

DEVELOPMENT OF PROTOTYPE UCAV
AIRFRAME COMPONENTS USING
ADVANCED COMPOSITE MATERIALS

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Submitted in partial fulfilment of the academic requirements for the Degree
of Master of Technology in the Department of Mechanical Engineering at
the Durban Institute of Technology.

SUBMISSION APPROVED FOR EXAMINATION

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JUNE 2004

To my parents

DECLARATION

I declare that this dissertation is my own unaided work except where due acknowledgement is made to others. This dissertation is being submitted for the Degree of Master of Technology to the Durban Institute of Technology, Durban and has not been submitted previously for any other degree or examination.

Kenneth Gary Jordan

June 2004

ACKNOWLEDGEMENTS

I would like to thank my supervisors Prof. David Jonson and Prof. Mark Walker for the opportunity to undertake this research project and who encouraged me to achieve my objectives. I would like to give a special thanks to Prof. David Jonson again for his wisdom and guidance without whom this dissertation would not have been possible. I would also like to thank my colleagues for their assistance and my parents for their encouragement and support.

ABSTRACT

The study presented here addresses the design of the composite wing and canard structures for an uninhabited combat air vehicle. The design philosophy is based on a combination of finite element analysis and mathematical programming. The wings and canards were manufactured using advanced composite materials. the manufacturing methodology was based on a rapid prototyping approach using 3D computer models and CNC machining.

The theory of composite materials is covered in detail, attention is given to the properties of the separate constituents, composite material properties and manufacturing methods that are relevant to the project. The finite element method and sequential linear programming are discussed in the context of structural analysis and optimisation. An overview of the methodology and how it is implemented is presented. Numerical optimisation techniques are discussed with particular emphasis being placed on sequential linear programming. The optimisation problem formulation is presented in detail with attention paid to elements and their formulation as well as design variables, constraints and sensitivity analysis.

Two design concepts were considered for the wing and canard structures, the first being a conventional configuration and the second being a novel radial design. The development and evaluation of these structural concepts are presented in detail. The optimisation study done on the canard is also presented as well as the manufacture thereof. Details regarding the manufacturing methodology used in the construction of the canard for the uninhabited combat air vehicle are presented in detail with particular

emphasis placed on the rapid prototyping method employed. The approach used incorporates the use of 3D computer design and CNC machining to manufacture the tooling.

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CHAPTER 1

INTRODUCTION

In our highly competitive world it is becoming ever more important to introduce new products that out perform competitors as soon as possible. This is the fundamental driving force that has inspired the vast amount of research and development that has gone into the likes of predictive analysis, optimisation techniques and rapid prototyping methods. The predominant reason for the huge improvements rest with the availability of modern computers and associated software. Much effort has gone into material development, where the emphasis has shifted from metals to alternative materials. Although, many years ago, the mechanical properties of composite materials (one such alternative material) were proven, only recently have they received the attention and respect from both industry, and the public alike, that they deserve.

Composite materials are discussed in detail in Chapter 2. The current attitude towards composite materials is one of acceptance and open mindedness as their capabilities are becoming increasingly accepted. Composite materials are in the forefront of material research at present, after years of exponential growth of the industry, and are receiving much of the exposure formerly reserved for high-tech alloys and specialized ceramics. As a result composite materials are slowly gaining the respect from industry that they deserve and are finding their way into almost every aspect of modern living including automotive, sporting equipment and numerous other applications. This is largely due to their high specific strength and stiffness properties although there are a number of other

attractive properties. The composite industry's biggest technological advancements in the recent past have been with improved materials systems such as pre-impregnated and more recently resin film infusion systems. These systems ensure that the optimal material properties are consistently achieved by having more control over the system.

In Chapter 3 the finite element method and optimisation techniques are discussed fully. Mankind has always aspired to improve his environment with the design of tools, structures, machines and even elaborate mathematical theories. Over the last century in particular there have been many promising theories put forward, however it's been largely due to the advent of computers that the high computational expense of these mathematical procedures have become less of a hindrance. As a result both finite element analysis and optimisation techniques are being used extensively in industry today. Although finite element analysis and optimisation methods are becoming more popular there is a great deal of research being done to improve them and make them more user friendly. Of particular interest is the work being done on integrating the two processes into a seamless and easy to use process as the use of finite element analysis coupled with an optimisation technique makes for a powerful design tool. Once a model is optimised the results can be incorporated into the analysis to produce a design solution, ultimately minimising or maximising the design objective as required.

The study, discussed in Chapter 4, describes the methodology adopted in the development of prototype wing and canard structures for an uninhabited combat air vehicle. The study details the development of the wing from the conceptual design phase through to the optimisation phase. Two design concepts were considered for the wing design and were evaluated with the use of finite element analysis and sequential

linear programming. The canard's development was similar to that of the wing and a prototype was manufactured. The manufacturing methodology is based on a rapid prototyping approach utilizing 3D models and advanced CNC machining to manufacture the tooling. The tooling is required in the processing of advanced composite materials including resin film infusion (RFI) materials which require high temperature tooling. A bladder system of inflatable silicon bags were used in the manufacture of the canard webs.

CHAPTER 2

COMPOSITE MATERIALS

2.1 INTRODUCTION

As the world contemplates its move into the new millennium, many sectors of society are assessing what the future may hold. Nowhere is this more evident than in the composites industry [1]. Laminated composite materials are used with ever increasing frequency in various technical applications, particularly in the fields of automotive, aerospace and marine engineering. This is primarily due to the high specific strength and stiffness properties that these materials offer [2].

The definition of a composite material is flexible and can be tailored to suit specific requirements. In this text a composite material is considered to be heterogeneous as it consists of two or more distinct constituent materials or phases with significantly different macroscopic behavior and a distinct interface between each constituent (on the microscopic level) [3]. This definition would include bricks, concrete, wood, bone, however for the purposes of this dissertation the term “composite” will refer to modern synthetic fibre-reinforced plastics (FRP).

Composites, as with many technological advances throughout history, predictably have their origins in military applications both from manufacturing and analysis viewpoints. The importance of composites, both in military and civilian applications, have

experienced steady growth over the last 50 years and is projected to continue to increase through the next several decades [4].

This chapter serves as an introduction to composite materials. Particular attention is paid to the different constituents that make up composites and the properties that result from their amalgamation. The mechanics of composites as well as manufacturing processes are also discussed in depth.

2.2 TYPES OF COMPOSITE

There are two main types of composites, classified according to the type of reinforcement namely particulate and fibrous. A particulate composite is characterised as being composed of reinforcement particles suspended in a matrix. These particles can be of virtually any size or configuration. A general example of a particulate composite would include concrete and particle board however if we regard a composite to be of a more synthetic nature (as defined above) then examples would include industrial components such as machine parts, valves and gears. The latter examples may be constructed from very short chopped fibres, of carbon or glass and combined with a synthetic matrix and compression or injection-moulded. This process affords greater flexibility in the moulding of far more complex structures whilst retaining many of the properties of continuous fibre composites such as resistance to creep, corrosion and fatigue as well as retaining high specific strength and stiffness behaviour [3].

2.2.1 FIBROUS COMPOSITES

The second type of composite are fibrous composites which are considered to consist of either continuous (long) or discontinuous (chopped or whiskers) fibres suspended in a matrix material [3]. Composites in which the reinforcement fibres are discontinuous have a random configuration and are commonly used in the form of chopped strand mat. Discontinuous reinforcements are considered to produce a material response that is anisotropic, this is due to the random nature of the reinforcement. In many instances the response of the composite can be assumed to be isotropic, however this is a crude simplification. Continuous reinforcement fibres, which have a biased orientation, are generally found in the form of rovings, which can be unidirectional (woven or continuous). The material behaviour of a continuous fibre composite is orthotropic [3].

As previously stated a composite is a heterogeneous material consisting of two distinct constituents. The first component is the matrix material, which is a plastic material (usually a thermoset resin) used to provide shape, colour and finish. The second component is the reinforcement, which can be in strand or mat form and imparts mechanical strength [5]. A single layer of a composite is termed a lamina and layered composites are generally referred to as laminates. The final laminate's properties are sensitive to the orientation and stacking sequence of the individual laminae and when constructing the laminate each lamina is specifically orientated to achieve the desired mechanical response [4].

2.2.2 REINFORCEMENT

There are a wide variety of reinforcements commercially available, and the number is increasing as the technology is continuously evolving with the vast amount of fibre research being done. Glass fibre was first used in the 1930's however it was only in the 1950's that glass fibres were produced that displayed stiffness properties suitable for structural applications [3]. The most commonly used reinforcements today are glass, carbon and aramid. These fibres are generally used in the continuous or chopped form and have diameters ranging from 3 to 200 μm [4], and lengths ranging from 25 to 50 mm. The continuous form of reinforcement is a collection of fibres (up to 120) called rovings, the type typically used in the filament winding process (discussed later) where piping or large tanks are made. Reinforcement is most commonly used in the mat format where it is supplied as large sheets of either chopped strand mat (CSM), woven rovings or stitched. CSM is a random collection of fibres held together by a binding agent. Woven rovings are used to a great extent and has/have biaxial properties, as the rovings are woven together at 90 degrees ~~of~~ to each other as shown in Figure 2.1 below.

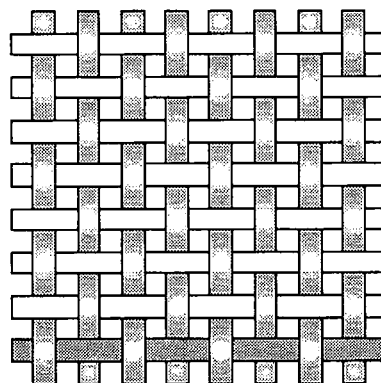


Figure 2.1 Plain weave woven roving.

Stitching is similar to woven but without any crimping of the rovings by effectively combining two layers of unidirectional materials without effecting the mechanical properties [6]. Typical properties of popular fibres are shown below in Table 2.1.

Material	Young's Modulus GPa	Tensile Strength GPa	Density g/cm³	Poisson's Ratio
E-Glass	71	2.4	2.54	0.22
Carbon	235	3.5	1.76	0.2
High Strength Carbon	295	5.6	1.74	0.2
High Modulus Carbon	690	3.3	2.17	0.2
Kevlar	124	3.6	1.45	0.34

Table 2.1 Material properties of fibre reinforcements. [3]

Refer to Figures 2.2, 2.3, 2.4, 2.5 and 2.6 showing material properties for various construction materials.

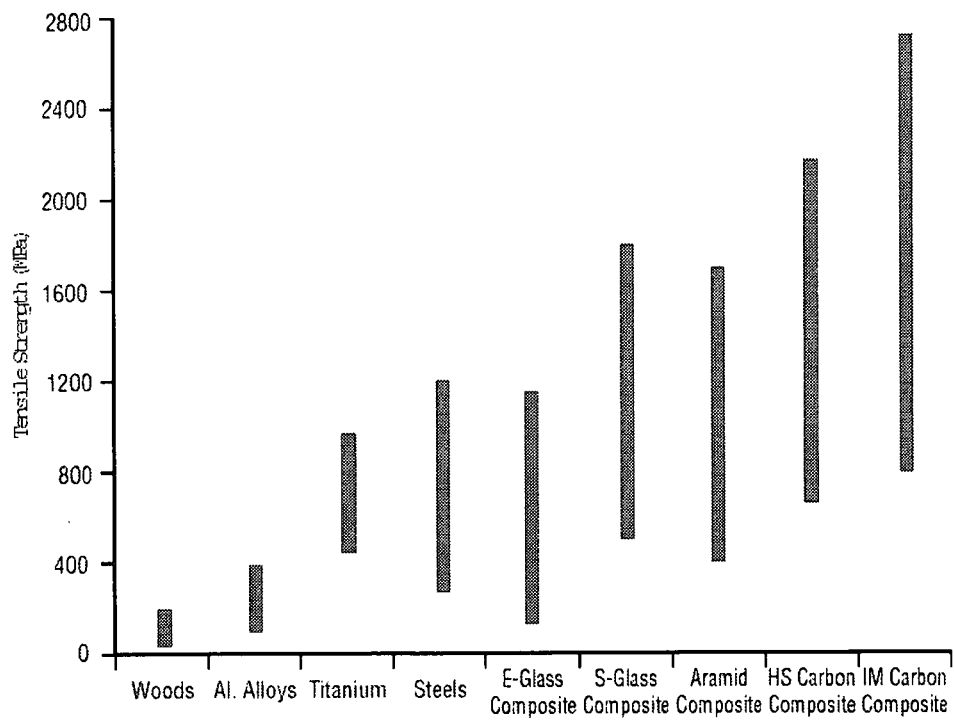


Figure 2.2 Tensile strength of common structural materials. [7]

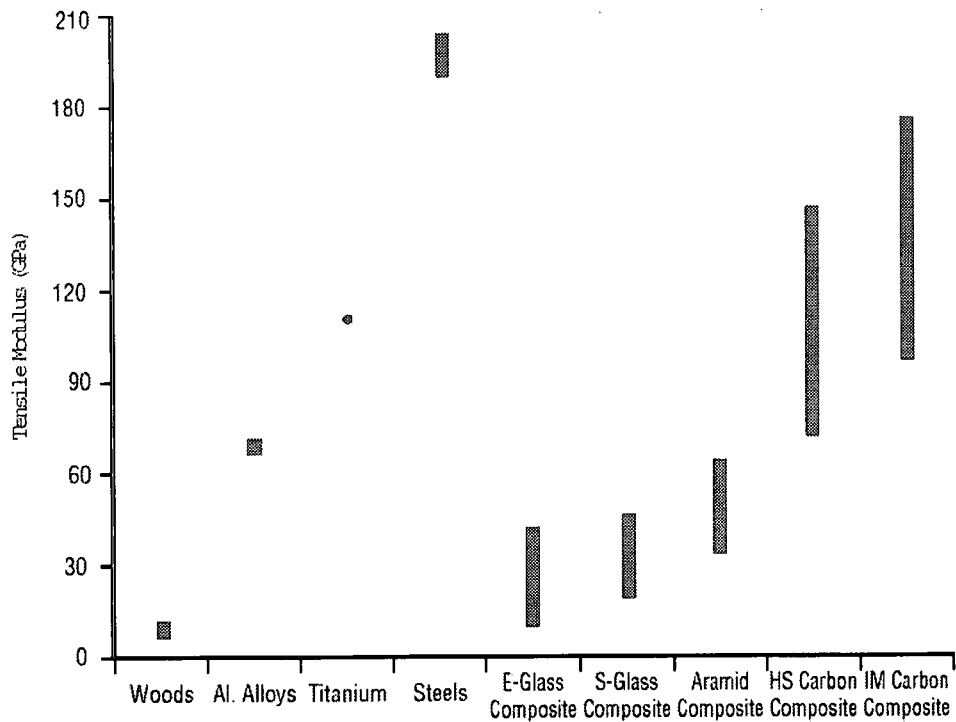


Figure 2.3 Tensile modulus of common structural materials. [7]

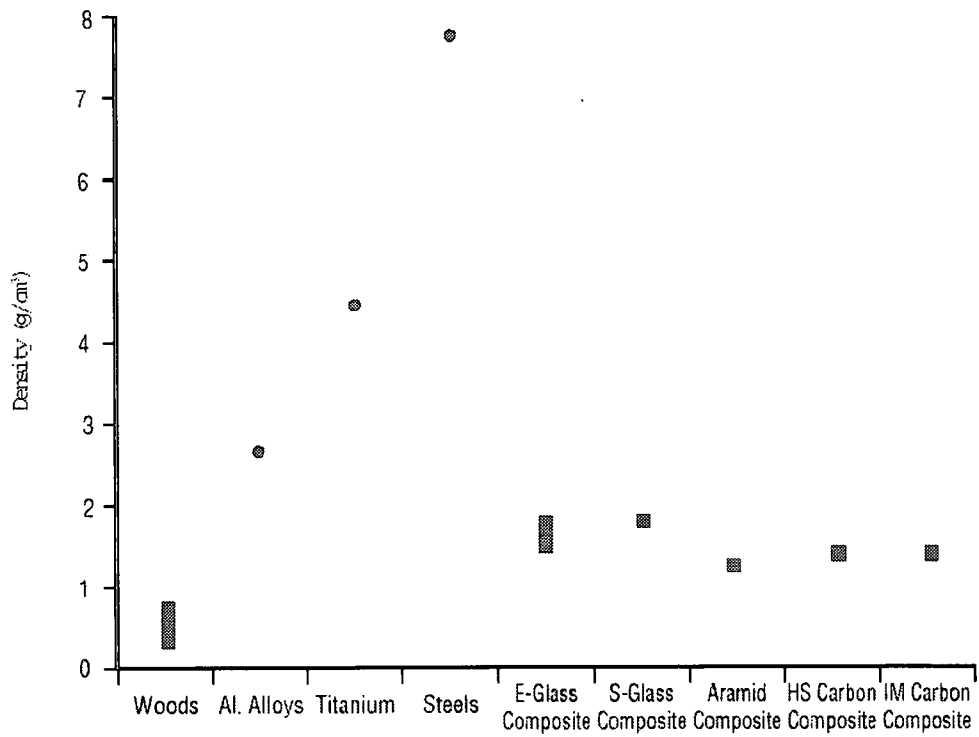


Figure 2.4 Density of common structural materials. [7]

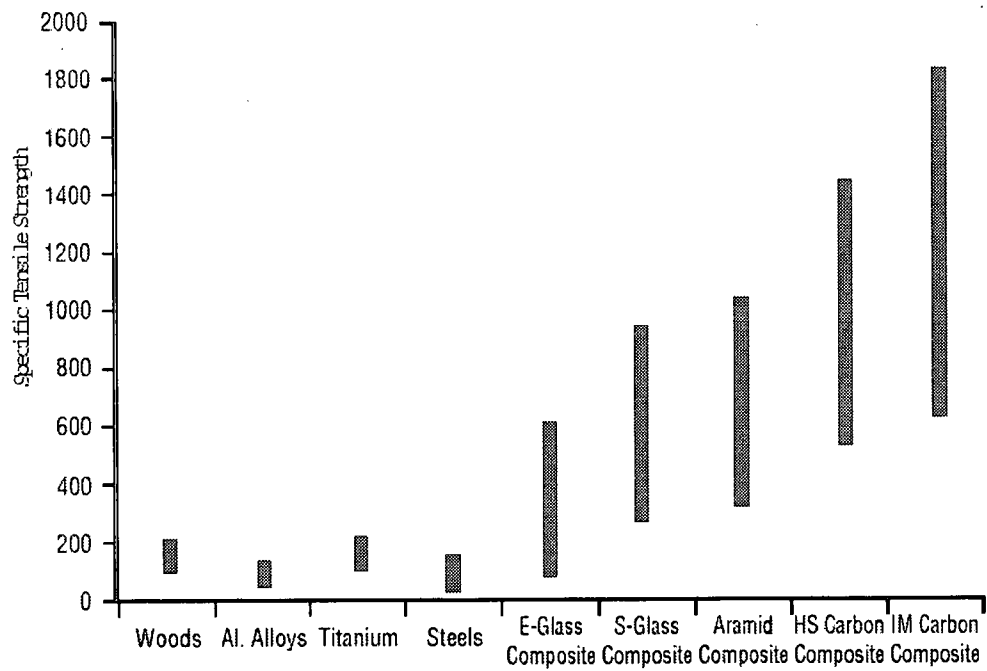


Figure 2.5 Specific tensile strength of common structural materials. [7]

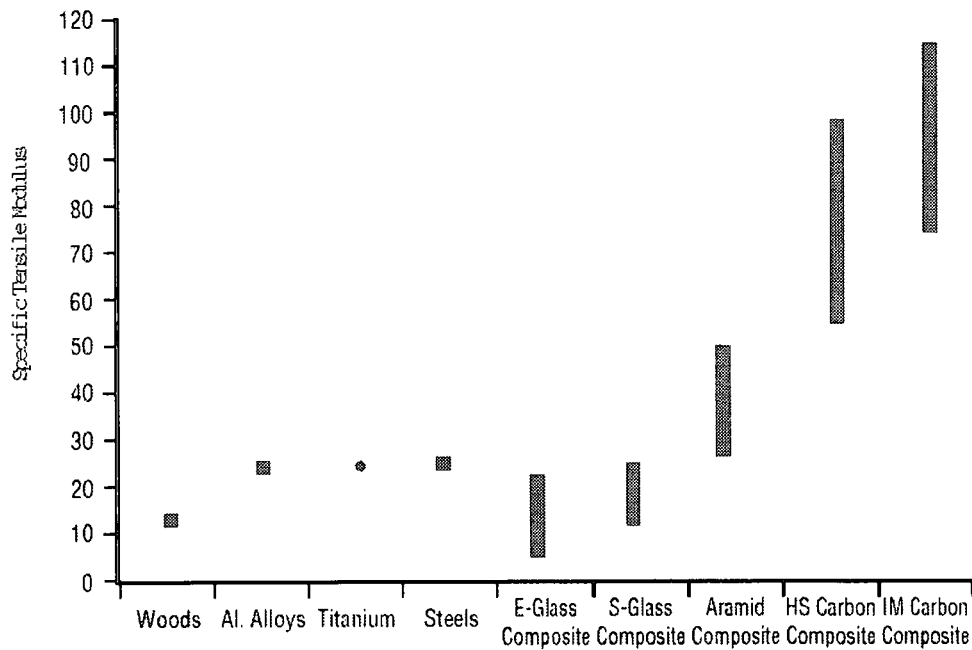


Figure 2.6 Specific tensile modulus of common structural materials. [7]

2.2.2.1 GLASS FIBRES

Glass fibre is a very important engineering material due to its high specific strength at low cost, however it has a relatively low modulus when compared with the other fibres available. As a result glass fibre is not considered to be an advanced composite. Glass fibres are available in a variety of forms providing desirable properties according to their intended use such as electrical resistivity, chemical resistance or stability under temperature [3], however it is the lower grade E-glass that is produced on the largest scale [8].

Glass fibre is produced by feeding molten glass through numerous holes in a gravity-fed tank. A chemical sizing (coating) is applied to the fibres to protect them as well as bind them together to form a continuous strand (or tow). The fibres that result are small in diameter, isotropic, and very flexible [3].

2.2.2.2 CARBON FIBRES

Carbon fibre is a broad definition which includes graphite fibres. The difference between carbon and graphite is the carbon content, carbon fibre contains 80-95% carbon whereas graphite fibres contains in excess of 99% [3]. For the purpose of this dissertation the general term carbon fibre will be used to denote both carbon and graphite fibres.

Carbon filaments are produced using a method that involves controlled pyrolysis (chemical decomposition by heat) of a precursor material in fibre form. There are three different precursors commonly used, which include polyacrylonitrile (PAN), rayon, and pitch fibres (made by spinning a petroleum-based substance). Fibre properties vary considerably with the fabrication temperature, which ranges from 1000 to 3000°C. Individual carbon filaments have a diameter range of 4 to 10 μm , as a result tows can consist of between 3000 to 30,000 filaments. The smaller filament size and tow arrangement is very flexible. The morphology of the fibres is highly dependent on the precursor used; with the PAN-based fibre for example an "onionskin" appearance is observed while the Pitch-based fibre has a radial appearance. As a result the properties of carbon fibre are anisotropic [3].

Carbon fibre has a higher modulus and lower density than glass fibre, and therefore has a higher strength to weight ratio. As a result carbon fibre is widely used as a reinforcement in polymeric materials for a wide range of lightweight, high strength applications. For example, carbon fibres are used in the manufacture of gas turbine fan blades, racing car body panels, and various high-performance sporting equipment [5].

Development in carbon CSM manufacture from hydrocarbon gases is in progress and is expected to result in a commercial product becoming available in the near future [8].

2.2.2.3 ARAMID FIBRES

Aramid fibres are organic fibres that are melt-spun from a liquid polymer solution. The Du Pont Company developed aramid fibres and sells their product under the trade name Kevlar, of which there are four grades with varying engineering properties. The morphology of the fibre consists of a radial configuration resulting in anisotropic properties. The filaments have a small diameter ($\sim 12 \mu\text{m}$) and as result, to some extent, are very flexible. Aramid fibres tend to have high tensile strength but only an intermediate modulus, as well as having a significantly lower strength in compression [8].

2.2.2.4 OTHER FIBRES

There are a number of other reinforcement fibres which can be used in advanced composite structures but their use is not widespread. These include polyester, polyethylene, quartz, asbestos, silicon carbide, phosphate, boron, ceramics, metals and natural fibres [6,8].

2.3 MATRIX

The matrix is the binding material that supports, separates, protects, and provides a load-path to the fibres. In the event of fibre breakage the matrix redistributes the load

amongst the remaining fibres. The density, stiffness, and strength of the matrix is/are typically lower than that of the reinforcement. The matrix material's properties vary considerably even within a given class and display brittle, ductile, elastic, plastic behaviour and either linear or non-linear stress-strain behaviour. In addition, the matrix material must have the correct viscosity so that the reinforcements may be penetrated sufficiently however still retain proper adhesion to the fibres ^A(chemical treatment to the reinforcement is often required to ensure this) [4].

Polymers, metals, and ceramics are all examples of matrix materials used to varying degrees in composites. Polymeric matrix materials are more commonly used and can be further divided into two subcategories namely thermoplastics and thermoset's. Thermoplastics are the name given to the group of polymers that soften and can be reshaped in the presence of heat and pressure. Thermoplastic composites typically possess high toughness and are suitable for high volume and low-cost processing. Thermoset polymers on the other hand become cross-linked during the curing process and as a result do not soften upon reheating. Thermoset's out perform thermoplastics in a number of areas including mechanical properties, chemical resistance, thermal stability and overall durability. Common thermoset polymers include polyesters, vinylesters and epoxies [3] which are discussed below.

2.3.1 POLYESTER RESINS

Polyester resins are by far the most common type of matrix material and are used extensively in conjunction with glass fibres as they are relatively inexpensive, lightweight, have a useful temperature range (up to 100°C), and are fairly resistant to

various environmental conditions [3]. The properties of polyester resin have great versatility and can vary considerably from strong and brittle to extremely flexible with each type customised to their own particular application [9].

Traditionally orthophthalic acid-based resins were and are still used extensively. More recently however, a slightly more expensive polyester resin based on isophthalic acid has proven to be superior in several areas such as water absorption, strength, flexibility, abrasive resistance, impact resistance and fatigue performance [10].

The polyesters referred to here are the unsaturated type and are produced by reacting a combination of saturated and unsaturated organic acids with a glycol and dissolving the mixture in a reactive monomer. The monomer (usually styrene) is used not only as a solvent for dissolving the polyester resin, but also reacting with the unsaturated acid during the curing process, thus cross linking the polyester molecules to form a copolymer of polyester and styrene, hence the monomer is an active part of the process. The curing process or the non-reversible chemical reaction referred to as cross-linking or 'polymerisation' results in a chemically resistant hard solid. By varying the acids, glycols and modifying agents it is possible to tailor the polyester for specific applications [9].

The idealised chemical structure of a typical polyester is shown in Figure 2.7.

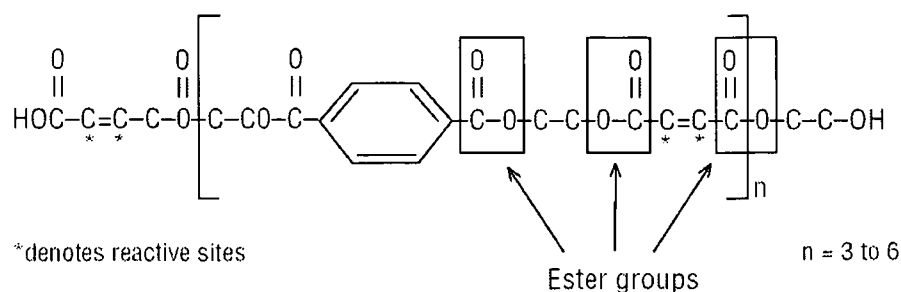


Figure 2.7 Idealised chemical structure of a typical isophthalic polyester. [7]

The molecular chains of the polyester resin can be represented as shown in Figure 2.8, where 'B' indicates the reactive sites in the molecule. With the addition of styrene 'S' in the presence of a catalyst, the styrene cross links the polymer chains at each of the reactive sites to form a highly complex three dimensional network [7].

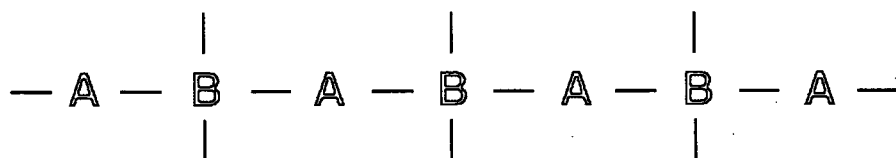


Figure 2.8 Schematic representation of polyester resin (uncured). [7]

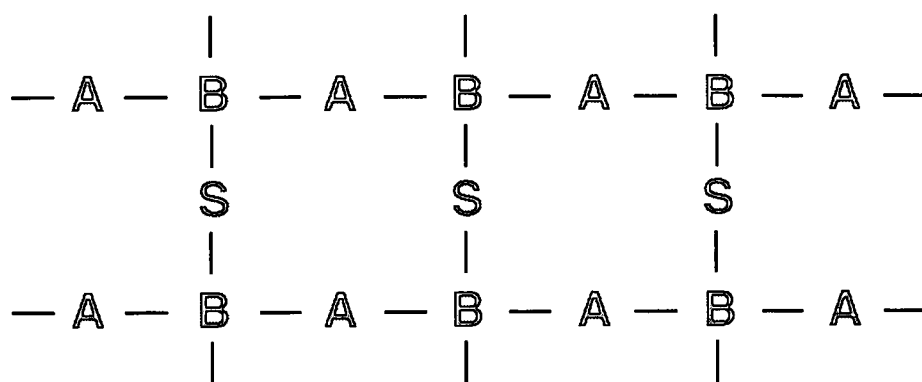


Figure 2.9 Schematic representation of polyester resin (cured). [7]

In order to initiate the exothermic curing process that will take the resin from liquid form to the final hardened state a catalyst must be added. The resin will harden relatively quickly however the maturing time on the resin can take hours or even days to achieve its full hardness and stability. Peroxide based catalysts are usually used with polyesters with the most common being Methyl Ethylketone Peroxide (MEKP) [9].

