity of SI texts :**

Familiarity with self-instructional (SI) texts.

Familiar with SI texts		50 (66,7%)
Not familiar with SI texts		14 (18,7%)
Not sure		11 (14,7%)
	TOTAL	75

Table 4.12

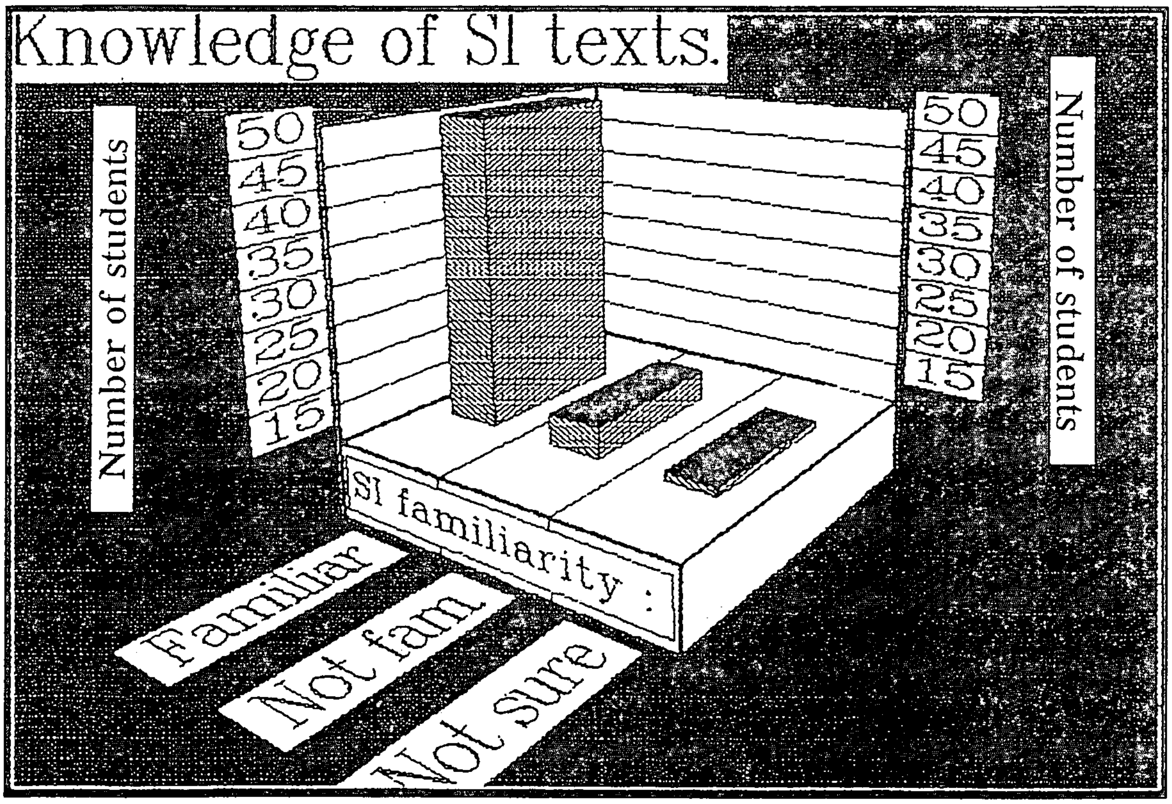


Figure 4.12

Comment: Table 4.8 and Figure 4.8 show student responses to this question before using the SI text. Whereas only ≈23% were familiar with SI texts prior to the experiment, ≈67% felt they were familiar with such texts afterwards. Presumably active prompting would have increased this latter percentage, but the students were merely asked to fill in the questionnaire, without comment by the presenter.

4.4.4 **Proportion of teaching time that can replaced by SLSI :**

Students' perception of the proportion of teaching time that can be devoted to self-instruction.

	Responses
Can replace traditional lectures completely.	12 (16,0%)
Can be used effectively ≈ ½ the time.	41 (54,7%)
Can be used effectively ≈ ¼ the time.	21 (28,0%)
Cannot be used effectively in Mathematics.	1 (1,3%)
TOTAL	75

Table 4.13

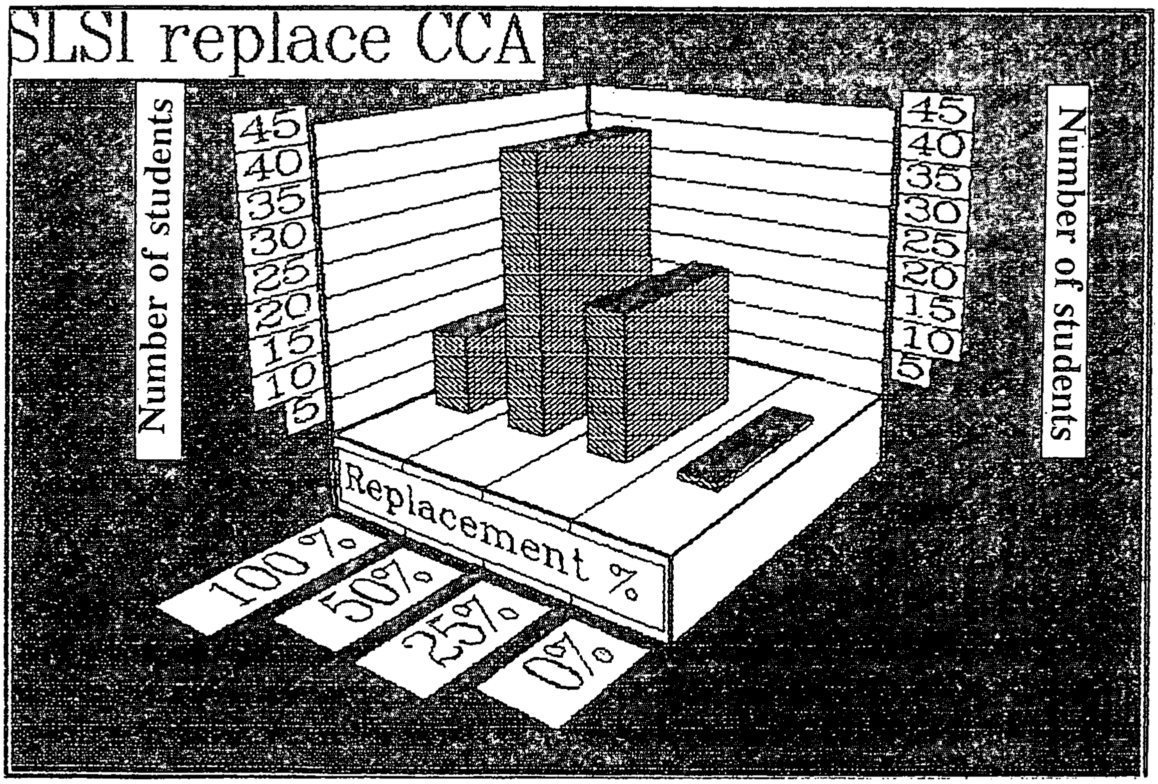


Figure 4.13

Comment: These responses were virtually unanimous in that the SLSI approach could partly replace the Conventional Classroom Approach. The majority of students felt that ≈ ½ the teaching time could be devoted to the self-instructional type approach.

