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Stars and junction conditions in Einstein–Gauss–Bonnet gravity

Byron P Brassel[1,2], **Sunil D Maharaj**[1,*] **and Rituparno Goswami**[1]

[1] Astrophysics Research Centre, School of Mathematics, Statistics and Computer Science, University of KwaZulu-Natal, Private Bag X54001, Durban 4000, South Africa

[2] Department of Mathematics, Faculty of Applied Science, Durban University of Technology, Durban 4000, South Africa

E-mail: maharaj@ukzn.ac.za

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Abstract

The junction conditions for a higher dimensional spherically symmetric charged and anisotropic static star are derived in Einstein–Gauss–Bonnet (EGB) gravity with nonvanishing cosmological constant. It is shown that for a timelike boundary hypersurface of zero thickness, the generalised matching conditions across this surface in EGB gravity are satisfied. A sufficient condition is that the Israel-Darmois conditions are valid. Therefore it is possible to generate a complete stellar model in EGB gravity. The interior matches to the exterior higher dimensional charged Boulware–Deser spacetime with cosmological constant. The barotropic radial pressure has to vanish at the boundary of the star which is also the case in general relativity.

Keywords: junction conditions, differential geometry, Einstein–Gauss–Bonnet gravity, stars

(Some figures may appear in colour only in the online journal)

1. Introduction

The second order Lovelock Lagrangian leads to additional structure related to the geometry of the spacetime manifold which involves sums of terms containing products of the Riemann tensor, the Ricci tensor and the scalar curvature. This is reflected in the Lovelock tensor which now appears explicitly in the field equations for spacetime dimensions $N \geqslant 5$. We refer to the resulting gravitational theory as Einstein–Gauss–Bonnet (EGB) gravity, and it has the desirable feature of containing Einstein gravity as a special case [1–8]. An important astrophysical

* Author to whom any correspondence should be addressed.

application is to model a dense static star in EGB gravity. The exterior spacetime can be taken to be the Boulware–Deser geometry [9]; the EGB analogue of the Schwarzschild solution in general relativity, and its extensions to include charge and the cosmological constant [10–18]. It is necessary to solve the nonlinear EGB field equations in the interior spacetime. This is not a trivial exercise due to the nonlinearity of the dynamics resulting from the appearance of the second order Lovelock tensor. In recent times certain families of exact solutions have been found in the interior. The constant density solution was found by Dadhich *et al* [19]. Exact solutions were obtained by several researchers [20–28] with isotropic pressures. For the simpler case of anisotropic pressures, particular exact models have been generated [29–39]. It should be noted, however, that the interior spacetime has to match across a comoving hypersurface Σ for a complete description of a stable stellar configuration.

In general relativity, the problem of matching across a comoving surface is paramount in the modelling of a relativistic star, and its eventual demise where it may collapse under its own gravity. It is with the presence of the junction conditions that the evolution of the stellar system can be analysed. Several exact solutions to Einstein's field equations, with various matter distributions, have been studied in astrophysical applications [40–47]. In the case of adiabatic systems, i.e. static stars, an interior spacetime metric with a matter distribution which is a perfect fluid (there is no heat flux) is matched across the timelike hypersurface to a vacuum spacetime. It is then expected that the pressure at the boundary of the star vanishes identically; there is no energy transfer across the surface. It is important to investigate whether this notion holds true in modified gravity theories, specifically EGB gravity. The embeddings of general hypersurfaces[3] which are null, timelike or spacelike were analysed by Mars and Senovilla [48].

The junction conditions at the stellar surface Σ are *not* known in EGB gravity for a spherical matter distribution. For stellar models in EGB gravity, researchers assume that the matching conditions from general relativity may be applied; this notion is true for the exact solutions in [19–39]. This results in an incomplete analysis and we do not have a full picture of the gravitational dynamics. It is therefore necessary to find the full set of junction conditions on Σ so that we can accurately describe a gravitating static star in EGB gravity. To achieve this we need to adapt, from the brane world scenario, the matching conditions for two spacetimes across Σ in the EGB theory, first presented by Davis [49]. Several other treatments have since analysed the Davis matching conditions, for example see [50–52]. The resulting expressions are complicated and involve terms containing the extrinsic curvature, the trace of the extrinsic curvature, and the divergence-free part of the Riemann tensor. A careful analysis of the Davis matching conditions has to be made; this is a more onerous procedure than is the case in general relativity, however the complications can be resolved. The Davis conditions arise by adapting the Einstein–Hilbert action with the additional Gibbons–Hawking–York term for the boundary to eliminate the normal derivatives of the metric variation so that this above-mentioned boundary term is generalised [53–56]. It is interesting to observe that the Davis conditions also arise independently by using a regularization procedure utilizing delta functions as a sequence of classical functions, as demonstrated by Chu and Tan [57]. It is necessary to show that the Davis conditions are satisfied and the interior of the star matches smoothly to the associated exterior spacetime.

The purpose of this paper is to obtain a full description of a higher dimensional static gravitating star in EGB gravity. This involves solving the Davis [49] conditions on the boundary surface Σ. This paper is organised as follows: In the following section we present the formalism

[3] Examples of null hypersurfaces are the light cone and the event horizon of a black hole. A timelike hypersurface can be considered as the boundary of a finite volume in a higher dimensional space; this is relevant to our study. An example of a spacelike hypersurface is a Cauchy surface.

of EGB gravity as well as the curvature corrected field equations. In section 3 we discuss the junction conditions in EGB gravity for a braneworld as presented by Davis [49] and prove that for a boundary of zero thickness, i.e. a timelike hypersurface separating two spacetimes, the EGB junction conditions are equivalent to the Israel-Darmois conditions of general relativity. We then utilise this fact to find the matching conditions for a charged static star with nonvanishing cosmological constant in higher dimensional EGB gravity in section 4.

2. EGB–Maxwell field equations

The action of EGB gravity in arbitrary spacetime dimensions N, with cosmological constant, is given by

$$S = \int_{\mathcal{M}} d^N x\sqrt{-g}(\alpha_0 + \alpha_1\mathcal{R} + \alpha_2\mathcal{R}^2) + \mathcal{S}_{\text{matter}}, \tag{1}$$

where $\mathbf{g}$ is the metric tensor on the spacetime manifold $\mathcal{M}$, $\alpha_0 = \Lambda$ is the cosmological constant and $\alpha_1 = 1$ is a constant term associated with the action ($\mathcal{R} = R$, the Ricci scalar) of general relativity. The term $\mathcal{S}_{\text{matter}}$ takes into account the matter content which includes the electromagnetic field or perhaps other exotic fluids, depending on the context. The coupling constant $\alpha_2 = \alpha > 0$ is affiliated with the Gauss–Bonnet curvature corrections in the term $\mathcal{R}^2$. Conventional Einstein gravity is regained as a limiting scenario when $\alpha \to 0$. In EGB gravity, the dimension of spacetime satisfies $N \geqslant 5$ with $N = 5$ presenting as a special dimensional case of the theory. In dimension $N = 4$ the EGB theory is indistinguishable from general relativity and the Gauss–Bonnet term $\mathcal{R}^2$ is merely a topological invariant; the two theories differ only for $N \geqslant 5$.

