

What are the strategies for teaching and learning mathematics that can be used effectively in a multilingual classroom?

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ABSTRACT— This paper reports on a study which explores the mathematics teaching strategies used in a multilingual classroom at technical and vocational education and training colleges in South Africa. Effective teaching and learning strategies are required that will motivate students to study and understand mathematical concepts, as well as engage in both procedural and conceptual discourses. Brijlall showed that in mathematics classes collaborative learning has a higher rate of success than does individual learning. He recommends further study on this teaching strategy. This study seeks to draw up some strategies which lecturers can use to teach mathematics effectively in a multilingual classroom. Effective strategies should lead to better understanding of mathematical concepts with subsequently improved performance in mathematics examinations. Hence, this study is important in addressing the call for greater research in this area.

KEYWORDS: teaching strategies, learning strategies, higher education.

1. INTRODUCTION

Language plays an important role in the development of deep conceptual understanding of mathematical ideas. Students learning mathematics in a language that is not their home language need explicit and deliberate linguistic support. Different strategies used by teachers to help improve students' learning in such settings have been highlighted by writers such as Clarkson [1]. Clarkson's studies came up with the following four ideas 1) Teachers should encourage different types of language such as informal talk in students' first language leading to more formal mathematical talk in the language of teaching. 2) Tracing the language paths of students in the complex multilingual situations 3) Informal talk inevitably occurs in students' first languages. This can lead to 'broken communication' when the teacher does not share the students' first language. Hence helping the students to move to more formal mathematical talking and writing, which often involves switching to the language of teaching can have unknown linguistic setbacks 4) Teachers need to use academic mathematical language and promote an expectation that students will come to use such language. Encouraging students to discuss ideas in their own language has proven to have a positive impact on students' conceptual understanding of mathematics [2]. However, oversimplifying the language of instruction has been shown to inhibit students' acquisition of mathematical concepts by obscuring them. [2] proposed more comprehensive ways of dealing with the language issues in mathematics classrooms. These include focusing on students' mathematical reasoning, moving away from simplifying language, recognising and supporting students to engage with the language complexities in mathematics, treating everyday language as resources and not as obstacles and finally uncovering the mathematics in what students say or do. Multilingualism in mathematics classrooms is not only a South African problem. Most countries around the world share that problem. For example, Sweden, America, Catalonia and Cyprus, to mention only a few, have multilingual classrooms due to demographic changes and international migration. To elaborate, in Swedish multilingual classrooms studies have shown that students with foreign backgrounds are less successful in mathematics than their Swedish classmates [3]. A second language learner needs scaffolding to make progress in both linguistic and mathematical skills [2]. Previous research has emphasised the importance of teachers taking responsibility for

enabling students' development of both language and mathematical skills. But one might question whether lecturers at TVET colleges, especially those who teach the N1 to N6, short three months engineering courses, have time to cater for this. In mathematics classrooms there is a complex mix of everyday language and discipline specific language and mastering both is the key to success. Being able to shift between the two is regarded by some researchers to be the key to developing mathematical understanding.

2. Literature Review

2.1 Some Teaching Strategies for Mathematics

The process of teaching and learning is very challenging, as it requires a teacher to ensure that learning takes place [3]. According to [4], the teacher has to design an appropriate learning environment and acquire and utilise appropriate teaching approaches to incur genuine interest among the students, so they participate in their own learning. Most teachers tend to teach content at the expense of understanding. They concentrate on the delivery of the textbook content, whether the students understand it or not. Teachers should prioritise learner understanding over covering content. According to [5], there are both traditional and innovative methods, individual and group methods, and teacher-centred and learner centred methods that could be used to teach effectively. Traditional teaching methods are mainly teacher centred, content laden, whereby students are passive learners, promoting rote learning and being exam oriented [6]. According to [3] the lecture method, sometimes referred to as the talk and chalk method, is the most popular among traditional methods, and is the most widely used at all levels; primary, secondary, or high schools, and at tertiary institutions. Students taught using traditional methods usually forget what they have been taught and cannot apply what they have learnt across contexts. For example, at TVET they learn the laws of logarithms from N1 to N4, but in N5 when they encounter logarithmic differentiation, they cannot apply these laws. At N4 they study complex numbers in both Mathematics and Industrial Electronics modules, but they cannot apply what they learn in one subject to another subject.