4.5 **Statistics relating to the post-test Experiment**
4.5.1 **Experimental and Control group Mathematics I results :**

EXPERIMENTAL GROUP				CONTROL GROUP			
Mathematics I exam %				Mathematics I exam %			
Anders	52	Badenhorst	58	Stieger	52	Ziehl	58
Carr	55	Coleman	73	Ralfe	56	Collyer	74
Constable	Ex	Crocker	92	Cronje	Ex	Govender	93
Davey	58	Daw	67	Olivier	58	Nel	67
Emery	69	Fleming	73	Munro	69	Knowler	71
Fraser	78	Halse	53	Venter	78	Abbot	52
Hudson	51	Koekemoer	52	Kuboni	50	Dimitres	51
Latimer	68	Rosa	83	Turner	71	Taylor	83
Street	52	Symington	62	Perkins	53	Plowes	62
Walker	67	Woodburn	50	Welsford	66	Ezra	50
Burring	74	Cave'	55	Culverwell	74	Sibiya	57
Cloete	54	Engelbrecht	50	du Plessis	54	Savy	50
Labuschagne	52	McAlisdair	67	Carstensen	53	Shazi	66
McKechnie	65	Morule	Ex	Crichton	64	Poane	Ex
Phaidi	Ex	Topham	55	Nair	Ex	Bouwer	54
Trauner	71			Wensley	71		

Table 4.14

Ex = exemption.
Source : Student records

$\overline{X}_1 = 62,71$
$s_1 = 11,14$
$n_1 = 28$

$\overline{X}_2 = 62,75$
$s_2 = 11,25$
$n_2 = 28$

4.5.2 **Achievement in Numerical Methods of the common examination paper :**

EXPERIMENTAL GROUP (1)				CONTROL GROUP (2)			
N.M. % in Maths II exam				N.M. % in Maths II exam			
Anders	27	Badenhorst	73	Stieger	40	Ziehl	47
Carr	33	Coleman	87	Ralfe	7	Collyer	60
Constable	13	Crocker	80	Cronje	67	Govender	40
Davey	40	Daw	53	Olivier	80	Nel	80
Emery	93	Fleming	40	Munro	0	Knowler	47
Fraser	33	Halse	47	Venter	73	Abbot	33
Hudson	47	Koekemoer	60	Kuboni	13	Dimitres	33
Latimer	53	Rosa	67	Turner	80	Taylor	67
Street	53	Symington	53	Perkins	20	Plowes	60
Walker	73	Woodburn	73	Welsford	53	Ezra	47
Burring	60	Cave'	73	Culverwell	67	Sibiya	13
Cloete	73	Engelbrecht	53	du Plessis	47	Savy	67
Labuschagne	80	McAlisdair	73	Carstensen	87	Shazi	40
McKechnie	13	Morule	0	Crichton	53	Poane	47
Phaidi	40	Topham	47	Nair	40	Bouwer	53
Trauner	60			Wensley	33		

Table 4.15

$\overline{X}_1 = 53,87$
$s_1 = 22,39$
$n_1 = 31$

$\overline{X}_2 = 48,19$
$s_2 = 22,49$
$n_2 = 31$

4.5.3 Hypotheses, Decision rules and Conclusions :

In order to test the effectiveness of the SLSI approach used in the project that led to this dissertation, the following statistical hypotheses were tested :

Null Hypothesis H_0: $\mu_1 = \mu_2$
There is **no** difference between the examination percentages obtained by the experimental group who were taught using the SLSI method and the control group who were taught by a Conventional Classroom Approach, in the numerical methods section of the common Mathematics II exam paper (i.e. there is no difference between the two instructional methods in the topic taught).

Alternative Hypothesis (Case 1) H_{a1}: $\mu_1 \neq \mu_2$
There is a difference between the examination percentages obtained by the experimental group who were taught using the SLSI method and the control group who were taught by a Conventional Classroom Approach, in the numerical methods section of the common Mathematics II exam paper.

Alternative Hypothesis (Case 2) H_{a2}: $\mu_1 \succ \mu_2$
The examination percentages obtained by the experimental group, who were taught using the SLSI method, are higher than the exam percentages of the control group who were taught by a Conventional Classroom Approach, in the numerical methods section of the common Mathematics II exam paper (i.e. SLSI method in this experiment was better i.t.o. exam results than the CCA method).

Degrees of freedom = $n_1 + n_2 - 2 = 31 + 31 - 2 = 60$

Level of significance $\alpha = 0,01$ (Both cases)
(i.e. a 1% chance of making a Type 1 error -- rejecting H_0 when it is true.)

Two tailed t-test when using H_{a1} (Case 1)
One tailed t-test when using H_{a2} (Case 2)

Decision rule :

Case 1 : Reject H_0 if $-2,660 \succ t_{test} \succ +2,660$

Case 2 : Reject H_0 if $t_{test} \succ +2,390$

Test statistic (t_{test})

$$t_{test} = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\dfrac{s_1^2(n_1 - 1) + s_2^2(n_2 - 1)}{n_1 + n_2 - 2}} \cdot \sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$$

$$= \frac{53,87 - 48,19}{\sqrt{\dfrac{22,39^2 . 30 + 22,49^2 . 30}{60}} \cdot \sqrt{\dfrac{1}{31} + \dfrac{1}{31}}}$$

$$\frac{5,68}{22,440056 . 0,2540002}$$

$= 1,00$ *correct to* 2 *decimal places.*

Conclusion :

Case 1 : Accept H_0 and conclude there is no difference between the two methods.

Case 2 : Accept H_0 and conclude that the statistical evidence does not indicate that the SLSI method is better than the CCA method.

Hence, although the mean score ($\overline{X}_1$ = 53,87%) of the experimental group of students who used the SLSI approach was higher than that

($\overline{X}_2$ = 48,19%) of the control group of students who were taught by the CCA, there is not sufficient statistical evidence to conclude that the SLSI method is better than the CCA. Had this research included task analysis or examination of the intrinsic factors the results could well have been different. The fact that the mean percentage obtained by the experimental group was higher than that obtained by the control group clearly indicates that the SLSI method is at least as good as the CCA, and hence deserves further investigation. The subjective responses by participating students also indicated that they had enjoyed the SLSI approach.