Although polyesters are the most common type of resin there are drawbacks when considering them for use in high performance applications, polyesters generally display poor elongation at break, high shrinkage on cure and inadequate adhesive and water-resistant properties. Elongation at break should be equal to that of the reinforcement in order to eliminate the failing of one component prematurely leaving the other to resist all the loading. Adhesion between the reinforcement and the binder is similarly important in that the two are required to work together if optimal properties are to be achieved. One of the reasons for this is the relatively high shrinkage on curing as well as the built-in stresses that result [9].

2.3.2 VINYLESTER RESINS

Vinylester resins are very similar to polyester resins in that they react with styrene by a chain-reaction process initiated by the addition of a catalyst, however they are tougher and more resilient. Unlike polyesters, vinylester's reactive positions on the molecular chain occur only at the ends of the chain and as a result this configuration is able to elongate to absorb shock loading more efficiently. The vinylester molecule also features less ester groups that are susceptible to water degradation by hydrolysis. It therefore stands to reason that vinylesters exhibit improved water and chemical resistance, and

are frequently chosen for pipelines in chemical plants. To fully benefit from the slightly more expensive vinylester (compared with polyester) it is necessary that the resin be cured at an elevated temperature [7,9].

The idealised chemical structure of a typical vinylester is shown in Figure 2.10.

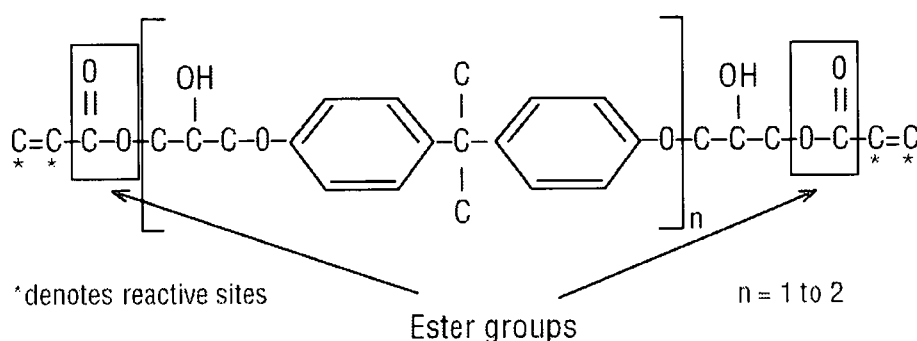


Figure 2.10 Idealised chemical structure of a typical epoxy based vinylester. [7]

Figure 2.11 clearly illustrates, the location of the reactive sites at the end of the molecular chain, the 'B' molecules represent the active sites of the molecular chain and 'S' represents the styrene monomer introduced in the presence of the catalyst.

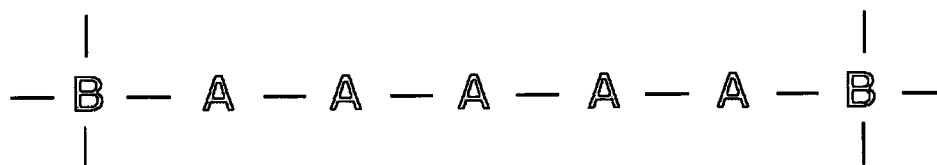


Figure 2.11 Schematic representation of vinylester resin (uncured). [7]

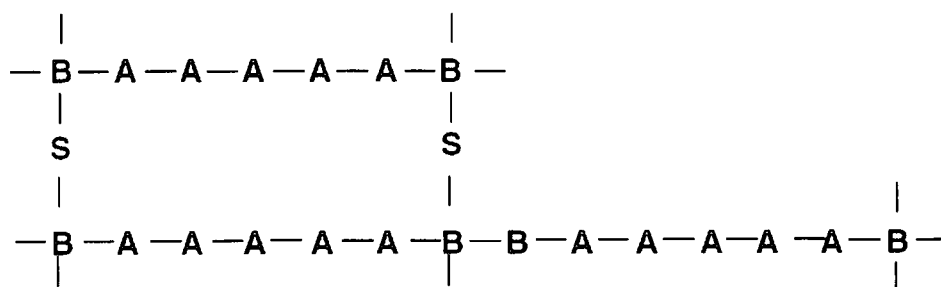


Figure 2.12 Schematic representation of vinylester resin (cured). [7]

2.3.3 EPOXY RESINS

Epoxy resins are extremely versatile and in one form or another are used as adhesives, sealants, paints and varnishes, casting compounds as well as laminating resins for a variety of industrial applications. Laminating epoxies are more expensive than polyesters and vinylesters, but generally possess superior mechanical properties when compared with most other resins, which leads to their extensive use. One of the most advantageous properties of epoxy resins is the low shrinkage experienced during the cure cycle, which minimises fabric print-through and far more importantly reduces the introduction of internal stresses [7,9]. Other mechanical properties that are worth mentioning are resistance to environmental degradation, good adhesion, electrical insulation and a maximum useful temperature in the vicinity of 175°C [3].

Epoxy resins are formed from a long molecular chain, similar to that of vinylester, with reactive sites at either end. However with epoxy resins the reactive sites are formed by epoxy groups instead of ester groups. The epoxy groups have a distinctly better water resistance to that of the ester groups. The epoxy molecule contains two ring groups at its centre that are capable of absorbing both mechanical and thermal stresses efficiently thus providing a resin that has very good stiffness, toughness and heat resistant

properties [7,9]. Shown below in Figure 2.13 is the idealised chemical structure of a typical epoxy molecular chain.

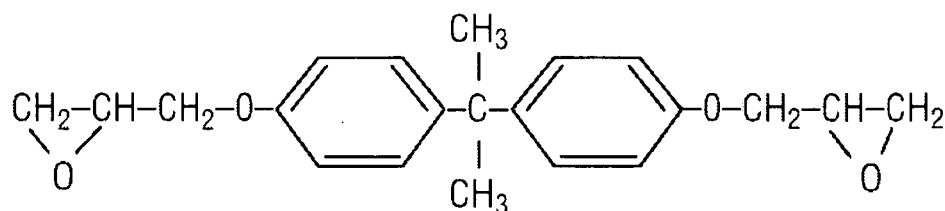


Figure 2.13 Idealised chemical structure of a typical epoxy (diglycidyl ether of bisphenol-A). [7]

Epoxy resins process differently from polyester and vinylester resins in that they are cured by a hardener (often amine) instead of a catalyst. The chemical reaction that cures the epoxy is achieved by an 'addition reaction' where both hardener and resin bind to form the product as shown in Figure 2.14 below [7].

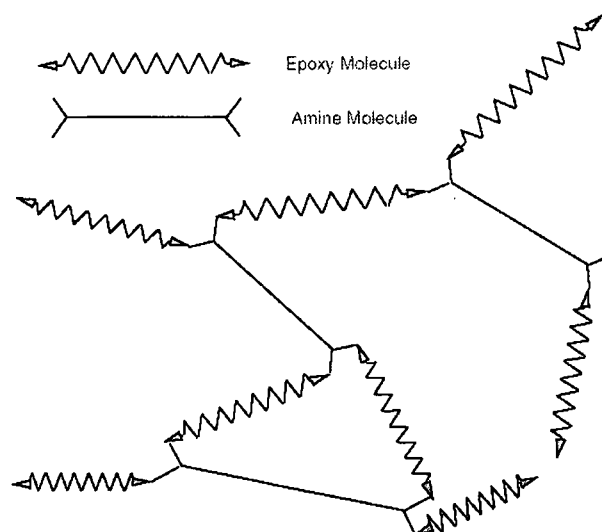


Figure 2.14 Schematic representation of epoxy resin (cured 3-D structure). [7]

Since the reaction requires both the hardener and the resin to bind together to form the product it is imperative that the mix ratio is correct in order to ensure a full cure takes place and the optimal properties are achieved [7].

2.3.4 OTHER MATRIX MATERIALS

Thermoset polyimide resins have shown great potential in combating tough stress and temperature challenges in engineering structures (for example aircraft) due to the advanced properties such as superior corrosion resistance, excellent chemical resistance, good dimensional stability and excellent dielectric properties [11]. Polyimide resins have a higher operating temperature (300°C to 350°C) but are more difficult to fabricate, and consequently are not used extensively and are considered to be more of a specialist material [3]. There are various other matrix materials available such as phenolics, cyanate esters, silicones, polyurethanes, and bismaleimides (BMI) however these materials have a small market share and are typically associated with specialist applications.

2.3.5 PROCESSING

Although material properties have been quoted for different resin systems these properties are dependent on many factors such as the concentration of various additives. However, the way in which a composite is processed also has an effect on the properties. For example, a composite processed at 80°C will present better mechanical, chemical and water-resistive properties to a composite cured at 25°C [10].

The maturing or postcure process is an equally important part of the process as this is the time when the thermoset resin reaches its full cure and ultimately its maximum hardness and stability. The postcuring process increases the amount of cross-linking of the molecules and the process may take hours or days depending on the temperature that the product is retained at. Generally the higher the temperature at which postcure takes place the quicker the time taken to reach full cure. Many resin systems will never reach their ultimate mechanical properties unless the resin is given this postcure. This is particularly true of the material's softening point or glass transition temperature (which is the temperature where a cured thermoset's mechanical properties will change significantly although it can not become liquid) which, up to a point, increases with increasing postcure temperature [7].

2.4 COMPOSITE PROPERTIES

Undoubtedly the main advantages of composites are their high specific stiffness and high specific strength as compared with conventional engineering materials. The initial development and application of composites were pursued in the early 1960's predominantly in transport and military applications and particularly in the aerospace sectors where weight critically affects fuel consumption, performance, and payload [3]. This is however a limited view of the potential of composites, as they are often found to be superior to metals and other conventional engineering materials due to their mechanical, physical and chemical properties. Composites also have interesting electromagnetic properties and due to this glass fibre-reinforced polyester (GRP) is used to construct mine-counter-measures and naval vessels, the requirement being a non-magnetic material. As another illustration, carbon fibre-reinforced polyester (CRP) is

used in medical applications because it is transparent to X-rays; this property is also exploited in military applications where stealth technology is applied [12].

As mentioned previously a composite is a material composed of one or more discontinuous phases (or reinforcement) which are embedded in a continuous phase (or matrix), both are required to accomplish specific tasks so that the composite that results will exhibit superior properties when compared with the individual constituents. The reinforcements are strong and stiff and act as the primary load-carrying component [3], whereas the continuous binder (matrix material) must surround each fibre so that they are kept distinctly separate from adjacent fibres to toughen the composite. The physical and mechanical properties (as shown in Table 2.2 below) of the final composite are directly dependent on the properties, geometry, and concentration of each individual constituent, and careful design is required if the composite is to meet all the performance requirements.

Material	Young's Modulus GPa	Tensile Strength GPa	Fibre Vol. Fraction V_f	Density g/cm^3	Poisson's Ratio
High Strength Carbon Epoxy	148.0	2.137	0.62	1.52	0.30
Carbon Epoxy	132.0	1.513	0.62	1.54	0.24
Kevlar Epoxy	76.8	1.380	0.55	1.38	0.34
S-Glass Epoxy	43.5	1.724	0.60	2.00	0.27

Table 2.2 Typical material properties of unidirectional composites. [3]

There are many factors to be considered when designing with composite materials which must all be taken into account [4], this is unlike dealing with traditional engineering materials which are typically isotropic and the choices are more limited; composites are generally directionally dependent and are usually fabricated into their final shape. As a result composites can be designed to meet the demands of each individual application by having the desired properties in specific directions (the optimal composite properties are associated with the fibre's axial orientation) without over-designing in other directions. Some of the other more important design options available include the choice of materials (fibre and matrix), the fibre volume fraction, fabrication method, number of layers in a given direction, thickness of individual layers, type of layer (unidirectional or woven rovings, or chopped strand mat), and the layer stacking sequence. With this vast array of design variables more efficient structures can be fabricated with less material wastage. With the large number of variables available when dealing with composites the use of computers, optimisation, expert systems, and artificial intelligence should be incorporated into the design process to improve designs.

2.4.1 DIRECTIONAL PROPERTIES

As already mentioned the strength and stiffness of composites are dependent on the orientation of the laminate. Materials that exhibit such directional bias are termed 'anisotropic', opposed to isotropic materials, which exhibit the same properties in all directions. Unidirectional (UD) and woven rovings (WR) are examples of anisotropic materials that are directionally dependant where as chopped strand mat (CSM) under in-plane loading can be considered an isotropic material [4].

The effect orientation has on the different laminate types is illustrated in Figures 2.15 and 2.16 below. The effect of angle on unidirectional (UD) rovings is quite pronounced with excellent properties inline with the fibre direction and very low properties in the transverse direction. Woven rovings (WR) have a biaxial configuration with equally good properties in the 0° and 90° case. The trend, as expected, is symmetric about 45° which results in the lowest properties. Chopped strand mat (CSM) due to its random nature displays isotropic behaviour under in-plane loading.

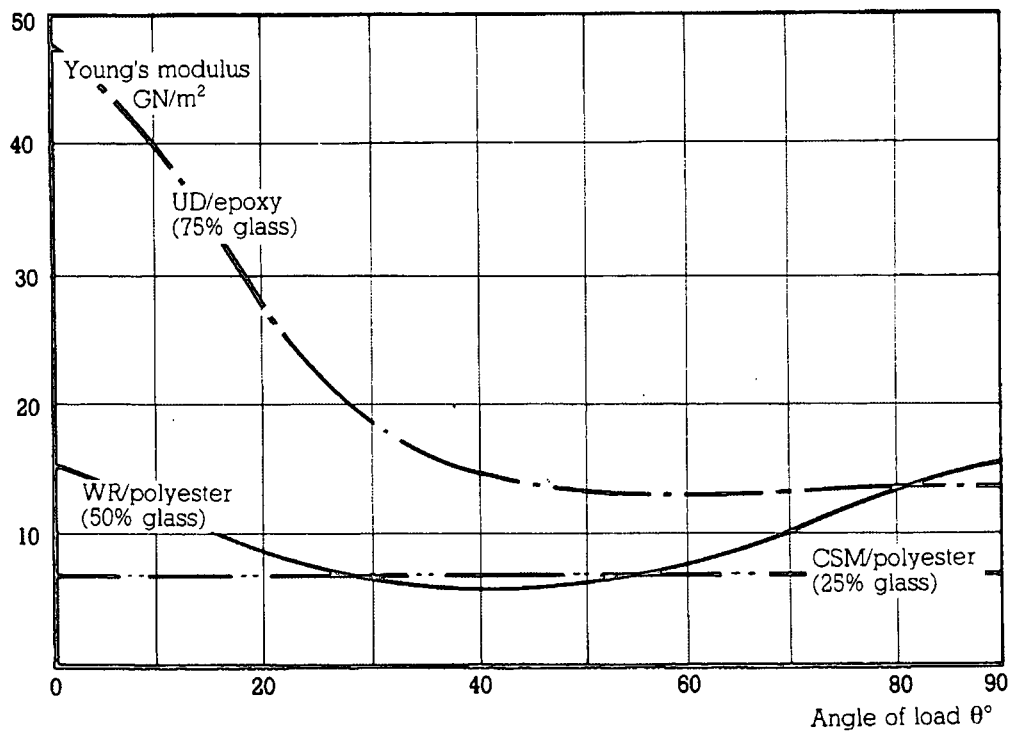


Figure 2.15 The effect of angle of load relative to the principle fibre direction, on the tensile modulus of UD glass/epoxy, WR glass/polyester and CSM/polyester. [6]

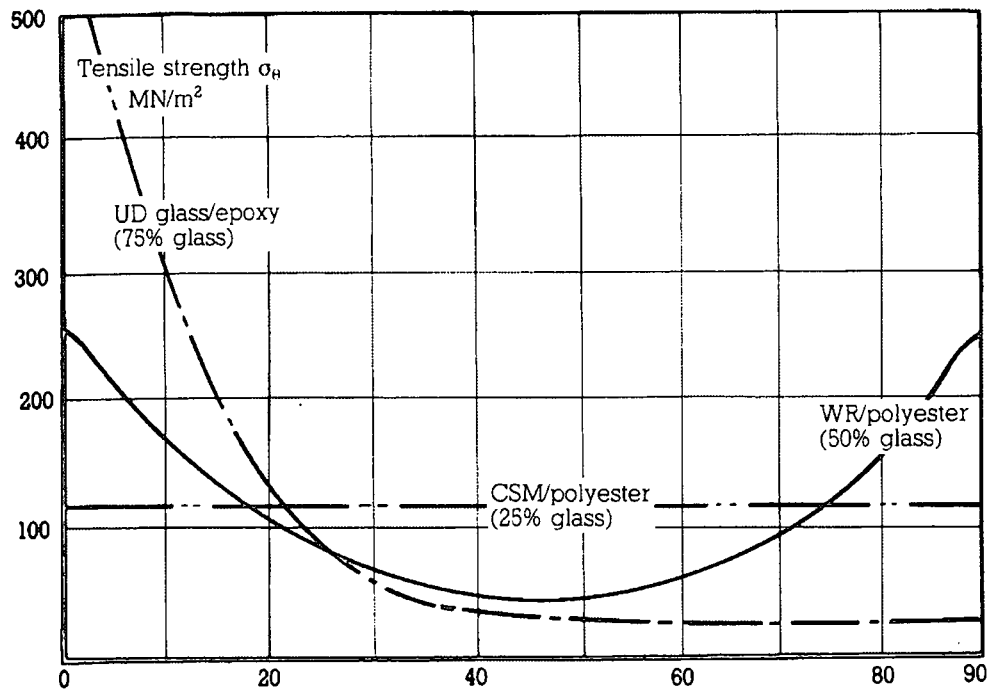


Figure 2.16 The effect of angle of loads on the tensile strength of UD glass/epoxy, WR glass/polyester and CSM/polyester. [6]

2.4.2 FATIGUE LIFE

Improved fatigue life in composites has been one of the contributing factors that have seen the increased use in aircraft where aluminium was previously used. The fatigue life of advanced composites is shown by Herakovich [3] to be far superior to that of aluminium. Similarly, composites have found favour in other structures that experience cyclic loading such as in transportation, bridges, industrial components and structures exposed to variable wind or water loading.

2.4.3 DIMENSIONAL STABILITY

Almost all structures experience temperature changes through their lifetime and the associated changes in shape or size, increased friction and wear, and thermal stresses that can result. Through careful design it is possible to engineer a structure that has a zero coefficient of thermal expansion (such as the carbon/epoxy optical bench for a space telescope [3]) or if required a thermal coefficient that matches neighbouring components and hence limits thermal mismatch and the resulting stresses.

2.4.4 CORROSION RESISTANCE

Composite materials are generally resistant to corrosion from moisture, certain specialist composites are also resistant to a variety of chemicals. Composites that have been driven by corrosion considerations include filament-wound underground storage tanks, structural members for offshore drilling platforms, chemical plants, sucker rods used in pumping oil from wells, pipe, and domestic applications including doors, window frames, and deck furniture in coastal regions where saltwater corrosion is a major problem. The corrosion resistive qualities of composites directly reduce maintenance costs, which can represent substantial savings and should be considered in all total cost evaluations. Unfortunately, cost decisions are often based primarily on the initial capital expenditure without consideration for the longevity of the structure, maintenance costs, and reduced replacement costs.

2.4.5 COST-EFFECTIVE FABRICATION

Certain composite structures can be fabricated quickly and efficiently through the use of automated methods such as filament winding, pultrusion, and tape laying. Composite components can be fabricated into the final product exactly to size specifications with little or no waste material. This is in stark contrast to traditional materials, such as metals, where the final component is often machined down to arrive at the final product leaving large portions of waste material. Generally fabrication costs are also directly proportional to the structures parts count. The use of composites can substantially reduce the number of parts due to the capability to fabricate to the final shape and because of the use of bonded rather than riveted joints. As an example investigated by Herakovich [3], firstly two sections of an aluminium fuselage were riveted together and then similar composite sections were adhesively bonded together. There was a tenfold reduction observed in the parts count of the composite fuselage, which represents a significant saving in both the cost of components and the cost of assembly.

2.4.6 CONDUCTIVITY

It is often desirable that an engineering structure be electrically non-conductive or occasionally conductive. Excellent examples of non-conducting composites are the glass/polyester ladders and booms that have been introduced to replace steel and aluminium alternatives in order to reduce the possibility of electrocution. In contrast, copper matrix composites are now being used for high temperature applications where the high thermal conductivity of copper can serve as radiators where it is necessary to maintain lower temperatures.

2.4.7 OVERALL COST CONSIDERATIONS

When evaluating the cost competitiveness of composite structures the longevity of the product should be taken into consideration. Although, per weight, composites are generally more expensive than traditional engineering materials, there are many other factors that must be considered before a decision is taken as to which material is preferred. A composite structure may well cost more than one fabricated from traditional materials such as metal. This is largely due to having a more integrated construction (i.e. a lower 'parts count') although this depends significantly on the level of automation in the manufacturing process [12]. Some other considerations are listed below.

- A lighter structure can be fabricated due to the higher specific stiffness and strength of the composite material.
- Fabrication costs can be lower.
- Transportation and erection costs are generally lower for composite structures.
- Composite structures are essentially maintenance free when compared with traditional engineering materials this is true primarily due to their corrosion resistive qualities.

Composite materials have proven to be cost competitive in a wide variety of aerospace, automotive, industrial, domestic, oil drilling, and electronic applications amongst others.

2.5 COMPOSITE MECHANICS

2.5.1 FIBRE VOLUME FRACTION

The fibre volume fraction is the fraction of fibre to resin by volume and plays a very important part in the strength and stiffness properties. Since it is the fibre that imparts the strength and stiffness, a greater fibre content for a given volume is desirable [9]. Typically unidirectional or woven reinforcement have a greater fibre volume fraction than chopped strand mat (CSM) for instance, as CSM requires a larger amount of resin for a full wet-out. The fibre volume fraction should typically be between 20% and 60% depending on the application. Anything over 50% and above can be considered very good, with 60% and up being considered high performance. A high fraction (of about 70%) is possible with Resin Transfer Moulding (RTM) or “prepreg” systems. Anything over 70% is undesirable as the matrix will be unable to wet-out fully, and will not ensure that the fibres do not entangle and keep the reinforcements separate, hence lowering the mechanical properties drastically.

2.5.2 RULE OF MIXTURES

The simplest model for predicting unidirectional continuous fibre composite properties is termed the rule of mixtures. This ‘rule’ estimates the material properties of a composite by evaluating the individual constituents contribution to the overall material by volume. The method employs the use of the fibre volume fraction to estimate the properties of the composite. For a given fibre volume fraction of V_f and a matrix volume fraction V_m , the following must be satisfied

$$V_f + V_m = 1 \quad (2.1)$$

Based on the rule of mixtures, a certain property, p , may be estimated from the constituent properties, p_f and p_m , of the fibre and matrix respectively as

$$p = p_f V_f + p_m V_m \quad (2.2)$$

The rule of mixtures has two forms, the one concerns the properties in the longitudinal direction (parallel to the fibres), and can be represented by two springs connected in parallel as seen in Figure 2.17 [13].

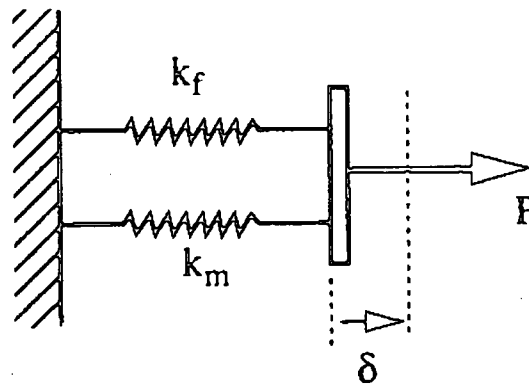


Figure 2.17 Stiffness representation for the rule of mixtures model in the longitudinal direction of a composite.

For example, the longitudinal stiffness property, E_l , of a composite may be calculated from the Young's moduli of the constituents E_f and E_m , using the rule of mixtures to give

$$E_l = E_f V_f + E_m V_m \quad (2.3)$$

The second form relates to the transverse direction (properties perpendicular to the fibres), where the problem may be represented by two springs connected in series as shown in Figure 2.18 [13].

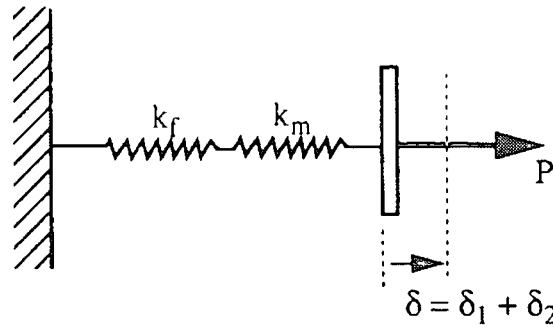


Figure 2.18 Stiffness representation for the rule of mixtures model in the transverse direction of a composite.

If the transverse modulus, E_2 , is required it may be found using the following expression

$$\frac{1}{E_2} = \frac{V_f}{E_f} + \frac{V_m}{E_m} \quad (2.4)$$

2.5.3 STIFFNESS AND STRENGTH OF COMPOSITES

2.5.3.1 BASIC STRESS-STRAIN RELATIONS

As previously discussed, the superior specific properties that can be obtained from composite materials make for an attractive engineering material. These properties are however direction sensitive as has been shown previously, for example, a unidirectional fibre-reinforced material will display higher properties in the direction of the fibre axis. When considering the perpendicular properties of the same specimen the stiffness and strength will be much lower and will be determined by the matrix and fibre-matrix interface. [8]

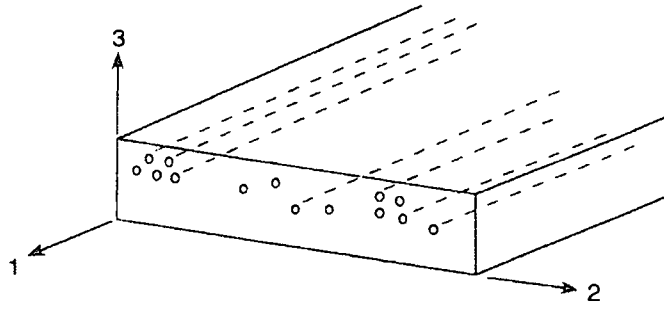


Figure 2.19 Orientation of principle material axes.

The stress-strain relationship for a unidirectional material can be found if the above mentioned direction bias is taken into account. Considering Figure 2.19, it can be seen that the applied stresses coincide with the principal material axes, the strains in terms of the stresses are given by

$$\begin{aligned}\epsilon_1 &= \frac{\sigma_1}{E_{11}} - \nu_{21} \frac{\sigma_2}{E_{22}} \\ \epsilon_2 &= -\nu_{12} \frac{\sigma_1}{E_{11}} + \frac{\sigma_2}{E_{22}} \\ \gamma_{12} &= \frac{\tau_{12}}{G_{12}}\end{aligned}\tag{2.5}$$

where E_{11} is the elastic modulus in the '1', or longitudinal direction

E_{22} is the elastic modulus in the '2', or transverse direction

G_{12} is the shear modulus in the 1-2 axes

ν_{12} is the 'major' Poisson's ratio

ν_{21} is the 'minor' Poisson's ratio

ν_{12} provides the transverse ('2' direction) strain caused by the strain applied in the longitudinal ('1') direction; conversely for ν_{21} . Due to the high stiffness in the

longitudinal direction v_{12} would be expected to be larger than v_{21} . This is confirmed by a fundamental law of elasticity which shows that [3,14]

$$\frac{v_{12}}{E_{11}} = \frac{v_{21}}{E_{22}}$$

or
$$v_{12} = v_{21} \frac{E_{11}}{E_{22}} \quad (2.6)$$

and as
$$\frac{E_{11}}{E_{22}} > 1, v_{12} > v_{21}$$

To attain stresses in terms of strains we rearrange equation (2.5) to give

$$\begin{aligned} \sigma_1 &= \frac{E_{11}\epsilon_1}{1 - v_{12}v_{21}} + \frac{v_{21}E_{11}\epsilon_2}{1 - v_{12}v_{21}} \\ \sigma_2 &= \frac{v_{12}E_{22}\epsilon_1}{1 - v_{12}v_{21}} + \frac{E_{22}\epsilon_2}{1 - v_{12}v_{21}} \end{aligned} \quad (2.7)$$

$$\tau_{12} = G_{12}\gamma_{12}$$

For convenience when dealing with laminates it is useful to rewrite equations (2.5) and (2.7) in matrix form. So (2.5) becomes

$$\epsilon_{12} = S\sigma_{12}$$

where
$$\epsilon_{12} = \{\epsilon_1 \quad \epsilon_2 \quad \gamma_{12}\} \quad (2.8)$$

and
$$\sigma_{12} = \{\sigma_1 \quad \sigma_2 \quad \tau_{12}\}$$

Note that $\{\}$ denotes a column vector written as a row vector. The compliance matrix is then defined as

$$S = \begin{bmatrix} \frac{1}{E_{11}} & -\frac{v_{21}}{E_{22}} & 0 \\ -\frac{v_{12}}{E_{11}} & \frac{1}{E_{22}} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix}$$

Note that $S_{12} = S_{21}$

and (2.7) becomes

$$\sigma_{12} = Q\varepsilon_{12} \quad (2.9)$$

where the stiffness matrix is defined by

$$Q = \begin{bmatrix} \frac{E_{11}}{1 - \nu_{12}\nu_{21}} & \frac{\nu_{21}E_{11}}{1 - \nu_{12}\nu_{21}} & 0 \\ \frac{\nu_{12}E_{22}}{1 - \nu_{12}\nu_{21}} & \frac{E_{22}}{1 - \nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix}$$

Note that $Q_{12} = Q_{21}$

It can be seen that the first column of S gives the strains caused by a unit value of σ_1 , the second column of Q gives the stresses needed to cause a unit value of ε_2 , and so on. As can be seen from the properties of the matrices $Q = S^{-1}$.