However, Innovative teaching methods are learner centred, they encourage collaboration, cooperation, contextual learning, and they are problem centred and include lots of activities [7], [6]. Innovative teaching methods include, amongst others, peer tutoring, problem-based learning, and discovery enquiry. A study done in Nigeria showed that at tertiary level mathematics and science and technology teachers seldom used discussion, question and answer, simulation, collaborative, or peer tutoring. They almost always used the lecture method or the rote learning method. In a few cases they used demonstration, experimental and project methods. The problem-based teaching method was one of the methods that was never used. To make an appropriate selection of teaching methods basic knowledge of the characteristics of a good teaching method is needed. Reading on work written on teaching methods will help teachers become familiar with the current trends in teaching methods or from attending seminars or workshops [8]. Lecturers at TVET colleges should benefit from such practices. [9] mentioned that, when adapting teaching strategies, the following should be taken into consideration: 1) learning styles 2) the skills and competencies to be developed, and 3) content of a subject. This study will investigate the teaching strategies the lecturers at some TVET colleges in South Africa use. Research carried out in Florida, United States of America showed that peer tutoring and cooperative learning were among the strategies that had proved to be effective in diverse classrooms of middle school classes. Peer tutoring is also effectively used at some universities in South Africa [1]. This strategy therefore seems to work right through the learning system. It would be worthwhile for the TVET colleges to also try this strategy.

At TVET colleges, the use of visuals could also be effectively used in teaching topics like mensuration and the application of differentiation and integration. To make sense of problems from these topics drawing a

diagram helps and sometimes the teacher needs to have a model or something concrete to illustrate the problem. Visuals have been found to be helpful when teaching students whose first language is not English [10]. Visuals enhance learning as they engage different senses, accommodate students who learn visually, and help reinforce key ideas. [36] notes that using visual teaching aids ensures that students attach meaning and images to words using concrete instructional materials. Different types of visuals can help stimulate the love to learn mathematics, including at tertiary level.

2.2 Explicit or Direct Instruction

Explicit instruction is characterised by clear descriptions and demonstration of a skill followed by supported practice and feedback given on time [11]. In this strategy the teacher leads in solving problems on the board, guiding the students through the questions and gains the students' attention by asking probing questions [12]. Students can discuss ideas with each other in any language. The teacher checks all the work and re-teaches any concepts with which students have difficulties. Due to the limited lecture time, lecturers tend not to allocate enough time, if at all they do, for discussion among students during class time. Informed discussions with fellow TVET lecturers indicate that the only feedback they give students is after the tests, but there are only two tests, which determine whether a student qualifies to write the exam or not.

According to Archer & Hughes, 2011, explicit instruction follows six key functions. The teacher:

1. Reviews relevant learning and pre-requisite skills and knowledge.
2. Presents instruction by stating the learning intentions and presents new material in small steps; modelling, providing examples and non-examples, using clear language.
3. Provides guided practice that requires a high frequency of responses, ensures high rates of success, provides timely feedback and prompts while students continue to practice until they are accurate.
4. Provides positive and corrective feedback.
5. Provides independent practice that is monitored during initial practice attempts and continuous practice as students build fluency. This can be achieved through monitoring of classwork and homework.
6. Provides spiral reviews of previously learned concepts over time.

When done properly Explicit instruction gives students a chance to practice and apply newly acquired knowledge in the future. This method would be beneficial to TVET lecturers and students.

2.3 Peer Tutoring

Peer tutoring allows students to work in pairs, giving each other chances or turns to lead in answering questions and explaining and can also be one tutor to many students. According to [13], the teacher facilitates the learning process and provides supportive information. This is a very effective strategy in a multilingual classroom because it gives the students an opportunity to discuss ideas in their own language. Students of different abilities and backgrounds become resources and teachers to each other. They end up relating better to each other than they do with the teacher. The strategy promotes communication, motivates the students, and helps students improve in the subject knowledge [14], [15]. The tutor gains confidence and appreciation and understanding of others. The other student learns to speak out and ask questions and seek clarification. With proper planning and execution, this strategy helps students to understand the subject content better. Peer tutoring positively affects the goal of students and increases their self-esteem and confidence. It helps students build relationships and provides emotional support and positive role models [16]. [17] in their study state that participation in peer tutoring markedly improved engineering students' success in a first-year calculus course. TVET students could greatly benefit from peer tutoring models operating after lecturer time.