The Einstein–Gauss–Bonnet-Maxwell (EGBM) field equations are derived by varying the above action (1), i.e. $\delta S = 0$ with respect to the metric $\mathbf{g}$, and can be given in general as

$$\mathcal{G}_{ab} + \Lambda g_{ab} = \kappa_N(T_{ab} + E_{ab}), \tag{2a}$$

$$F_{[ab;c]} = 0, \tag{2b}$$

$$F^{ab}{}_{;b} = \mathcal{A}_{N-2}J^a, \tag{2c}$$

where the Lorentzian signature is $(-,+,+,\ldots,+)$. Here κ_N and $\mathcal{A}_{N-2}$ are the N-dimensional Einstein coupling constant and surface area of the $(N-2)$-sphere respectively. These are given by

$$\kappa_N = \frac{2(N-2)\pi^{\frac{N-1}{2}}}{(N-3)\Gamma\left(\frac{N-1}{2}\right)}, \qquad \mathcal{A}_{N-2} = \frac{2\pi^{\frac{N-1}{2}}}{\Gamma\left(\frac{N-1}{2}\right)},$$

in terms of the gamma function $\Gamma(\ldots)$. The tensor $\mathcal{G}_{ab}$ in (2a) is represented by

$$\mathcal{G}_{ab} = G_{ab} - \frac{\alpha}{2}H_{ab},$$

and T_{ab} is the stress energy tensor. Note that $\mathcal{G}_{ab}$ is expressed in terms of the Einstein tensor G_{ab} and the second order Lovelock tensor

$$H_{ab} = g_{ab}L_{GB} - 4RR_{ab} + 8R_{ac}R^c{}_b + 8R_{acbd}R^{cd} - 4R_{acde}R_b{}^{cde}, \tag{3}$$

which are connected by the coupling constant α. When $N < 5$ the above curvature tensor (3) is identically zero. The Lovelock invariant is given by

$$\mathcal{R}^2 = L_{GB} = R^2 + R_{abcd}R^{abcd} - 4R_{cd}R^{cd}, \tag{4}$$

which involves the squares of the Riemann tensor, Ricci tensor and scalar curvature. When $N = 4$ this term (4) is merely a quadratic diffeomorphism invariant, *id est* a surface term. For dimensions $N < 4$, we have that (4) vanishes identically by the Chern–Gauss–Bonnet theorem [58, 59]. The electromagnetic field tensor is given by

$$E_{ab} = \frac{1}{\mathcal{A}_{N-2}}\left(F_a{}^c F_{bc} - \frac{1}{4}F^{cd}F_{cd}g_{ab}\right), \tag{5}$$

which is expressed in terms of the Faraday[4] tensor $F_{ab} = \Phi_{b;a} - \Phi_{a;b}$ which is antisymmetric, the electric gauge potential Φ and the surface area $\mathcal{A}_{N-2}$ covering the unit $(N-2)$-sphere. Equation (2*b*) is the Gauss-Faraday law, a combination, in spacetime, of Faraday's law of induction and Gauss's law for magnetism. The Gauss-Ampère law (2*c*) is essentially a combination of Gauss's flux theorem and Ampère's circuital law on the spacetime manifold. We have that $J^a = \sigma u^a$ is the current density, which is expressed in terms of the proper charge density σ. Equation (2*c*) implies charge conservation on the spacetime, i.e. $J^a{}_{;a} = 0$.

3. Junction conditions in EGB gravity for two spacetime manifolds

The Israel-Darmois junction conditions [60] for the matching of two spacetime manifolds $\mathcal{M}^{\pm}$ across a comoving boundary surface layer Σ of zero thickness are given by

$$(\mathrm{d}s^2_-)_\Sigma = (\mathrm{d}s^2_+)_\Sigma = \mathrm{d}s^2_\Sigma, \tag{6a}$$

$$K^-_{ij} = K^+_{ij} = K_{ij}\big|_\Sigma. \tag{6b}$$

In the above the extrinsic curvature can be written as

$$K^{\pm}_{ij} \equiv -n^{\pm}_a \frac{\partial^2 \chi^a_\pm}{\partial \xi^i_\pm \partial \xi^j_\pm} - n^{\pm}_a \Gamma^a{}_{bc} \frac{\partial \chi^b_\pm}{\partial \xi^i_\pm}\frac{\partial \chi^c_\pm}{\partial \xi^j_\pm}, \tag{7}$$

and ξ^i are coordinates on Σ. Here, the unit vectors normal to the boundary Σ are $n^{\pm}_a(\chi^b_\pm)$. The quantities $\chi^a_\pm = \chi^a_\pm(\xi^i_\pm)$ are the coordinates of the two spacetimes, and they are indeed functions of the induced coordinates ξ^i on the boundary surface Σ. The Riemann–Christoffel connections of the second kind are given by $\Gamma^a{}_{bc}$. These conditions (6) hold in general relativity as well as various modified theories of gravity in which the Ricci scalar is present in the gravitational action. In this regard, certain extra conditions may need to be satisfied. For example in $f(R)$ gravity, the continuity of the Ricci scalar and its derivative across the boundary also need to be satisfied [61, 62]. In this section we prove geometrically that if the matching conditions (6) hold in general relativity, they will hold in EGB gravity.

3.1. Braneworld scenario

The conventional Gauss–Bonnet brane world Universe consists of a $(N-1)$-dimensional brane which is embedded into a *single* N-dimensional bulk spacetime and this brane has a particular thickness and hence, matter content S_{ij}. Shiromizu *et al* [63] studied the gravitational field equations on a 3-brane with the Z_2 symmetry; the four dimensional world is described by a 3-brane which is essentially a domain world within a five dimensional bulk manifold.

[4] The Faraday tensor F_{ab} is always trace-free, i.e. $F^a{}_a = 0$. However, the electromagnetic field tensor E_{ab} is trace-free only in four dimensions.