2.5.3.2 OFF-AXIS LOADING OF A UNIDIRECTIONAL COMPOSITE

Often laminates are constructed by assembling a number of layers, usually unidirectional and positioned with different orientations. At this stage a single unidirectional lamina with the principal axes offset by θ is considered as shown in Figure 2.20.

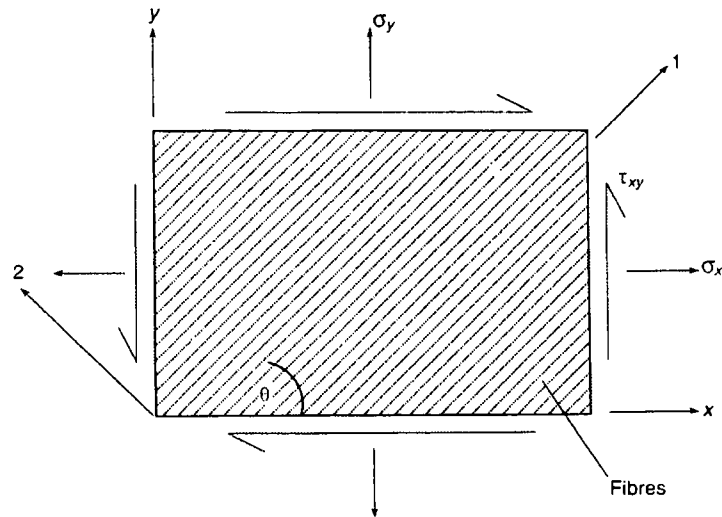


Figure 2.20 Unidirectional lamina with principal axes rotated by θ relative to the x-y axes. [14]

The situation that is illustrated can be seen to correspond to the analysis of principal stress for isotropic materials. We can then write

$$\sigma_{12} = T \sigma_{xy} \quad (2.10)$$

and
$$\bar{\epsilon}_{12} = T \bar{\epsilon}_{xy} \quad (2.11)$$

where

$$\sigma_{12} = \{\sigma_1 \quad \sigma_2 \quad \tau_{12}\}$$

$$\sigma_{xy} = \{\sigma_x \quad \sigma_y \quad \tau_{xy}\}$$

$$\bar{\epsilon}_{12} = \{\epsilon_1 \quad \epsilon_2 \quad \frac{1}{2} \gamma_{12}\}$$

$$\bar{\epsilon}_{xy} = \{\epsilon_x \quad \epsilon_y \quad \frac{1}{2} \gamma_{xy}\}$$

and the transformation matrix, T , is given by

$$T = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & (m^2 - n^2) \end{bmatrix} \quad (2.12)$$

with $m = \cos \theta$ and $n = \sin \theta$.

The stresses for a set of known strains (or visa versa) can be determined when the elastic properties are known. If the properties are known for the 1-2 axes they need to be referred to the x-y axes, this is done with some mathematical manipulation which leads to:

$$\sigma_{xy} = \bar{Q} \epsilon_{xy} \quad (2.13)$$

The transformed stiffness matrix is \bar{Q} , the elements of which are

$$\begin{aligned} \bar{Q}_{11} &= Q_{11}m^4 + 2(Q_{12} + 2Q_{33})n^2m^2 + Q_{22}n^4 \\ \bar{Q}_{22} &= Q_{11}n^4 + 2(Q_{12} + 2Q_{33})n^2m^2 + Q_{22}m^4 \\ \bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{33})n^2m^2 + Q_{12}(m^4 + n^4) \\ \bar{Q}_{33} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{33})n^2m^2 + Q_{33}(m^4 + n^4) \\ \bar{Q}_{13} &= (Q_{11} - Q_{12} - 2Q_{33})nm^3 + (Q_{12} - Q_{22} + 2Q_{33})n^3m \\ \bar{Q}_{23} &= (Q_{11} - Q_{12} - 2Q_{33})n^3m + (Q_{12} - Q_{22} + 2Q_{33})nm^3 \end{aligned} \quad (2.14)$$

It can be seen that knowledge of orientation (θ) and unidirectional properties (Q) in the principal directions enables the calculation of stiffness of the rotated lamina. If strains are required then we invert equation (2.13) to give

$$\epsilon_{xy} = \bar{Q}^{-1} \sigma_{xy}$$

or

$$\epsilon_{xy} = \bar{S} \sigma_{xy} \quad (2.15)$$

where \bar{S} is the transformed compliance matrix, the elements of which can be obtained by a similar process to that used for finding the elements of \bar{Q} , i.e.

$$\begin{aligned} \bar{S}_{11} &= S_{11}m^4 + (2S_{12} + S_{33})n^2m^2 + S_{22}n^4 \\ \bar{S}_{22} &= S_{11}n^4 + (2S_{12} + S_{33})n^2m^2 + S_{22}m^4 \\ \bar{S}_{12} &= (S_{11} + S_{22} - S_{33})n^2m^2 + S_{12}(m^4 + n^4) \end{aligned}$$

$$\begin{aligned}
\bar{S}_{33} &= 2(2S_{11} + 2S_{22} - 4S_{12} - S_{33})n^2m^2 + S_{33}(m^4 + n^4) \\
\bar{S}_{13} &= (2S_{11} - 2S_{12} - S_{33})nm^3 + (2S_{12} - 2S_{22} + S_{33})n^3m \\
\bar{S}_{23} &= (2S_{11} - 2S_{12} - S_{33})n^3m + (2S_{12} - 2S_{22} + S_{33})nm^3
\end{aligned} \tag{2.16}$$

2.5.3.3 STIFFNESS OF LAMINATES

A laminate is comprised of a number of laminae each with independent orientations and thicknesses to give the desired stiffness and strength properties. The stacking sequence has an important effect on the flexural performance of the laminate.

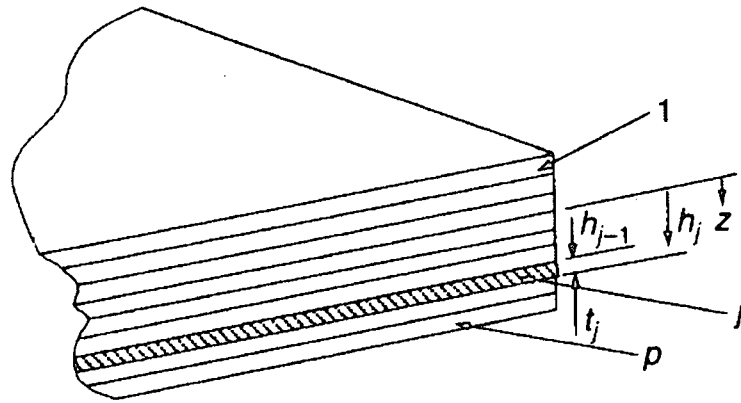


Figure 2.21 Definition of plies within a laminate. [14]

A laminate is usually subjected to both in-plane and transverse loading, which means that each layer will experience both stretching and bending. Both these effects need to be taken into account as is shown in the Figure 2.22.

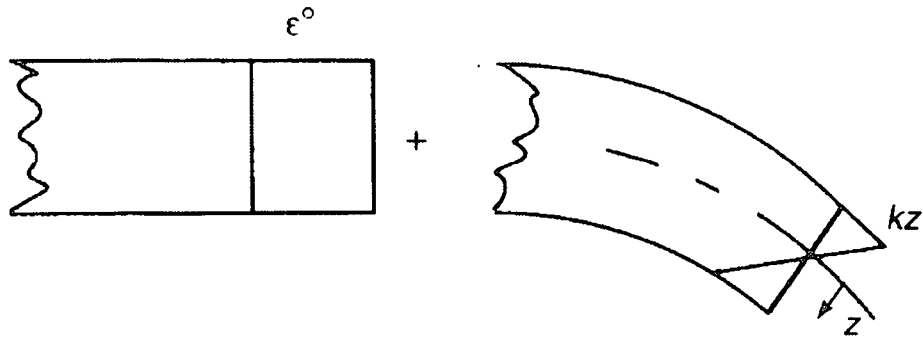


Figure 2.22 The two components of laminate strain: ϵ^o , in-plane, constant over the thickness; $\epsilon_x = zk_x$, bending, linear variation over the thickness. [14]

The relationship between the lamina in-plane and bending strain is given as

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \epsilon_x^o \\ \epsilon_y^o \\ \gamma_{xy}^o \end{bmatrix} + z \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \quad (2.17)$$

or
$$\epsilon_{xy} = \epsilon^o + z\kappa \quad (2.18)$$

Due to the nature of the manufacturing process it can be assumed that the layers are bonded together and hence the in-plane strains and curvatures are the same on all layers.

So, if considering an arbitrary layer j

$$\sigma_{xyj} = \bar{Q}_j \epsilon^o + z \bar{Q}_j \kappa \quad (2.19)$$

\bar{Q}_j being the transformed stiffness matrix for the j^{th} layer.

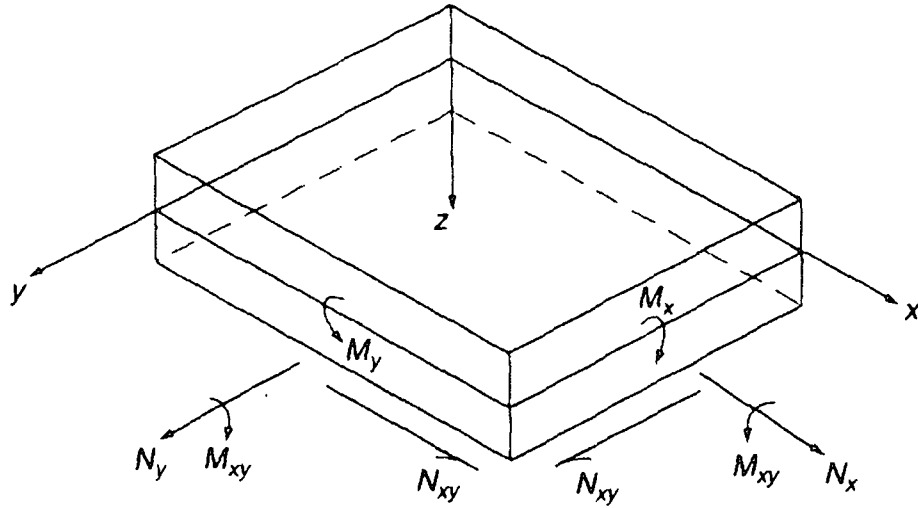


Figure 2.23 Loads acting on a laminate. [14]

It must be noted that the stresses act in the plane of the laminate. These stresses can be converted into equivalent forces, so from σ_x the load $N_{xj} = \sigma_{xj} t_j$ results, where t_j is the thickness of layer j . Likewise, the equivalent moments can be found on any particular layer, so again from σ_x we get $M_{xj} = \sigma_{xj} t_j z_j$, where z_j is the distance from the laminate mid-plane to the mid-thickness of the ply. If all the equivalent components are added they will equal the external value, this is true for both the equivalent forces and moments.

The stress resultants and the in-plane strains and curvatures are related as:

$$N = A\varepsilon^0 + B\kappa \quad (2.20)$$

and

$$M = B\varepsilon^0 + D\kappa \quad (2.21)$$

The above equations are known as the 'plate constitutive equations', and the associated analysis as 'Classical Laminated Theory'.

The elements of A, B and D matrices are

$$\begin{aligned}
A_{rs} &= \sum_{j=1}^p \bar{Q}_{rsj} [h_j - h_{j-1}] = \sum_{j=1}^p \bar{Q}_{rsj} t_j \\
B_{rs} &= \frac{1}{2} \sum_{j=1}^p \bar{Q}_{rsj} [h_j^2 - h_{j-1}^2] \\
D_{rs} &= \frac{1}{3} \sum_{j=1}^p \bar{Q}_{rsj} [h_j^3 - h_{j-1}^3]
\end{aligned} \tag{2.22}$$

where $r, s = 1, 3$. Matrix A represents the in-plane stiffness, B represents the bending-stretching coupling and C represents the bending stiffness. [3,6]

2.6 MANUFACTURING METHODS

2.6.1 TOOLING

The need for tooling in the plastics industry has grown substantially over the last century firstly for un-reinforced polymers and more recently for reinforced plastics (composites). Predictably, the aerospace industry has pioneered tooling technology required for processing composites. There are several parameters that need to be taken into consideration before a tooling system can be decided upon, as discussed by Middleton [8]:

- Number of parts to be made
- Temperature and pressure requirements of part cure
- Timescales
- Cost boundaries
- Curvature complexities
- The component dimensional tolerances

There is a vast array of manufacturing and fabrication techniques that are currently available. For example, dimensionally stable, high temperature tooling is required for the fabrication of RFI materials as the curing process involves vacuum bagging at elevated temperatures. Several manufacturing procedures currently exist, a few methods are discussed in detail below.

2.6.1.1 FABRICATED METAL TOOLING

Fabricated metal tooling usually consists of a system of metal profiles (headers) that are manufactured and positioned on a base. The mould surface is then rolled to shape and fastened to the profiles. This type of tooling can have a long life span. Allowances must be made for the differences in thermal expansion between the tooling and the composite product, which can result in fibre lock-on. Fibre lock-on is when, due to the shrinkage of the composite on cure, the component shrinks tightly onto the tool. (The temperature at which fibre lock-on occurs is not always the cure temperature but rather the temperature where the fibres and matrix become bound together and act as a composite). [8]

2.6.1.2 MACHINED METAL TOOLING

Machined metal tooling requires 3D models in conjunction with CNC machining facilities if the geometry is complex, which is usually the case. This type of tooling is widely used and generally has a long life span. As with fabricated metal tooling (or any other metal tooling for that matter) the difference in thermal expansion between the tool and product must be taken into consideration as fibre lock-on can result [8]. Metal

tooling is accurate and expensive however the life span is much greater and the capital cost can be recovered with mass production. Aluminum is often chosen as it is easily machined, relatively light and a corrosion resistant metal.

2.6.1.3 COMPOSITE TOOLING

Initially composite tools were introduced to combat the problem of differences in thermal expansion coefficients between the composite product and metallic tooling, which lead to fibre lock-on and the associated induced stresses. Several composite tooling options are currently available from wet lay-up glass reinforced polyester to high performance carbon fibre reinforced epoxy made from tooling prepreg. Wet lay-up is discouraged, as the process is largely uncontrolled with poor control over resin content and distribution, which can have an adverse effect on the performance of the tool [8].

The tooling process adopted depends on a number of factors, such as whether the tool needs to be male or female and the amount of information that is available about the surface geometry. Usually the process begins with the manufacture of a pattern (or master model) that is constructed from a variety of materials including foams, (wet layed) composite and wood. These materials are used as they are easily shaped into the complex geometry that is often required, however, they are often fragile and as a result will not result in robust tooling. Once a pattern is available the “splash” can be manufactured (from composites) which is an intermediate stage in the process used to transfer the intended shape from the female to male configuration or visa versa. The actual tool will be made from the splash. There are many material options to consider in

tooling construction with the basic requirement being that the tooling retains its dimensional stability through the various curing cycles, including high temperature and pressure applications. The working life of the tool is hard to define as there are a number of overriding influences to take into account such as the materials to be cured and often more important is the treatment and handling of the tooling. Composite tooling offers advantages in several key areas over traditional metal tools:

- Dimensional stability as the coefficient of thermal expansion between the tool and product to be made can be tailored and closely matched.
- Low weight results in ease of handling as well as rapid heat-up and cool-down of tools due to the low thermal mass.
- Uniform heat-up due to uniform wall thickness of tool.
- Unlimited size.
- Ease and speed of construction.

To take advantage of the dimensional stability it must be noted that glass fibre should be used for GRP components and carbon fibre should be used for CRP and aramid components. Aramid is not generally used for tool manufacture as it has a propensity for absorbing moisture, which can have numerous negative effects on the tool. Carbon has a closer coefficient of thermal expansion to aramid than glass so it follows that carbon tooling should be used for aramid products. Composite tools achieve an even temperature distribution due to the uniform thickness of the tool, thus minimizing thermal stresses being induced in the part to be cured. Metal tools however generally do not have a uniform thickness and therefore, are known to suffer localized 'hot' and 'cold' spots that induce residual thermal stress [8].

2.6.2 MANUFACTURING PROCESSES

An important factor to remember when dealing with composites is that, unlike with traditional engineering materials, the material is created at the same time as the component. Although this gives increased freedom to the designers, it also presents more opportunity to introduce errors hence attention must be paid to the manufacturing process. It is also important to understand that the component made with the same constituents can show very different properties purely by changing the manufacturing process. The designer must also keep the manufacturing capabilities in mind when designing, for example it is not possible to maintain a constant fibre angle on a tapered cylinder using filament winding. It is therefore imperative that there is good knowledge of all aspects of the manufacturing process including design, fabrication and evaluation [12]. In this section a few commonly used production methods are presented. It should be noted that although spray lay-up, filament winding and pultrusion methods are not discussed in the text (as they are of little importance in this dissertation) they are of importance in other areas of industry.

2.6.2.1 HAND LAY-UP / WET LAY-UP

The traditional method of processing a composite material is by means of hand lay-up (or wet lay-up). Hand lay-up is the most elementary method and involves impregnating the resin into the reinforcement with the use of brushes and consolidation rollers. Laminates are typically left to cure under atmospheric conditions.

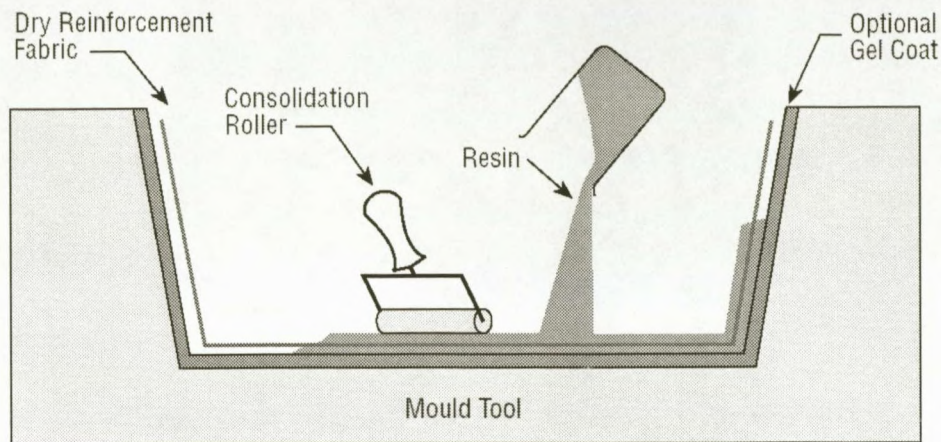


Figure 2.24 Hand lay-up / wet lay-up process. [7]

Hand lay-up has been the most widely used method for many years and this is largely due to its simplicity, low cost tooling and materials. There are many disadvantages associated with this method such as low quality laminates with poor fibre volume fractions and high quantities of voids. The quality of the laminate is directly related to the laminator's skill. Other disadvantages that are largely unavoidable are high styrene emissions from polyester and vinylester resins and compromised mechanical properties due to the need for a lower viscosity resin to aid wetting out [7].

You can do wet lay-up w/ epoxies!

2.6.2.2 VACUUM BAGGING

The vacuum bagging process is an extension of the hand lay-up process where pressure is applied to the laminate for further consolidation. This is achieved by sealing a plastic vacuum bag over the wet laminate and onto the tool. Under the vacuum bag is a layer of peel ply followed by a layer of release film and finally the breather as shown below.

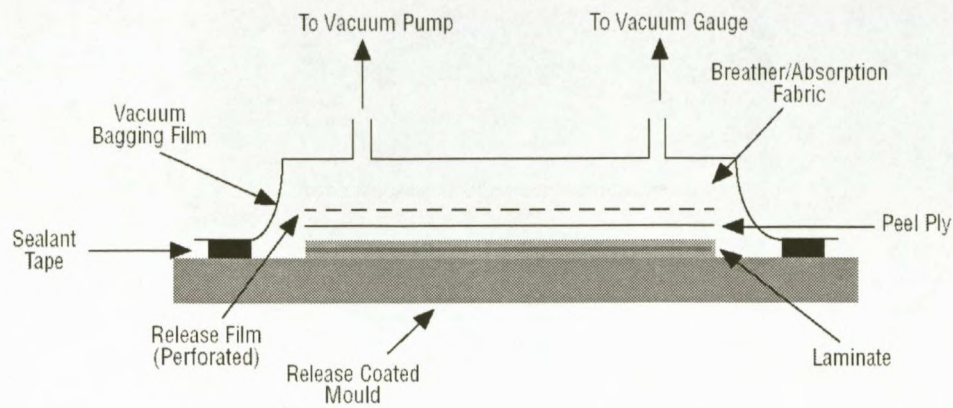


Figure 2.25 Vacuum bagging process. [7]

Vacuum bagging can improve the fibre content, lower the void content, improve fibre wet-out and limit volatile gas emissions during the cure cycle. There are however extra process costs of both labour (higher skill level is required) and disposable materials, and the mixing and control of the resin is still largely determined by the laminator [7].

2.6.2.3 RESIN TRANSFER MOULDING (RTM)

Resin transfer moulding (RTM) involves the use of two matching tools into which the reinforcement is placed and then the resin is drawn through the fibre. The creation of a preform is incorporated into the process, which consists of shaping the reinforcement into the desired shape, and placing it in the tool. Once the preform is placed in the tool and the matching tool is clamped down the resin can be injected into the cavity as shown below. Vacuum may be used to assist the resin flow through the reinforcement, this is known as vacuum assisted resin injection (VARI). As soon as all reinforcement has been wet out the inlets are closed and the laminate is allowed to cure. The curing of the laminate may take place at ambient temperature or at an elevated temperature.

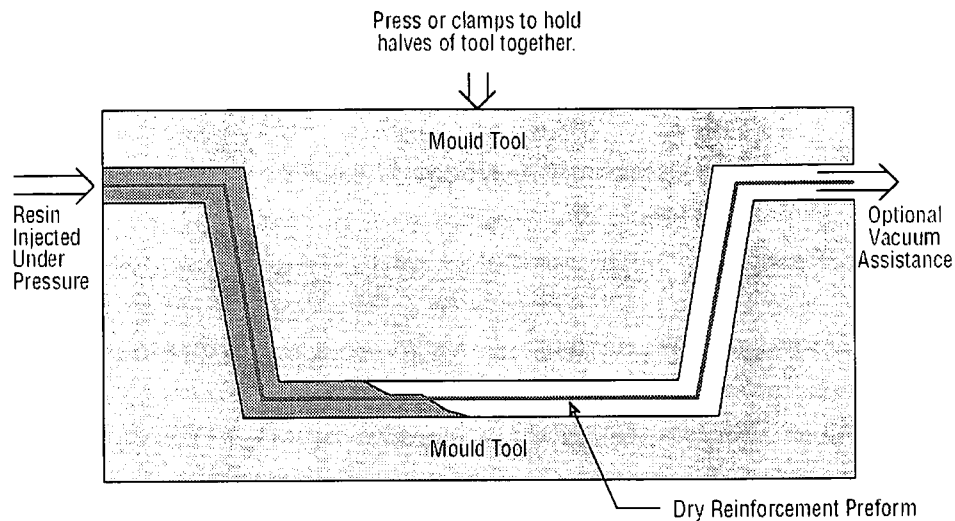


Figure 2.26 RTM process. [7]

The advantages of resin transfer moulding include efficiency (low wastage), suitability for complex shapes, superior surface definition on both sides, better reproducibility, relatively low clamping pressure, low volatile gas emissions, ability to include inserts, low void content and high fibre volume fraction [3,7]. The tooling however can be expensive and heavy in order to withstand the pressures exerted. Generally, the process is limited to smaller components and unimpregnated areas do occur if the design of the tooling is not done correctly [7].

2.6.2.4 OTHER INFUSION PROCESSES – SCRIMP, RIFT, VARTM ETC.

In these processes the reinforcement is placed in the tool as with resin transfer moulding. The reinforcement is then covered with peel ply and a knitted non-structural fabric. The dry reinforcement is then covered with a vacuum bag and, once sealed, resin is drawn into the laminate. The non-structural fabric aids the resin distribution over the entire laminate. This process is illustrated below.

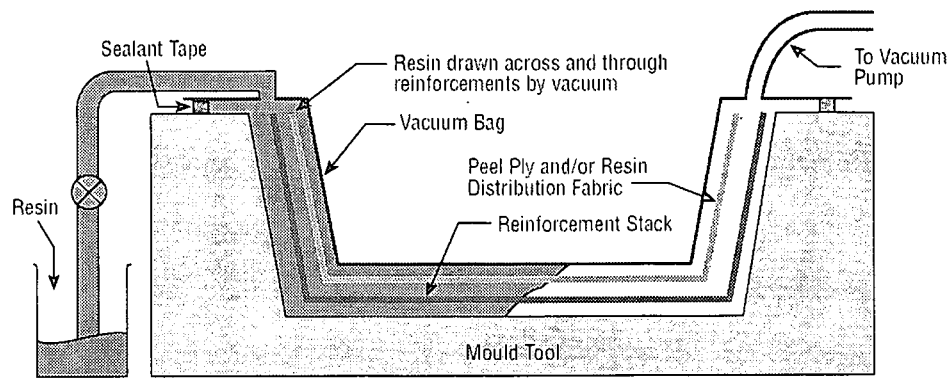


Figure 2.27 Infusion process. [7]

This system shares many of its advantages and disadvantages of resin transfer moulding. Due to the nature of the system the tooling costs are far less, it is possible to fabricate larger components, standard hand lay-up tools may be easily modified and cored structures may be produced in one operation. This process can be complex to perform and only one side of the product has a moulded finish.

2.6.2.5 PREPREG

Pre-impregnated materials (commonly referred to as a “prepreg”) are renowned for their high performance properties [8]. Prepreg’s are comprised of reinforcement fibres or cloth fabrics into which a pre-catalysed resin system is impregnated by the manufacturer [15]. The pre-catalysed resin system in these materials only react very slowly (several weeks or months after defrost) at room temperature, and require heat and pressure to cure fully [8]. The resin is almost solid at room temperature and has a light tacky feel to it. The prepreg is layed up in the tool, vacuum bagged, and then heated to typically 120-180°C. Additional pressure and elevated temperatures are provided by an autoclave to effect the cure cycle as well as ensuring consolidation of the laminate. Low temperature

curing prepregs are available, which do not require an autoclave, however these systems have a substantially short self-life once defrosted.

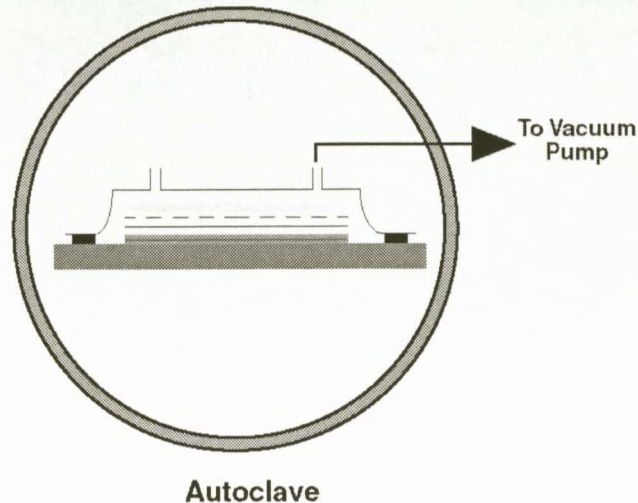


Figure 2.28 Pre-preg process. [7]

Prepregs are more expensive, however, they have many benefits that include high fibre volume fraction, low coefficient of thermal expansion and low void content [7,8]. Due to the elevated temperatures required for the curing process the tooling and any core materials need to withstand the temperatures involved.

2.6.2.6 RESIN FILM INFUSION (RFI)

The RFI concept offers a cost-effective alternative to traditional prepreg systems. Instead of impregnating resin into the fibres, the material consists of dry fibre reinforcements, into which a thin semi-solid precatalysed resin film has been inserted. When a vacuum bag is applied, the dry reinforcement provides a path for air to be extracted from the material, from between the reinforcement layers and from the mould

surface. During the heat cure cycle the resin film softens and is allowed to flow into the air-free reinforcement [15]. The process is illustrated below.

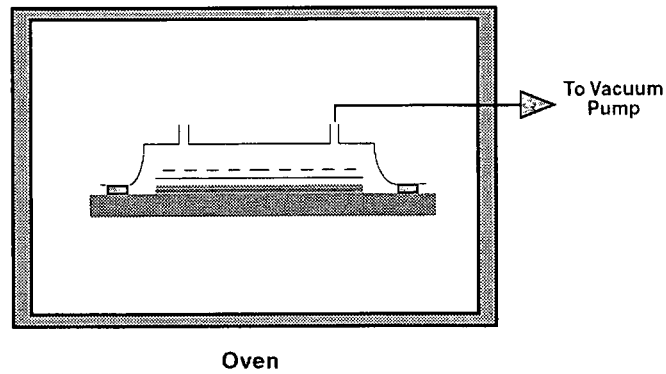


Figure 2.29 RFI process. [7]

Resin film infusion yields a composite component with high fibre content, very low void content (0-0.5%) and good surface finish. The material system is claimed to result in product quality and performance comparable with prepreg systems whilst minimising the associated manufacturing difficulties and processing costs [15]. As with the low temperature prepreg, resin film infusion requires the use of vacuum bagging in conjunction with an oven to achieve full cure. As a result all tooling and core materials are required to withstand the temperatures experienced during the cure cycle [7].