2.4 Cooperative Learning

In Cooperative learning the teacher provides small mixed-ability groups with problems that give them

opportunities to practice different mathematical skills and concepts. Working in groups helps to develop the students' level of self-esteem and motivates them to learn. Research shows that this method has shown some improvement among students who are non-English speaking and among students with learning disabilities. According to [18], it assists students from different backgrounds in developing intellectual autonomy. [19], implemented the 'Learning Together' model of cooperative learning and found a significant positive effect of cooperative learning models on mathematics achievement. Students can be given individual tasks, discuss their solutions in pairs and come out with one good solution which they will then present to the class. Cooperative learning models contribute to the improvement of students' performance [20]. [21] shows that students working in groups solve problems better than those working individually. [22] allude that cooperative learning is more effective in acquiring cognitive behaviours, and positively affects the students' relationships, increasing their confidence and improves their attitude towards school. Cooperative learning offers opportunities for positive social interaction and interpersonal communication, which are important for providing positive learning outcomes [23], [24].

2.5 Problem Solving

[25] recommends [26] problem solving for use during problem solving. In problem solving students can work towards a solution through the guidance of a teacher. The following activities make up the problem-solving model: read and think, explore, and explain, select strategies, find answers, reflect and extend. These activities help in developing students' positive attitudes towards the subject and improve their creative thinking abilities [27]. During the problem-solving approach students get the opportunity to construct their solution strategies for solving problems using their own understanding of the mathematical concepts involved. Equipping Students at school with problem-solving skills builds them in solving complex real-life problems [28]. Students with better prior knowledge tend to have better problem-solving abilities [27]. [28] also argued that one of the best ways to help students to acquire better content understanding is providing them with the opportunity to learn through problem solving and inquiry learning because both include critical thinking.

3. Conceptual Framework

The basis of this study is Bruner's theory of cognitive constructivism. In constructivism a teacher is a facilitator who monitors, guides and encourages critical thinking, including encouraging teamwork. Constructivism allows students to develop skills and confidence to analyse the world around them, create solutions and justify their words and actions. Bruner proposes a cognitive constructivist approach, in that learning is an active social process whereby new knowledge is constructed by students through exploration of their world and through the filter of their prior knowledge. According to [29], Bruner's ideas on cognitive psychology examine the thoughts and reasoning of people in addition to how they respond to stimuli. Bruner's cognitive theory of instruction encourages educators to create instruction that leads the student through a sequence of statements until the student masters the content. The teacher's role in a constructivist model includes setting up conditions that enable students to discover relationships between concepts. Bruner believes a learner can be encouraged and supported with assistance [30]. It also provides greater motivation, so students subsequently remember more.

4. Methodology, participants

This section focuses on the research design, participants and ethical issues that were used in the study.

4.1 Research Design

Since the aim of this study was to craft effective teaching and learning strategies in mathematics in a multilingual classroom at TVET college level, the research was carried out at a TVET college in KwaZulu-Natal. A qualitative case study was carried out. In order to collect qualitative data, interviews and class

observations were conducted, participants were respected and given a chance to answer freely without the interviewer's influence [31]. Interview and observation schedules were prepared beforehand to ensure that the same general areas of information were collected from the interviewees and from the observations. The emphasis of the interviews was on the effective teaching strategies in a multilingual classroom.

4.2 Population

In this study, the population consisted of three lecturers and 120 students from two campuses of a TVET College in Kwazulu-Natal comprising of N5 mathematics lecturers and their students. Both campuses have mainly IsiZulu speaking students but there are a few who speak a variety of other languages. The lecturers helped with the selection of the students for the focus groups. Students were chosen according to their home languages. From Campus 1 we selected three Xhosa speaking students, two Sotho speaking students, two Swati speaking students and three isiZulu speaking students. From Campus 2, six IsiZulu speaking students, three Swati speaking students and one Xhosa speaking student were selected.

4.3 Interviews and Observations

Lecturers were interviewed individually since it was impossible to have them at the same time. Class observations were also done with the same three lecturers with their full classes. Focus groups were used for interviewing the students. The interviews were audio recorded and transcribed. The transcribed data was given to the participants to verify if it was an accurate reflection of the interview. Thematical analysis was used to analyse the data. For the observations, observation schedules were drawn which looked for the teaching methods, the language used during the lessons and the effectiveness of the teaching strategy and students' success rate. Same themes created from the interview data were used for the observations.