CHAPTER 5

NB

GENERAL RECOMMENDATIONS AND SUGGESTIONS

The results of this research project as well as findings by other researchers (reported in Chapter 2) in the field of self-instruction, bearing in mind the problems facing education (specifically mathematics education at present and in the foreseeable future), lead to certain specific and general recommendations and suggestions. These do not only pertain to technikon education, but also to the senior secondary phase of school education and other tertiary education.

Often the best is left for last; however, in this case, the author regards the first suggestion as crucial in attempts to cope with the expected increase in tertiary-student numbers.

5.1 The possible **Rolling-over** of students

One of the biggest problems likely to face mathematics education at a post-school level will probably be increased numbers; resources such as space and lecturers will not increase proportionately.

In this project, students were given a 10 to 15 minute lecture before they worked through each lesson. This was instead of a 40 to 45 minute formal lecture. Hence up to three groups of students could be "rolled-over" within one normal period and in the same venue by the same or different lecturers. Students could spend the "free-time" working through the applicable SI work in the library or other venues. As indicated in the student responses to a questionnaire (Appendix F), almost without exception they enjoyed the short-lecture-self-instruction (SLSI) approach in the project. With the increase in student numbers over the last 20 years, the lecturer's load has increased significantly; although student contact-times may have stayed the same (or even come down slightly), the marking, assisting of students out of normal lecture/tutorial time, and the administrative load, have all increased dramatically. At technikons, staff are expected to pursue research, enhance their qualifications, keep up in their subject fields, improve their lecturing competence, and become involved in community projects. Use of self-instruction could lead to a reduction in contact-time for lecturing staff, enabling them to devote more of their energy to some of the personal (and social) growth areas mentioned above.

In an article entitled *Instructional Technology - Fifteen Years Later - What Has Happened, What Has Not*, Anandam (1989) contends that educational institutions still use student contact time for accountability and that with lecturers spending time in addition to student-contact hours to mark papers, help students, and prepare classes, there is very little energy left to pursue innovative ideas in teaching.

Obviously the initial preparation of suitable SI material would require considerable time and effort. The long term benefits would surely make the effort worthwhile. A department (say a technikon mathematics department) could set the development of such material as a short term goal (for instance suitable topics in the Mathematics I syllabus could be the object of an exercise for a team of lecturers for one year). Variations, such as co-operative learning (where students work in pairs or groups), could also be introduced to avoid the loneliness of working completely on one's own. Hoogstraten (1976), after research to see if programmed material that was normally studied individually could be studied in pairs without resulting in inferior achievement, found that besides the advantages of social contact when studying in pairs, this co-operative learning did not result in inferior performance. Other problems mentioned by Lewis (1981:13), such as feeling isolated and finding it hard to keep going, getting discouraged when stuck, and a lack of necessary self-discipline to work alone, could also be partially alleviated by studying in pairs.

5.2 **Academic support - school to technikon**

Well-written SI material in certain school mathematics topics could be used as part of a support programme for students who have not reached the necessary standard in this subject.

Recently, students entering engineering and applied science departments at technikons generally seem to have achieved better symbols in Standard Ten mathematics, but their basic mathematical knowledge seems to have been inferior to that of students of a decade ago. The problem is possibly related to the trend of increasing subject-content at school-level by adding topics previously covered at tertiary level to the high school syllabi, without an increase in teaching time. Apple & Jungck (1990) are of the opinion that the increase in quantity is at the expense of quality and further state that "intensification leads people to cut corners"(1990:234). Teachers, in order to get through the syllabus, spend less time on the basics, and students entering the technikons find it difficult to cope with mathematics, which pre-supposes a good school grounding. Topics that have been identified as problem areas by the mathematics department at Technikon Natal include : Exponents and logarithms, manipulation of formulae, and trigonometry.

Academic Support Programmes (ASP) are being run by various tertiary institutions. Mathematics and English are the subjects that receive the most attention. The writer's suggestion here is that those students who do not obtain a prerequisite score in mathematics in an entrance examination (for prospective Engineering and Applied Science students), should attend a "support" programme which covers the above topics, using SI. The writer further suggests that the entrance examination be written at least a month before the commencement of formal instruction, and that the support course be one to two weeks long. This would enable mathematics I lecturers to start lectures without having to spend hours going over the basics.

5.3 Technikon mathematics

Responses by students (Chapter 4) indicate that they feel that the SLSI approach would not be suitable for all the mathematics topics. Further research could well show that this is not the case; some of the self-instructional courses run at universities (reported in Chapter 2) were in advanced mathematical topics. The recommendation arising from this study is that initially only some of the topics in the Mathematics I & II syllabuses be run using the SLSI approach. Based on the experience gained in developing the SI material for numerical methods, topics that seem to lend themselves to this approach include : Statistics ; Matrix Algebra and Linear Programming.

5.4 Remedial and revision use

SI material, where available, would be extremely useful in the above-mentioned areas. Students who experience difficulty with a particular topic could be given an appropriate SI text, and a follow-up session could be arranged with the lecturer if the student still had a problem. SI material would need to be developed for topics in the syllabi covered by traditional means, as well as additional basic school-mathematics topics not included in the support programme. Such texts would also be useful for revision.

Often technikon students who have to rewrite mathematics are employed by firms in remote areas. These students find it difficult to do the necessary work on their own using material that is not specifically designed for self-study; they also do not have the opportunity of consulting fellow-students or lecturers when they experience difficulties. SI material that is specifically written for self-study of topics in the syllabus could be the answer to these students.

SI material could also serve as a back-up for students who miss lectures, or who do not understand a lecture. Lecturers are only human and often the approach of certain lecturers does not suit a section of the class. With timetable and logistic constraints, students cannot easily change to another lecturer who would suit them better, and even if they could this might result in chaos. Students in such a situation who had access to SI material would be able to get a different perspective on the work and would not fall further behind.

From the above it is clear that there are many possible ways in which the SLSI or SI approach can be used. In all the mentioned suggestions, the interest of the student is served, the efficiency of the educational system is enhanced, and the personal development of the academic staff is promoted.

Academic staff at technikons who become involved in the development of "custom made" SI material, its use and evaluation, will in a practical way be contributing to the development of an instructional method that could be utilized to overcome some of the problems mentioned in Chapter 1 of this dissertation.

Without denigrating the traditional lecture, which is by far the most common method of instruction at technikons (but which according to Jackson & Prosser (1989) does not develop creativity and discipline in students), the use of the SLSI method as outlined in this dissertation could possibly result in <u>less lecturing and more learning</u>, a phrase which Jackson and Prosser select as the title of their article.

In order to facilitate the introduction and use of self-instruction in the teaching of mathematics, it is suggested that besides changing teacher-training programmes to include provision of training in SI, courses should be run for lecturers and other parties interested in the development of SI materials. More publicity, including presenting and discussing the findings of this research project at mathematics forums, as well as libraries obtaining literature and circulating information on self-instruction in mathematics would also promote the use of this teaching method.