SPRINT™ - SP Resin Infusion Technology - is an example of a commercially available product based on the resin film infusion concept. **SPRINT™** profits from the advantages associated with RFI and allows aerospace-quality glass and carbon fibre components with high mechanical properties to be manufactured at high production rates and with considerably reduced cost [16].

The steps involved in the manufacturing of SPRINT™ are listed below. This process is illustrated with the aid of Figure 2.30.

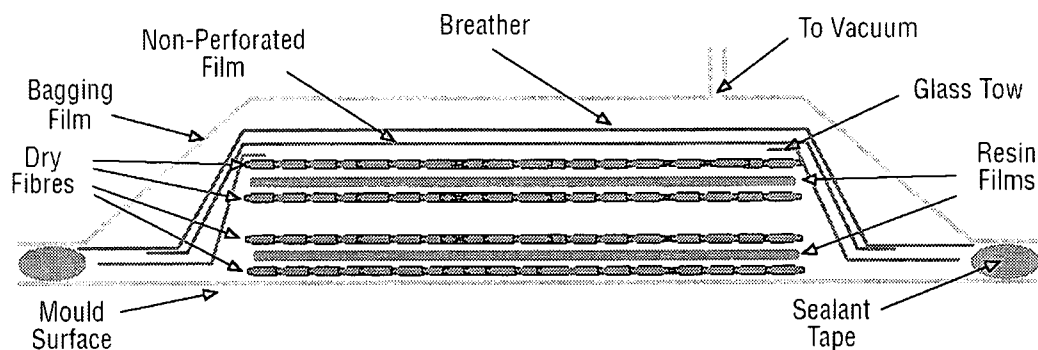


Figure 2.30 Typical Processing of the SPRINT™ Material System. [41]

The Basic process is relatively simple and involves the following steps [6]:

- Preparation of mould surface with appropriate release interface.
- Lay-up of SPRINT™ layers with fibre tows between each layer.
- Placement of peel ply.
- Placement of non-perforated release film.
- Placement of breather.
- Placement of vacuum bag.
- Application of vacuum for 15 minutes at ambient temperature.
- Application of appropriate cure cycle.
- Product de-mould.

2.6.2.7 CORE MATERIALS

Engineering theory shows that the flexural stiffness of a panel is directly proportional to the cube of its thickness. The purpose of a core in a composite laminate, as with any

sandwich panel for that matter, is to increase the stiffness by effectively separating the outer layers (where the highest tensile and compressive stresses occur) by inserting a low-density core material. This “thickening” of the laminate provides a large increase in the stiffness for very little additional weight.

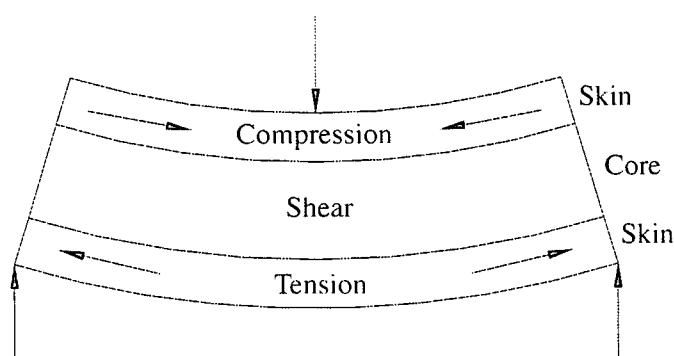


Figure 2.31 Core laminate in bending. [41]

Figure 2.31 above illustrates a cored laminate under a bending load. The sandwich laminate presented can be compared to an I-beam, in that the laminate skins act as the I-beam flanges, and the core materials act as the beam's web. In this mode of loading it can be seen that the upper skin is put into compression, the lower skin into tension and the core experiences shear. It therefore follows that one of the most important properties of a core is its shear strength, stiffness and most importantly light weight.

In addition to the above mentioned properties required in a core material and particularly when considering a thin laminate skin, the core must be capable of withstanding a compressive load without failure. This aids to prevent the thin skins from wrinkling, and failing prematurely in a buckling mode. There are numerous core materials on the market including the following [41]:

- Foams - Foams are very popular core materials and can be manufactured from a variety of synthetic polymers. They are most commonly supplied in densities ranging from 40 to 200kg/m³ and thicknesses from 5 to 50 mm.
- Honeycomb structures - Honeycomb cores are available in a variety of materials such as paper, plastic or aluminium and typically range from 3 to 50 mm thick.
- Woods - Wood has a very similar structure to that of honeycomb however it is susceptible to moisture absorption. Typical woods include balsa and cedar.
- Low-density mats - Low-density mats consist of a fabric containing density-reducing hollow spheres. These mats are typically 1 to 3 mm thick and are used as a layer in a laminate.

One core material variant is a relatively new product on the market called Parabeam® 3D Glass Fabric, which is a fabric woven out of E-glass yarn and consists of two deck layers bonded together by vertical piles in a sandwich structure. These piles are woven into the deck layers thus forming an integral sandwich structure. When the Parabeam® is impregnated with a thermoset resin, the fabric absorbs the resin and due to the capillary forces of the piles, the fabric rises to the preset height. In this one-step process a lightweight and strong sandwich laminate is formed that offers excellent mechanical properties [13].

2.7 CONCLUSION

The growth in the composites industry has been remarkable and the possibilities for the future are endless with the advancements in technology being the driving factor. Various military aerospace divisions have undertaken much of the research and

development in composites, as well as taking the risks in applying them. The civil market has followed suit with extensive use of composites not only in aircraft but also in transportation, automotive, and sporting equipment to name but a few. The future promises increased composite use in an ever-increasing variety of applications. As knowledge and acceptance of composites increases so the growth is projected to increase exponentially.

CHAPTER 3

THE FEM AND OPTIMISATION

3.1 OVERVIEW OF THE FEM AND OPTIMISATION

The need for finite element analysis (FEA) stemmed from the necessity for a predictive analysis method [15]. The finite element method (FEM) has now become the predominant structural analysis and design tool, which is used routinely by structural engineers. The finite element method is a numerical analysis technique for obtaining approximate solutions to engineering problems. The use of finite element analysis coupled with an optimisation technique makes for a powerful design tool. Once a model is optimised the results can be incorporated into the analysis to produce a solution, ultimately minimising or maximising the design objective as required. Design optimisation (or mathematical programming) refers to the process of generating improved designs. An optimisation problem can be attempted using any of a variety of mathematical techniques. One such technique is linear programming (LP) where the objective and constraint functions are linear functions of the design variables. For problems that cannot be formulated as linear programming problems, algorithms that deal with non-linear programming problems can be used [13]. A general approach to structural optimisation, which has received considerable attention of late, is that of sequential linear programming. It represents one of the most flexible approaches to structural optimisation. The general nature of the solution methodology involves an iterative procedure where linearisation of the problem is carried out for a succession of

trial solutions. The technique effectively converts the problem into a series of linear programming problems, which allows the solution to converge to within an acceptable range of the true optimum [18].

3.2 FINITE ELEMENT METHOD (FEM)

3.2.1 INTRODUCTION TO THE FEM

Traditionally the use of classical (or continuum) methods have been used to solve stress analysis problems in engineering structures. These methods, which have evolved over many decades, provide the designer with displacement and stress information using a combination of equilibrium, compatibility and stress-strain behaviour.

When using these classical methods to solve a problem, assumptions predictably need to be made such as, materials are often considered to be isotropic, and beam and plate structures are commonly treated as one- or two-dimensional respectively. These simplifications are required as the governing equations become increasingly more complicated in more complex structures and therefore require increasingly sophisticated mathematical techniques to solve them [14], this approach generally results in inaccuracies or incorrect results. As a result of the highly complicated calculations many engineers in the past have designed components that were not stress analysed. When a designer did any calculations it was generally an approximation steeped in assumptions that only provided a starting point to the design process. A prototype would invariably need to be stress-tested or run through accelerated service tests to determine, by trial-and-error, what the final design would be.

As an alternative method to solving these problems, numerical techniques may be used to find an approximate solution whilst retaining the complexity of the problem. One of the most commonly used techniques is the general finite difference scheme. Although very useful in solving fairly difficult problems, if irregular or unusual boundary conditions are considered the finite difference scheme becomes difficult to implement [18].

The finite element method (FEM) is an alternative to the classical methods and an evolution of the finite difference scheme of solving the governing equations of a structural problem [14]. The introduction of finite element method (as an alternative approach) to industry has transformed the market, by allowing complicated analysis of intricate components, and as a result has become routinely used by engineers as the structural analysis and design tool of choice. Engineering problems are not usually represented by analytical functions, therefore the finite element analysis technique for obtaining approximate numerical solutions to structural engineering problems have become popular. The finite element method was initially developed to study stresses in aerospace structures, however the method has diversified into areas such dynamic structural analysis, fluid flow and heat transfer and in fields as diverse as economics and social development [18].

The finite element method is described by Cook [19] as being a piecewise polynomial interpolation. This, simply put, involves splitting the structure up into a finite number of elements (or parts), treating each element independently, and then reassembling the structure using the nodes as a link between elements. As a result, a set of simultaneous algebraic equations is generated. As the structural problems become increasingly more

complicated so the number of equations become overwhelming, hence the need for computers [19].

The finite element method was initially developed for materials that displayed isotropic behaviour, as these were the preferred construction materials of the time. To apply the technique to composites, which exhibition anisotropic, or orthotropic behaviour, requires different element formulations that adequately represent their, stiffness and strength, as well as the laminated form of construction often used [14]. As a result, much of the skill in applying the finite element method is in selecting appropriate elements of the correct type, size and distribution (the FE 'mesh') to represent material behaviour. The success of the analysis also largely depends on the approximations made by the user regarding the modelling, loading and boundary conditions.

3.2.2 A SHORT HISTORY OF THE FEM

The FEM was initially developed through a combination of applied mathematics and engineering from as early as 1941 where Hrenikoff [20] explored a continuum structure that was comprised of elements. Relevant work done in the applied mathematics field (albeit unnoticed at the time) started in 1943 with Courant [21] who investigated the use of a piecewise polynomial solution for a torsional problem. At the time this process was impractical due to the extensive mathematical processing required. In the 50s work in the aircraft industry prompted the practical introduction of the FEM to designers with work done by people such as Turner, Clough, Martin and Topp [22] who presented a solution of plane stress analysis of delta wings based on the equations of elasticity theory. There has been a large amount of work done on the subject over the years with

vast improvements being the net result, as well as different applications such as static and dynamic analysis, heat transfer, fluid flow, and in various other areas. However it wasn't until 70s that the availability of computers and FE software became available so that the process could be practically implemented. By the late 80s the software had progressed and was available on microcomputers complete with relatively easy to use pre- and postprocessors. By the 90s the FEM had been well documented with many papers and books being published on the subject. Today the finite element method has become a mainstream tool for engineers and becoming even more popular for designers on a daily basis.

3.2.3 THE FINITE ELEMENT METHOD CONCEPT

The finite element discretisation procedure involves representing the solution region by a finite number of small elements. The final solution is an approximation defined in terms of the results of the functions for each element. The approximation functions are defined at the nodes of each element. Nodes are commonly located at node boundaries (such as vertices and mid-side nodes) however interior nodes are also possible. The degree of approximation is dependent on the size, configuration and number of elements as well as the formulation of the interpolation function. The function needs to be chosen carefully as the formulation satisfies certain criteria. Unlike with other approximate numerical methods, the finite element method provides the formulated expressions for individual elements that combine to present the overall result [23]. The implementation of the finite element method is discussed in detail below.

3.2.3.1 PRACTICAL FEM IMPLEMENTATION

3.2.3.1.1 Pre-Processing and Post-Processing

When using a FEA computer package there are numerous procedures that occur when analysing such as matrix manipulations, numerical integration, and equation solving to name a few, of which the user often witnesses little of. The user mainly deals with the pre-processor and postprocessor, which are computer interfaces that make the process more user-friendly. The pre-processor deals with the model generation that consists of describing loads, constraints, materials, and generating the FE mesh whereas the postprocessor entails evaluating the outputs, and listing and plotting the results [19].

3.2.3.1.2 Model Generation

Modeling is a numerical simulation of a physical structure or process, as stated by Cook [19], and as a result care must be taken to fully represent the structural problem when creating the geometric model so that the mathematical process can produce an accurate result. In some cases limited simplification of a complex problem is acceptable as long as the resulting model sufficiently represents the problem the user wishes to analyse, this is where the skill of the user is important. The model is made up of regions that may be lines, surfaces or solids. Various checks and blending routines are necessary to prevent artificial cracks or other discontinuities appearing [19].

Often user manuals will provide information on which elements work best under particular conditions. A decision has to be made whether to use linear (first or low

order) or parabolic (second or high order) elements and which element type to use, whether to use a 1D beam, 2D shell, or 3D solid. A parabolic element has mid-side nodes whereas a linear element does not, this increases the accuracy of the model, as a generalisation, the more nodes, the more accurate the model.

X X

3.2.3.2 MESH AND ELEMENTS TYPES

3.2.3.2.1 General Definition

Elements, as previously mentioned, are interconnected at imaginary points on boundaries or surfaces of the elements called nodes. In general elements do not deform or change shape; it's rather defined as a region of space where a displacement field exists. The nodes of an element are located in space where the displacements or its derivatives are known or sought. Once the element mesh has been determined, the approximate behaviour of the unknown field variable is established over each element by continuous functions expressed in terms of the nodal values. These functions are also known as interpolation functions, shape functions, or field variable models. The solution is established by assembling the interpolated functions into a piecewise approximation to the field variable [18].

3.2.3.2.2 Domain Discretisation

Once the model of the problem is completed and the continuum or solution region (curves, surfaces or solids) has been subsequently created the user can discretise the continuum by dividing it into a series of elements. This simply put is the meshing

process, and can also be described as subdividing the structure up into a finite number of pieces which resemble segments of the structure [14]. It is often desirable to use different element types and sometimes necessary. Discretisation problems are often introduced at this stage where the mesh density needs to be refined. Care must be taken to maintain a fine mesh in high stress areas as well as in regions where complicated geometry requires full representation. Convergence testing (refer below) is required at this point in order to establish how many elements are required to fully represent the structure.

Care must be taken to keep the elements as compact and as regular as possible so that the model can give an accurate representation of the design's behaviour. If a long narrow element is used inaccurate results may occur due to the resulting direction bias. For example, a square shell element will give the most accurate results when considering a quadrilateral element. It is not always possible to avoid element distortion completely in that case it is advantageous to keep it to a minimum or at least confine it to a zone where low stress variation is expected. The choice of element type is critical, as certain elements are only suitable under specific loading conditions and should be chosen to suit the problem under investigation. For example, a very wide or short deep beam is not suited to beam theory as the formulation only takes axial normal stress into account [19]. Often only one type of element is sufficient to represent the structure, however circumstances can dictate the use of two or more element types. A good example would be an elastic body supported by pin connected bars. Here it would be effective to use one dimensional elements to represent the pins whereas the elastic body could be modelled with three dimensional solid elements such as bricks [18]. when considering two-dimensional elements, the most popular and versatile type due to the

ease with which they conform to complex geometry is triangular elements. The number and type of element to be used is ultimately left to the engineer's discretion.

3.2.3.2.3 Basic Element Shapes

The continuum or solution domain of arbitrary shape can be accurately modelled by a collection of elements, which are usually of a simple geometric shape.

When considering a one-dimensional problem with only one independent variable, the use of line segment elements is the rational choice. The number of nodes assigned to a one-dimensional element is usually two, however this can vary depending on the degree of accuracy required. For problems such as framework analysis the use of the one-dimensional finite element analysis is adequate.

The three-node flat triangular element is the simplest two-dimensional element and it is recognised as being the most often used element. The reason for this distinction is that the triangular elements conform to complex geometry without much distortion relatively easily when compared to other two-dimensional elements. Another very popular two-dimensional element is the four node rectangular element. These elements are easily constructed by the finite element program however they are limited when defining curved boundaries. In addition to these basic elements high order multi-node triangular and quadrilateral elements are also fairly common.

The simplest and most common three-dimensional element is the four-node tetrahedron element. Other common three-dimensional elements include the simple right hand prism and the hexahedron type elements that are constructed from five tetrahedral elements.

Isoparametric elements are very useful due to their curved boundary construction. These elements have proven popular in reducing computational effort (by using less elements) whilst retaining accuracy when considering three-dimensional problems.

3.2.3.2.4 Convergence Testing

Convergence testing is required to ensure that there are sufficient elements to provide an accurate answer without wasting analysing time by over-refining the mesh. This method will by no means tend towards a real-life answer, but rather a more reliable FE solution. Convergence testing involves successive “runs”, each time altering the number of elements that make up the structure, until the results starts to converge towards a particular value. When convergence is achieved you are left with a structure that is sufficiently represented with reliable results without excess elements that translate into unnecessary calculation time.

3.2.3.3 THEORETICAL FEM FORMULATION

3.2.3.3.1 Continuum Problems

When considering the continuum approach to nature, all processes are characterised by field quantities that are defined at every point in space. In continuum problems the independent variables are the co-ordinates of space and time that are concerned with areas of temperature, stress, mass concentration, displacement, electromagnetic and acoustic potentials, etc. These problems arise from the phenomena in nature that are

approximately characterised by partial differential equations and their boundary conditions.

Continuum problems of mathematical physics are often referred to as boundary value problems because their solution is sought in some domain defined by a given boundary, on which certain conditions called boundary conditions, are specified. The boundary is said to be closed if conditions affecting the solution of the problem are specified everywhere on the boundary and open if part of the boundary extends to infinity and no boundary conditions are specified on the part at infinity [18].

3.2.3.3.2 Problem Statement

If ^{one} (you) consider some domain D bounded by the surface Σ . Let ϕ be a scalar function defined in the interior of D such that the behaviour of ϕ in D is given by

$$L(\phi) - f = 0 \quad (3.1)$$

Where f is a known scalar function of the independent variables and L is a linear or non-linear differential operator. It is assumed that the physical parameters in the differential operator are known constants or functions. In n dimensions, second-order differential operators can usually be reduced, by a suitable transformation, to the form

$$L(\cdot) = \sum_{i=1}^n A_i \frac{\partial^2(\cdot)}{\partial x_i^2} + \sum_{i=1}^n B_i \frac{\partial(\cdot)}{\partial x_i} + (\cdot)C + D \quad (3.2)$$

where coefficients A_i , B_i and C and the term D may be functions. The operator as given in equation (3.2) is linear if A_i , B_i and C and D are functions of x_i ; and the dependent parameter, as well as first derivatives of the dependent parameter. An operator is linear only if

$$L(f + g) = L(f) + L(g) \quad (3.3)$$

The general definition of the operator $L()$ in equation (3.1) precludes a discussion of appropriate boundary conditions. However, without conditions, equation (3.1) does not describe a specific problem.

It can be seen from equation (3.1), the general problem is to find the unknown function ϕ that satisfies the equation and the associated boundary conditions applied to surface Σ . There are many methods that can be used to solve this type of problem, these approaches can be broken up into two categories namely analytical and numerical methods. The numerical method, which has gained the most interests over recent years and the subject of this chapter, is the finite element method [18].

3.2.3.3 The Variational Approach

Often continuum problems have different, but equivalent, differential and ^{SP} variational formulations. When considering the classical variational formulation, the problem is to find the unknown function or functions that extremize or make stationary a functional such as $I(\phi)$ or system of functionals subject to the boundary conditions. Variational methods are among the oldest means of obtaining solutions to problems in physics and engineering. One general method for obtaining the approximate solutions to these problems, expressed in variational form, is known as the Ritz method [18,24].

3.2.3.3.4 Relation Of The ~~Fem~~ To The Ritz Method



The finite element method is closely based on the Ritz method and hence they are essentially equivalent. The finite element method is in fact a special case of the Ritz method where the interpolation functions satisfy certain continuity criteria. The main difference between the two methods is that the assumed trial functions in the finite element method are not defined over the whole solution domain as with the Ritz method and they do not have to satisfy boundary conditions only certain continuity conditions. As the Ritz method uses functions defined over the entire domain, only relatively simple geometrically shaped problems may be solved. When considering the finite element method, the same limitations exist however only each element. As a result of this the finite element method is far more versatile and flexible than the Ritz method, capable of analysing far more complex geometries [18,24].

3.2.3.3.5 Element Equations From The Variational Principle

At this stage the finite element method takes over by determining the matrix equations to express the properties of the individual elements from the information provided by the user. Subsequently the element properties are assembled to obtain the system equations. The matrix equations are combined to express the behaviour of the elements and formed to express the behaviour of the entire solution system. Finally the system of equations is solved.

The finite element solution to the problem involves determining the nodal values of ϕ so as to make the functional $I(\phi)$ stationary. To make $I(\phi)$ stationary with respect to the nodal values of ϕ , it is required that

$$\frac{\partial I(\phi)}{\partial \phi_i} = \sum_{i=1}^n \frac{\partial I}{\partial \phi_i} \delta \phi_i = 0 \quad (3.4)$$

where n is the total number of discrete values of ϕ assigned to the solution domain.

Since the $\delta \phi_i$'s are independent, equation (3.4) can be satisfied only if

$$\frac{\partial I}{\partial \phi_i} = 0, i = 1, 2, \dots, n \quad (3.5)$$

The functional $I(\phi)$ may be written as a sum of individual functions defined for all elements of the assemblage, that is,

$$I(\phi) = \sum_{e=1}^M I^{(e)}(\phi^{(e)}) \quad (3.6)$$

where M is the total number of elements and the superscript (e) denotes an element.

From equation (3.6), it follows that

$$\delta I = \sum_{e=1}^M \delta I^{(e)} = 0 \quad (3.7)$$

where the variation of $I^{(e)}$ is taken only with respect to the nodal values associated with the element (e). Equation (3.7) implies that

$$\left\{ \frac{\partial I^{(e)}}{\partial \phi} \right\} = \frac{\partial I}{\partial \phi_j} = 0, j = 1, 2, \dots, r \quad (3.8)$$

where r is the number of nodes assigned to element (e). Equation (3.8) comprises a system of r equations that characterise the behaviour of element (e). Equation (3.8) for element (e) can always be written as [18]

$$\left\{ \frac{\partial I^{(e)}}{\partial \phi} \right\} = [K]^{(e)} \{\phi\}^{(e)} - \{F\}^{(e)} = \{0\} \quad (3.9)$$

where $[K]^{(e)}$ is a square matrix of constant stiffness coefficients, $\{\phi\}^{(e)}$ is the column vector of nodal values and $\{F\}$ is the vector of resultant nodal actions. Symbolically, the complete set of equations can be written as

$$\frac{\partial I}{\partial \phi_i} = \sum_{e=1}^M \frac{\partial I^{(e)}}{\partial \phi_i} = 0, i, 1, 2, \dots, n \quad (3.10)$$

$$\left\{ \frac{\partial I}{\partial \phi} \right\} = \{0\} \quad (3.11)$$

The problem is solved when the set of n equations (3.10) is solved simultaneously for the n nodal values of ϕ . If there are Q nodes in the solution domain where ϕ is specified by boundary conditions, there will be $n - q$ equations to be solved for the $n - q$ unknowns.

3.2.3.3.6 Requirements For Interpolation Functions

There are two requirements that need to be considered when dealing with the interpolation functions namely the compatibility and completeness. Compatible elements refer to those elements whose interpolation function satisfy the first requirement below. The second point refers to complete elements whose interpolation functions satisfy the second points requirements. Both requirements are stated below by Felippa and Clough [9] and justified by Oliveria [25].

At element interfaces (boundaries) the field variable ϕ and any of its partial derivatives up to one order less than the highest order derivative appearing in $i(\phi)$ must be continuous.

X

All uniform states of ϕ and its partial derivatives up to one order less than the highest order derivative appearing in $i(\phi)$ should have representation in $\phi^{(e)}$ when, in the limit, the element size shrinks to zero.

Once the number, type and distribution of elements to be used has been established and the nodes have been assigned to each element the user is required to select the interpolation function that will represent the variation of the field variable over the element. Polynomials are the usual choice as they are easily integrated and differentiated. As a generalisation, the more nodes there are the more accurate the results.

3.3 OPTIMISATION

3.3.1 INTRODUCTION TO OPTIMISATION

When considering a structural design, an increased number of design variables can have both positive and negative implications. The designer now has more control over the design, however at the same time these variables become a hindrance as identifying their value in providing an ideal design becomes more difficult. Traditionally the process of improving a design was achieved through the evaluation and comparison of alternative designs. Although mathematical methods related to optimisation existed for many decades, as with the FEM, they have generally been too complex for manual calculation. Hence the design was left up to the designer's intuition and experience. With the advent of personal computers there has been a marked increase in research into optimisation procedures and as a result there are many techniques available today.

Achieving a design that is safe but still satisfies many constraints makes mathematical optimisation the natural choice when considering composite design in particular [13].

3.3.2 DESIGN ORIENTATED STRUCTURAL ANALYSIS

Design orientated structural analysis (or DOSA) represents optimisation techniques for structural analysis used in conjunction with the finite element method. The realisation that design optimisation and finite element analysis were fundamentally linked resulted in a great deal of work done to increase the efficiency of the finite element method in particular. The DOSA system generally falls into three broad categories. Firstly the sensitivity of the problem's response to the design variables needs to be established, in other words data that defines the rate of change of the design response to the design variables is required [26,27]. The second area of interest is the application of approximate optimisation functions that will adequately represent the solution of the problem with the minimum of computational effort [28,29]. The final category of interest deals with the integration of the finite element method code with the optimisation strategy [30]. Efforts have been made in this area to develop an efficient interface between the two systems. The flexibility of the DOSA system has been a direct result of the general applicability of the components namely the finite element method and the optimisation techniques used. Shown below in Figure 3.1 is a typical structural optimisation process.

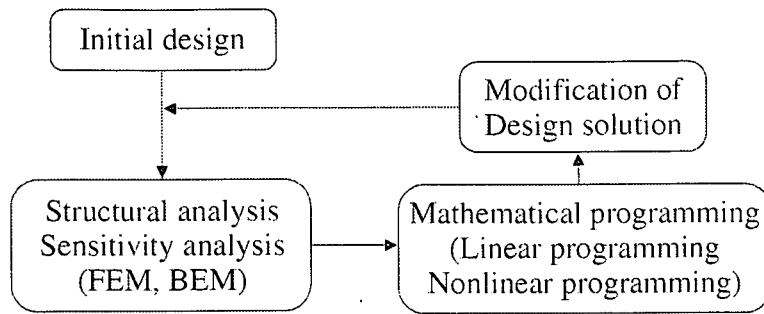


Figure 3.1 A typical structural optimisation process. [31]

3.3.3 PROBLEM FORMULATION FOR OPTIMUM DESIGN

A structural system can be complex and difficult to define, however it can be idealised by describing it in terms of a hierarchy of descriptors which specify the type of structure, the type of components, the materials, the geometric layout and the dimensions. By defining the structure in this way enables the formulation of the equations required for the optimisation problem to be developed.

The mathematical statement is defined by identifying the variables that govern the concept, called the design variables. The design variables may theoretically consist of any quantifiable aspect of the structural system, however the most commonly used variables include material thickness, fibre angle, material failure and system response. These variables are given numerical values so a solution may be reached, this is not the optimum, merely a starting point of the process. Without any limitations there would be nothing to govern the design hence constraints are imposed on the variables. There are three types of constraint functions, the first and most common type is the inequality constraint denoted by G_j , the second type is the equality constraint denoted by H_k and the last type is the side constraints (or move limits or step size). The side constraints are

placed on each individual design variable X_i , these are represented by the lower bound, X_i^l , and the upper bound X_i^u . The first two constraints guide the overall design whereas the side constraints relate to specific design variables. If all the constraints are satisfied then we have a feasible solution. In most practical problems the design constraints are of a non-linear nature however there are several techniques that deal with this.

A feasible solution is not necessarily the best feasible solution, or optimum which is the ultimate aim. Mathematical optimisation as previously mentioned consists of the minimisation (or maximisation) of the objective function with respect to the limits or constraints. Hence, an objective (or cost) function, which defines the criterion that guides the design through alternatives, must be introduced in order to ascertain whether the solution is better than another [32]. The objective function is normally defined in the form of a scalar function and is usually performance or cost related in terms of the design variables. It follows that the objective function is also represented in the design space as a hypersurface or hyperplane.

The standard mathematical statement representative of an optimisation problem is stated below [32]:

$$\text{Minimise:} \quad F(\mathbf{X}) \quad (3.12)$$

$$\text{Subject to:} \quad G_j(\mathbf{X}) \leq 0 \quad j = 1, m \quad (3.13)$$

$$H_k(\mathbf{X}) = 0 \quad k = 1, l \quad (3.14)$$

$$X_i^l \leq X_i \leq X_i^u \quad i = 1, n \quad (3.15)$$

$$\mathbf{X} = (X_1, X_2, X_3, \dots, X_n) \quad (3.16)$$

where $F(\mathbf{X})$ represents the design objective function. G_j the j th inequality constraint, and H_k the k th equality constraint, which are used to place limits on the structural behaviour such as displacement and stress. \mathbf{X} is the set of n design variables.

An important concept when considering the design variable vector is the design space. For an optimisation problem containing n design variables, the design space will have n dimensions and is termed a hyperspace. It stands therefore that any point located within the design space has n components.

The mathematical statement presented above represents the standard format of the process. A non-linear problem is generally the result, and there are many techniques that many be used to solve it. A common technique to solve a complex non-linear problem is to transform it into a simpler linear approximation that is easier to find the solution to. The solution is typically found using the simplex method.