4.4 Ethical Issues

Full ethical considerations were adhered to in order to address research bias. Participants' privacy was taken into consideration; pseudonyms were used for all participants. Ethical clearance was granted by Unisa and the college authorities gave the research permission to carry out the study.

5. Analysis and Discussion

Teaching Strategies

Both lecturers and students have their own methods, which they think are more effective.

Teaching strategies students think will help them improve:

Students want to be taught for understanding not to finish the syllabus.

Student C1S7 says "you must teach so that I can understand, so that I can pass, that's the purpose". According to the students, the purpose of classes should be teaching, learning, and understanding; not simply to finish the syllabus. C1S4 says "He (the lecturer) just explains what he has done on the board, not explaining what is really happening". This comment shows that students want to understand conceptually, rather than simply getting to the answer.

Another idea is put forward by C1S5, who says "... lecturers don't understand, we lack basics, that's the problem". This student implies that the lecturers are assuming too much prior knowledge. These are N5 students who want to be taught the basics involved in every chapter before embarking on a new chapter. C1S5 goes on "... and my answer is different, and I don't know how to simplify". These students struggle with simplifying addition, subtraction, multiplication, and division of fractions. C1S5 also believes that if they are taught the basics they will improve. He says "If we get the basics, basics are essential in mathematics, right

and we forget the basics. And lecturers don't understand that we forget basics, you know. Yeh they need to emphasise on basics like this and that yeh, that's it". To explain and justify why they lack basics, C1S10 says "... we spend two months sitting at home, we do not study so we forget". It should be noted that, when the Nated Report, 191 students have completed their trimester examinations, they return their books to the college and go on holiday. Those who pass the examinations, on return for the next trimester they proceed to the next level. Therefore, they have no textbooks to help them study during the break. However, if they were serious about their studies, they could revise using their notebooks. Due to the lack of revision or practice while on holiday, they remember very little that was taught during the previous trimester as mentioned by student C1S10. According to [30], to avoid knowledge gaps, Bruner's discovery learning can be used to help students to be creative and motivated to remember. By implementing optimal sequencing of material to lead students logically through the content, lecturers can help the students can master the content. Praise can also be used to motivate students [32].

Lack of basics does play a major role in the performance of these students. For example, before learning differentiation of logarithmic functions, they need to be taught or to at least revise the laws of logarithms before asking them to differentiate something like $y = \ln e^{2x} 3^x$.

Students think the following will help them understand mathematics better:

- Group Work

Student C1S7 would prefer more group work: "Lecturers should also ask us to do some group work stuff". When working in groups, the students use their home language. Asked how speaking in their home language helps them to understand mathematics the students responded in the following manner:

C1S4 says "I understand better because that person he or she just explains more further, she goes deep when explaining in my language."

C1S5 says "Some mix Zulu and English. Like for example, when dealing with a question with product and quotient and chain rule, the individual will explain to me that is simpler when you look at it like a function of a function. The individual can say 'a function within a function. So, they use Zulu plus a bit of English'. I kind of get it much better because the individual is simplifying better and that helps a lot".

Research has shown that group work is one of the methods that enhances students' understanding and grasping of concepts. Students are empowered, as indicated in Polya's problem solving method. Working in groups gives them an opportunity to discuss ideas in their own language, translate these for one another and present their work in the language of teaching and learning. According to [4], learning and using additional languages lead to structural adaptations in the brain. Lecturers can use teaching strategies such as cooperative learning, which make use of group learning, to develop students' self-esteem and level of understanding. [19] allude that students who learn mathematics through cooperative learning models do better than those who learn using other forms of instruction. Cooperative learning is student centered and helps lecturers in classroom management.

- Clear Explanations

Commenting on the lecturers' explanations, student C1S7 says "... I feel lecturers all they need to do is explain the procedure on how to do things, sums. That affects us a lot because if you don't know the procedure, you can't know the sum. But if he or she explains the procedure very well, then we can be able to attempt everything he or she gives".

In the same light, C1S9 says "... there is that teacher, I would love all teachers to do. That educator, I won't say the name. She did one which specifically deals with common factor, one which deals with difference of two squares and one quadratic. What the other lecturers they would do, they would teach you quadratic only

... That lecturer teaches us that way and I would like all lecturers to do that. Maybe we will improve". According to this student, lecturers are teaching part of a concept, they are not covering all aspects involved. Step by step explanations of each concept seem to be lacking. Some lecturers expect students to fill in the gaps, which they find difficult due to the conceptual gaps in their mathematical knowledge.