If SI is used wholly or partially, it would become necessary to revise syllabi, as well as the examination system, to reflect and incorporate the aims of the method (e.g. continuous assessment and mastery).

In conclusion, there appears to be a need for similar research to be conducted on a broader front using a more integrated approach. The use of, and interest in, alternative teaching methods, such as the one used in the research project that led to this dissertation, could result in more effective and efficient instruction in mathematics.

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Unit 3: Thinking about objectives by Gillham B.

Unit 4: Writing objectives by Gillham B.

Unit 5: Assessment by Lewis R.

Unit 8: Production 3: editing and production by Jenkins J.

Unit 9: Production 4: how to choose and use different media by Jenkins J.

Unit 10: The problem of evaluation by Gillham B.

Unit 11: Setting up an evaluation by Gillham B.

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APPENDIX A
Questionnaire to be filled in prior to commencement of self instruction lessons.

Instructions:
Please complete by **filling in spaces** and by **ticking the applicable boxes.**
Date: (DD-MM-YY) ____________________

****

SECTION A: Personal Details (will be kept confidential).

Surname : ______________________________

First names: ______________________________

Age : ____________ years

Home language :______________________(Specify)

Accommodation :Home ☐ Hostel ☐ Private ☐

Sex : Female ☐ Male ☐

Employed : Yes ☐ No ☐

Do you enjoy Mathematics (please rate on scale 5 (very much) to 0 (not at all) ☐

****

SECTION B: Mathematics information.

B1 Final school level Mathematics

B1.1 In which **year** did you write Standard 10 or equivalent? 1 9 ☐☐

B1.2 At which **grade** did you pass? H=Higher; S=Standard; O=Other H ☐ S ☐ O ☐

B1.3 **Symbol** obtained A ☐ B ☐ C ☐ D ☐ Other ☐

B2 **Mathematics** I or **Engineering Mathematics T1 information**

B2.1 In which **year** and **semester** did you pass 1 9 ☐☐

Semester 1 ☐ Semester 2 ☐

B2.3 Final **percentage obtained** [ ] %

P.T.O.

SECTION C : Self Instruction perception and attitude.

C1 Are you familiar with self instruction texts ?

Yes [ ] No [ ] Not sure [ ]

This Numerical Methods self instruction course will follow the following format :
Eight lessons : Each with a ten to fifteen minute introduction (brief lecture), followed by you having to work through the applicable self instruction text on your **own.**

C2 **At this stage** <u>rate</u> your attitude towards this self instruction procedure on the scale :

5 (very excited and looking forward to doing the work)
to 0 (not looking forward to having to work on my own)

[ ]

Note : After completing this series of self-instruction lessons, you will be asked to fill in another questionnaire.
Your response to this and the follow up questionnaire will be used in the evaluation of this self-instruction research project.

THANKS for your time and effort.
GOOD LUCK WITH NUMERICAL METHODS.

****
****

<u>Finally</u>

If you have any queries or would like to comment personally on the self-instruction lessons as given in this series, **please** come and see me (G Hunter) in S8 314.

APPENDIX B

Questionnaire to be filled in after the self-instruction lessons have been completed.

Instructions:

Please complete by **filling in spaces** and by **ticking the applicable boxes.**

Date: (DD-MM-YY) ____________________

****

SECTION A: Personal Details (will be kept confidential).

Surname : ____

First names: ____

****

SECTION B: Your views and impressions on the Self-Instruction series of lessons on Numerical Methods you (hopefully) have just completed.

B1: Did you enjoy doing this work (please rate on scale 5 (very much) to 0 (not at all)

PLEASE be honest .... ☐

B2: Do you think that the short introductory lectures helped you to understand the new concepts?

Yes ☐ **No** ☐

B3: Do you think that you will do better in the Numerical Method section of the exam paper than in the other sections ?

Yes ☐ **No** ☐

B4: Do you prefer this approach to the normal lecture, tutorial system.

Yes ☐ **No** ☐

Please (in a few lines), indicate your reason for your answer.

....
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B5: Do you think that similar self-instruction texts would be helpful in other sections of Mathematics II ?

Yes ☐ **No** ☐

B6: Please comment on the Self-Instruction text you have worked through the past two weeks, and make suggestions as to how you think the lessons can be improved on.

....
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....

B7: Are you familiar with self-instruction texts ? (repeat question)

Yes ☐ No ☐ Not sure ☐

B8: <u>Rate</u> your attitude towards self-instruction texts on the scale :

4 (can replace traditional lectures in Mathematics completely)
3 (can be used effectively $\approx \frac{1}{2}$ the time)
2 (can be used effectively $\approx \frac{1}{4}$ the time)
1 (cannot be used effectively in Mathematics)

☐

THANKS for your time and effort.
GOOD LUCK WITH YOUR EXAMS.

****
****

<u>Finally</u>

If you have any queries or would like to comment personally on the self-instruction lessons as given in this series, **please** come and see me (G Hunter) in S8 314.

APPENDIX C

Technikon Natal

Questionnaire on the use of self-instruction in Technikon Mathematics. (In condensed form)

Please fill in by ticking [✓] the appropriate box or commenting briefly. If you are prepared to respond in more detail please attach your additional response to this questionnaire.

Technikon : ______________________________

Name of respondent : ______________________________

Position (eg. Senior lecturer) : ______________________________

1. Is self-instruction in mathematics being used at your Technikon ?

   Yes [ ]   No [ ]   Not sure [ ]

   If **no** or **not sure** go to question 2. If **yes** please answer following questions as well as question 2:

   1.1 Subject & level :

   1.2 Topic(s) :

   1.3 How successful has this form of instruction been on a scale of 0(not successful at all) to 5 (very successful)

   0[ ]   1[ ]   2[ ]   3[ ]   4[ ]   5[ ]

2.

   2.1 Do you think that a short lecture self-instruction method can be used effectively in Technikon mathematics courses.

   Yes [ ]   No [ ]

   2.2 Please list topics that you think would lend themselves to this approach :

   2.3 Are you interested in this type of instructional method?

   Yes [ ]   No [ ]   Unsure [ ]

   If you are unsure, is it because you aren't familiar with the method.

   Yes [ ]   No [ ]

   2.4 Please give any comments you think are relevant to this method of instruction.

THANK YOU.

APPENDIX D

Relevant part of the t-distribution
This table gives the values of $t^{(P)}_n$ where n is the number of degrees of freedom.

	ONE-TAIL TEST		Z
	0,010	0,005	
P	TWO-TAIL TEST		
	0,02	0,01	
DF			
60	2,390	2,660	

Source : Willemse (1990)

Statistics Formula Sheet.