The junction conditions for the embedding of a brane into a bulk manifold were developed by Davis [49]. The Davis junction conditions accommodate the existence of a brane and are given by

$$2\langle K_{ij} - Kh_{ij}\rangle + 4\alpha\langle 3J_{ij} - Jh_{ij} + 2\hat{P}_{iklj}K^{kl}\rangle = -\kappa_N^2 S_{ij}, \tag{8}$$

where $\langle \mathcal{X}\rangle = \frac{1}{2}[\mathcal{X}(\Sigma_+) + \mathcal{X}(\Sigma_-)]$ represents the average of the quantity $\mathcal{X}$ across the brane. Note that K_{ij} and K are the extrinsic curvature and its trace respectively, and h_{ij} is the induced metric[5]. In the above J is the trace of

$$J_{ij} = \frac{1}{3}\left(2KK_{ik}K^k{}_j + K_{kl}K^{kl}K_{ij} - 2K_{ik}K^{kl}K_{lj} - K^2K_{ij}\right), \tag{9}$$

and P_{ijkl} is the divergence-free part of the Riemann tensor, given by

$$P_{ijkl} = R_{ijkl} + 2R_{j[k}g_{l]i} - 2R_{i[k}g_{l]j} + Rg_{i[k}g_{l]j}. \tag{10}$$

The caret '$\hat{}$' indicates tensors associated with the induced metric, therefore the term $\hat{P}_{iklj}$ acts on this metric h_{ij}. Explicitly we can write

$$\hat{P}_{ijkl} = \hat{R}_{ijkl} + 2\hat{R}_{j[k}h_{l]i} - 2\hat{R}_{i[k}h_{l]j} + \hat{R}h_{i[k}h_{l]j}. \tag{11}$$

The quantity S_{ij} is the energy momentum tensor associated with the $(N-1)$-brane in the treatment of Davis [49]. It is important to note that these conditions hold on the Gauss–Bonnet brane world, and will have to be adapted for the case of a boundary hypersurface layer with zero thickness which is the case for a stellar model.

Such a surface is taken to be a hypersurface Σ with vanishing thickness. For our case we are matching two N-dimensional bulk spacetimes $\mathcal{M}^-$ and $\mathcal{M}^+$ across an $(N-1)$-dimensional hypersurface Σ with zero thickness (so there is no radial contribution $\mathrm{d}r = 0$). Therefore this $(N-1)$-dimensional hypersurface is embeddable into both of the bulk spaces $\mathcal{M}^\pm$, as opposed to a single bulk in the case of a brane. These features are presented in figure 1 respectively. For the Gauss–Bonnet brane world, we have an embedding map

$$\Phi : \Sigma_{\text{brane}} \longrightarrow \mathcal{V},$$

with the function Φ taking the $(N-1)$-brane isometrically into the bulk manifold $\mathcal{V}$, and all of the matter content S_{ij} is contained within this brane Σ_{brane} as it moves in the bulk. In the case of the two N-dimensional bulk spacetimes matching across the hypersurface $\Sigma_{\text{hypersurface}} = \Sigma$ [48, 60, 64–72], we have the following isometric embedding map

$$\Psi^\pm : \Sigma \longrightarrow \mathcal{M}^\pm,$$

where the coordinates on the boundary are mappable to either bulk spacetime via $\xi^i_\pm \mapsto x^a_\pm = \Psi^i_\pm(\xi^i_\pm)$. The quantities $x^a_\pm$ are individual points on the manifolds $\mathcal{M}^\pm$. This embedding map takes the boundary hypersurface of zero thickness (hence $S_{ij} = 0$) into both bulk manifolds $\mathcal{M}^\pm$, in which case the two bulk spaces are essentially *glued* together with $\Sigma\,(\hookrightarrow \mathcal{M}^\pm)$ acting as an interface.

[5] The general induced metric h_{ij} arises from the action

$$S = \int_\Sigma d^{N-1}\xi\sqrt{-h}K,$$

where $\mathbf{h}$ is the induced metric tensor, and K is the trace of the extrinsic curvature on the boundary Σ. This action is the Gibbons–Hawking–York term.

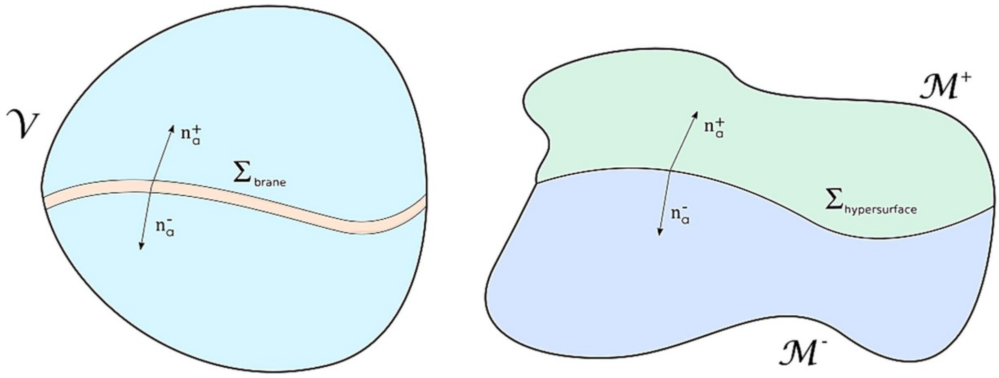

Figure 1. On the left we have an $(N-1)$-brane Σ_{brane}, with a nonzero thickness, which is embedded into a single N-dimensional bulk manifold $\mathcal{V}$ with the unit normal vectors $n_a^{\pm}$ which are orthogonal to the surface of the brane. On the right we have two N-dimensional bulk spacetime manifolds $\mathcal{M}^{\pm}$ which are matched across a single $(N-1)$-dimensional hypersurface layer $\Sigma_{\text{hypersurface}} = \Sigma$ with the unit normal vectors $n_a^{\pm}$ orthogonal to Σ with $\mathrm{d}r = 0$.

3.2. Stellar surface

Consequently for a stellar model in EGB gravity, the Davis conditions at the stellar surface become

$$[K_{ij} - Kh_{ij}]^{\pm} + 2\alpha\left[3J_{ij} - Jh_{ij} + 2\hat{P}_{iklj}K^{kl}\right]^{\pm} = 0, \tag{12}$$

across the hypersurface Σ. We have established the important result:

Theorem 1. *For the matching of two Lorentzian manifolds $\mathcal{M}^{\pm}$ across a hypersurface Σ, the junction conditions are given by*

$$(\mathrm{d}s_-^2)_{\Sigma} = (\mathrm{d}s_+^2)_{\Sigma} = \mathrm{d}s_{\Sigma}^2, \tag{13a}$$

$$[K_{ij} - Kh_{ij}]^{\pm} + 2\alpha\left[3J_{ij} - Jh_{ij} + 2\hat{P}_{iklj}K^{kl}\right]^{\pm} = 0, \tag{13b}$$

in EGB gravity.

We observe that the Israel-Darmois conditions (6*b*) in general relativity contain only the extrinsic curvature K_{ij}. The adapted Davis conditions (13*b*) in EGB gravity contain in addition, the trace K, the EGB coupling constant α and the divergence-free part of the Riemann tensor P_{ijkl}. It is also important to note that we have not directly utilised the matter content in the manifolds $\mathcal{M}^-$ and $\mathcal{M}^+$ to generate the condition (13*b*). The matching conditions therefore permit a variety of forms for the matter distributions in a stellar model.