In a similar vein, C1S10 says "... Sometimes what kills us is the lecturer goes and writes up the sum on the board, then dot, dot, dot (...), you know how it is completed.... This kind of does affect us. Do best by completing the sum so that by the end of the day I know how this sum is completed." Due to their lack of basic knowledge, these students cannot complete most of these exercises.

Bruner's theory of instruction puts emphasis on clear explanation of the content. Although these are tertiary students they seem to need to be taught in a simplified sequential way. Lecturers need to complete their statements and not leave students to fill in the gaps. Over estimation of students' prior knowledge, forces lecturers not to complete their examples. According to [33], sequential teaching of concepts helps students understand and master mathematical content. These TVET students, given proper guidance and encouragement can develop critical thinking skills.

- **Student Centred Approach**

Students want to be in control of their learning, and they want time to explore and do problem solving on their own.

Student C1S10 proposes that the lecturers should give them a day in a week where students work on problems on their own in groups in the absents of the lecturer. C1S7 agrees, saying "... Lecturers need to give us some time... Maybe we explain some of the problems using our competent knowledge. It can help other students. They will have the thing that they will have passion on it. They will ask even that thing they can't ask the lecturer because we are so open on our own." He also suggests that the lecturer could be present monitoring their work, as they discuss it after having worked on their own. These students are advocating use of Polya's method. [25] recommends the use of Polya's problem solving model and the use of any language chosen by the students during group work.

Student C1S4 and C1S5 say they understand better when they are working in groups because they go into deeper explanations when using their mother tongue. Switching to students' own language helps when students do not understand [33]. Shifting between the mother tongue and the mathematical language is a key to success (ACARA,2012; [34]. [32] suggest that code switching can play an important role in improving the quality of classroom discussions and interactions.

Student C2S5 says "Sometimes it's not about we have to rely on the lecturer, usually we help one another..., like we are different. I can understand the other chapter better than my friend and the friend can understand the other chapter better so like share information and help one another". This student is advocating Peer tutoring. Peer tutoring gives students a chance to practise while receiving feedback from the lecturer [13]. In Peer Tutoring self-esteem and confidence in students are increased.

Students are eager to understand mathematical concepts, but they want the lecturers to change their teaching strategies and adopt strategies that would help them to understand. C2S4 emphasises this when she says teaching techniques and speed of teaching are crucial in determining the levels of understanding. According to the study by [17], peer tutoring improved the students' success in calculus. In South Africa Peer tutoring is an important intervention utilised at university level [17]. TVET colleges could also benefit from this type of tutoring.

Cognitive constructivism talks about the effective manner by which teachers should present material to the learner, which is just what these students are asking for. A child's level of intellectual development is determined by the extent to which he or she has been given instruction and practice [35].

What lecturers think will help improve students' understanding.

Lecturers believe that using laptops, overhead projectors and computer software would help improve the teaching process and foster students' understanding. However, the college had removed all overhead projectors from the classrooms because quite a number of them had been stolen.

Lecturer L2C1 is of the opinion that laptops will save him time and will make teaching easier. He believes that with a laptop and access to the internet one can use tutorials from YouTube to clarify some concepts. He says, "At the beginning of the lesson give or show five minutes of a You Tube video of a Topic... because they might have better illustrations and staff...". He is of the opinion that use of the laptop and the overhead projector bridges the gap caused by difference in languages. According to [36] visuals can help stimulate the love for Mathematics. Currently the college is in the process of procuring laptops for all its lecturing staff.

Lecturer L3C2 thinks the current N5 curriculum is too extensive for the time that is currently allocated for its teaching. He also thinks the textbook in use lacks a lot of detail and clear explanations, which makes it difficult for students to follow on their own.

L1C1 believes in analysing problems with the students, using the students' language in conjunction with the language of teaching and learning. This is in line with the problem-solving method, where, according to [12] the lecturer leads in by asking guided questions. According to [37] code switching plays an important role during such discussions. The study by [20] shows that students do better when working in groups like this.

Learning Skills***What students think will help Improve their understanding.***

Most students think they need to "Put more effort" and "Practise" if they have to improve in Mathematics.