3.3.3.1 LINEARISATION OF A NON-LINEAR PROBLEM

The linearisation of the objective function and constraint functions is achieved with the use of the first order Taylor series expansion about the present design \mathbf{X}^0 resulting in a statement of the form

$$\text{Minimise:} \quad F(\mathbf{X}) \approx F(\mathbf{X}^0) + \nabla F(\mathbf{X}^0) \cdot \delta \mathbf{X} \quad (3.17)$$

$$\text{Subject to:} \quad G_j(\mathbf{X}) \approx G_j(\mathbf{X}^0) + \nabla G_j(\mathbf{X}^0) \cdot \delta \mathbf{X} \leq 0 \quad j = 1, m \quad (3.18)$$

$$H_k(\mathbf{X}) \approx H_k(\mathbf{X}^0) + \nabla H_k(\mathbf{X}^0) \cdot \delta \mathbf{X} \leq 0 \quad k = 1, l \quad (3.19)$$

$$X_i^l \leq X_i + \delta X_i \leq X_i^u \quad i = 1, n \quad (3.20)$$

where $\delta \mathbf{X} = \mathbf{X} - \mathbf{X}^0$ (3.21)

The above equations represent linear combinations of the design variables, contained in vector $\delta \mathbf{X}$. It is evident that the gradients of the structural responses are required to construct the objective and constraint approximations. This data may be obtained by sensitivity analysis (which is discussed in detail later). For example, finite difference methods may be used to obtain changes in response for small changes in the design variable. This is computationally expensive, as at least two additional analyses for each design variable are required to obtain sufficient data. A more efficient approach involves implicit differentiation of the spatially discretised equilibrium equation, which yields the gradients of the displacements directly [13]. The displacement gradients may in turn be used to provide gradient information for other responses via the constitutive relationships.

Shown below in Figure 3.2 is a graphical representation of the linearisation method in the design space with two design variables.

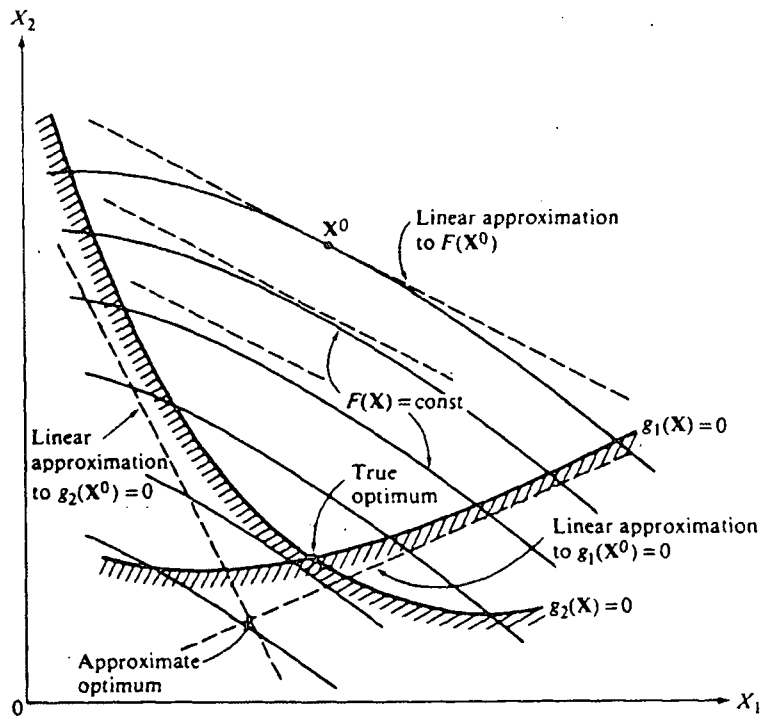


Figure 3.2 Linearisation of a non-negative design space. [18]

3.3.3.2 THE STANDARD FORM OF THE LINEAR PROGRAMMING PROBLEM

The standard form of the linear programming problem is stated as follows:

$$\text{Minimise:} \quad F(X) = \sum_{i=1}^n c_i X_i \quad (3.22)$$

$$\text{Subject to:} \quad \sum_{i=1}^n a_{ij} X_i = b_j \quad j = 1, m \quad (3.23)$$

$$X_i \geq 0 \quad i = 1, n \quad (3.24)$$

The problem statement is defined in terms of equality constraints and non-negative design variables. Two problems with the problem statement presented above are that inequality and negative design variables are not catered for. To take care of the negative design variables each negative variable is replaced with the difference of two positive variables

$$X_i = X_i' - X_i'' \quad (2.25)$$

$$X_i' \geq 0 \leq X_i'' \quad (3.26)$$

therefore two extra variables X_i' and X_i'' are introduced the design problem for each negative design variable. The inequality constraints are of the form

$$\sum_{i=1}^n a_{ij} X_i = b_j \quad (3.27)$$

are treated by adding a *slack* variable to the equation i.e.

$$\sum_{i=1}^n a_{ij} X_i + X_{n+1} = b_j \quad (3.28)$$

The slack variable must be added to each inequality constraint in the optimisation process. The resultant problem statement represents a basic feasible solution to the optimisation problem. The solution may not be the optimum however.

3.3.3.3 REVISED SIMPLEX METHOD

The *revised simplex method* was developed by Danzig [33] and is based on the original *simplex method*. The simplex method is a systematic process whereby the basic feasible solutions are evaluated by continually decreasing the objective function in a 'downhill' direction until a minimum is found. The minimum (or optimal) will naturally occur at the lowest vertex within the LP problem. A basic feasible solution is found at an

arbitrary vertex of the feasible domain of a convex polyhedron. In order to execute the simplex algorithm one requires a set of equations, which includes the objective function along with the equality constraints in canonical form. The canonical form is obtained by introducing artificial variables into the standard form of the linear programming problem. If the solution does not satisfy the convergence criteria then the problem is re-linearised about X_{i+1} , and the process is repeated until the optimum is found [32,34].

3.3.3.4 SEQUENTIAL LINEAR PROGRAMMING (SLP)

Most practical engineering design problems as one would expect are non-linear in both design objective and behavioural response when considering the usual design variables such as cross-sectional area and plate thickness. An area that has received a lot of research and development and become common practice is that of problem linearisation. This technique linearises the non-linear functions that compose the problem about the present design to produce a simplified approximation of the original problem. The approximation is computationally easier to solve when compared to other techniques that solve the complex non-linear problem directly.

The sequential linear programming (SLP) solution methodology consists of an iterative procedure to find the solution to a complex non-linear problem by solving a series of linear programming (LP) trial solutions that allows convergence to within a range of the true optimum. The linear programming problems are generated using a linear approximation technique, usually the first order Taylor series expansion about the design vector (X_i) is utilised as explained earlier [32]. This process converts the feasible design space into a polyhedral region bound by linear approximations of the inequality

constraints of the problem. Once the objective and constraint functions have been linearised they are subsequently solved using the *revised simplex method* to find the new design vector (X_{i+1}).

As has been discussed the linearisation of the objective and constraint functions with respect to the initial design X^0 produces the straight line approximation of the original non-linear problem as shown in the Figure 3.2. Minimisation of the objective function in the design space will result in the optimal design occurring at a vertex defined by the linear approximations of the constraint functions. It is possible that the optimum will lie in the infeasible region of the original problem however the iterative re-linearisation of the problem about each new design will force the approximate to within an acceptable range of the true optimum. Depending on the degree of the non-linearity of the original problem, move limits may be imposed on the design variables to limit the extent that the approximate solution extends into the infeasible region. Figure 3.3 illustrates the execution of the sequential linear programming algorithm in the form of a flow chart.

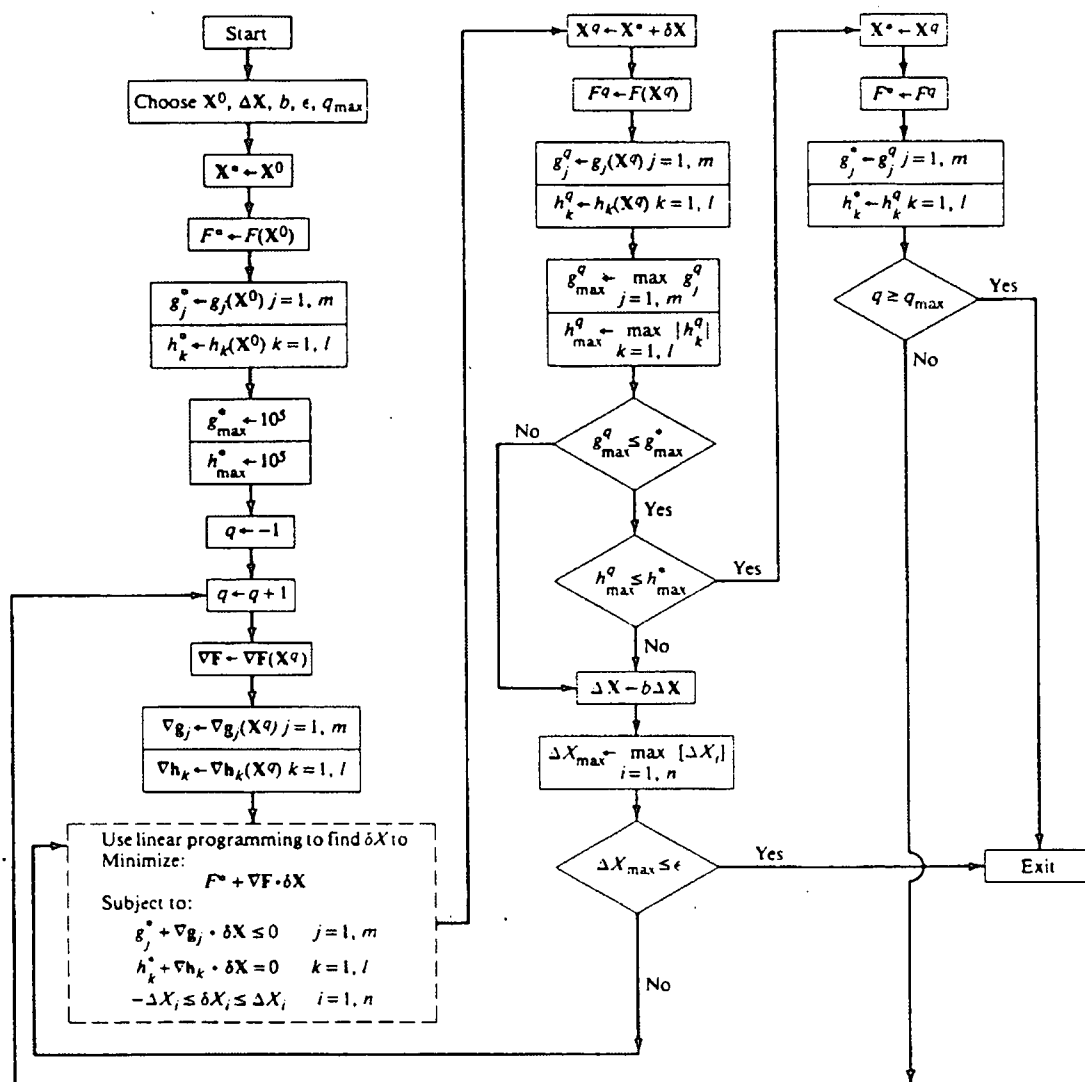


Figure 3.3 The execution flow of the sequential linear programming method. [18]

3.3.3.5 SENSITIVITY ANALYSIS

The design sensitivity analysis is what drives the analysis toward an improved design by indicating the relationship between the variables and the design's response. In other words, if there is a large change in the design's response for a small change in the design variable it is considered to be sensitive. There are basically three methods that

can be used to calculate the sensitivity information namely, finite difference approximations, variational methods and implicit differentiation. These methods essentially provide gradient information for driving the optimisation process from the finite element analysis. The finite difference method is conceptually simple however as the problem size becomes larger so the computational effort becomes excessively expensive. The last two methods differ in that the differentiation is carried out before and after the discretisation process respectively. The variational approach is highly advantageous in this respect as the sensitivity information for a wide range of analysis types and structural systems is available without having to specifically create it. This process is substantially quicker than the implicit differentiation approach however the code has to be specifically written to include this method. When considering implicit differentiation method, the sensitivity information is acquired from the stiffness equation and as a result can be applied to all finite element codes as an add-on.

3.3.3.5.1 Implicit Differentiation

As discussed above the implicit differentiation method is not as efficient as variational methods however it is substantially easier to apply. The equations used in the FEA are in the following form:

$$Ku = P \quad (3.29)$$

Where u represents nodal displacements, P represents nodal loads and K is the global stiffness matrix of the structural system. The stiffness matrix, load and displacement vectors are usually expressed in terms of the design variables X . Implicit differentiation of the governing equation (3.29) with respect to the design variables results in the following:

$$K \frac{\partial u}{\partial X_i} = \frac{\partial P}{\partial X_i} - \frac{\partial K}{\partial X_i} u \quad (3.30)$$

from which the displacement sensitivities may be obtained as

$$\frac{\partial u}{\partial X_i} = K^{-1} \left(\frac{\partial P}{\partial X_i} - \frac{\partial K}{\partial X_i} u \right) \quad (3.31)$$

The load vector rarely is a function of the design variables and generally the equation may be written in the form

$$\frac{\partial u}{\partial X_i} = K^{-1} \frac{\partial K}{\partial X_i} u \quad (3.32)$$

The decomposed stiffness matrix is usually available as a result of the displacement solution. The displacement sensitivities may therefore be solved for once the derivative of the reduced global stiffness matrix has been assembled. This method is referred to as the direct method of implicit differentiation and has been used extensively in structural optimisation [35].

An alternative to the direct method is the adjoint variable method, which is similar to the direct approach in that it relies on the differentiation of the finite element state equation. However this method uses the adjoint variable to reduce the computational effort in the calculation of the design sensitivities. The adjoint variable is defined as the solution to the following equation

$$K \lambda = u \quad (3.33)$$

The equation above is pre-multiplied by $z^T K^{-1}$ which results in:

$$z^T \frac{\partial u}{\partial X_i} = z^T K^{-1} \left(\frac{\partial P}{\partial X_i} - \frac{\partial K}{\partial X_i} u \right) \quad (3.34)$$

If an adjoint variable λ is defined as being the solution to the following system

$$K\lambda = z \quad (3.35)$$

where z is a vector of the intermediate gradients of the constraints g with respect to the response quantity u , i.e.

$$z_j = \frac{\partial g}{\partial u_j} \quad (3.36)$$

The derivatives representing the response sensitivities may then be calculated from the following expression

$$z^T \frac{\partial u}{\partial X_i} = \lambda^T \left(\frac{\partial P}{\partial X_i} - \frac{\partial K}{\partial X_i} u \right) \quad (3.37)$$

Once λ has been calculated substitution into the above equation will give displacement sensitivities that may be used to determine the gradients of other constraints via constitutive relations.

The two techniques discussed above, for the determination of sensitivity data using the implicit differentiation method differ in the number of equations that require solving. The adjoint variable approach is more efficient as the number of equations and the number of the active constraints to be solved are the same. In the direct method however the number of equations equals the number of design variables and hence if the number of design variables is greater than the number of constraints then the adjoint variable method should be used and vice-versa [18].

3.3.3.6 MOVE LIMITS

A problem may occur in the linear programming problem in that the changes in design becoming too large, therefore presenting an invalid linear approximation or unbounded solution [32]. Move limits (or step size) are imposed on the design variables to restrict

the range of change during one design cycle (or iteration). The move limit is a function of the sensitivity and can be different depending on the particular variable.

3.4 CONCLUSION

A brief introduction into the concepts of the finite element method has been presented. It has been shown that the process is applicable to any structure as long as there is sufficient knowledge of the problem and the components that comprise them such as materials, elements and interpolation functions. It therefore presents an approach that is flexible and may be applied to a variety of design situations.

A theoretical basis for optimisation techniques have been discussed in detail. A computationally efficient approximation method has been presented in the form of sequential linear programming. Optimisation methods have not gained the acceptance that has been afforded to finite element analysis however with the increased importance of developing high performance optimised structures it is inevitable that sophisticated integrated design systems will become common place in the near future.

CHAPTER 4

DEVELOPMENT OF THE WING AND CANARD STRUCTURES FOR THE UCAV-TD

4.1 INTRODUCTION TO MATERIAL SELECTION IN AVIATION

In the design and manufacture of aircraft there is a great need for increased payload size, this emphasises the pressing need for lightness in the interest of economical operation. Further, there is often a need for great range, which will mean the allocation of space and weight for the accommodation of fuel. When considering commercial aircraft, this will inevitably reduce the capacity to carry paying passengers. In military aircraft light-weight equates to performance therefore material selection in these heavily loaded aircraft airframes has always been (and still is) a search for materials with improved strength to weight ratios and stiffness.

Early airframes consisted of fabric covered wooden frames, by the First World War aluminium alloy was being introduced to increase durability. It was not until 1926 that William B. Stout designed the first all metal aircraft. Much has changed in the aircraft industry since then however all-metal construction is still favoured in most cases.

It has been established that an aircraft needs to be as light as possible, yet strong enough to withstand the effects of the loading experienced by the aircraft. This loading includes changing air pressure associated with altitude changes and cabin pressurization, constant buffeting from turbulence in the airflow over the wings, body, engine housings, control surfaces as well as impact loading associated with takeoffs and landings [5]. Rigidity is another important consideration as alignment of the control surfaces under varying load conditions is imperative to the performance of the aircraft. All construction materials must be compatible so as to avoid electrolytic corrosion where the materials come together. As can be expected corrosion is of particular concern when considering construction materials due to extreme temperature and pressure changes that are experienced by the aircraft as well as operation in humid regions. It is also advantageous that the aircraft's skin is constructed from large sheets in order to limit the number of joints in the interests of strength and an uninterrupted surface airflow. Sheet material chosen for this purpose must be easy to flow form in the cold condition to the shape of the aircraft. The requirements may be summarised as follows [5]:

- Strength and stiffness.
- Toughness.
- Low density.
- Availability and cost.
- Ease of production.
- Corrosion resistance.
- Cyclical stressing life span (fatigue).

There are several candidate materials that meet the requirements as stated above. In addition to what has already been mentioned, material choice will include factors such

as structural efficiently, high yield stress, high elastic modulus and low density.

Candidate materials include the following

- Aluminium alloys.
- Titanium.
- Magnesium.
- Composites (glass and carbon fibre-reinforced plastics).

All the materials listed here have their particular strengths and limitations and are suitable materials for aircraft manufacture, however composites are of particular interest here [5].

Generally high strength metals have too high a density and many are highly corrosive, as a result these materials are limited to high stress components. Although composites are increasingly finding their way into stressed members they are not as yet widely accepted in passenger-carrying airliners, however this is not the case in small aircraft, competition gliders and military aircraft. Synthetic adhesives are increasingly used in highly stressed members as the stresses are transmitted uniformly along the joint. Composites are also becoming popular as their use often translates into smaller parts count and the parts that are present can often be joined effectively using adhesives.

It has been well documented that there has been a large increase in the use of composites in an ever-increasing variety of applications. However it is in aerospace that the advantages of composites are most often exploited, and there are many examples where composite materials have been used to advantage in both military and civilian aircraft. The advantages of using composites in these applications, as previously

established, includes specific stiffness and specific strength, design tailorability, fatigue resistance, and dimensional stability. Many aircraft would not have been possible were it not for the availability of advanced composites [3], and the use of composites in the aerospace industry is becoming even more significant.

An example of the advantage of composite components is in the B2 stealth bomber. Many features of the aircraft, particularly the stealth technologies, would not have been possible were it not for the availability of advanced composites, both fibreglass and graphite fibres were used with epoxy matrix and polyimide matrix. The space shuttle is another example, many structural components are constructed from composites; carbon/epoxy, Kevlar, and carbon/carbon are among those used to advantage. On the leading edges and nose cone of the space shuttle carbon/carbon is used as temperatures exceeding 1500°C are experienced during re-entry [3]. There are numerous examples of the use of composites in today's aircraft including the Boeing 757 where composites are used extensively [4], and more recently the SAAB Gripen where a variety of composites were used to meet performance requirements beyond the capabilities of metals.

4.2 INTRODUCTION TO THE UCAV-TD CONCEPT

The concept of an Uninhabited Combat Air Vehicle (UCAV) exploits the design and operational freedoms of relocating the pilot outside of the vehicle. A variety of cost and weight penalties are associated with the presence of a human pilot and the aircraft's manoeuvre capabilities are also limited by the pilot's physiological limits. Removing the pilot from the vehicle eliminates man-rating requirements, pilot systems, and

interfaces. The UCAV offers new design freedoms that can be exploited to produce a smaller, more manoeuvrable and simpler aircraft.

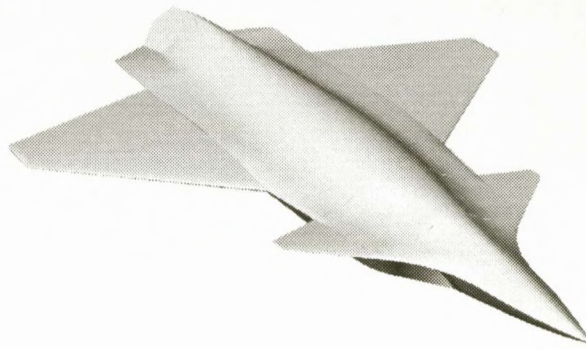


Figure 4.1 Rendered image of the UCAV-TD.

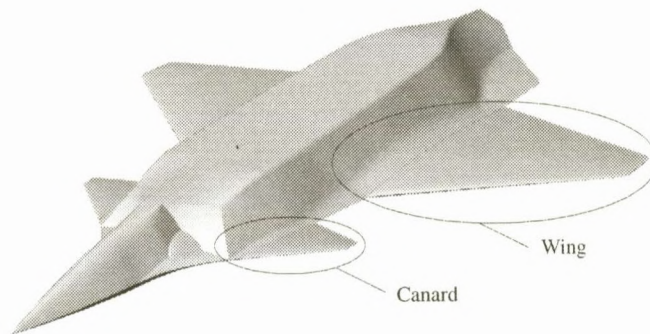


Figure 4.2 A reverse rendered view of the UCAV-TD.

The Uninhabited Combat Air Vehicle–Technology Demonstrator (UCAV-TD), is a X
prototype research vehicle that is being used to investigate the design possibilities associated with the UCAV concept. The vehicle has a tailless configuration with all-moving wings and all-moving forward canards to affect altitude control as shown above. Advanced composite materials are used almost exclusively in the manufacture of

the airframe. This is primarily due to the high specific strength and stiffness properties that these materials offer.

Detailed design and optimisation studies were carried out for the wings and canards of the UCAV-TD in order to produce a structure of minimal weight whilst retaining reasonable deflection, allowable stress limits at maximum loading conditions were also imposed. The design and optimisation of the prototype wings and canards were developed utilising finite element analysis and mathematical programming methods.

The following chapter presents the results of two studies, the first aimed at investigating the conceptual design of a composite wing for the prototype uninhabited combat air vehicle technology demonstrator (UCAV-TD). The second study was aimed at the optimisation of the canard structure. The relaxation of design constraints due to the removal of the pilot enables increased performance targets to be specified as discussed previously. This implies that the structures are expected to endure larger loads whilst still ensuring that performance is not compromised. The primary objective of the first study was to determine the optimal structural configuration that will result in a wing of minimum weight. The study conducted on the canard was aimed at establishing a structure of minimum weight. The optimisation methodology used in both cases is based on the integration of sequential linear programming with finite element analysis.

Manufacture of the prototype canard was completed to specifications determined by the analysis and optimisation techniques. A rapid prototyping method was investigated in the manufacture of the canard. Extensive use was made of a new generation of pre-impregnated materials known as SPRINT™ (SP Resin Infusion Technology). The

material is based on the resin film infusion concept, which offers a cost-effective alternative to traditional pre-impregnated systems. High temperature tooling was produced to construct the prototype canards as the curing process for SPRINT™ involves vacuum bagging at elevated temperatures. The cost-effective and accurate rapid prototyping method for the processing of composite materials in the manufacture of a canard for the UCAV-TD is discussed in detail. The manufacturing method adopted has proven effective by Jonson, Jordan and Mann in the design and manufacture of high performance composite structures [36]. A novel radial web configuration was selected as it was found to give improved performance in terms of weight, stiffness and improved moment of inertia characteristics. The principle objective is the minimisation of the costs of the prototype manufacturing process without affecting the accuracy and performance of the final product. This was achieved by using solid geometry computer models, advanced computer numerical control (CNC) machining and easily machinable materials to manufacture the tooling, thereby saving fabrication time and limiting inaccuracy. The method investigated in the manufacture of the canard is being used to evaluate the rapid prototyping process and can be adopted in the manufacture of any similar radial configuration.

4.3 DESIGN AND OPTIMISATION OF THE WING STRUCTURE

4.3.1 STRUCTURAL DESIGN CONCEPTS FOR THE WING

Two structural design concepts were considered for the wings of the UCAV; the first being a conventional configuration using a system of longitudinal ribs and transverse

spars as shown in Figure 4.3 and the second being a more unconventional configuration using a radial system of spars as seen in Figure 4.4. Although the conventional configuration includes composite upper and lower skins as well as composite ribs and spars, it also includes a high strength steel control shaft attached to the root and mid ribs. The radial configuration is all-composite with the control shaft being an integral part of the root rib.

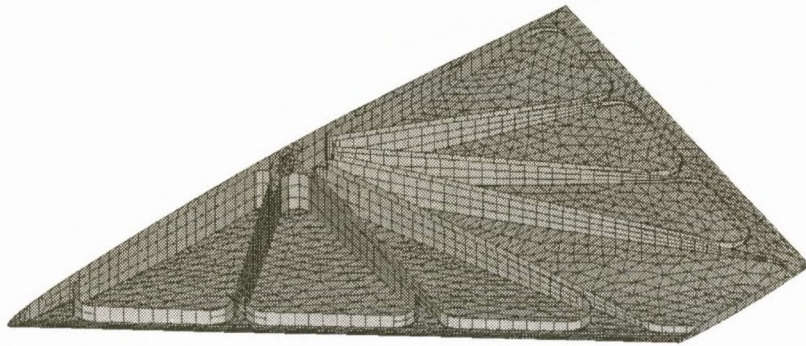


Figure 4.2 Radial configuration (without upper wing skin).

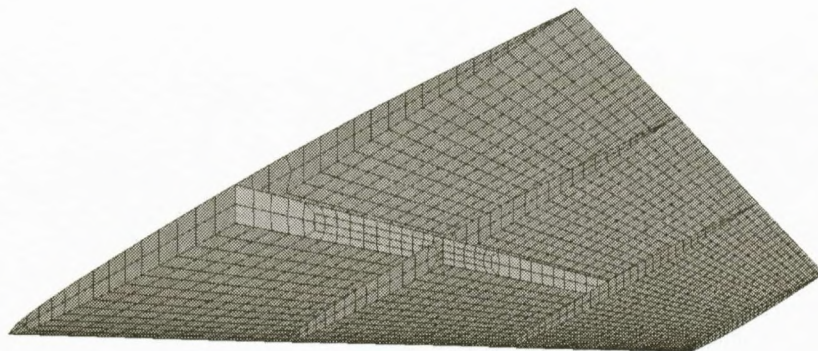


Figure 4.3 Conventional configuration (without upper wing skin).

4.3.2 DESCRIPTION OF PROBLEM

The optimal design of the UCAV wing structure is considered for the case where the wing is subjected to aerodynamic pressure loading. The loading varies depending on whether the vehicle is operating at subsonic, transonic or supersonic Mach numbers. The aim of the problem is to determine the optimal structural design that will result in a wing of minimum weight whilst satisfying design limitations imposed to maintain aerodynamic performance and structural integrity. The aim encompasses two objectives; firstly a choice needs to be made between two candidate structural concepts and secondly a greater understanding of the changes in the optimal design as a function of pressure distribution is required. In order to achieve these objectives, a series of optimisation problems have been configured for each design concept, these problems have been solved for various pressure distributions corresponding to operation within the subsonic, transonic and supersonic regimes. The structural response of the wing is determined using finite element analysis and the optimisation problem is solved using sequential linear programming.

4.3.3 FINITE ELEMENT MODELLING

The finite element method was used to provide response data for driving the optimisation process which is described in more detail below. The finite element models for the conventional and radial configurations are shown above in Figures 4.2 and 4.3 respectively. In order to simplify the analysis, characteristic 2D orthotropic material properties were input for the entire laminate rather than on a ply-by-ply basis as the optimisation module available in the MSC.Nastran code does not allow for optimisation

on a ply-by-ply level. Material orientation was modified on a particular component if required, for example, all spars and ribs use a biaxial material orientated at 45 degrees to the chord line due to the dominance of shear stresses in these components. A [0/90]₂ weave orientated to the root rib was used for the skins as the stress paths (which were established in a previous study using isometric materials) dictated. Detailed material property data used in the FEA is presented in the Table 4.1 (It must be noted that the units of the input values presented in the table were used because the modelling was done in millimetres and all stress values that would result would then be in MPa's). This approach is adequate for the conceptual design study described here as the format of the SPRINTTM material used in the manufacture consists of pairs of balanced carbon fibre fabrics knitted together which effectively results in a laminate with a [0/90/90/0]₂ configuration. Unidirectional carbon tapes are used in the radial configuration for spar caps due to the dominance of tension and compression in these members. Material modulus and strength data were also available for the material in its supplied format.

Property	Unidirectional [0]	Biaxial Weave [0/90/90/0]	Biaxial Weave [45/-45/-45/45]	Aluminium Shaft
E ₁ (MPa)	129200	59175	25310	155000
E ₂ (MPa)	9312	59175	25310	-
G (MPa)	3745	7122	47900	65000
Y	0.034	0.056	0.764	0.192
Density (kg/mm ³)	1.45 x 10 ⁻⁶	1.45 x 10 ⁻⁶	1.45 x 10 ⁻⁶	4.2 x 10 ⁻⁶

Table 4.1 Material property data used in the FEA.