The junction conditions in the form (13*b*) are difficult to analyse or evaluate due to their complicated structure. It is not possible to show analytically that it can be satisfied in general. However we can show that (13*b*) does admit solutions. To show this suppose that the Israel-Darmois conditions hold in EGB gravity so that

$$K_{ij}^- = K_{ij}^+. \tag{14}$$

This implies that the trace of the extrinsic curvature is satisfied

$$K^- = K^+, \tag{15}$$

across Σ. Since the quantity J_{ij} is defined in terms of K_{ij} and K, we have from (9), (14) and (15), that

$$J_{ij}^{-} = J_{ij}^{+}, \tag{16}$$

implying the condition on the trace

$$J^{-} = J^{+}. \tag{17}$$

We are now in the position to establish an important result for the trace-free part of the Riemann tensor.

Theorem 2. *If the Israel-Darmois conditions hold on the comoving hypersurface Σ, then the matching of the divergence-free part of the Riemann tensor is identically satisfied on the surface, i.e.*

$$P_{ijkl}^{-} = P_{ijkl}^{+} = \hat{P}_{ijkl}. \tag{18}$$

Proof. Davis [49] has written the Gauss–Codazzi equations in the form

$$R_{pqrs}h^{p}{}_{i}h^{q}{}_{j}h^{r}{}_{k}h^{s}{}_{l} = \hat{R}_{ijkl} + K_{jk}K_{il} - K_{ik}K_{jl}, \tag{19a}$$

$$n^{i}R_{iqrs}h^{q}{}_{j}h^{r}{}_{k}h^{a}{}_{l} = D_{l}K_{jk} - D_{c}K_{jl}, \tag{19b}$$

$$\hat{R}_{jl} = R^{i}{}_{qkp}h^{c}{}_{i}h^{q}{}_{j}h^{p}{}_{l} + KK_{jl} - K_{jk}K^{k}{}_{l}, \tag{19c}$$

where D represents the covariant derivative. Using the system (19) the normal vectors acting on the divergence-free part of the Riemann tensor on the hypersurface Σ, $n_{\pm}^{i}\hat{P}_{ijkl}$, can be expressed in terms of the extrinsic curvature $K_{ij}^{\pm}$, its trace $K^{\pm}$ and the induced metric h_{ij} as

$$\begin{aligned} n_{\pm}^{i}\hat{P}_{ijkl} = {} & 2D_{[l}K_{k]j}^{\pm} + 2D_{m}K_{\pm}^{m}{}_{[l}h_{k]j} + 2h_{j[l}D_{j]}K^{\pm} + 2\hat{G}_{j[k}n_{l]}^{\pm} \\ & + 2\left(K^{m}{}_{j} - Kh^{m}{}_{j}\right)^{\pm}K_{m[k}^{\pm}n_{l]}^{\pm} + \left(K^{2} - K_{im}K^{im}\right)^{\pm}h_{j[k}n_{l]}^{\pm}, \end{aligned} \tag{20}$$

where $\hat{G}_{ij}$ is the Einstein tensor on Σ. Using the condition that $K_{ij}^{-} = K_{ij}^{+}$ on Σ from (6b), as well as equation (19b), it can be shown that the terms containing covariant derivatives of the extrinsic curvature must match across Σ, and so are satisfied on Σ. Consider the first term in equation (20). It can be written as

$$2D_{[l}K_{k]j}^{\pm} = [D_{l}K_{kj} - D_{k}K_{lj}]^{\pm},$$

and invoking (6b), this is satisfied across Σ due to the Gauss–Codazzi equation (19b). A similar argument holds for the other two terms containing covariant derivatives. The remaining terms in equation (20) all match across the hypersurface Σ from (19b). We note that $\hat{G}_{ij}$ is contracted from $\hat{R}_{ijkl}$ which is given in terms of the extrinsic curvature K_{ij} by (19b). Therefore we have that

$$n_{\pm}^{i}\hat{P}_{ijkl} = 0,$$

on Σ. Hence equation (13b) is identically satisfied if and only if

$$P_{ijkl}^{-} = P_{ijkl}^{+} = \hat{P}_{ijkl},$$

on the hypersurface[6] Σ.

□

[6] We note that the divergence-free part of the Riemann tensor is nonzero in general.

We can then see that the EGB junction condition (13*b*), using the results (14)–(18), is identically satisfied. Hence, we have the following existence result:

Theorem 3. *If the Israel-Darmois conditions hold, then the EGB junction conditions for the matching of two spacetime manifolds* $\mathcal{M}^{\pm}$ *across a comoving hypersurface* Σ *are satisfied in general.*

In other words, a sufficient condition for the Davis junction condition (13*b*) at the hypersurface Σ to be satisfied is that the Israel–Darmois condition (6*b*) admits a solution. This is a physically important result as theorem 3 ensures that matching across a comoving boundary hypersurface Σ, connecting two manifolds, is possible in EGB gravity. It is therefore possible to model stellar structure in EGB gravity, and generate a complete model for a relativistic star.

4. Static star

We now consider the model for a charged spherically symmetric static and anisotropic star in EGB gravity with nonvanishing cosmological constant. Since the junction conditions in the previous section are for the general matching of two spacetime manifolds, we emphasise that the static star is a special case.

4.1. Interior spacetime

For the interior spacetime we make use of the N-dimensional spherically symmetric static line element

$$\mathrm{d}s_-^2 = -A^2\mathrm{d}t^2 + B^2\mathrm{d}r^2 + r^2\mathrm{d}\Omega_{N-2}^2, \tag{21}$$

in Schwarzschild coordinates where $A = A(r)$, $B = B(r)$. The metric for the unit $(N-2)$-sphere is

$$\mathrm{d}\Omega_{N-2}^2 = \sum_{i=1}^{N-2}\left(\left[\prod_{j=1}^{i-1}\sin^2(\theta_j)\right](\mathrm{d}\theta_i)^2\right). \tag{22}$$

The barotropic energy momentum tensor $\mathbf{T}$ consists of an imperfect fluid configuration of matter

$$T_{ab}^- = (\rho + p_{\|})u_a u_b + p_\perp g_{ab} + \left(p_{\|} - p_\perp\right)V_a V_b, \tag{23}$$

where ρ is the energy density, $p_{\|}$ is the radial pressure and $p_\perp$ is the tangential pressure. The fluid N-velocity $\mathbf{u}$ is given by $u^a = A^{-1}\delta^a{}_0$ and $\mathbf{V}$ is a unit radial vector orthogonal to $\mathbf{u}$. Isotropic matter is obtained when $p_{\|} = p_\perp$. The electromagnetic potential is chosen as

$$\Phi_a = (\varphi(r), 0, 0, \ldots, 0). \tag{24}$$

The only surviving Faraday tensor components are

$$F_-^{01} = -F_-^{10} = \frac{\varphi'(r)}{A^2B^2}. \tag{25}$$

Here, the prime indicates differentiation with respect to the radial coordinate r. From the respective Gauss-Faraday and Gauss-Ampére laws (2*b*) and (2*c*) we acquire

$$\varphi'' - \left(\frac{A'}{A} + \frac{B'}{B} - (N-2)\frac{1}{r}\right)\varphi' = \mathcal{A}_{N-2}\sigma AB^2. \tag{26}$$

This equation can be integrated to yield

$$\varphi' = \frac{AB}{r^{N-2}} l(r), \tag{27}$$

where

$$l(r) = \mathcal{A}_{N-2} \int^{r} \sigma B r^{N-2} \mathrm{d}\tilde{r},$$

giving the total charge within the hypersurface Σ.