- Practise Mathematics

Student C1S9 is of the opinion that making a personal study timetable to utilise all the free time they have could be beneficial. He said "There are so many hours you can use for study. That's our problem, we don't study. We just wait for the lecturer to come and lecture. We should utilise that time we have. Midnight try to do something, tomorrow try to do something and do something...". C1S4 agrees with him, and says "After cooking you have extra time, so we should be taking that time to equip yourself as an individual".

C1S10 suggests "...set an alarm. When you wake up you work till morning, until you are tired. Just make time for yourself". C1S7 has the view about taking responsibility: "I guess we can have some similar mindset, to work hard to study even though we have limited time, but put yourself under pressure to push, push, push. Because all you need is to push till you make it". Most of these students are living away from home for the first time and are renting rooms close to the college.

These responses show us that students know they must practise, but two problems arise. Some of the students simply do not practise; they need a push from somewhere or from someone and some need appropriate practice. Students need assistance that will help them to work on their own without becoming discouraged. Student C1S5 says: "Oh, they say practice makes perfect. But we practise, sometimes you open the book, the first sum that you are doing will crush you and you don't know what to do.... That sum crushed your mind, you could not go forwards". Similarly, C2S3 says "Practise... I have been practising day by day but somewhere, somehow eesh". There should be a relationship between the students and their lecturers such that when the students practise and fail, they should approach their lecturers for clarification and help to identify where they are going wrong.

- Attitude to learning Mathematics.

A positive mindset is necessary to foster success, as suggested by student C1S9. He says "If you love mathematics, you will be able to do it. Love is the key, love Maths." According to a study by Yesildere-Imre (2017), both students and mathematics teachers suggested that a good mathematics teacher should make students love mathematics. Student C1S9 agrees with them that there is a positive outcome if students love

mathematics. Other students, however, think their attitude is affected by lecturer's comments and attitude. C1S2 says "Can lecturers stop saying "You did this in N3, or you did this in N4". C1S10 says "... if a lecturer is teaching and he or she sees one student outstanding or understanding, he or she must not assume that we all are understanding... The lecturers should not judge us according or based on this person." According to [8] Low expectations of some students make them think that only a select few can succeed in mathematics. Lecturers should avoid comparison among students and rather put more effort into fostering learning goals [17]. Lecturers should avoid stereotypical judgements of students. Lecturers should know their students' individual barriers to learning and help them to overcome them. They should teach the positive role of failure, mistakes, and challenges in learning [33]. Lecturers should support students' individual learning processes by giving them positive remarks, not protecting them from challenges and give critical constructive feedback. [37] allude that understanding the reasons behind students' behaviour helps to motivate the students.

Analysis of data from observations

- Teaching Strategies

From the observations we can see that the three lecturers use mainly the lecture method. In their interviews they did state that they use the lecture method and question and answer. There was not any formal group work planned in any of the four lessons observed, although they said they normally used group work. Lecturer L2C1 teaches by using worked examples, which does not go down well with the students. During the interview, student C1S10 says "If the lecturer teaches us worked example" while making facial expressions of dissatisfaction. A variety of teaching methods and approaches should motivate students and achieve better results. Research shows that teaching strategies that involve the participation of the students, where the students are active in solving problems, are more effective in inculcating a love for the subject, which then leads to better understanding of concepts. [25] recommends the use of Polya's problem solving model and the use of any language during group work. [16] argue that structured cooperative learning has a significant beneficial effect on students' achievement and attitude in mathematics. Lecturer L3C2 teaches also uses group work in addition to the lecture method. The group work did not seem to have been carefully planned. If he put more attention to his group formation, he might have achieved his goals. He asked students to work in pairs. Students just paired themselves with the person sitting next to them. This was not very helpful because in some of the pairs both individuals were weak in mathematics, they had little understanding of what was going on and thus achieved nothing by the end of the lessons. If he knew his students, he was going to pair students with different abilities for better results. Marking students' work would help lecturers to know their students and it would also enable them to help themselves appropriately.