The distribution of the aerodynamic pressure loading was calculated based on the required total lift and the location of the mean centre of aerodynamic pressure. The pressure loading was applied to the upper surface of the wing as per the industry standard. Six load cases were defined representing the movement of the centre of pressure from 25% to 50% along the mean aerodynamic chord in 5% increments. Pressure load distributions for the 25% and 50% cases are shown in Figures 4.5 and 4.6 respectively.

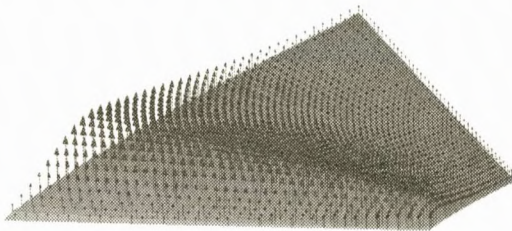


Figure 4.5 Pressure distribution CP = 25% of mean aerodynamic chord. [15]

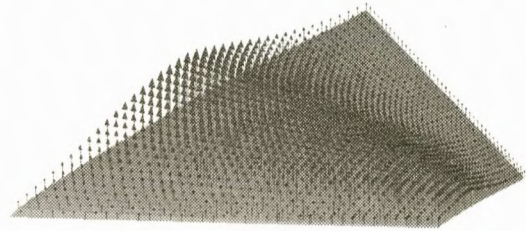


Figure 4.6 Pressure distribution CP = 50% of mean aerodynamic chord. [15]

The boundary conditions used in the FEA included a fixed constraint on the control shafts. For the conventional configuration this meant a single fixed node where the control shaft meets the root rib in a rigid element. However for the radial configuration this included a fixed circular perimeter where the control shaft is to be positioned on the root rib. Additional constraints used to guide the optimisation procedure are discussed in detail later in this chapter.

4.3.4 OPTIMISATION PROBLEM

The optimisation studies of the conventional and radial structural configurations for the UCAV-TD wing were carried out using SLP with the response and sensitivity data being supplied by the FE analysis. The objective of the optimisation was to determine the structural configuration that would result in a wing of minimum weight for each of the pressure distributions considered. The design variables used to optimise the designs for the conventional and radial configurations are shown in Figures 4.7 and 4.8 respectively. Optimisation of the design variables shown below was conducted by varying their plate thicknesses.

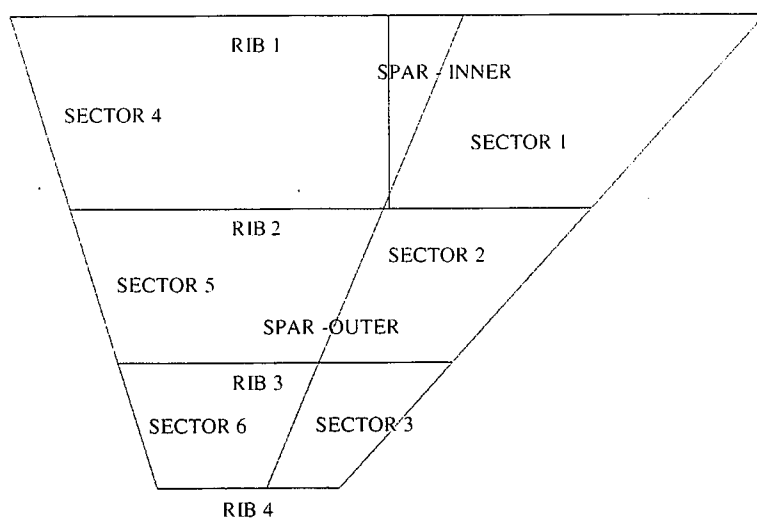


Figure 4.7 Conventional configuration.

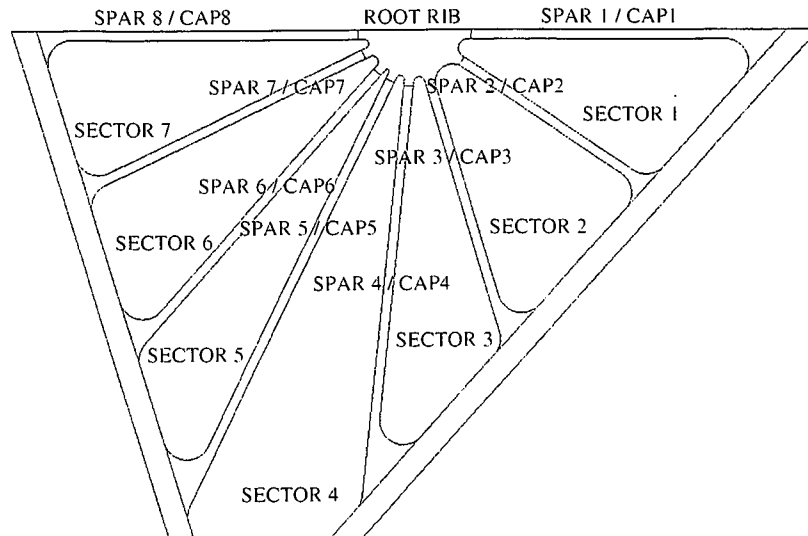


Figure 4.8 Radial configuration.

In order to retain an acceptable aerodynamic shape, deflection everywhere on the wing surface was limited to 5 mm. In addition, in order to prevent the wing root fouling the fuselage a horizontal deflection limit of 1 mm was specified for the vertical wing root surface (rib 1 or root rib). Stress limits were applied using the maximum stress criteria, which essentially limits the stress values in the material directions. A lower bound of 0.7 mm was also placed on all thickness variables to limit the thickness to a single ply of the SPRINTTM material used in the manufacture of the wing skins and internal structure while an upper bound of 15 mm was placed on all skin thicknesses. A fixed constraint was also placed on the control shafts as discussed previously.

In the case of the conventional configuration the design variables included the wing skin thickness for sectors 1 to 6. The thickness of the ribs 1 to 4 were considered independently and the thickness of the central spar was divided into an inboard and outboard portion separated by the mid-span rib. The initial design of the conventional

wing skin (in sectors 1 to 6), rib and spar thicknesses were arbitrarily specified with a value of 8 mm.

Property	Type of Constraint	Initial	Minimum	Maximum
Wing surface	Vertical deflection	0 mm	-5 mm	5 mm
Root rib	Horizontal deflection	0 mm	-1 mm	1 mm
All properties	Major principle stress	0 MPa	-500 MPa	500 MPa
Skin, ribs and spars	Thickness	8 mm	0,7 mm	15 mm
Control shaft	Fixed	-	-	-

Table 4.2 Constraints applied to the conventional configuration.

The radial configuration considered a larger set of design variables compared with the conventional configuration. The skin thickness was divided into 7 sectors, with each sector defined between the radial spars. The thickness of the each spar was considered independently. The thicknesses of the spar-caps were also included in the optimisation studies. The initial design of the radial wing was specified with skin thickness values of 5 mm in sectors 1 to 7 and spar and spar-cap thickness values of the same.

Property	Type of Constraint	Initial	Minimum	Maximum
Wing surface	Vertical deflection	0 mm	-5 mm	5 mm
Root rib	Horizontal deflection	0 mm	-1 mm	1 mm
All properties	Major principle stress	0 MPa	-500 MPa	500 MPa
Skin, spars and spar-caps	Thickness	5 mm	0,7 mm	15 mm
Control shaft	Fixed	-	-	-

Table 4.3 Constraints applied to the radial configuration.

4.3.5 RESULTS

Note all figures under discussion in this section are presented in appendix A.

4.3.5.1 CONVENTIONAL CONFIGURATION

The iteration history showing the weight reduction for the case where the aerodynamic centre of pressure is located at 25% of the mean aerodynamic chord is shown in Figure 4.9. The iteration history for the wing skins, rib thickness values and spar thicknesses for the same load-case are shown in Figures 4.10, 4.11 and 4.12 respectively. Results are presented in terms of a normalised weight, this being the current weight divided by the initial weight of the wing. The variable thickness values are presented in a similar way, in this case referenced against the initial value of the variable. The change in the optimal weight of the wing as a function of the location of the centre of pressure is shown in Figure 4.13, the corresponding changes in the optimal wing skin thicknesses are shown in Figure 4.14.

4.3.5.2 RADIAL CONFIGURATION

The change in the optimal weight of the radial configuration as a function of the location of the centre of pressure is shown in Figure 4.15, the corresponding changes in the optimal wing skin, spar and spar-cap thickness values are shown in Figures 4.16, 4.17 and 4.18 respectively. As with the conventional configuration, results below are presented in terms of normalised weight and thicknesses, referenced against the original weight of the radial configuration the initial value of the thickness variable respectively.

4.3.6 DISCUSSION

A comparison of the weights of the conventional and radial configurations as a function the location of the centre of pressure is shown in Figure 4.19. As can be seen the optimal weight of the wing with the conventional spar and rib construction is consistently heavier than for the radial configuration. This is largely due to the conventional wing being required to carry a larger portion of the load on the wing skins in sectors 4 to 6. Thicker wing skins are therefore required to ensure that the deflection remains below the specified 5 mm maximum. This is reflected in the data shown in Figure 4.14 where the rear wing skins represented by sectors 4 to 6 become increasingly thicker while those on forward portions of the wing become thinner as the centre of pressure moves towards the 50% case.

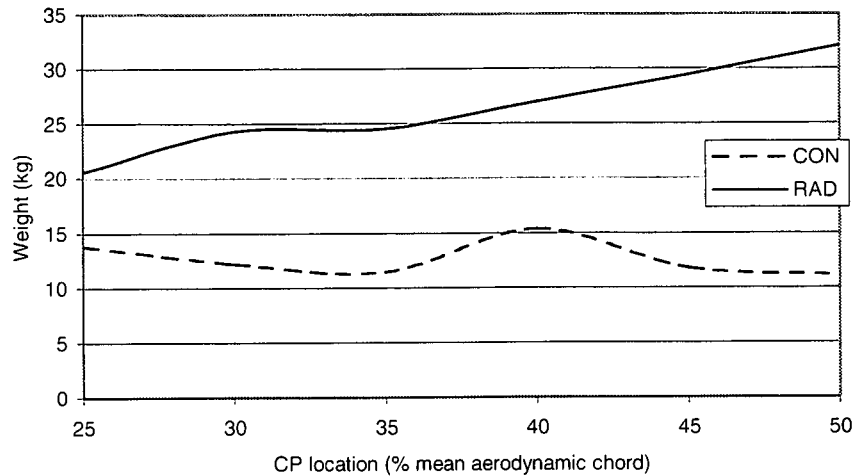


Figure 4.19 Comparison of optimal weights for conventional and radial concepts CP = 25% to 50%. [15]

The multi-spar arrangement of the radial configuration is more efficient in limiting the deflection of the wing, thereby allowing for a reduction in wing skin thickness. This is also demonstrated by the observation that the optimal weight of the radial wing remains relatively consistent as the centre of pressure moves towards the 50% case whereas the conventional configuration becomes progressively heavier. *This isn't visible in the fig*

4.4 DESIGN AND OPTIMISATION OF THE CANARD STRUCTURES

4.4.1 STRUCTURAL DESIGN CONCEPT FOR THE CANARD

Development of the canards of the UCAV-TD follow on from the results of the previous study, as a result the structural design would adopt a similar radial spar

configuration. The radial arrangement has been altered slightly from the wing design due to manufacturing reasons. The alterations included two sectors that were found to be too small and were therefore combined into one larger sector, and the smallest vertical wall (in the web design) was also limited to 8 mm. The construction of the canard, as with the wing, would be an all-composite design with the control shaft being an integral part of the root rib.

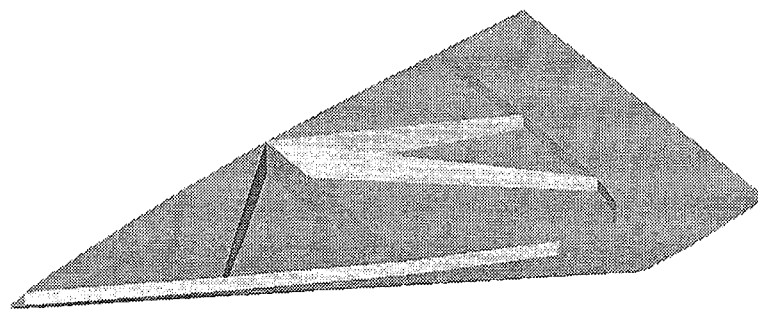


Figure 4.20 Canard configuration (without upper skin).

4.4.2 DESCRIPTION OF PROBLEM

The optimal design of the UCAV-TD canard structure is considered for the case where the canard is subjected to aerodynamic pressure loading similar to that experienced by the wing. Hence the same design and optimisation approach would be followed to that previously presented. The aim of the problem is to determine the optimal structural design that will result in a canard of minimum weight whilst satisfying design limitations imposed to maintain aerodynamic performance and structural integrity. As before the structural response of the wing is determined using finite element modelling and the optimisation problem is solved using sequential linear programming.

4.4.3 FINITE ELEMENT MODELLING

The finite element model for the canard is shown in Figure 4.21. The material orientation was modified on a particular component if required, for example all the ribs have their material orientated at 45 degrees to the chord line due to the dominance of shear stresses in these components. This approach is adequate for the design study described here as the format of the SPRINTTM material used in the manufacture consists of pairs of balanced carbon fibre fabrics knitted together which effectively results in a laminate with a $[0/90/90/0]$ configuration. Unidirectional carbon tapes are used for the spar caps. The skin lay-up as before is $[0/90]$ weave orientated to the root rib [38,39,40]. The material property data is shown in the Table 4.4.

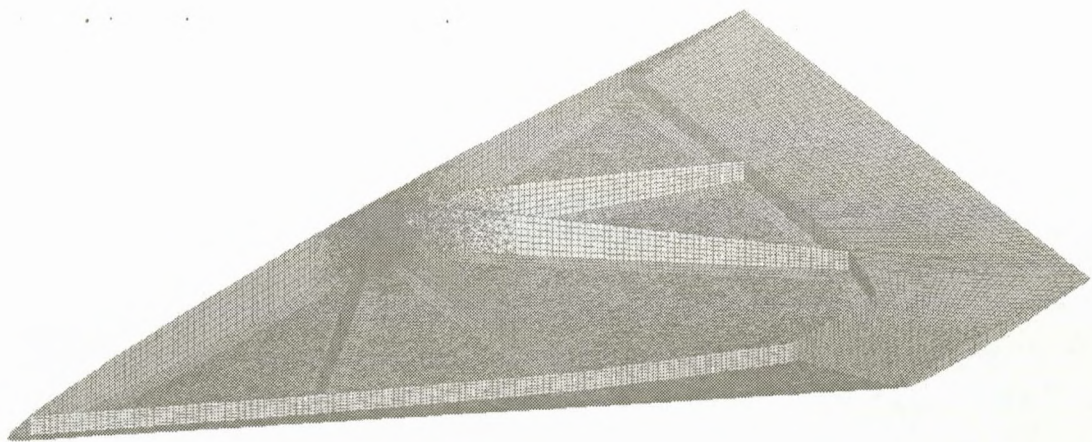


Figure 4.21 FE model of canard (without upper skin).

Property	Unidirectional [0]	Biaxial Weave [0/90/90/0]	Biaxial Weave [45/-45/-45/45]
E_1 (MPa)	129200	59175	25310
E_2 (MPa)	9312	59175	25310
G (MPa)	3745	7122	47900
δ	0.034	0.056	0.764
Density (kg/mm ³)	1.45×10^{-6}	1.45×10^{-6}	1.45×10^{-6}

Table 4.4 Material property data used in the FEA for the canard.

The distribution of the aerodynamic pressure loading was calculated based on the required total lift and the location of the mean centre of aerodynamic pressure. The pressure loading was applied to the upper surface of the wing. Six load cases were defined representing the movement of the centre of pressure from 25% to 50% in 5% increments along the mean aerodynamic chord. Pressure load distributions for the 25% and 50% cases are shown below.

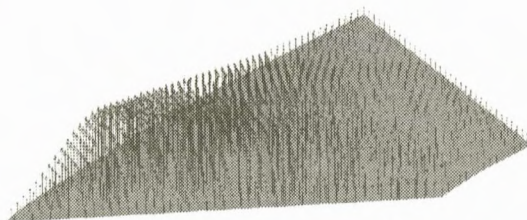


Figure 4.22 Pressure distribution CP = 25% of mean aerodynamic chord.

[37,38,39]

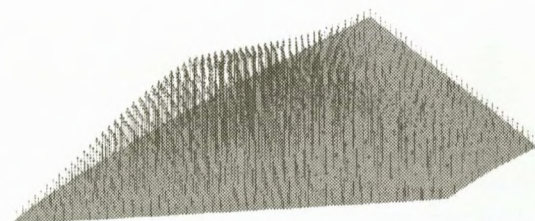


Figure 4.23 Pressure distribution CP = 50% of mean aerodynamic chord.

[37,38,39]

4.4.4 OPTIMISATION PROBLEM

The optimisation studies of the structural configuration for the UCAV-TD canard were carried out using SLP with the response and sensitivity data being supplied by the FE analysis. The objective of the optimisation was to determine the structural configuration that would result in a wing of minimum weight under all pressure distributions considered. The design variables used to optimise the designs are shown below.

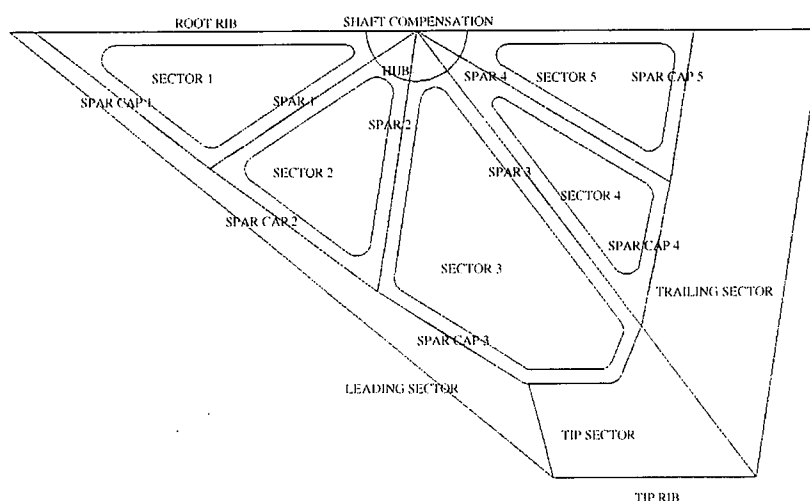


Figure 4.24 Canard configuration.

The skin was divided into 8 distinct sectors, 5 of these sectors were separated by radial ribs. The thickness of the each rib/spar was also considered independently. The model also included spar-caps surrounding each of the 5 sectors. The spar-caps made up part of the skin. The thickness of the spar-cap's were also included in the optimisation studies. The initial design of the canard was specified with thickness values of 5 mm for all variables.

In order to retain an acceptable aerodynamic shape, deflection everywhere on the skin surface was limited to 5 mm. In addition, in order to prevent the wing root fouling the fuselage a horizontal deflection limit of 1 mm was specified for the vertical root rib. Stress limits were applied using the maximum stress criteria, which essentially limits the stress values in the material directions. A lower bound of 0.1 mm was also placed on all thickness variables to ensure that no property would 'bottom out' and leave the structural integrity of the canard in question. The thickness was not limited to one layer of SPRINTTM, as it was for the wing optimisation, because it was found that the optimisation procedure was limited by on the lower bounds. An upper bound of 15 mm was placed on all skin and spar cap thickness variables with a limit of 5 mm on the spar thicknesses. As previously discussed there was a fixed constraint applied to the root rib to account for the control shaft.

Property	Type of Constraint	Initial	Minimum	Maximum
Wing surface	Vertical deflection	0 mm	-5 mm	5 mm
Root rib	Horizontal deflection	0 mm	-1 mm	1 mm
All properties	Major principle stress	0 MPa	-500 MPa	500 MPa
Skin, spars and spar-caps	Thickness	5 mm	0,7 mm	15 mm
Control shaft	Fixed	-	-	-

Table 4.5 Constraints applied to the canard.

4.4.5 RESULTS

Note all figures under discussion in this section are presented in appendix A.

The iteration history showing the weight reduction for the case where the aerodynamic centre of pressure is located at 25% of the mean aerodynamic chord is shown in Figure 4.25. The results presented are in terms of a normalised weight, this being the current weight divided by the initial weight of the canard. The iteration history for the canard skins, rib thickness and spar-cap thicknesses for the same load-case are shown in Figures 4.26, 4.27 and 4.28 respectively. The variable thickness values are presented as normalised values, in each case referenced against the initial thickness of the variable.

The change in the optimal weight of the wing as a function of the location of the centre of pressure is shown in Figure 4.29 below. The corresponding changes in the optimal wing skin, rib and spar-cap thicknesses are shown in Figures 4.30, 4.31 and 4.32 respectively. The results are once again presented in terms of a normalised weight.

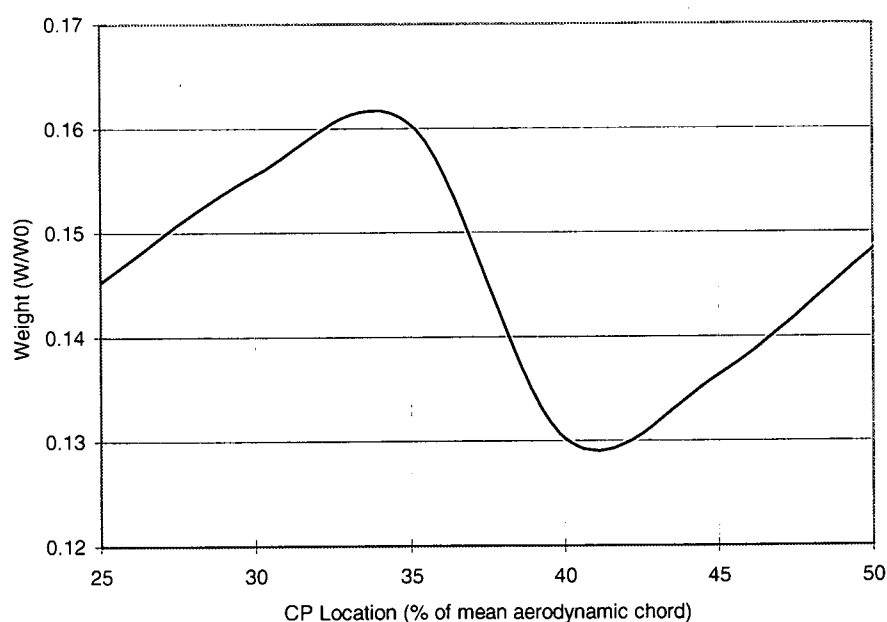


Figure 4.29 Canard weight CP = 25% to 50%.

4.4.6 DISCUSSION

The iteration history for the 25% load case is shown in Figures 4.25, 4.26 and 4.27, which are included in appendix A, as an example. It can be seen from these figures that a good convergence was achieved with all the properties. It is interesting to note that the spar-caps have very little work to do in this design as can be seen from Figure 4.28. This could be due to the skins orientation being efficient enough to carry the load, and possibly the spar-caps were designed too short. The spar-caps were designed in this manner due to manufacturing reasons.

The first observation that can be made with regards Figure 4.29 is that, like the radial wing configuration, the weight of the canard remains relatively consistent over the various loading regimes. Although both the radial wing design and the canard have radial configurations the placement of the ribs/spars are quite different and as a result the trends of the optimal weights are very different but no less efficient.

On the inspection of the results of the optimal skin, rib and spar-cap thicknesses as a function of the location of the centre of pressure (refer Figures 4.30, 4.31 and 4.32 in the appendix A), it was noted that most of the properties appeared to show a similar trend to that of the optimal weight in Figure 4.29. It is observed that sector 3, rib 2 and spar-cap 3 do not follow this trend. These properties are consistently heavier than other properties, particularly in the 40%-50% cases. This is largely due to the absence of a rib running from the control shaft constraint to the tip, which was relegated from the design due to manufacturing concerns. This has two implications the first is that the canard is less stiff and hence rib 2 and spar-cap 3 have to compensate for this by thickening up.

The second implication is that the canard skin in sector 3 is a comparatively large (and unsupported) surface and as a result a larger portion of the load has to be carried therefore the sector 3 has to be thicker.

4.5 MANUFACTURING OBJECTIVES

The processing of composite material systems generally requires the construction of a rigid tool to ensure that the finished product conforms to the desired geometry within specified tolerances. Tooling may be manufactured using many different methods, and each may have several variations [40,8]. Tooling costs are not inconsiderable and are typically recovered in mass production. This is, however, not the case for prototyping or limited run projects where the tooling costs constitute a high percentage of the overall cost of the project. In this situation, the costs of tool manufacture needs to be carefully considered and, although there are a number of issues involved, these are generally dependent on the manufacturing method selected and the degree of accuracy required. These specifications will dictate the processes and materials used as well as the skill-level and the time period required to complete the tooling.

A concept for the manufacture of tooling aimed at providing an accurate, rapid and cost-effective option specifically for prototype development or one-off projects is described here. The method was developed as part of a project involving the one-off manufacture of a half-scale radar model of a prototype uninhabited combat air vehicle (UCAV). The project requirements stated that all the external surfaces were to conform to the specified geometry as accurately as possible whilst minimizing the fabrication cost. The proposed method was therefore developed to provide accurate cost-effective tooling for

the moulding of the canards. The approach relies on a combination of computer-aided design and machining as well as the use of low cost, easily machined materials.

The tool manufacturing time was to be minimized by leveraging computer-based modelling and machining technologies coupled with materials that could be machined at fast feed-rates. Costs were also to be reduced by minimizing the requirement for skilled labour and by using low cost materials.

4.5.1 PROCESS DEVELOPMENT

The processing of composite material systems generally requires the construction of a rigid tool to ensure that the finished product conforms to the desired geometry within specified tolerances. Tooling may be manufactured using many different methods, and each may have several variations [8,40]. Tooling costs are not inconsiderable and are typically recovered in mass production. This is, however, not the case for prototyping or limited run projects where the tooling costs constitute a high percentage of the overall cost of the project. In this situation, the costs of tool manufacture needs to be carefully considered and, although there are a number of issues involved, these are generally dependent on the manufacturing method selected and the degree of accuracy required. These specifications will dictate the processes and materials used as well as the skill-level and the time period required to complete the tooling.

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The tool manufacturing time was to be minimized by leveraging computer-based modelling and machining technologies coupled with materials that could be machined at fast feed-rates. Costs were also to be reduced by minimizing the requirement for skilled labour and by using low cost materials.

In our case, high temperature tooling would be produced in order to withstand the increased temperature and vacuum required for the manufacture of the canard skins using the SPRINTTM system. In order to accelerate the manufacture of the canards, the required female tooling for the canard webs would be manufactured directly thereby eliminating the need for a pattern. The tooling would be milled from SupaWood and subsequently treated to present a high-quality finish. In addition, computer-based models generated using CAD techniques and tools would be used exclusively for geometry definition and for machining.

4.5.2 PROCESS DEVELOPMENT

There are many manufacturing and fabrication techniques that are available these days, which are predominantly expensive and incorporate an exhaustive processes involving

skilled labour and metallic tooling. A rapid prototyping method was implemented in this paper. The emphasis was on a quick, easy and cost effective approach.

A rapid prototyping approach involving computer modelling and CNC machining was utilized in the manufacture of the tooling for the canard. A standard wet-lay-up and vacuum bagging process was used to manufacture the skins. An innovative approach was implemented in the fabrication of the webs.

4.5.2.1 COMPUTER-AIDED DESIGN (CAD) MODELS

As mentioned previously CAD models were used exclusively for defining the external geometry of the canards. These models have been developed as part of the overall UCAV-TD project and were therefore available without having to specifically create them. The CAD model of the canard is shown in Figure 4.33. The CAD models for the canard webs were created from the original external geometry.

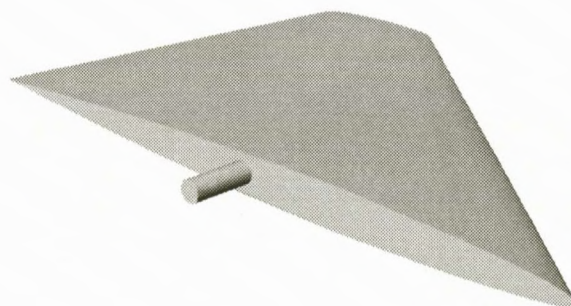


Figure 4.33 External CAD geometry model of the canard.

4.5.2.2 TOOLING CONCEPT AND MANUFACTURE FOR THE CANARD SKINS

A program of process evaluation and development was previously investigated to gain experience with the SPRINT™ material system, in order to manufacture the canard skins [41].

As mentioned previously the pattern was milled from SupaWood using solid geometry computer models and a CNC milling machine. Once the male pattern was milled, it was primed with oil wood primer. Two layers of white universal undercoat were subsequently applied each time being sanded down sufficiently in order to create a smooth surface. Care was taken in this process to not alter the dimensions of the pattern by sanding the undercoat sufficiently in order to prevent build up. Black high-gloss enamel was used for the contact surface. The release interface satisfied the strength and resilience requirements as well as being relatively inexpensive.



Figure 4.34 Canard skin pattern during the milling process.

The pattern was treated with Frekote release agent prior to lay up. The mould was processed using standard wet-lay-up and vacuum bag procedures. The laminating schedule for the canard skin mould consisted of 14 uniform layers of glass fibre twill weave reinforced with SP690 high temperature tooling resin, which gave an effective thickness of 5 millimetres. A uniform lay up was maintained in order to minimize any distortion due to the increased temperature and vacuum that is required for curing process of SPRINT™ [41].



Figure 4.35 Completed female tooling for the canard skins
and the wooden pattern.