The EGBM field equation (2) then take the form

$$\kappa_N \rho = \frac{N-2}{r^4 B^4}\left[r^3 B B' + \frac{N-3}{2} r^2 B^4 - \frac{N-3}{2} r^2 B^2\right] + \frac{\hat{\alpha}(N-2)(B^2-1)}{r^4 B^4}\left[2r\frac{B'}{B} + \frac{N-5}{2}(B^2-1)\right] - \frac{\kappa_N l^2}{2\mathcal{A}_{N-2} r^{2N-4}} - \Lambda, \tag{28a}$$

$$\kappa_N p_{\|} = \frac{N-2}{r^4 B^4}\left[r^3 B^2 \frac{A'}{A} - \frac{N-3}{2} r^2 B^4 + \frac{N-3}{2} r^2 B^2\right] + \frac{\hat{\alpha}(N-2)(B^2-1)}{r^4 B^4}\left[2r\frac{A'}{A} - \frac{N-5}{2}(B^2-1)\right] + \frac{\kappa_N l^2}{2\mathcal{A}_{N-2} r^{2N-4}} + \Lambda, \tag{28b}$$

$$\begin{aligned}\kappa_N p_{\perp} = {} & \frac{(N-3)(N-4)}{2r^2 B^2}(B^2+1) + \frac{A''}{AB^2} - \frac{A'B'}{AB^3} + \frac{(N-3)}{rB^2}\left(\frac{A'}{A} - \frac{B'}{B}\right) \\ & + \frac{\hat{\alpha}}{r^2 B^2}\left[2\left(\frac{A''}{A} - \frac{A'B'}{AB}\right) - 2\frac{A''}{A} + \frac{2(N-5)}{rB^2}(B^2-1)\left(\frac{A'}{A} - \frac{B'}{B}\right)\right. \\ & \left. + 6\frac{A'B'}{AB^3} - \frac{(N-5)(N-6)}{2r^2B^2}(B^2-1)\right] - \frac{\kappa_N l^2}{2\mathcal{A}_{N-2} r^{2N-4}} + \Lambda,\end{aligned} \tag{28c}$$

$$\sigma = \frac{l'}{\mathcal{A}_{N-2} B r^{N-2}}. \tag{28d}$$

In the above we have set $\hat{\alpha} = \alpha(N-3)(N-4)$ for convenience. We also note that if $N = 4$ these field equations reduce to the conventional four dimensional field equations for general relativity; the Gauss–Bonnet corrections cease to contribute to the gravitational dynamics.

4.2. Exterior spacetime

For the exterior spacetime, we take the higher dimensional vacuum Boulware–Deser-Wiltshire-(anti) de Sitter [9, 10, 17] solution

$$\mathrm{d}s_+^2 = -f(\mathrm{r})\mathrm{d}v^2 - 2\mathrm{d}v\mathrm{dr} + \mathrm{r}^2 \mathrm{d}\Omega_{N-2}^2, \tag{29}$$

in Eddington–Finkelstein coordinates. Here $d\Omega_{N-2}^2$ is given by (22) and

$$f(\mathrm{r}) = 1 + \frac{\mathrm{r}^2}{2\hat{\alpha}}\left(1 - \sqrt{1 + \frac{4\hat{\alpha}}{N-3}\left(\frac{2M}{\mathrm{r}^{N-1}} + \frac{2\Lambda}{(N-1)(N-2)} - \frac{\kappa_N Q^2}{(N-2)\mathcal{A}_{N-2}\mathrm{r}^{2N-4}}\right)}\right). \tag{30}$$

The use of the above coordinate system is not unique. Several prior works make use of these coordinates as they make subsequent calculations easier. The use of conventional Schwarzschild coordinates will yield the same end result. In the above, we recall that

$\hat{\alpha} = \alpha(N-3)(N-4)$ and M is the constant mass which will encompass the entire star upon completion of the matching. We note the presence of both the cosmological constant Λ and the charge contribution Q. The function $f(\mathsf{r})$ is the negative branch solution to the vacuum EGB field equations with cosmological constant and charge. The positive branch solution has been omitted as it does not contain the general relativistic limit, and so exists only in the EGB corrected theory. The above solution (29) satisfies the EGBM field equations and reduces to the N-dimensional Reissner–Nordström-(anti) de Sitter geometry when $\alpha \to 0$. A generalisation of Birkhoff's theorem was proven by Wiltshire [10]: *the only spherically symmetric solutions to the EGBM field equations are the Boulware–Deser-Wiltshire (29) ($\Lambda = 0$) and Bertotti-Robinson type solutions* [73, 74]. The metric (29) with $\Lambda = 0$ can be transformed to the Bertotti-Robinson solution; see [10] (page 38) for details. Further, when $N = 4$ the above solution reduces to the four dimensional Reissner–Nordström-(anti) de Sitter metric. When the charge vanishes, in both cases, we acquire the relevant exterior Schwarzschild geometries.

4.3. Matching

The $(N-1)$-dimensional induced metric to the boundary surface Σ which takes into account the representation of the interior spacetime $\mathcal{M}^-$ in comoving coordinates is of the form

$$\begin{aligned} ds^2_\Sigma &= h_{ij} d\xi^i d\xi^j, \\ &= -d\tau^2 + \mathcal{R}^2 d\Omega^2_{N-2}, \end{aligned} \tag{31}$$

where the unit $(N-2)$-sphere is given by (22). In the above, $\mathcal{R} = \mathcal{R}(\tau)$ and we have coordinates $\xi^i = (\tau, \theta_1, \theta_2, \ldots, \theta_{N-2})$. The coordinate τ is defined only on Σ. The reason for utilising a comoving boundary Σ is the fact that the mass function on Σ is no longer constant, but a function of the interior radial coordinate. The mass of the star contains all of the matter from the interior across any point on the comoving boundary. The unit spacelike normal vectors take the form

$$n^-_a = [0, B(r_\Sigma), 0, 0, \ldots, 0], \tag{32a}$$

$$n^+_a = [-\grave{\mathsf{r}}, \grave{v}, 0, 0, \ldots, 0], \tag{32b}$$

where we have that $\grave{} = \frac{d}{d\tau}$.