- Learning Strategies

Students learnt by asking questions and working on the exercises given during classwork. There is evidence of lack of willingness to think and lack of motivation on the part of the students in some of their solutions. In L2C1's classes students were passive learners since he was explaining worked examples from the book. These students needed to be part of the problem solving, answering probing questions instead of merely listening to explanations in every lesson. Students' learning activities affect the achievement of mathematics learning outcomes. With this lecturer, only a few students who were confident in their English were asking questions and participating. Students in L3C2's classes learnt through whole class participation and listening when their colleagues asked in their mother tongue and they discussed in their mother tongue. [15] argued that students benefit from working together, one gains confidence and appreciation of others while the other one learns to speak out and ask questions.

- Time

Lack of teaching time forced lecturers to use only one type of teaching method, which might not be the most

effective. More time allows them to use a variety of teaching strategies, gives them an opportunity to thoroughly plan and this could help enhance students' understanding of mathematical concepts. Some students mentioned lack of time during interviews, as a reason for lecturers teaching too quickly leading to the students' failure to grasp the mathematical concepts. Having more time allows the lecturers to be facilitators who monitor, guide, and encourage critical thinking among the students. L2C1's students appear to want to be spoon-fed all the way. During observations they kept on asking 'Can you please do number so and so for us.' This could be because they are lazy, or their behaviour might be driven by the teaching strategy used. The lecturer went through two examples on sine substitution and one on tan substitution when he was teaching integration by trigonometric substitution. Students struggled with the example when the coefficient of x^2 was 1 and when a^2 was not a perfect square, as in $\int \frac{dx}{\sqrt{4-x^2}}$ and $\int \frac{2dx}{7-x^2}$. The lecturer wrote these questions on the board and tried to relate them to the worked examples, while asking leading questions. At the end of the period, the lecturer did not have time to summarise his lecture due to the time taken up by the many queries from the students. Using appropriate, well planned teaching methods saves time in the long run. L3C2, tries to use pairing, but did not implement it effectively; perhaps because of lack of planning which could be due to lack of preparation time or inclination. He simply paired off the students who sat together. Ideally, one of the pair should be better in the subject than the other, or at least good enough at English to understand the lecturer, and then explain the problem to the other in their mother tongue. Time for planning, time for teaching and learning are all crucial in implementing a good teaching strategy.

- Class sizes

A reasonable or manageable class size allows the lecturer to explore teaching strategies that involve participation of students which empower the students. Each of the three classes observed had less than 30 students, which according to the lecturers, is a manageable number. Nevertheless, although the class sizes were manageable, the lecturers did not make effective use of the relatively small numbers. They only paired off the students, which was not very effective because there were no criteria used in pairing the students. Students paired themselves with the person sitting next to them. So, some pairs worked with no communication between them. Other pairs could not assist each other, because both individuals were poor at mathematics. Some pairs discussed the work in their mother tongue. This type of pairing was not very productive. When lecturer L3C2 was teaching integration of trigonometric functions, he addressed leading questions to the students to encourage them to come up with the solutions. He went around checking the work and assisting those who needed help. He addressed common mistake or misconception on the board. The lecturer did not manage to reach all the pairs and he did not get time to summarise his lesson. This shows that even if the class size is manageable, planning how to execute one's lesson is important.

- Teaching Strategies

A lot of planning and attention to detail is needed to make a teaching strategy effective. Teaching by explaining worked examples only, and leaving new examples incomplete, demotivated the students. A variety of teaching strategies helps to liven up the lectures and inculcate a love for the subject. Lecturer L3C2 tried to use a variety of strategies, although they were not properly planned. Giving more time to planning would have produced better lessons and benefited the students. Our lecturers in this study seem to need to go the extra mile. The way they teach shows that they have an idea of what to do to get good results in terms of better understanding of mathematical concepts by the students. However, they appear to not be giving it their best effort. They try pairing up students, but no criteria were used to pair off the students, so they end up with some pairs not working together and others not knowing how to even start answering the given questions. Using the same method of teaching for every topic and concept is boring and demotivating for the students, thus we find that the same students participate. Good effective methods should motivate students to be active in their learning would be more appropriate.

6. Conclusion

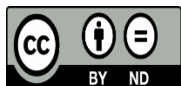
The study showed that teaching without understanding forces students to aim for the answer without understanding the procedure. Teaching for completion of syllabus and not for understanding contributes greatly to students' lack of conceptual understanding. Some students were suggesting lecturers should not aim to teach to finish the syllabi at the expense of their understanding the concepts. Appropriate teaching strategies, well planned and well executed, help in developing the language and the understanding of mathematical concepts.

7. References

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