Lay up of the canard skins consisted of two uniform layers of SPRINT™, which equates to an effective 1,5 millimetres thickness. The curing process of SPRINT™, as discussed above, involves vacuum bagging at elevated temperatures. The skins were cured at 80°C for seven hours under full vacuum. A [0/90] orientation (relative to the root rib) was selected due to the dominance of stress in these directions as previously established.

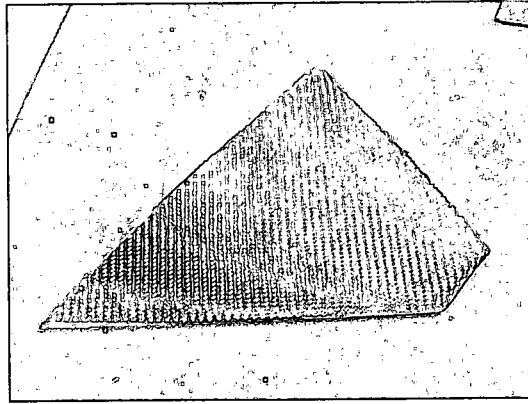


Figure 4.36 Completed canard skin.

4.5.2.3 TOOLING CONCEPT AND MANUFACTURE FOR THE CANARD WEBS

Unlike with the canard skins, the web tooling would be manufactured directly. Hence the need to produce separate patterns, as with the canard skins, would be negated. The female tooling for the webs, as with the skin patterns were milled from SupaWood using solid geometry computer models and a CNC milling machine [41]. The tooling was treated with oil wood primer and subsequently painted with undercoat and finally black gloss enamel, as with the canard skin pattern, described above. Care was taken once again not to alter the dimensions of the contact interface. Holes were drilled to allow the pressure ports to penetrate.

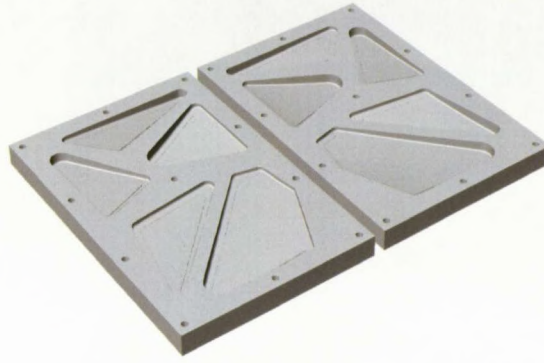


Figure 4.37 Solid geometry computer model of the tooling for the canard webs.

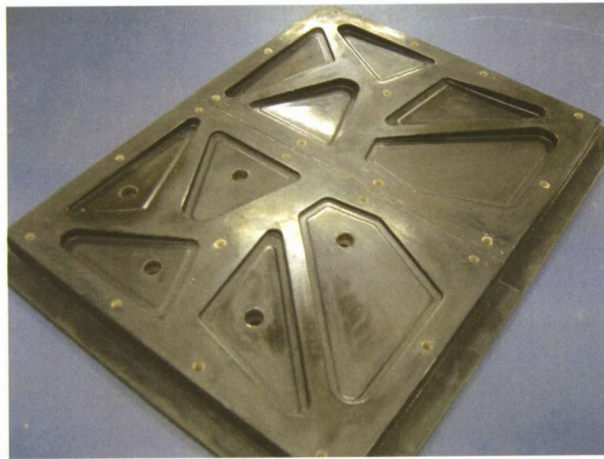


Figure 4.38 Tooling for the canard webs.

A bladder system of inflatable silicon bags was incorporated into the tooling for the manufacture of the webs. The bladder system consisted of building up a silicon layer, of approximately 1,5 millimetres thickness, in the two separate halves of the tool. The silicon bladder's vertical walls were built up to approximately 3 millimetres thickness in order to have a larger contact surface in order to join the two halves. The silicon layer was built up by painting on several layers of silicon until the desired thickness was achieved. Polypropylene templates were used to ensure an even wall thickness as shown in Figure 4.39.



Figure 4.39 Silicon bladder manufacturing process.

Once the silicon had been built up sufficiently and the excess was trimmed off, the two halves were joined to one another (using silicon) and an airtight bladder was the end result. The pressure port was integrated with the silicon bag so that the bladders could be inflated to a pressure in excess of 100 KPa.



Figure 4.40 Completed silicon bladder.

The pressure port design included two flanges that act to clamp the bladder to form an airtight seal. A M10 nut and bolt was used to ensure sufficient clamping pressure was

applied. The bolt was drilled and a standard bicycle tyre valve (presta type) was integrated into the pressure port design to allow pressurisation of the bladder.

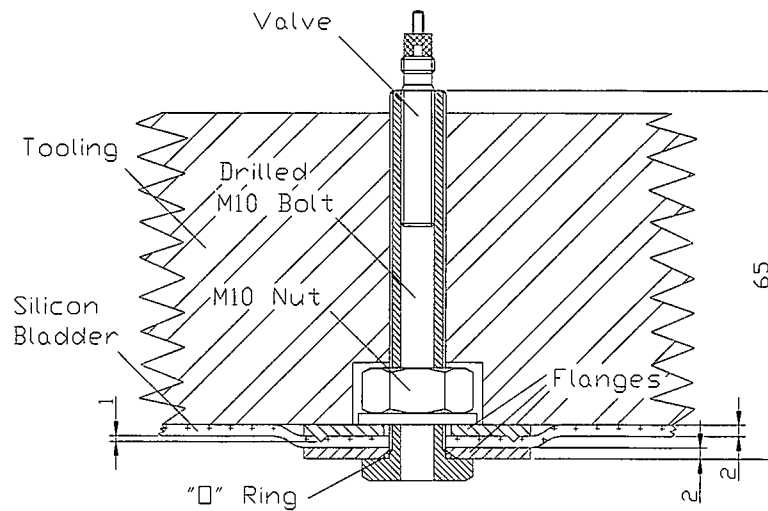


Figure 4.41 Pressure port design.

The webs were constructed from standard carbon fibre reinforced epoxy using the wet-lay-up technique. A preform was used in the manufacture the webs for the following reasons:

- To allow for precise positioning of the fibre in order to retain the [45/-45] orientation specified.
- To position the unidirectional fibres.
- To prevent fraying of the fragile twill weave particularly where 'darts' were used.
- Less waste material.
- Easier bladder extraction in the demoulding process.

The procedure involved the use of an inflated silicon bladder used to position the carbon fibre preform in place, to ensure that it conformed to the geometry of the fully enclosed female tooling, and to consolidate the composite.



Figure 4.42 A close-up of the perform.

The process proved to be successful in producing accurate webs that matched well together to produce the radial web configuration. Shown below is the radial web configuration with the 5 webs fixed in place.



Figure 4.43 Top view of radial web design.

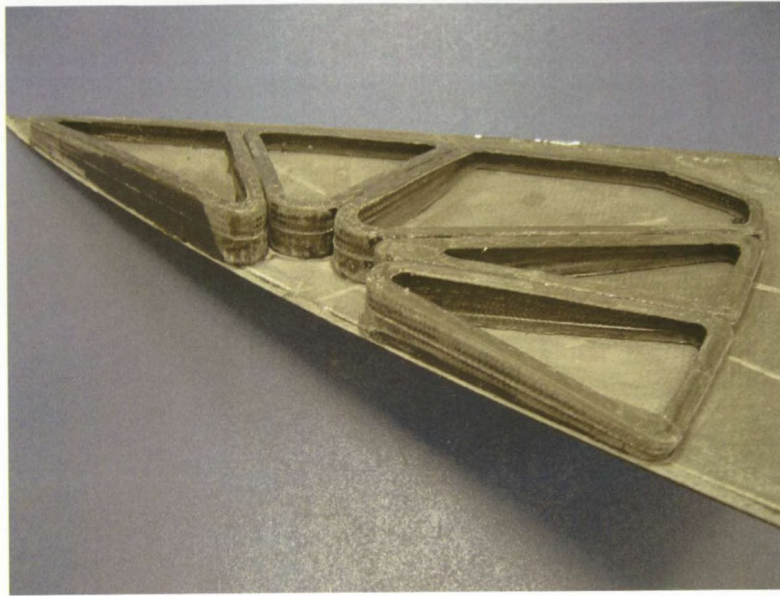


Figure 4.44 Another view of the radial configuration .

4.6 CONCLUSION

The results of the analyses indicate that the SLP strategy adopted was effective in the carrying out the optimisation studies to examine the design of the composite UCAV-TD wing and canard under high loading regimes.

Optimisation iterations between 4 and 14 resulted for both studies, which is reasonably efficient in terms of computational effort required. With regards the wing design, it is noted that a more detailed and accurately represented model would be required to fully understand it's performance however, the study was aimed at the conceptual stage of the design process. The study is considered successful in the evaluation of the different wing configurations and establishment of the wing with the optimal design. The radial configuration for the canard design was observed to be efficient as the optimal weights were found to be fairly consistent through the different loading regimes. The value of

the optimal weights were found to be low and resulted in a canard with a weight of 1.2 kg.

The primary objective of the project was to manufacture the prototype UCAV-TD canard using SPRINTTM in a rapid and cost-effective manner. The method is based on the use of computer-aided modelling and machining as well as inexpensive materials and processes. An effective manufacturing process was implemented successfully in order to produce the canard skins. A bladder system was employed to produce the canard webs. The advantages of the process included a quick, uncomplicated and accurate process in producing the webs.

CHAPTER 5

CONCLUSION

The development of the wing and canard structures were carried out with the use of finite element analysis integrated with sequential linear programming. The manufacture of the canard tooling was undertaken to establish a rapid prototyping method which involved the use of 3D modelling and CNC machining. The processing of the RFI skins was conducted on high temperature tooling. The use of inflatable silicon bags was used in the construction of the canard webs.

A study was presented on the development of the wing structure. Two structural configurations were evaluated, the first being a conventional configuration of longitudinal ribs and transverse spars the other being a more unconventional radial system of spars. The results reflected that the radial configuration handled the varying loading regimes more efficiently than the conventional configuration. Both wing structures were an all-composite construction except for the integral aluminium control shaft that was used in the conventional configuration. The design philosophy was based on finite element analysis integrated with sequential linear programming. As a result of the optimisation procedure a laminating schedule was established that resulted in a weight that was approximately half of the conventional configuration. A carbon fibre reinforced epoxy was specified as the material of construction.

The canard has similar external geometry, constraints and loading regimes to the wing and as a result a radial configuration was chosen based on the results of the previous study that was conducted on the wing. A modified radial configuration was adopted for the canard taking manufacturing concerns into consideration for the smaller wing structure. An optimisation study was conducted on the canard to establish the optimal laminating schedule of the carbon epoxy composite that was specified as the construction material.

Manufacture of half scale canards for the UCAV was undertaken as part of the project. The tooling for the canard skins as well as for the webs were made using a rapid prototyping approach based on 3D computer modelling and CNC machining and easily machinable materials. High temperature tooling was produced for the canard skins, as the SPRINT™ material system used in the manufacture requires curing at an elevated temperature and under vacuum. A novel bladder system of inflatable silicon bags were used in the manufacture of the canard webs.

Future work could include the manufacture of a working model of the UCAV to full structural specification. More specifically, the canard's integral composite control shaft needs to be developed. High temperature tooling for the canard webs could expand the process allowing for the curing of SPRINT™.

REFERENCES

- [1] Bannister, M. (2001). Challenges for Composites into the Next Millennium – a Reinforcement Perspective. *Composites: Part A* 32, pp. 901-910.
- [2] Jones, R. M. (1975). *Mechanics of Composite Materials*. McGraw-Hill, New York.
- [3] Herakovich, C.T. (1998). *Mechanics of Fibrous Composites*. John Wiley & Sons Inc, New York.
- [4] Staab, G.H. (1999). *Laminar Composites*. Butterworth-Heinemann, Boston.
- [5] Timings, R.L. (1998). *Engineering Materials*. (Vol. 1, 2nd Edition). Addison Wesley Longman Limited, Singapore.
- [6] Sleight, S. (1985). *Modern Boat Building Materials & Methods*. Nautical Books, London.
- [7] Cripps, D. (2003). *SP Systems Guide To Composites*. SP Systems, United Kingdom.
- [8] Middleton, H.D. (1990). *Composite Materials in Aircraft Structures*. Longman Scientific & Technical, England.

- [9] Felippa, C.A. and Clough, R.W. (1970). The Finite Element Method. *Proceeding of SIAM-AMS Vol. 2*, American Mathematical Society, Providence.
- [10] Jonson, D., Daya, N., Jordan, K. and Walker, M. (2002). The Design and Fabrication of 20 ton GRP Sugarbins. *Proceedings of Composites Africa 2002 Conference*, South Africa.
- [11] Ranganathaiah, C., Sprinkle, D. R. and Pater, R. H. (1996). A Method For Characterizing Thermoset Polyimides. NASA Technical Memorandum 4707, Virginia.
- [12] Matthews, F.L. (2000). Finite Element Modelling of Composite Materials and Structure. Woodhead Publishing Limited, Cambridge.
- [13] Gurdal, Z., Haftka, R.T. and Hajela, P. (1999). Design and Optimisation of Laminated Composite Materials. John Wiley & Sons Inc., New York.
- [14] Matthews, F.L. (2000). Finite Element Modelling of Composite Materials and Structure. Woodhead Publishing Limited, England.
- [15] Jonson, D. and Smith, R. (2001). The Manufacture of a Prototype UCAV Wing Using the SPRINTTM Resin Film Infusion Process. *Proceeding of SAMPE'02*, pp 305-316, Paris.

- [16] Searle, M. (2001). Processing Notes for SPRINTTM. SP Systems Technical Services Report, TSR No. 070 0917 Rev. 6, SP Systems, United Kingdom.
- [17] Jan Kleef, J. (2001). Parabeam® Introduces ParaGlass Structural Core Laminates. Parabeam media release, Netherlands.
- [18] Jonson, D. (1995). Design Synthesis Through the Integration of Optimisation Techniques and Finite Element Analysis.
- [19] Cook, R.D. (1995). Finite Element Modelling for Stress Analysis. John Wiley & Sons Inc, New York.
- [20] Hrenikoff, A. (1941). Solution of Problems in Elasticity by the Framework Method. *J. Appl. Mech.*, Vol 8.
- [21] Courant, R. (1943). Variational Methods for the Solutions of Problems of Equilibrium and Variations. *Bulletin American Mathematical Society*, Vol 49.
- [22] Turner, M.J., Clough, R.W., Martin, H.C. and Topp, L.J. (1956). Stiffness and Deflection Analysis of Complex Structures. *Journal of the Aeronautical Sciences*, Vol 23, No. 9.
- [23] Huebner, K.H. and Thornton, E.A. (1982). The Finite Elements Method for Engineers. John Wiley and Sons, Toronto.

- [24] Zinkovitz, O.C. and Taylor, R.L. (1989). The Finite Element Method, Basic Formulation and Linear Problems. (4th ed.). McGraw-Hill Book Company, England.
- [25] Oliveira, E.R.A. (1968). Theoretical Foundations of the Finite Element Method, *Int. J. Solids Struct.*, Vol.4.
- [26] Fox, R.L. (1971). Optimisation Methods in Engineering Design, Addison Wesley, Massachusetts.
- [27] Fox, R.L. and Kapoor, M.P. (1968). Rates of Change of Eigenvalues and Eigenvectors. *AIAA journal*, Vol. 6.
- [28] Fox, R.L. and Muira H. (1971). An Approximate Analysis Technique for Design Calculations. *AIAA Journal*, Vol. 4.
- [29] Noor, A.K. and Lowder, H.E. (1974). Approximate Techniques of Structural Reanalysis. *Computer and Structures*, Vol. 4.
- [30] Bhatia, K.G. (1971). Rapid Iterative Reanalysis for Automated Redesign. NASA TN.
- [31] Vanderplaats, G.N., Muira H., Nagendra, G. and Wallerstein, D. (1989). Optimisation of Large Scale Structures using MSC/Nastan. *Proceedings of the first international conference*, Computational Mechanics Publications.

- [32] Arora, J.S. (1989). Introduction to Optimum design. (International edition). McGraw-Hill Book Company, Singapore.
- [33] Danzig, G.B. (1963). Linear Programming and Extensions. Princeton University Press, Princeton.
- [34] Rao, S.S. (1996). Engineering Optimisation Theory and practice. John Wiley & Sons Inc., New York.
- [35] Haug, E. J., Choi, K. K. and Komkov, V. (1986). Design Sensitivity Analysis of Structural Systems. Academic Press Inc., London.
- [36] Jonson, D., Jordan, K. and Mann, J. (2003). The Design and Manufacture of High-Performance Composite Structures for the UCAV-TD Using Cost-Effective Material Systems and Manufacturing Methods. *SAMPE*.
- [37] Jonson, D. and Jordan K. (2002). The Design and Optimisation for A UCAV Wing Manufactured Using Advanced Composite Materials. *Proceedings of the Third International Conference on Engineering Computational Technology*, pp 161-162.
- [38] Jonson, D., Jordan, K. (2002). Structural Design and Optimisation for a Composite Wing for an Unmanned Aircraft. *Fourth International Conference on Composite Science and Technology*.

- [39] Jonson, D., Jordan, K. (2004). Manufacture of a Novel Wing Structure using Advanced Composite Materials. *Proceedings of the International Conference of Competitive Marketing*.
- [40] Peters, S.T. (1998). Handbook of Composites. Chapman and Hall, London, pp 352-456.
- [41] Jonson, D. and Smith, R. (2001). The Manufacture of a Prototype UCAV Wing Using the SPRINTTM Resin Film Infusion Process. *Proceeding of SAMPE'02*, Paris, pp 305-316.

APPENDIX A

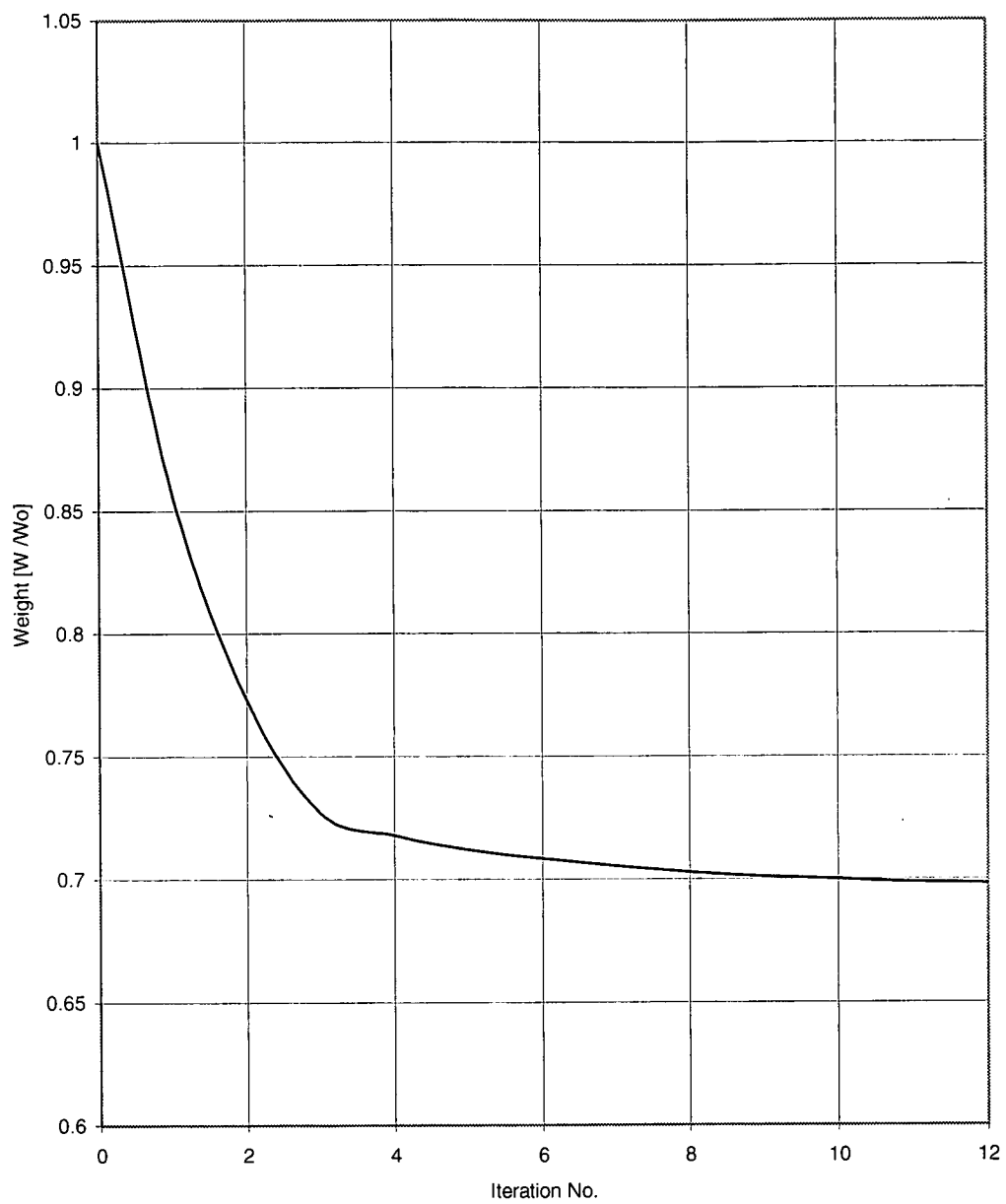


Figure 4.9 Iteration history for weight: conventional configuration (CP = 25%).

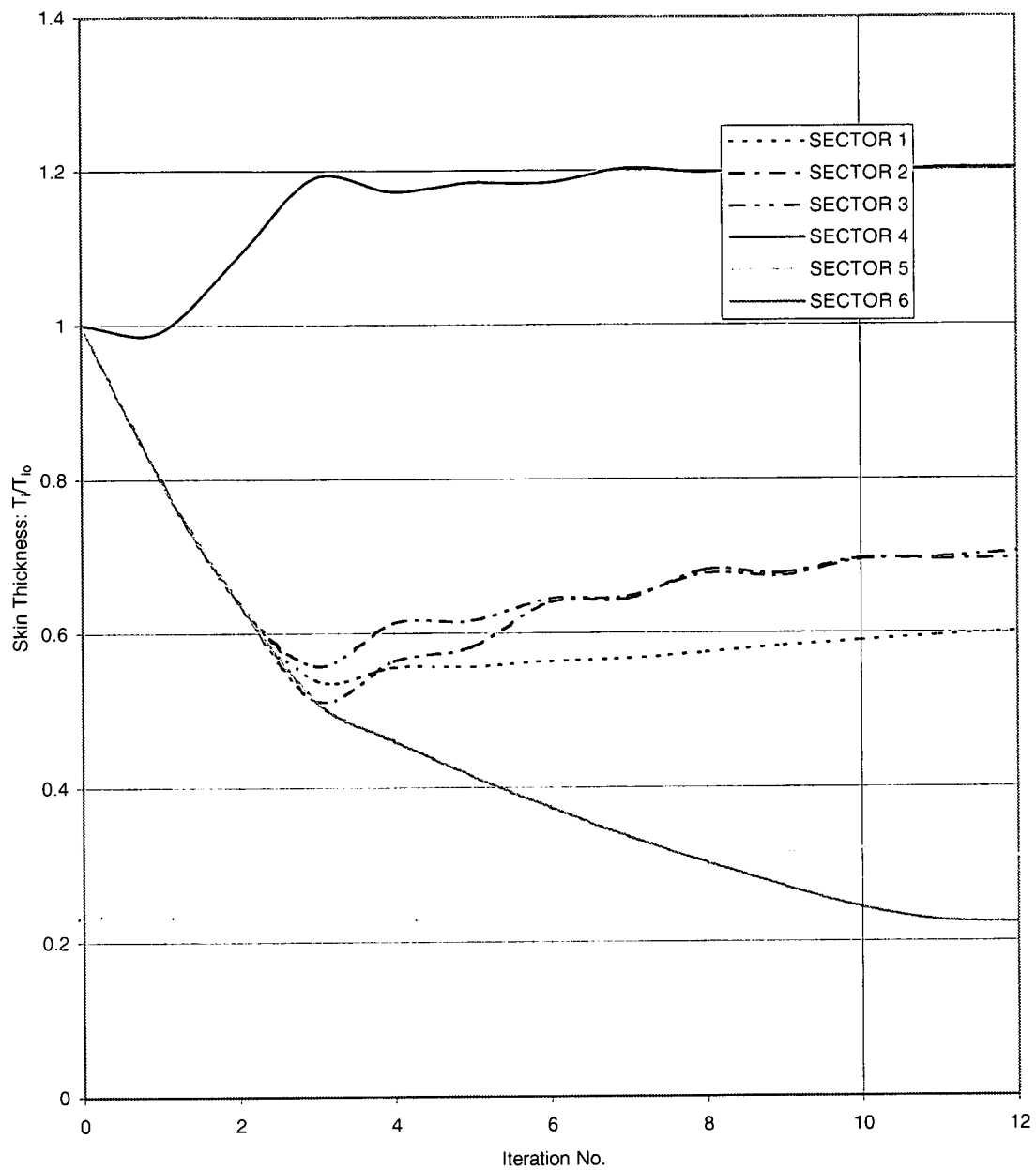


Figure 4.10 Iteration history for skin thickness: conventional configuration (CP = 25%).

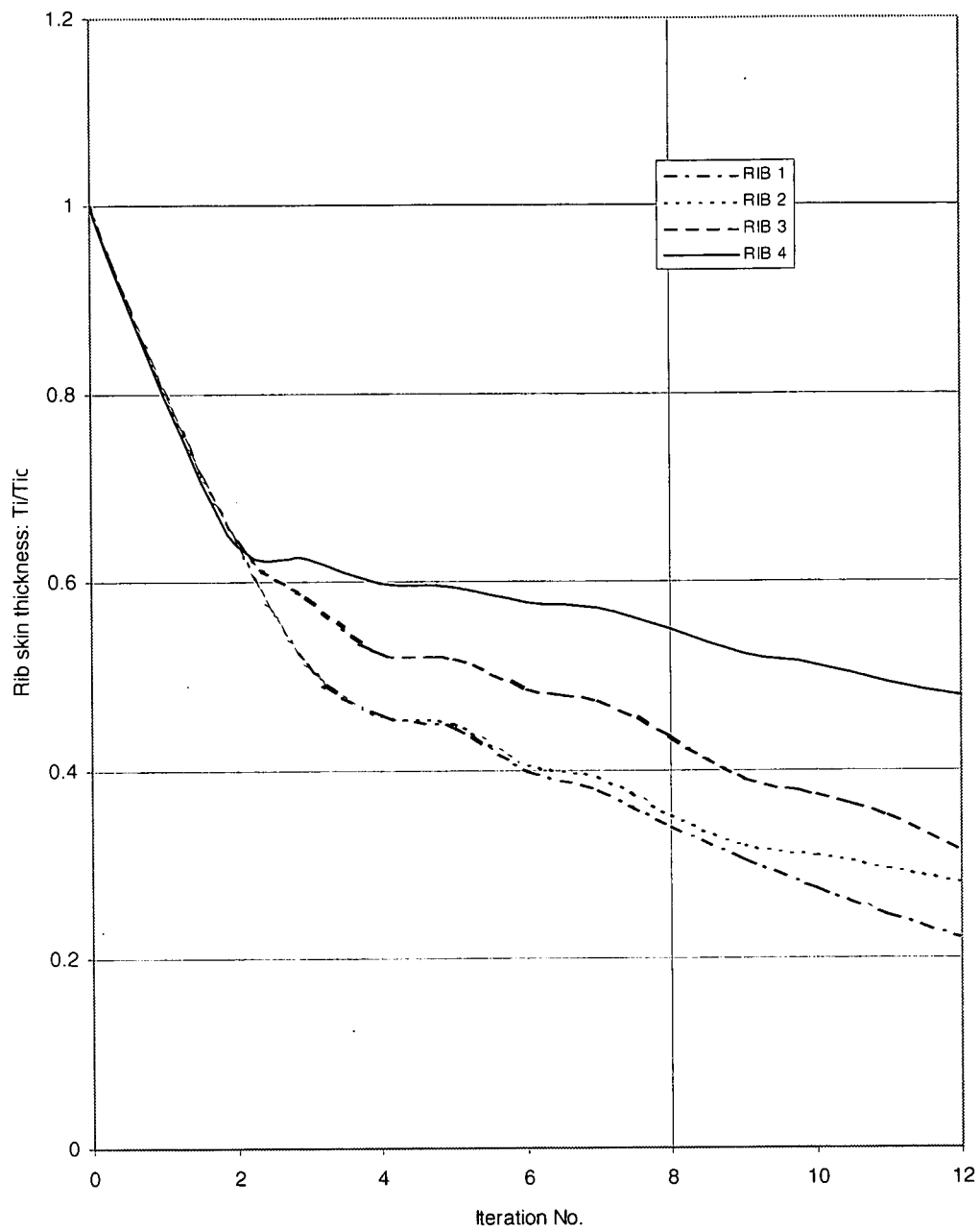


Figure 4.11 Iteration history for rib thickness: conventional configuration (CP = 25%).