For grouped data :

$$\overline{X} = \frac{\sum X.f}{n} \quad or \quad \overline{X} = \frac{\sum X.f}{\sum f}$$

$$M_e \text{ (value of } \frac{n}{2}\text{th item)} \quad ; \quad M_o = L + \frac{\Delta_1}{\Delta_1 + \Delta_2}.c$$

$$s = \sqrt{\frac{\sum (X - \overline{X})^2 . f}{n - 1}}$$

$$\text{correction factor [finite population] : } \sqrt{\frac{N - n}{N - 1}}$$

$$\text{Difference between means : } z = \frac{\overline{X_1} - \overline{X_2}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$t = \frac{\overline{X_1} - \overline{X_2}}{\sqrt{\frac{s_1^2.(n_1 - 1) + s_2^2.(n_2 - 1)}{n_1 + n_2 - 2}} . \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \qquad DF = n_1 + n_2 - 2$$

Source : Technikon Natal Mathematics Department

TECHNIKON NATAL

INTERNAL EXAMINATIONS - JUNE 1992

SUBJECT: **MATHEMATICS II (CHEM ENG) (1604016220)**
MATHEMATICS II (1604019220)

DATE:
TIME:

DEPARTMENT: **MATHEMATICS**

NO OF PAGES: 6
ANNEXURE: TABLE OF INTEGRALS

QUALIFICATIONS:
NAT DIP: ELECTRICAL ENG (HC) (3208123)
NAT DIP: ELECTRICAL ENG (HC) (3208596)
NAT DIP: ELECTRICAL ENG (LC) (3208124)
NAT DIP: ELECTRICAL ENG (LC) (3208597)
NAT DIP: MARINE ENG (3208127)
NAT DIP: MARINE ENG (3208602)
NAT DIP: MECHANICAL ENG (3208129)
NAT DIP: MECHANICAL ENG (3208605)
NAT DIP: SURVEYING (3208133)
NAT DIP: SURVEYING (3208607)
NAT DIP: PULP & PAPER TECH (3208136)
NAT DIP: PULP & PAPER TECH (3208609)
NAT DIP: CIVIL ENGINEERING (3208475)
NAT DIP: CIVIL ENGINEERING (3208610)
NAT DIP: CHEMICAL ENGINEERING (3208417)
NAT DIP: CHEMICAL ENGINEERING (3208593)
NAT DIP: ANALYTICAL CHEM (3215393)
NAT DIP: ANALYTICAL CHEM (3215595)

REQUIREMENTS:
ANSWER BOOK,
GRAPH PAPER

EXAMINER: JC SMITH
MODERATOR: SA LEGGOTT

MAX MARKS: 100
TOTAL MARKS: 125

INSTRUCTIONS

1 Answer question 1-6 then either question 7 or 8.

2 Scientific calculators may be used but all intermediate working must be shown.

3 Give answers correct to 2 decimal places except where stated otherwise.

QUESTION 1:

Find $\frac{dy}{dx}$ *and* $\frac{d^2y}{dx^2}$

simplifying your answers given that:

1.1 $y = x \tan^{-1} x + \ln \sqrt{1 + x^2}$ (5)

1.2 Given that $x = e^t \sec t$; $y = e^t \operatorname{cosec} t$
Show that

$$\frac{dy}{dx} = \left(\frac{\tan t - 1}{\tan t + 1}\right) \cot^2 t$$

(5)

1.3 A piece of wire 36 metres long is cut into two pieces 12p metres and (36-12p) metres long. The 12p metres piece is bent into the shape of an equilateral triangle and the other piece into a square. Find p when the combined area of these shapes is a minimum and hence find the lengths of the two pieces of wire.
[HINT: use area of triangle = ½ a.b. sin C] (8)
[18]

QUESTION 2:

2.1 *If* $z = \ln \left[\frac{1}{\sqrt{x^2 + y^2}}\right]$ *show that* $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$

(4)

2.2 Using partial differential techniques find:

$\frac{dy}{dx}$ *given that* $y + \frac{x}{x^2 + y^2} = 0$

(4)

2.3 One end of a ladder 6m in length stands against a wall. If the bottom of the ladder is pulled away from the wall at a rate of 0,25 m/sec at what rate does the top of the ladder slide down when the bottom is 1,5 m from the wall? (5)
[13]

QUESTION 3:

3.1 Find the first four non-zero terms by Maclaurin's expansion of
$\ln(1 + e^x)$. (5)

Integrate the following with values where necessary:

3.2 $\int_0^{\frac{\pi}{4}} \cos^2 2x \sin^3 2x \, dx$

(5)

3.3 $\int_0^{\frac{5}{4}} \frac{dx}{16x^2 + 25}$

(4)

3.4 $\int \frac{2x + 5}{\sqrt{5 - 2x - x^2}} \, dx$

(6)

3.5 $\int \frac{\ln x}{\sqrt{x}} \, dx$

(4)

3.6 $\int \frac{(x^2 + 1) \, dx}{(x^2 + 2)(x + 1)}$

(6)

3.7 $\int_0^{\frac{3}{4}} \sqrt{9 - 16x^2} \, dx$

(4)
[34]

QUESTION 4:
Solve the following differential equations:

4.1 $(x^3 - y^3)\dfrac{dy}{dx} = x^2y$

(5)

4.2 $\dfrac{dy}{dx} + y\cot x = e^{\cos x}$

given that y = 1 when x = $\pi/2$ (5)

4.3 $\dfrac{dy}{dx} + y\tan x = y^2\sec^3 x$

[HINT: divide by y^2 and use substitution z = 1/y]. (5)
[15]

QUESTION 5:
5.1 Find the error on the following calculation

$$E = \left[(1{,}31)^2 + \frac{4{,}0}{3{,}8}\right]^{\frac{1}{3}} \text{ to 4 } D.P.$$

and hence the range for the value of E. (6)

5.2 Consider the data

x	0,300	0,350	0,400	0,450	0,500	0,550	0,600
f(x)	0,2955	0,3429	0,3894	0,4350	0,4794	0,5277	0,5646

a) Form and correct the finite difference table

b) Find f(0,7) by extending the table (6)

5.3 Using Simpson's rule find the numerical values of

$$\int_0^{\frac{\pi}{6}} \sin 3x \, dx$$

[HINT: use 5 strips] (3)
[15]

QUESTION 6:
A study of the fares paid by 40 people travelling into Durban from various areas by taxi revealed the following information:

Cost of fare in Rands:	0-4	5-9	10-14	15-19	20-24	25-29
Frequency :	3	12	15	5	3	2

6.1 Using this information calculate the mean fare and the standard deviation.

6.2 Form the cumulative frequency table and hence find the median fare by calculation.

6.3 How many people pay between R7 and R17 for their fare. Explain your method.

[10]

ANSWER EITHER QUESTION 7 OR 8:

QUESTION 7:
7.1 Find the volume when the area bounded by $y = \cos x + 2 \sin x$ from $x = 0$ to $x = \pi/3$ is rotated about the x axis. (7)

7.2 Find the r.m.s. value of $i = \cos x + 2 \sin x$ over the range

$x = 0$ to $x = \pi/3$ (3)

[10]

QUESTION 8:
8.1 Find the area under the curve defined by the following parametric equations:
$x = \Theta - \sin \Theta$ $y = \cos \Theta$ when $\Theta = 0$ to $\Theta = \pi$ (5)

8.2 Hence find the y co-ordinate of the centroid of the curve defined by
$x = \Theta - \sin \Theta$ $y = \cos \Theta$ when $\Theta = 0$ to $\Theta = \pi$ (5)

[10]

APPENDIX F (Part A)

Student responses to Question B4 "Do you prefer this approach (SLSI) to the normal lecture, tutorial system ?"...**Please (in a few lines), indicate the reason for your answer** (see annexure B) :
In each case **Y** indicates that they prefer the SLSI approach, while **N** indicates that they do not. This is followed by their comment.

Anders P **Y** "A better interest was created within the class. The work was easy but was made more interesting by the way in which it was presented. The dedicated interest and enthusiasm of the lecturer was much appreciated. Mathematics would be a lot more interesting and pleasurable if all lecturers and lectures were presented as in the past week"

Badenhorst Q **Y** "We were taught what we needed to know and afterwards completed the tut. Which enabled me to understand the concept"

Cave' C **Y** "There are a range of worked examples that are easy to follow. The self study is clear & concise. We knew where we were going & how we were going to get there. Everything leads into each other in an interesting way."

Christie S **Y** "I understood much more in the lectures with Mr Hunter, than in the rest of the semester."

Cloete L **Y** no comment.

Coleman J **Y** "I prefer an introductory lecture ; then to read through the work myself and then to go for help afterwards if needed."

Crocker M **N** "The reason I will do better in Numerical Methods than in the other sections in the exam is that the work in this section is much easier. I prefer the normal lecture system, as one does not tend to lag behind too much when overloaded with work in other subjects."

Davey G **Y** "This section was a lot easier to understand. Handout is very useful."

Elliott A **Y** "Each section was divided up into separate lessons with tuts on each lesson this makes it easier to revise the work."

Emery M **Y** "Lessons are presented in an understandable manner & at a reasonable speed, the worked examples help considerably."

Englebrecht M **Y** "In this section I was first shown the different rules, where they came from and then how to use them thus I understood it better."

Fleming R **Y** "The worked examples were easy to follow and well set out."

Fraser M **Y** "The notes state clearly what is required and the method of study is easier as the individual can work through the lesson himself at home, and immediately knows if he has misunderstood some concepts during the lecture. Also the notes point out problem areas, which benefits the student."

Hudson G **Y** "Having to do the work on your own gives one a better understanding of the work."

Koekemoer M **Y** "Given notes and don't have to miss out something in case you miss a point/lesson. Helpful in studying to have notes + to know what the lecturer expects from you."

Labuscagne G **Y** "I liked the numerous examples and the orderly manner and sequence of the work."

Latimer D **Y** "For a section like Numerical methods this system is suitable as it is not that difficult but I don't think this system can be used for any of the other sections of the course."

Lupton-Smith P **Y** "Yes, as the work is put out in a clear way, thus one can see where one is going right and wrong. As long as one is determined to pass the subject. Unlike "Joubert et al." which is set out badly i.e. very easy examples & very hard examples, with no real explanations about the subject."

McAlistair R **Y** "The self study notes are easier to understand than notes received with lectures."

McKechnie S **Y** "I would prefer a normal lecture to gain a good understanding but notes are helpful for reinforcing this and reteaching anything not understood."

Morule W **Y** no comment

Nel J **Y** "For numerical systems it is a great idea and works well. It also ensures you do some home work."

Phaidi G **Y** "I don't think there is any wrote, because this ratings will somehow improve our maths and this will also help our lecturers to improve the lessons" (?)

Rosa E **Y** "After a short introductory to the work we feel it necessary to do the work while it is still fresh in our minds instead of having to wait till we get home (where we usually loose interest)"

Street K **Y** "It forces you to do the work on your own but I don't think that all the sections could be dealt with in a similar manner"

Symington D **Y** "The given notes are much more explanatory than notes taken from the board or from the text book."

Trauner J **Y** "I prefer working on my own and struggle when we don't get a good lecturer. this way a lot of it is self-study."

Walker D **Y** "Work is made easier to understand."

Wescott D **Y** "You get fully worked examples in the text which have all the steps included. In normal classes some lecturers leave out steps making revision harder come test/exam time."

Woodburn J **Y** "What do the pro's reckon. I think they say twice your age is your concentration span, that's 46 mins. I can concentrate well for about 20 mins and then I start wandering. As long as the information is well portrayed, then 20 min lectures are more effective."

Phillips D **Y** "The concept behind the lectures makes them interesting and enjoyable. Although a lot depends on the calibre of the lecturer."

Responses from pilot sessions :

Addinall R **Y** "Because it gives you more worked examples & answers so you can see where you are going wrong."

Armour C **Y** "Basics are explained, however certain sections may require a longer initial lecture to get the point across."

Anderson N **Y** "The descriptive notes prior to doing the tuts helped me greatly in understanding any of the work."

Baillie M **N** "The explanations don't seem thorough enough."

Bensley B **N** no comment

Booer S **N** "The section has been covered at the wrong time of the year. I have given Maths the last position as I have a course mark and learning for other tests has taken preference. marks for tut work is a good idea."

Borchers S **N** "This method can only be used in certain sections others need more explaining. Lectures are easy to miss."

Bowman T **Y** "Time to do tutorials during the lecture period"

Burden R **Y** "One is able to understand the work + then apply your understanding immediately to the tut work."

Cahi S **Y** "It is far easier to understand, and if it so happens that you slip-up on something in class, it is easy to follow up at home."

Collett D **Y** no comment

Cribbins I **Y** "These are better notes, with good worked examples that show you what you actually have to know instead of to much theory explanations which I feel do not really help me."

Dawson C **Y** "Having the lessons to work through being left to yourself one tends to understand the work to a greater extent."

Du H.D.Rauville M **Y** "In lectures you are normally to busy writing to absorb all the information given to you, but the text helps put your full attention on the lecture. The lecturers also go much quicker and thus time is saved."

Frank P **Y** "It was a good way of lecturing but with the book you tend to think you can miss lectures,. This is your downfall - maybe take register."

Gilbert T **Y** "Short lectures are less confusing or mind straining, making you less tired when getting home. Therefore able to work longer."

Goudie J **N** "More used to the "traditional" lecture. Possibly give text to other sections in addition to lectures."