The first junction conditions (6a) then yield

$$A(r_\Sigma)\grave{t} = 1, \tag{33a}$$

$$r_\Sigma = \mathcal{R}(\tau), \tag{33b}$$

$$\mathsf{r}_\Sigma(v) = \mathcal{R}(\tau), \tag{33c}$$

$$\left(1 + \frac{\mathsf{r}^2}{2\hat{\alpha}}(1-\mathcal{F}) + 2\frac{d\mathsf{r}}{dv}\right)_\Sigma = \left(\frac{1}{\grave{v}^2}\right)_\Sigma, \tag{33d}$$

where have set

$$\mathcal{F}(\mathsf{r}) = \sqrt{1 + \frac{4\hat{\alpha}}{N-3}\left(\frac{2M}{\mathsf{r}^{N-1}} + \frac{2\Lambda}{(N-1)(N-2)} - \frac{\kappa_N Q^2}{(N-2)\mathcal{A}_{N-2}\mathsf{r}^{2N-4}}\right)},$$

for neatness. Therefore, the necessary and sufficient conditions for the matching of the two metrics across Σ are

$$(Adt)_{\Sigma} = \left(1 + \frac{\mathsf{r}^2}{2\hat{\alpha}}(1-\mathcal{F}) + 2\frac{\mathrm{d}\mathsf{r}}{\mathrm{d}v}\right)^{\frac{1}{2}}_{\Sigma}, \tag{34a}$$

$$r_{\Sigma} = \mathsf{r}_{\Sigma}(v). \tag{34b}$$

For the interior spacetime manifold $\mathcal{M}^-$, the extrinsic curvature components are calculated (using (7) and (21)) as

$$K^-_{\tau\tau} = \left(-\frac{1}{B}\frac{A'}{A}\right)_{\Sigma}, \tag{35a}$$

$$K^-_{\theta_1\theta_2} = \left(\frac{r}{B}\right)_{\Sigma}, \tag{35b}$$

$$K^-_{\theta_2\theta_2} = \sin^2\theta_1 K^-_{\theta_1\theta_1}, \tag{35c}$$

$$\vdots$$

$$K^-_{\theta_{N-2}\theta_{N-2}} = \left[\prod_{j=1}^{N-2}\sin^2(\theta_j)\right]K^-_{\theta_1\theta_1}. \tag{35d}$$

Using (7) and (29) with (30), the extrinsic curvature components for the exterior manifold $\mathcal{M}^+$ can be written, after a lengthy calculation, as

$$K^+_{\tau\tau} = \left(\frac{\grave{\grave{v}}}{\grave{v}} - \grave{v}\left[\frac{\mathsf{r}}{2\hat{\alpha}}\left(\frac{\mathcal{F}-1}{\mathcal{F}}\right)\right]\right)_{\Sigma}, \tag{36a}$$

$$K^+_{\theta_1\theta_1} = \left(\grave{v}\mathsf{r}\left[1 + \frac{\mathsf{r}^2}{2\hat{\alpha}}(1-\mathcal{F})\right] - \mathsf{r}\grave{\mathsf{r}}\right)_{\Sigma}, \tag{36b}$$

$$K^+_{\theta_2\theta_2} = \sin^2\theta_1 K^+_{\theta_1\theta_1}, \tag{36c}$$

$$\vdots$$

$$K^+_{\theta_{N-2}\theta_{N-2}} = \left[\prod_{j=1}^{N-2}\sin^2(\theta_j)\right]K^-_{\theta_1\theta_1}. \tag{36d}$$

The necessary and sufficient conditions for the validity of the second set of junction conditions (6*b*) to be satisfied are therefore

$$\left(-\frac{1}{B}\frac{A'}{A}\right)_{\Sigma} = \left(\frac{\grave{\grave{v}}}{\grave{v}} - \grave{v}\left[\frac{\mathsf{r}}{2\hat{\alpha}}\left(\frac{\mathcal{F}-1}{\mathcal{F}}\right)\right]\right)_{\Sigma}, \tag{37a}$$

$$\left(\frac{r}{B}\right)_{\Sigma} = \left(\grave{v}\mathsf{r}\left[1 + \frac{\mathsf{r}^2}{2\hat{\alpha}}(1-\mathcal{F})\right] - \mathsf{r}\grave{\mathsf{r}}\right)_{\Sigma}. \tag{37b}$$

It is possible to obtain the mass function M by eliminating the variables $\grave{v}$, r and $\grave{\mathsf{r}}$ from equation (37*b*). A tedious calculation yields the mass function to be

$$
\begin{aligned}
M = & \left\{ \frac{N-3}{2} \left[r^{N-3} \left(1 - \frac{1}{B^2} \right) - \frac{\Lambda r^{N-1}}{(N-1)(N-2)} \right. \right. \\
& \left. \left. + \frac{\kappa_N Q^2}{(N-2)(N-3)\mathcal{A}_{N-2} r^{N-3}} + \hat{\alpha} r^{N-5} \left(1 - \frac{1}{B^2} \right)^2 \right] \right\}_\Sigma \\
= & M_E + M_{GB},
\end{aligned} \tag{38}
$$

where we have defined

$$
\begin{aligned}
M_E = & \frac{N-3}{2} \left[r^{N-3} - \frac{r^{N-3}}{B^2} - \frac{\Lambda r^{N-1}}{(N-1)(N-2)} \right. \\
& \left. + \frac{\kappa_N Q^2}{(N-2)(N-3)\mathcal{A}_{N-2} r^{N-3}} \right],
\end{aligned} \tag{39a}
$$

$$
M_{GB} = \frac{N-3}{2} \hat{\alpha} r^{N-5} \left(1 - \frac{1}{B^2} \right)^2. \tag{39b}
$$

The first quantity M_E is the mass of the star in the general relativity limit [75–77]. The second quantity M_{GB} is the additional contribution from the EGB corrections. The total mass function (38) can be interpreted as the gravitational mass M of the star within the boundary hypersurface Σ.