Griffin K **Y** "I prefer this approach as I find that I can work when I feel motivated to do so."

Harcourt M **Y** "This approach gives you a chance to do your tut work straight away."

Hockley G **Y** "By having the notes set out the way they are in the numerical methods section it helps to give me a better understanding of the work."

Human J **Y** "Since the lectures were shorter, one could do the tut work straight after the lecture while the work was still fresh in our mind, rather than to leave it for a tut period and having to go through it again."

Hutton G **Y** "I enjoy self study more than being lectured to as I find I understand the work more teaching myself."

James L **Y** "A person seems to be closer to the problems, therefore a person will understand the work. It also depends on the person to do the work, and not wasting the lecturers time."

Jones P **Y** "I believe a combination of the two is fine. The notes are well set out and I find that working numerically is more fundamental in the Civil Eng. field."

Loumeau C **Y** "Yes, but it varies in different section of work."

Messenger A **N** "I learn more by writing down what we have to do."

Mills W **Y** "They are short and brief and not very hard to understand. The notes are good as to help you if you have difficulties."

Mullan J **N** "The lecturing of a full period avoids the rushing of the lesson by the lecturer and a slower possible better explanation could be given."

Pfeifer J **Y** "Because you have an explanation of the work at hand the notes are very well put and easy to understand this will make studying much easier."

Pitrakou P **Y** "Because I get to do a few examples after the lecture while it is still fresh in my mind."

Price G **N** no comment

Reeves B **Y** "Explanations are set out in the yellow book and are useful for referring back. Normal lectures would involve copying down from the board."

Rodrigues V **N** "The allocation of tutorial time during the school day ensures that time can be set aside for maths without conflicting with other subjects."

Roscoe E **Y&N** "You have to spend more time on the subject. The time spend working through the book will definitely benefit you but you then tend to spend less time on your other subjects."

Smith I **Y** "Tut lessons are used mainly for catching up work. If a introductory lecture was included more interest would be shown + work would be done while still fresh in you head."

Swanepoel J **N** "I do like this system (for certain sections) - but would prefer the normal system for the other lectures."

Tarr G **Y** "It is a lot more interesting as concentration can last the duration of the lesson and we can put into practice what one has learnt immediately."

Thring D **Y** "Information comes in at a slower rate, and if the student takes the time to follow up and do the tut, in the specified way, he/she gets a good understanding of the work."

Volker V **Y** "Better notes for study purposes."

Von Fintel R **Y** "The notes to self study modules are generally better and easier to work through, so finding your problem areas and working through them becomes much easier."

Walden S **Y** "Its a good approach to easier sections but to harder sections a lot of effort would have to go into making up the notes. Personally I feel that the students wouldn't have the self discipline to work through the difficult sections."

Walford R **Y** "Because the work was easily understood through the lecture and the notes were also easily understood. This approach is effective in this section but I do not think it would be as effective in other sections."

Willson R **N** "These two methods are incomparable, Numerical Methods is easy compared to other sections, full lectures are not needed but other sections such as Integration require longer lectures."

Wright C **Y** "The way the book is written is step by step and well explained. It gives one an opportunity to try examples oneself. If one goes wrong one can see exactly where you've gone wrong. On top of that, it is generally a logical and interesting section."

Yelseth R **Y** "If something was not fully understood in the lecture -there were good notes to look up on how to do it. The notes could be worked through at my own pace & not that of the class."

Ziesing K **Y** "A lot of work is covered in a short period of time, thus leaving more time .. for revision."

****
****

ANNEXURE F (PART B)

Students responses to *Question B6* **"Please comment on the Self Instruction text you have worked through the past two weeks, and make suggestions as to how you think the lessons can be improved on."**(see annexure B) :

Anders P *"Excellent and easy to follow. The word explanations are very useful and help to explain very easily. Useful examples are a good idea too. Maybe a few more tuts can be added."*

Badenhorst Q
No comment

Cave' C *No comment*

Christie S *No comment*

Cloete L *"Generally good. Introductory parts of notes excellent as an aid too further problems in the lesson."*

Coleman J *"I don't think improvements need to be made on the way the work was presented."*

Crocker M *"All was run very smoothly and successfully."*

Davey G *No comment*

Elliott A *"There are lots of examples and each section is well explained."*

Emery M *No comment*

Englebrecht M *"The texts given to me is good. However, more examples would help more."*

Fleming R *No comment*

Fraser M *"It would help to have important formulae written in the summary at the end of each section."*

Hudson G *No comment*

Koekemoer M *"Very good and quite enjoyable."*

Labuscagne G *"I found the work most enjoyable and found the new section much easier to comprehend. I liked the numerous examples."*

Latimer D *No comment*

Lupton-Smith *No comment*

McAlistair R *"The notes are well set out. Maybe a few more complicated tutorials which combine the different methods can be used."*

McKechnie S *"In addition to the tuts possible references to the text books where more examples can be found."*

Morule W *"No comments lecture. Lecturer 100% sure of his work."*

Nel J *"I think the lessons are compiled very well. The BUSH method really appealed to me because I could test all my answers. The lecture also gave a good introduction to each lesson and presented it very well."*

Phaidi G *"Well, It was quite interesting section. I have enjoyed it and hopefully I gonna pass it on exams. The lessons can be improved on by just doing more Exercises. i.e. (10-15 problems)"*

Rosa E *"no improvements need to be made."*

Street K *No comment.*

Symington D *No comment.*

Trauner J *"I enjoyed the work because it was up to me, how well I would do, and didn't rely on the quality of the lecturer."*

Walker D *"I found the notes to be excellent but too bulky, not so many examples are required."*

Wescott D *"Tutorials could be more clearly marked. One can sometimes miss the T notation."*

Woodburn J *"It depends entirely on the type of section. Graphical work might take longer to get across to us. I'm very pro this new method, however it would take a lot to break some of the maths sections down to 20 min lectures."*

Phillips D *No comment.*

Responses from pilot sessions :

Addinall R *"I think this text was very helpful when studying and learning the basic concepts of the work."*

Armour C *"If the number of lectures were to decrease, & no. of tutorials increase then problems could be viewed individually."*

Anderson N *"No suggestions."*

Baillie M *"Longer explanations should be given."*

Bensley B *No comment.*

Booer S *"The lessons were very clear and understandable. But I feel if work was more intricate it would be harder to grasp."*

Borchers S *No comment.*

Bowman T *"I think it is fine and well set out."*

Burden R *No comment.*

Cahi S *"It was very helpful in respect i.t.o. time because it is easy to understand there is no way to improve on it."*

Collett D *No comment.*

Cribbins I *"I found it very easy to work through. The only complaint is that the work could be more spread out on the page. More spaces will make it easier to read."*

Dawson C *"Not sure how to improve. Seem fine the way they are."*

Du H.D.Rauville M *"It is very simple and straightforward to understand, has many examples to work through."*

Frank P *"As above- take register during lecture. Students get lazy and miss lectures because they have the book."*

Gilbert T *"System is pretty good, no improvement necessary."*

Goudie J *"Good. More tut eg.s."*

Griffin K *"I feel the presentation and the lectures were good and enjoyed the new approach to the lessons."*

Harcourt M *"It will be effective in a few of the sections but not for the whole mathematics course."*

Hockley G *"I found that there were very few problems in the tuts. I would like it if the tuts had more problems to do so that I could make sure that I knew what was going on."*

Human J *"The lessons were set out very good. It could be clearly understood. Can't think of improvements."*

Hutton G *"I found the instruction booklet very helpful and instructive and tutorials were well placed, testing you on work just done."*

James L *"The self instruction text was less pressure that before, if one does not follow in class, one can always go back to the beginning of the lesson and start all over again. Maybe a few more eg.s in the more difficult sections."*

Jones P *"Everything is on the golden platter it is up to the student."*

Loumeau C *No comment.*

Messenger A *No comment.*

Mills W *"Quite enjoyable the understanding and learning was made easy the lectures were not uninteresting because they were short and precise."*

Mullan J *"I think the idea is a good one but one must do the work that day and not it be left for another day since the lessons get done so quickly that one gets left behind. I don't think that the lessons can be improved much but a possible increase in past papers so that one can see the standard."*

Pfeifer J *"The text is very good - could be improved by adding more tuts."*

Pitrakou P *"The work is great but the layout could be made clearer eg. where a tut starts instead of just T and a no. how about writing out TUTORIAL and underline it. Otherwise it is great."*

Price G *No comment.*

Reeves B *"Maybe more examples at the end of the booklet."*

Rodrigues V *"Actual text is presented in understandable and concise way. Although time at the end of each lecture is available to work the pressures of catching up on other subjects leads to a false sense of comfort in that the work can be caught up at a later stage as lecture time gives the impression that there is not much work to be done."*

Roscoe E *"I think that the self instruction text was very clear and the worked examples were very helpful."*

Smith I *"Think the work was productive as it makes you think for yourself and use basic logic."*

Swanepoel J *"No improvements."*

Tarr G *"I think they have been going well."*

Thring D *"I think they are good as they are."*

Volker V *"No improvements necessary. Maybe expand the method into other parts of the syllabus."*

Von Fintel R *"The notes are generally very good, one suggestion would be to print the answers to the tuts at the back of the book."*

Walden S *"Notes are set out very well. A lot of hard work went into them, if the student could put ½ the amount a distinction would be a walk in the park."*

Walford R *No comment.*

Willson R *No comment.*

Wright C *"I enjoyed this approach to teaching. I think if all lessons were done this way, it would make life easier. Maths is a difficult subject and new methods of teaching should be tried until everyone does well. The ultimate way of teaching should be found rather that people failing - life is too short to spend 3 years trying to pass T2 maths."*

Yelseth R *"Lessons are fine as they are - can see no improvement."*

Ziesing K *"I think that everything that has been covered in this text is basically all that a student needs. I don't there is any room for improvement."*

APPENDIX G

Nine responses received from Technikons with regard to the use of self-instruction at their institutions. (see Appendix C)

1. Is self-instruction in mathematics being used at your Technikon?
   Yes [**2**] No [**7**] Not sure [**0**]

   If **no** or **not sure** go to question 2.
   If **yes** please answer following questions as well as question 2:

   1.1 Subject & level : **Mathematics I ;**

   1.2 Topic(s) : **Algebra ; Trigonometry ; All Levels ;**

   1.3 How successful has this form of instruction been on a scale of 0(not successful at all) to 5 (very successful)
   0[**-**] 1[**-**] 2[**-**] 3[**2**] 4[**-**] 5[**-**]

2.
   2.1 Do you think that a short lecture self-instruction method can be used effectively in Technikon mathematics courses.
   Yes [**6**] No [**2**] Not sure [**1**]

2.2 Please list topics that you think would lend themselves to this approach :
Algebra - Quadratic Equations, Logarithms, Simultaneous equations
Trigonometry -
Some topics from Calculus
Any of the topics in the mathematics syllabus - Differentiation,
Integration, D.E. etc.
Determinants, Radian measure, Complex numbers, graphs.
Mathematics I - Exp. & Logs, Trig, Binomial, Partial fractions
Mathematics II - Integration of Trig. functions

2.3 Are you interested in this type of instructional method?

   Yes [**7**] No [**0**] Unsure [**2**]
   If you are unsure, is it because you aren't familiar with the method.
   Yes [**2**] No [ ]

2.4 Please give any comments you think are relevant to this method of instruction.

"From my experience this method will work if we get students with a good basic knowledge in mathematics. Here we get students with a very weak mathematical background. I am very much interested in this method."

"I can see many areas for application of this style of instruction.
...
I am interested in any method/technique which might be of benefit to the development of this course (and myself)."

"Maths syllabi are very full and lecturers must inevitably resort to this kind of instruction to be able to finish the syllabi in time. TEST AND Exam Questions are set on these topics to ensure that the relevant subject matter was covered by the students."

"The book available to the students must be in programmed form (frames) as in "Stroud""

"We as a Maths Dept have reservations as to the effectiveness (as opposed to efficiency) in applying this approach to students from educational backgrounds on a lower level than the "norm". In our Bridging program the students show an uncanny ability to overcome these academic problems when subjected to intensive teaching & tutorial sessions."

"I think that many parts of topics can be suitable for this method."

APPENDIX H

List of supporting documents and other relevant material the author has in his possession (Contact W.G. Hunter, Technikon Natal, Mathematics Department, P.O.Box 953, Durban, 4001)

HSRC literature search printout.

Photocopies of most of the journal articles mentioned in the Literature References.

Responses to all questionnaires.

Printouts of student details.

Relevant quotations from reference books.

Initial SI text.

APPENDIX I

Continuous assessment as used at Technikon Natal for the subjects Mathematics IA and Mathematics IB

For Mathematics IA (1604026120) and Mathematics IB (1604029120) the final result a student obtains will be based on continuous assessment determined as follows :
The average of the best 5 out of 7 class tests and/or assignments will be converted to a mark out of 20.
Two control tests will each count 40 (i.e. 2x40 = 80)
The final result will be the sum of the above out of 100.
A final result of 50 or more is required to pass Mathematics IA (1604026120) or Mathematics IB (1604029120) (source: Technikon Natal Mathematics I Study Guide, 1993).

The class tests and/or assignments are conducted fortnightly and cover the work done in lecturers and tutorials the previous two weeks. The control tests each cover two modules (the subjects are split into 4 four week modules).