From (33*a*)–(33*c*) we have that $\grave{\mathsf{r}} = 0$. Using this fact and substituting the mass function (38) into (37*b*) yields an expression for $\grave{\nu}$ as

$$
\grave{\nu} = \left[B \left(1 + \frac{r^2}{2\hat{\alpha}} (1 - \mathcal{D}) \right) \right]^{-1}, \tag{40}
$$

where we get

$$
\begin{aligned}
\mathcal{D} = & \left(1 + \frac{8\hat{\alpha}}{r^{N-1}} \left[\frac{r^{N-3}}{2} - \frac{r^{N-3}}{2B^2} - \frac{\Lambda r^{N-1}}{(N-1)(N-2)} \right. \right. \\
& \left. \left. + \frac{\kappa_N Q^2}{(N-2)(N-3)\mathcal{A}_{N-2} r^{N-3}} + \frac{\hat{\alpha}}{2} r^{N-5} \left(1 - \frac{2}{B^2} + \frac{1}{B^4} \right) \right] \right)^{\frac{1}{2}},
\end{aligned} \tag{41}
$$

and where we used the fact that $r = \mathsf{r}$ on Σ. Since the expression (40) depends solely on the radial coordinate r, we have that the second derivative $\grave{\grave{\nu}} = 0$. Therefore, upon substituting (40) into (37*a*), a lengthy calculation finally yields the boundary condition at the surface of the star

$$
\begin{aligned}
& \left\{ \frac{N-2}{r^4 B^4} \left[r^3 B^2 \frac{A'}{A} - \frac{N-3}{2} r^2 B^4 + \frac{N-3}{2} r^2 B^2 \right] + \Lambda + \frac{\kappa_N Q^2}{2\mathcal{A}_{N-2} r^{2N-4}} \right. \\
& \left. + \frac{\hat{\alpha}(N-2)(B^2-1)}{B^4} \left[\frac{2A'}{r^3 A} - \frac{N-5}{2r^4} (B^2 - 1) \right] \right\}_\Sigma = 0.
\end{aligned} \tag{42}
$$

This is a highly nonlinear differential equation involving the Gauss–Bonnet coupling constant $\hat{\alpha}$, the cosmological constant Λ and the charge Q from the exterior. It is important to realise that this result holds on the comoving surface Σ. Using the field equation (28*b*), the above expression is equivalent to writing

$$
\left(p_\| - \frac{l^2}{2\mathcal{A}_{N-2} r^{2N-4}} + \frac{Q^2}{2\mathcal{A}_{N-2} r^{2N-4}} \right)_\Sigma = 0. \tag{43}
$$

On the boundary Σ of the star, we must have that $l = Q$. Therefore the barotropic radial pressure satisfies the condition

$$p_{\|}\big|_{\Sigma} = 0. \tag{44}$$

Therefore the barotropic radial pressure $p_{\|}$ vanishes at the stellar boundary Σ in EGB gravity, as is the case in general relativity. However, it is important to realise that despite the fact that $p_{\|} = 0$ on Σ, it contains contributions associated with the second order Lovelock tensor through the Gauss–Bonnet coupling constant $\hat{\alpha}$ from the field equations (2). The corresponding differential equation to be solved at the boundary Σ is different in EGB gravity as can be seen by the EGBM field equation (28*b*).

The absence of our general analysis involving the Davis junction conditions may lead to incorrect physical models. We have shown that the Israel-Darmois conditions of general relativity imply that the Davis junction conditions in EGB gravity are satisfied. There are some other points to be observed from the boundary condition in EGB gravity. Firstly, the boundary condition depends on the Gauss–Bonnet coupling constant $\hat{\alpha}$, the spacetime dimension N, the charge Q and the cosmological constant. Secondly the spacetime dimension $N = 5$ is special as the barotropic pressure term in (28*b*) reduces to a simpler form. Note that ρ and $p_{\perp}$ in the EGBM field equation (28) also reduce to a simpler form when $N = 5$.

We can now state the following theorem:

Theorem 4. *Consider two N-dimensional manifolds connected by the $(N-1)$-dimensional hypersurface Σ. The interior spacetime is static with anisotropic matter, an electromagnetic field and cosmological constant. The exterior spacetime is the Boulware–Deser-Wiltshire-(anti) de Sitter metric. The boundary condition in EGB gravity at the stellar surface is then*

$$p_{\|}\big|_{\Sigma} = \left[p_{\|_E} + p_{\|_{GB}}\right]_{\Sigma} = 0,$$

which relates the Gauss–Bonnet coupling constant $\hat{\alpha}$, the spacetime dimension N, the charge Q and the cosmological constant Λ.

5. Special cases

The boundary condition (44) holds for a charged anisotropic matter distribution with cosmological constant in EGB gravity. It shows that the total radial pressure $p_{\|}$ on the surface is a vanishing quantity. This condition presents as a highly nonlinear differential equation which depends on the interior gravitational potentials $A(r)$ and $B(r)$, their derivatives, the cosmological constant Λ, the dimension N of spacetime and additionally, the Gauss–Bonnet coupling constant α. Some special external spacetimes in N dimensions which are of physical interest in EGB gravity and general relativity, are indeed special cases of our result.

5.1. EGB gravity

The interior spacetime is spherically symmetric and static, with a charged distribution of matter with anisotropy. The exterior spacetimes which form a subclass of our general result are then given by the following:

- Boulware–Deser ($\Lambda = Q = 0$):

$$ds^2 = -f(\mathrm{r})dv^2 - 2dv d\mathrm{r} + \mathrm{r}^2 d\Omega^2_{N-2},$$

with

$$f(\mathsf{r}) = 1 + \frac{\mathsf{r}^2}{2\hat{\alpha}}\left(1 - \sqrt{1 + \frac{4\hat{\alpha}}{N-3}\left(\frac{2M}{\mathsf{r}^{N-1}}\right)}\right).$$

- Boulware–Deser-(anti) de Sitter ($Q = 0$):

$$\mathrm{d}s^2 = -f(\mathsf{r})\mathrm{d}v^2 - 2\mathrm{d}v\mathrm{d}\mathsf{r} + \mathsf{r}^2\mathrm{d}\Omega^2_{N-2},$$

with

$$f(\mathsf{r}) = 1 + \frac{\mathsf{r}^2}{2\hat{\alpha}}\left(1 - \sqrt{1 + \frac{4\hat{\alpha}}{N-3}\left(\frac{2M}{\mathsf{r}^{N-1}} + \frac{2\Lambda}{(N-1)(N-2)}\right)}\right).$$

- Boulware–Deser-Wiltshire ($\Lambda = 0$):

$$\mathrm{d}s^2 = -f(\mathsf{r})\mathrm{d}v^2 - 2\mathrm{d}v\mathrm{d}\mathsf{r} + \mathsf{r}^2\mathrm{d}\Omega^2_{N-2},$$

with

$$f(\mathsf{r}) = 1 + \frac{\mathsf{r}^2}{2\hat{\alpha}}\left(1 - \sqrt{1 + \frac{4\hat{\alpha}}{N-3}\left(\frac{2M}{\mathsf{r}^{N-1}} - \frac{\kappa_N Q^2}{(N-2)\mathcal{A}_{N-2}\mathsf{r}^{2N-4}}\right)}\right).$$

5.2. General relativity

In the case when $\hat{\alpha} \to 0$, we then have that the charged interior with anisotropy is matched to the higher dimensional metrics from general relativity, which are given by:

- Schwarzschild:

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{(N-3)\mathsf{r}^{N-3}}\right)\mathrm{d}v^2 - 2\mathrm{d}v\mathrm{d}\mathsf{r} + \mathsf{r}^2\mathrm{d}\Omega^2_{N-2}.$$

- Schwarzschild-(anti) de Sitter:

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{(N-3)\mathsf{r}^{N-3}} - \frac{2\Lambda\mathsf{r}^2}{(N-1)(N-2)(N-3)}\right)\mathrm{d}v^2 - 2\mathrm{d}v\mathrm{d}\mathsf{r} + \mathsf{r}^2\mathrm{d}\Omega^2_{N-2}.$$

- Reissner–Nordström:

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{(N-3)\mathsf{r}^{N-3}} + \frac{\kappa_N Q^2}{(N-2)(N-3)\mathcal{A}_{N-2}\mathsf{r}^{2N-6}}\right)\mathrm{d}v^2 - 2\mathrm{d}v\mathrm{d}\mathsf{r} + \mathsf{r}^2\mathrm{d}\Omega^2_{N-2}.$$

Therefore the analysis in this paper includes both the boundary conditions of N-dimensional general relativity ($\hat{\alpha} = 0, p_{||_{GB}} = 0$) and the extensions to EGB gravity ($\hat{\alpha} \neq 0, p_{||_{GB}} \neq 0$). Ours is a unified treatment.

6. Matching

The results found in this paper can be used to match any static charged interior matter distribution to the exterior Boulware–Deser-Wiltshire spacetime in EGB gravity. To illustrate the matching across the stellar boundary Σ we consider the exact solution found by Bhar and Govender [29] with $N=5$ for a charged static sphere. The model satisfies the requirements for a physically acceptable relativistic sphere and has a quark equation of state.

The Bhar-Govender model has the interior metric potentials in (21) with the forms

$$A^2 = e^{\tilde{b}r^2+\tilde{c}}, \qquad B^2 = e^{\tilde{a}r^2}, \tag{45}$$

where $\tilde{a}$, $\tilde{b}$ and $\tilde{c}$ are constants. For the quark equation of state

$$p = \beta\rho - \gamma, \tag{46}$$

we obtain the matter variables

$$\rho = \frac{1}{3B^4(1+\beta)\pi^2r^2}\left[-12\alpha(\tilde{a}+\tilde{b}) + 3\gamma\pi^2r^2B^4 + 3B^2(\tilde{a}+\tilde{b})(4\alpha+r^2)\right], \tag{47a}$$

$$p_{\|} = \frac{1}{3B^4(1+\beta)\pi^2r^2}\left[-12\alpha\beta(\tilde{a}+\tilde{b}) + 3\gamma\pi^2r^2B^4 + 3\beta B^2(\tilde{a}+\tilde{b})(4\alpha+r^2)\right], \tag{47b}$$

$$l^2 = \frac{1}{2(1+\beta)r^2}\left[12\alpha(\tilde{b}-\tilde{a}\beta) + B^4(3+3\beta-3\pi^2\gamma r^2) -3B^2\left(1+\beta+(\tilde{b}-\tilde{a}\beta)(4\alpha+r^2)\right)\right]. \tag{47c}$$

The exterior spacetime is the Boulware–Deser-Wiltshire metric with $\Lambda=0$ and $N=5$. Matching at the stellar surface $r=R$ gives the conditions

$$e^{-\tilde{a}R^2} = 1 + \frac{R^2}{4\alpha}\left(1-\sqrt{1+\frac{8\alpha M}{R^4}-\frac{2\alpha Q^2}{R^6}}\right), \tag{48a}$$

$$e^{\tilde{b}R^2+\tilde{c}} = 1 + \frac{R^2}{4\alpha}\left(1-\sqrt{1+\frac{8\alpha M}{R^4}-\frac{2\alpha Q^2}{R^6}}\right). \tag{48b}$$

The stellar condition $p_{\|}=0$ then gives a value for $\tilde{b}$ in the form

$$\tilde{b} = \frac{\tilde{a}}{\chi}\left[4\alpha\beta + R^2e^{2\tilde{a}R^2}(1+2\beta+4\alpha\beta\tilde{a}) - \beta e^{\tilde{a}R^2}(4\alpha+R^2)\right], \tag{49}$$

with $\chi = -4\alpha\beta + e^{\tilde{a}R^2} + R^2e^{2\tilde{a}R^2}(1+4\alpha\tilde{a})$. From (48$a$) and (48$b$) we obtain

$$\tilde{c} = -(\tilde{a}+\tilde{b})R^2. \tag{50}$$

From the above we observe that $\tilde{a}$ follows from (48a). The constant $\tilde{b}$ is given by (49), and $\tilde{c}$ follows from (50) for specific choices of α, β and γ. The charge $Q=l(R)$ contains α, β, γ, $\tilde{a}$, $\tilde{b}$ and $\tilde{c}$. Hence, values for $\tilde{a}$, $\tilde{b}$ and $\tilde{c}$ are obtainable once the mass M and the radius R of the star are fixed. Alternatively it is possible to fix values of $\tilde{a}$ and R to obtain $\tilde{b}$, $\tilde{c}$ and M for different values of the coupling constant α. The latter approach is followed in [29] to study the behaviour of a quark star in EGB gravity.

7. Conclusion

In this article we have constructed a higher dimensional gravitating and electrically charged static star in the presence of Λ in EGB gravity. This involved solving the Davis [49] conditions on the surface Σ. We presented the formalism of EGB gravity as well as the modified field equations, and then analysed the Davis junction conditions in EGB gravity [49] for a braneworld. It was proved that for a timelike hypersurface separating two spacetime manifolds, i.e. a boundary of zero thickness, the Israel-Darmois conditions of general relativity imply that the Davis conditions are satisfied. We then made use of this fact to generate the matching conditions for a static and charged star with nonvanishing cosmological constant in EGB gravity in higher dimensions. It was shown that the barotropic radial pressure $p_{\|}$ on the boundary of the star vanishes so that

$$p_{\|}\big|_{\Sigma} = 0.$$

When the Gauss–Bonnet coupling constant $\hat{\alpha} \to 0$ we then regain the boundary condition of N-dimensional general relativity. This is an important physical result as it allows us to have a complete model of a gravitating static star in EGB gravity. It then becomes possible to test various physical properties and parameters like the energy conditions, sound speed, adiabatic index, surface redshift and the collapse rate at late times. This is the basis of ongoing work. The results in this paper should be extendable to the radiating star case where the pressure at the boundary does not vanish. The presence of the higher order curvature corrections from the second order Lovelock tensor should influence the evolution of the radiating star.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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ORCID iDs

Byron P Brassel https://orcid.org/0000-0002-7297-1004
Sunil D Maharaj https://orcid.org/0000-0002-1967-2849
Rituparno Goswami https://orcid.org/0000-0002-5355-3288

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