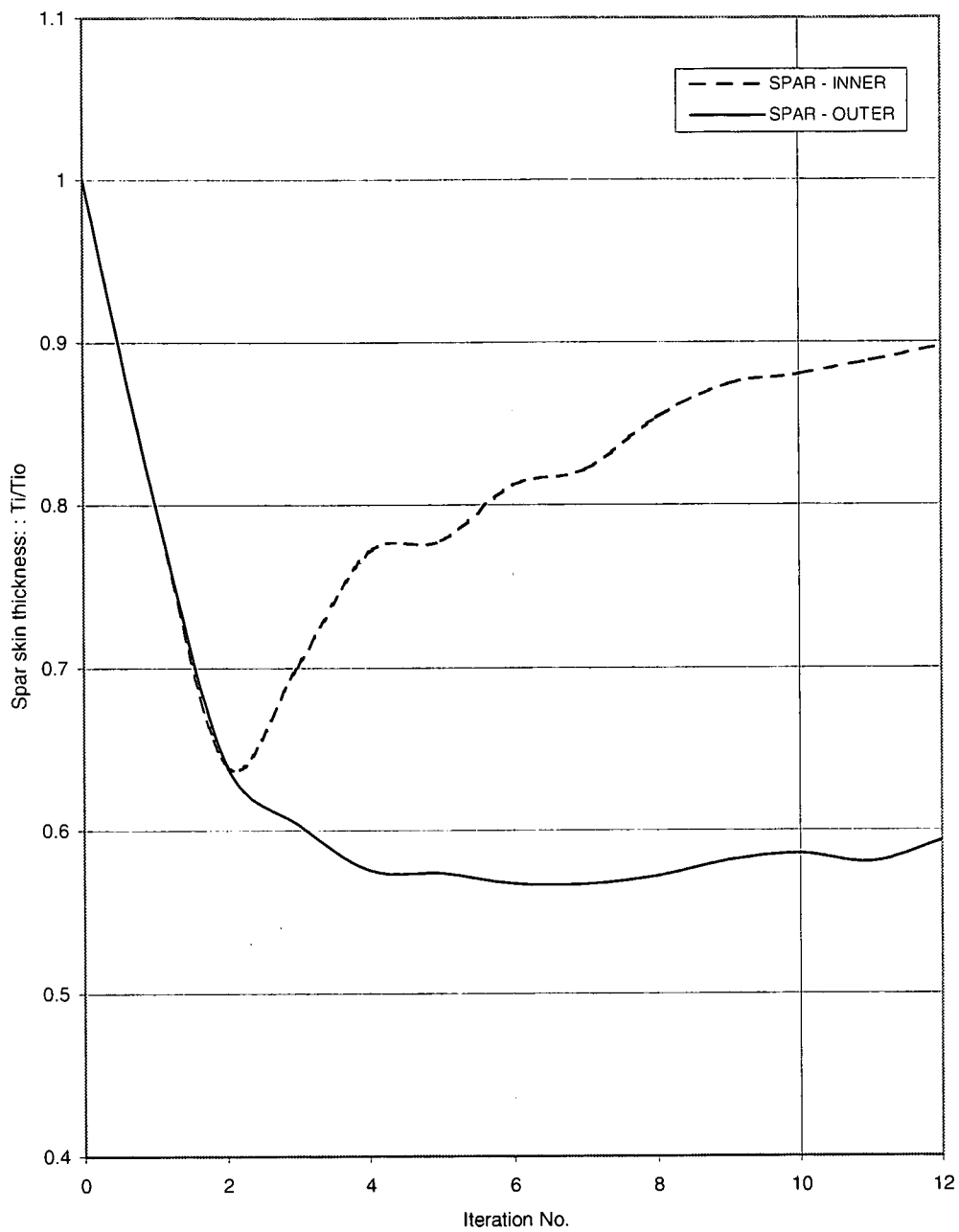


Figure 4.12 Iteration history for spar thickness: conventional configuration (CP = 25%).

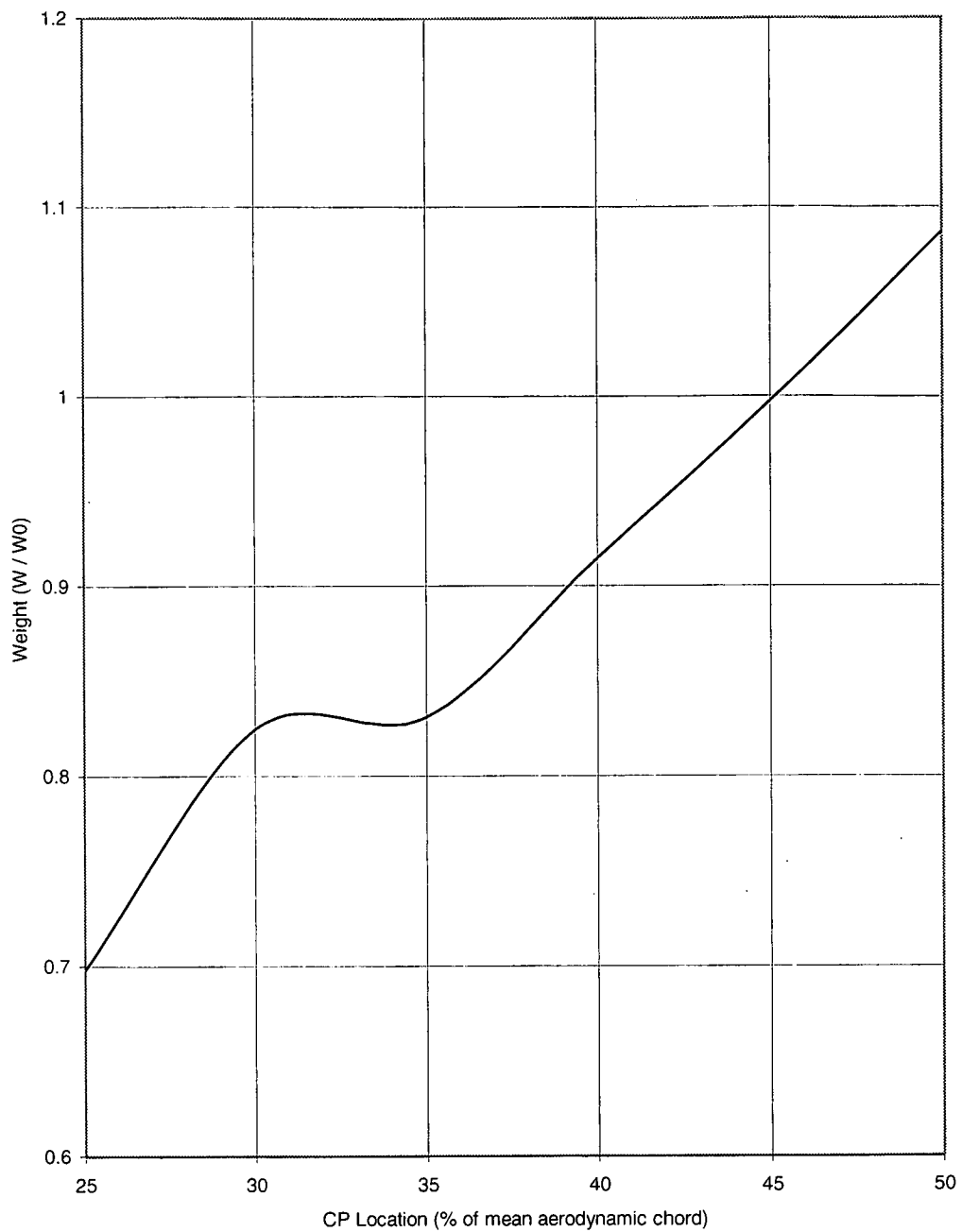


Figure 4.13 Optimal wing weight: conventional configuration CP = 25% to 50%.

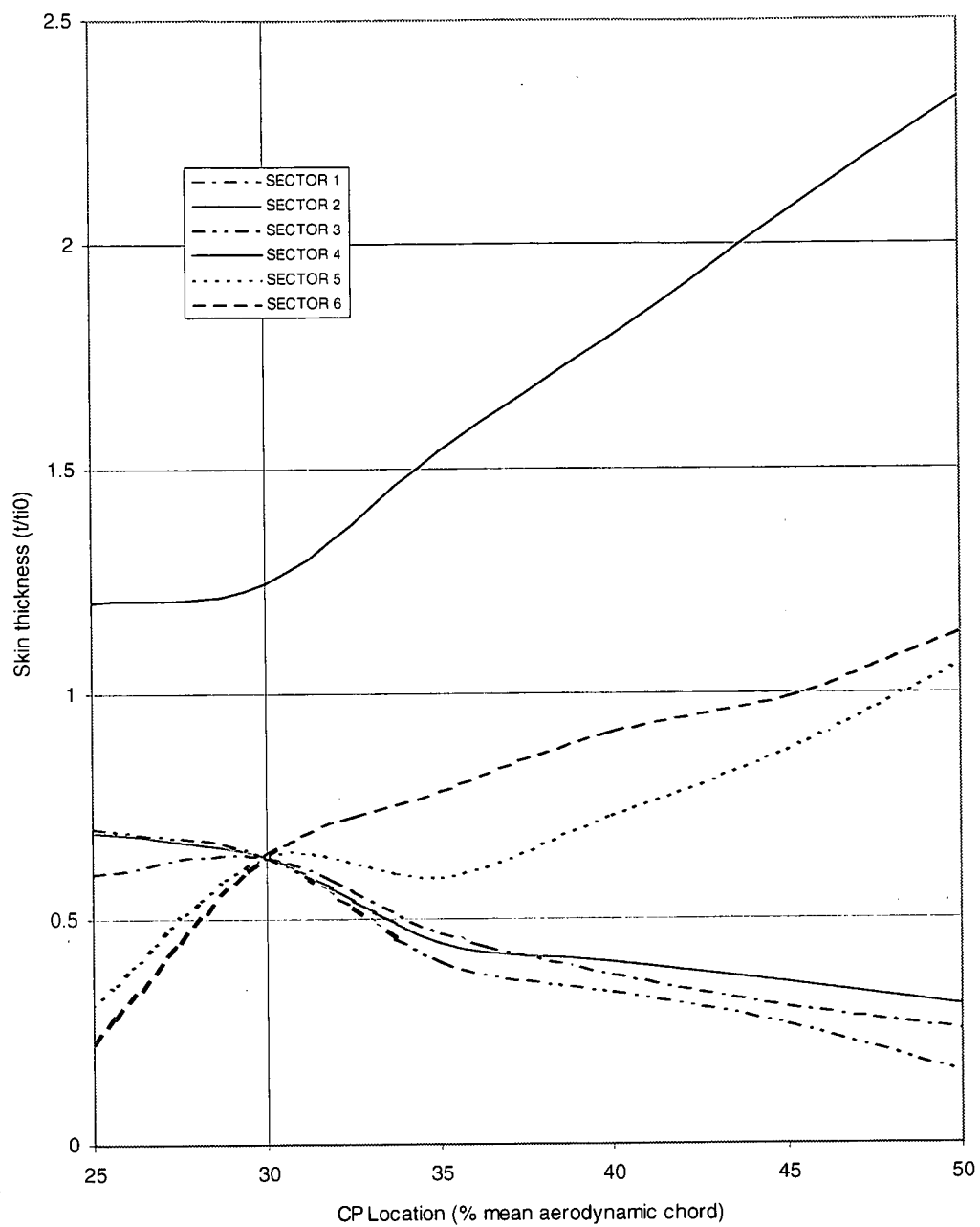


Figure 4.14 Optimal wing skin thicknesses: conventional configuration CP = 25% to 50%.

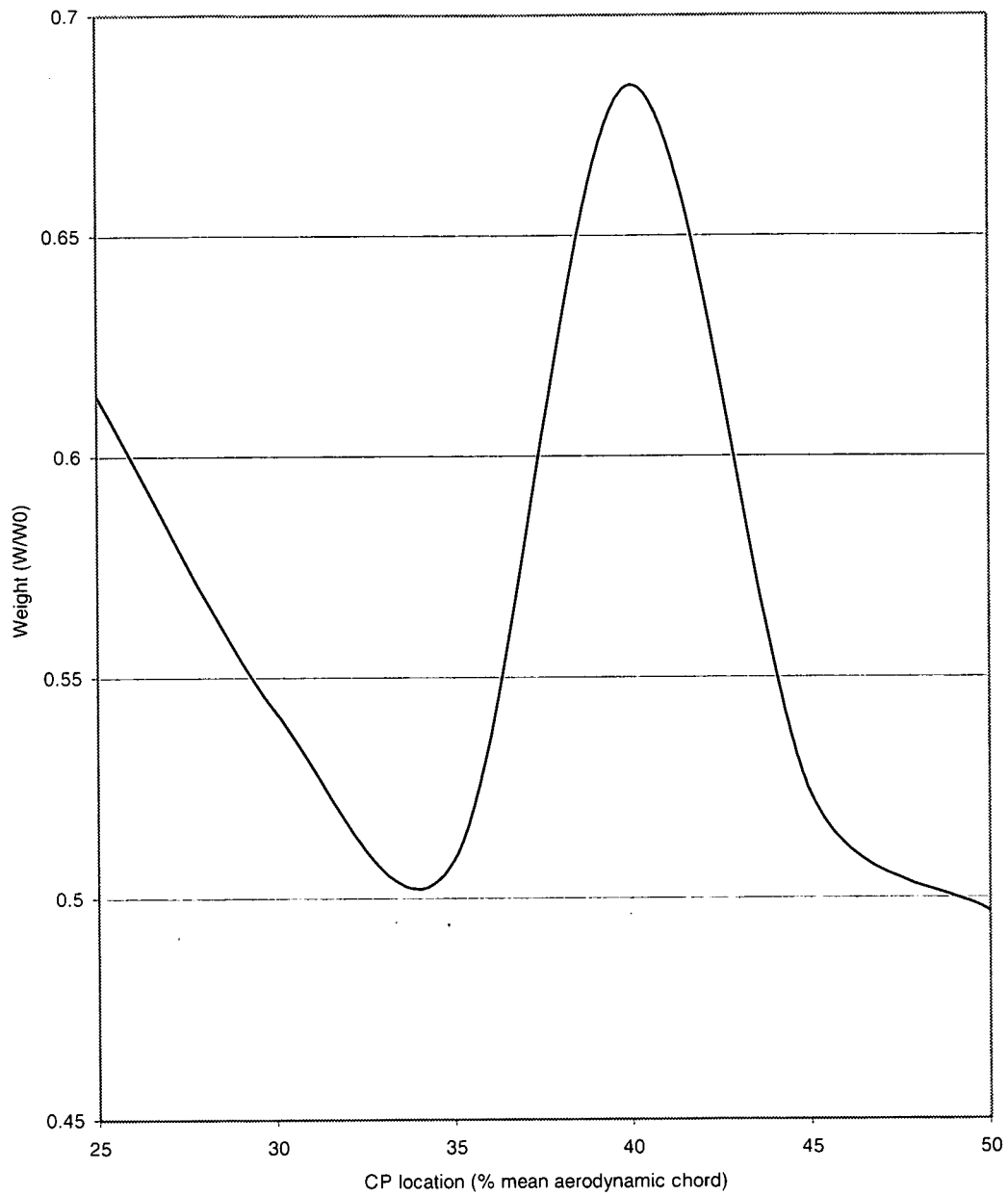


Figure 4.15 Optimal wing weight: radial configuration CP = 25% to 50%.

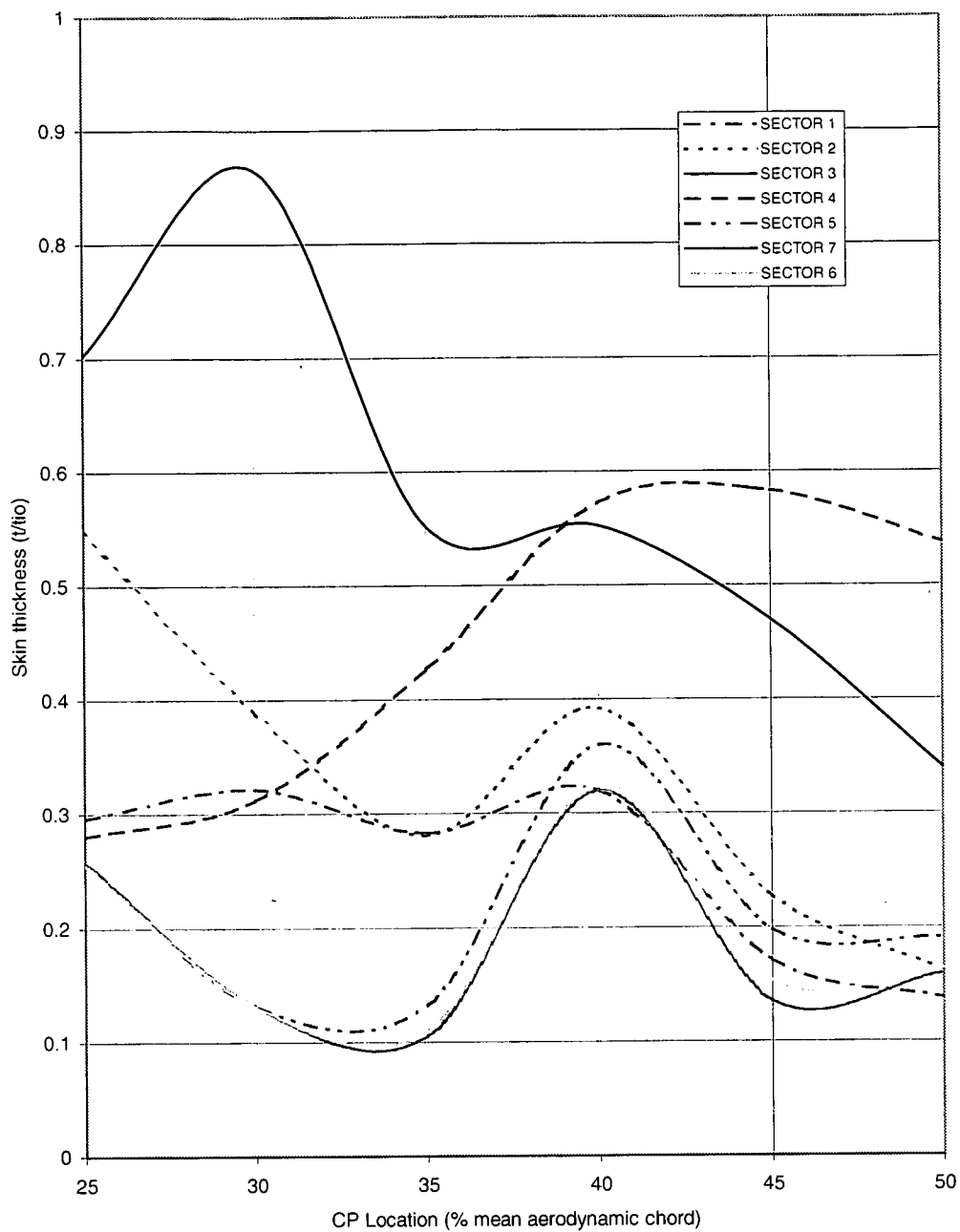


Figure 4.16 Optimal wing skin thicknesses: radial configuration CP = 25% to 50%.

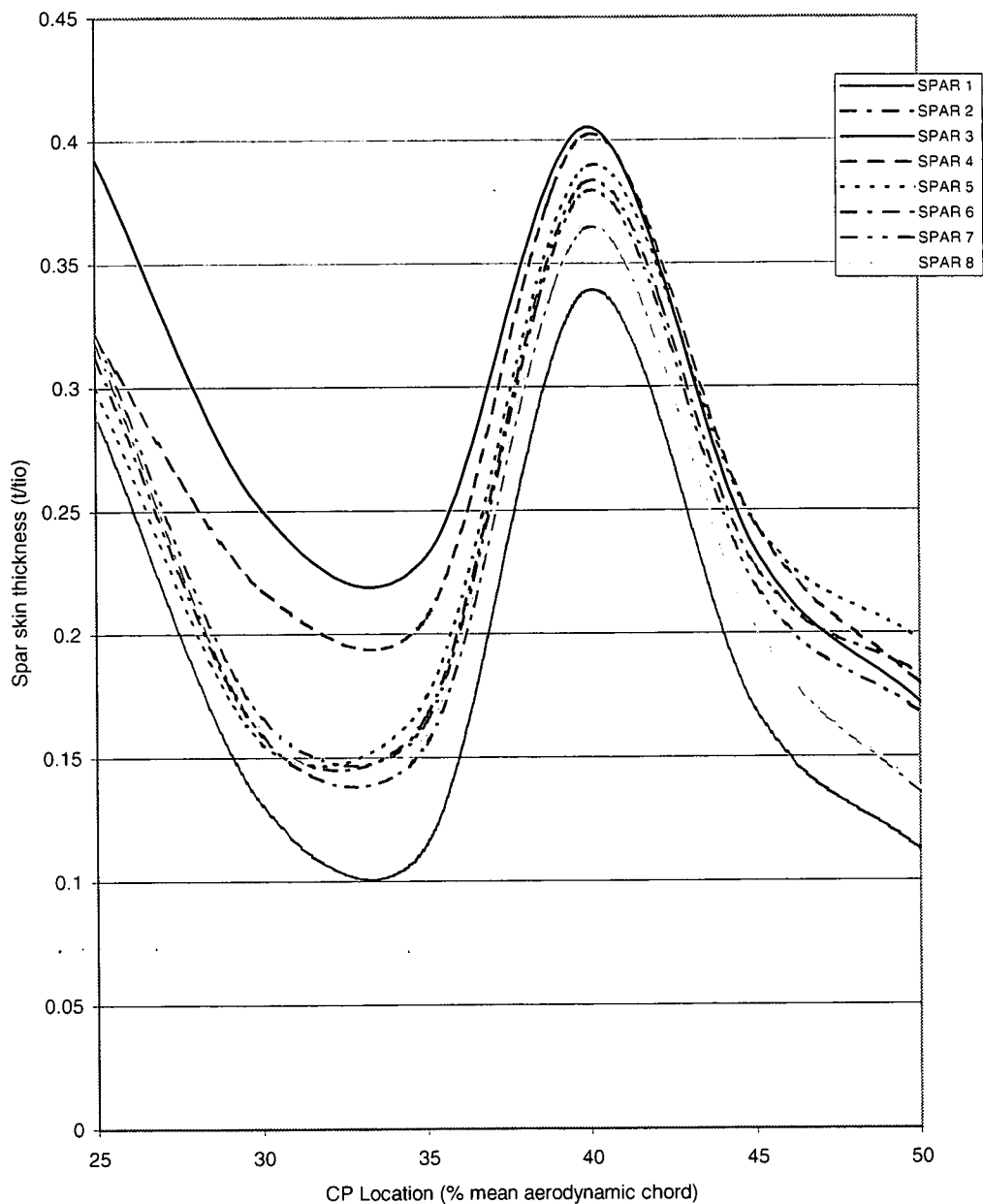


Figure 4.17 Spar thickness: radial configuration CP = 25% to 50%.

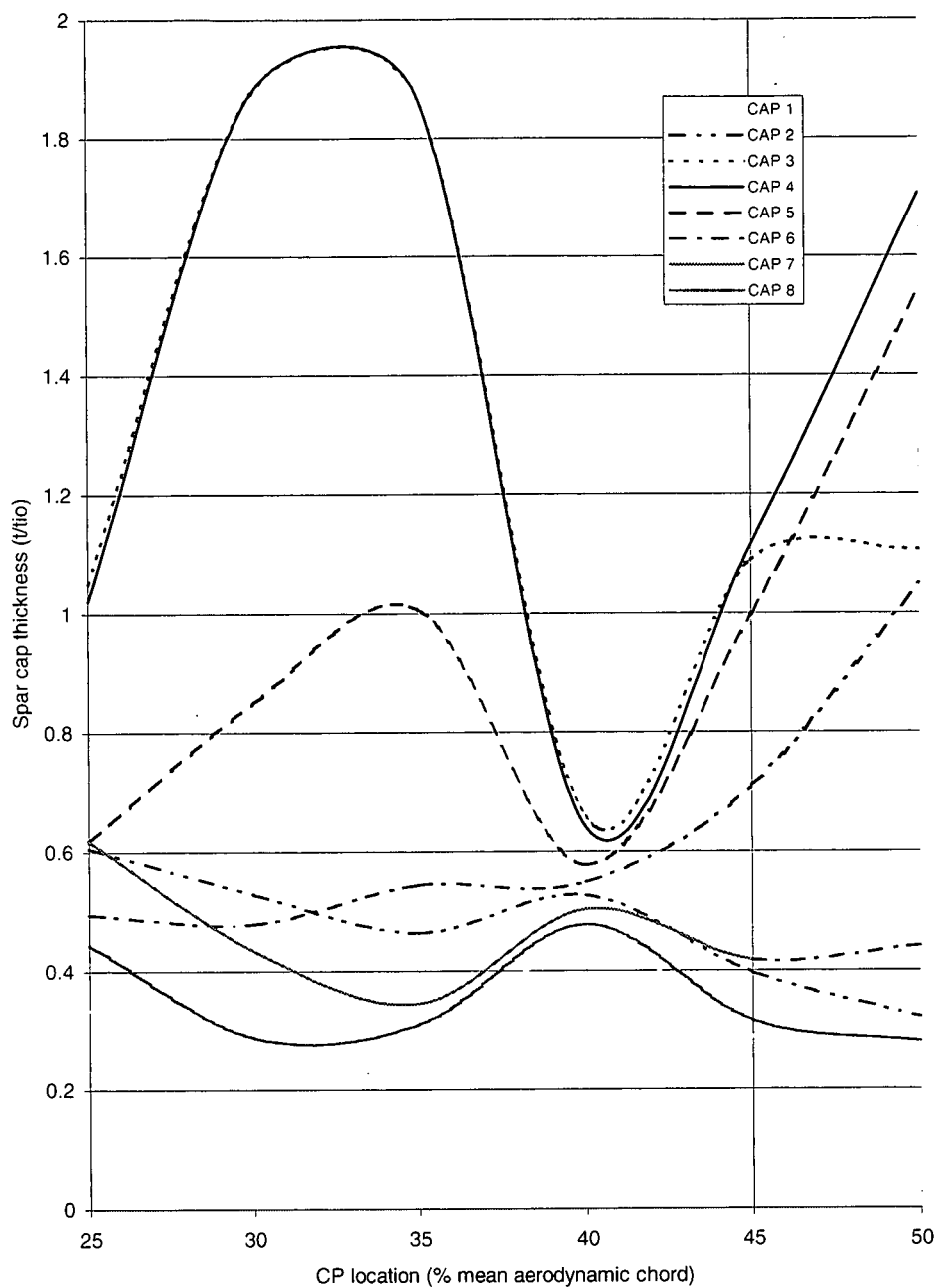


Figure 4.18 Spar-cap thickness: radial configuration CP = 25% to 50%.

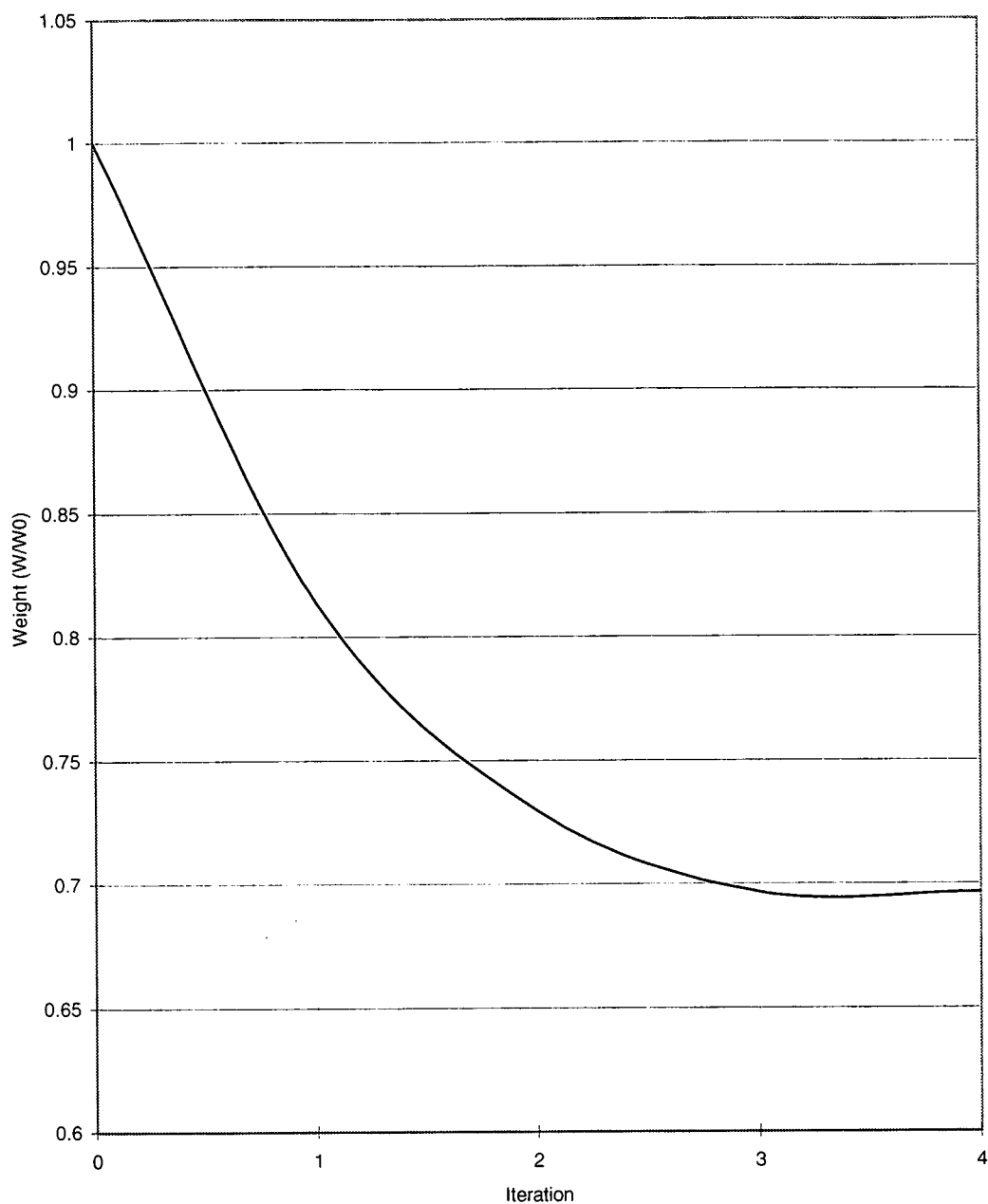


Figure 4.25 Iteration history for weight: CP = 25%.

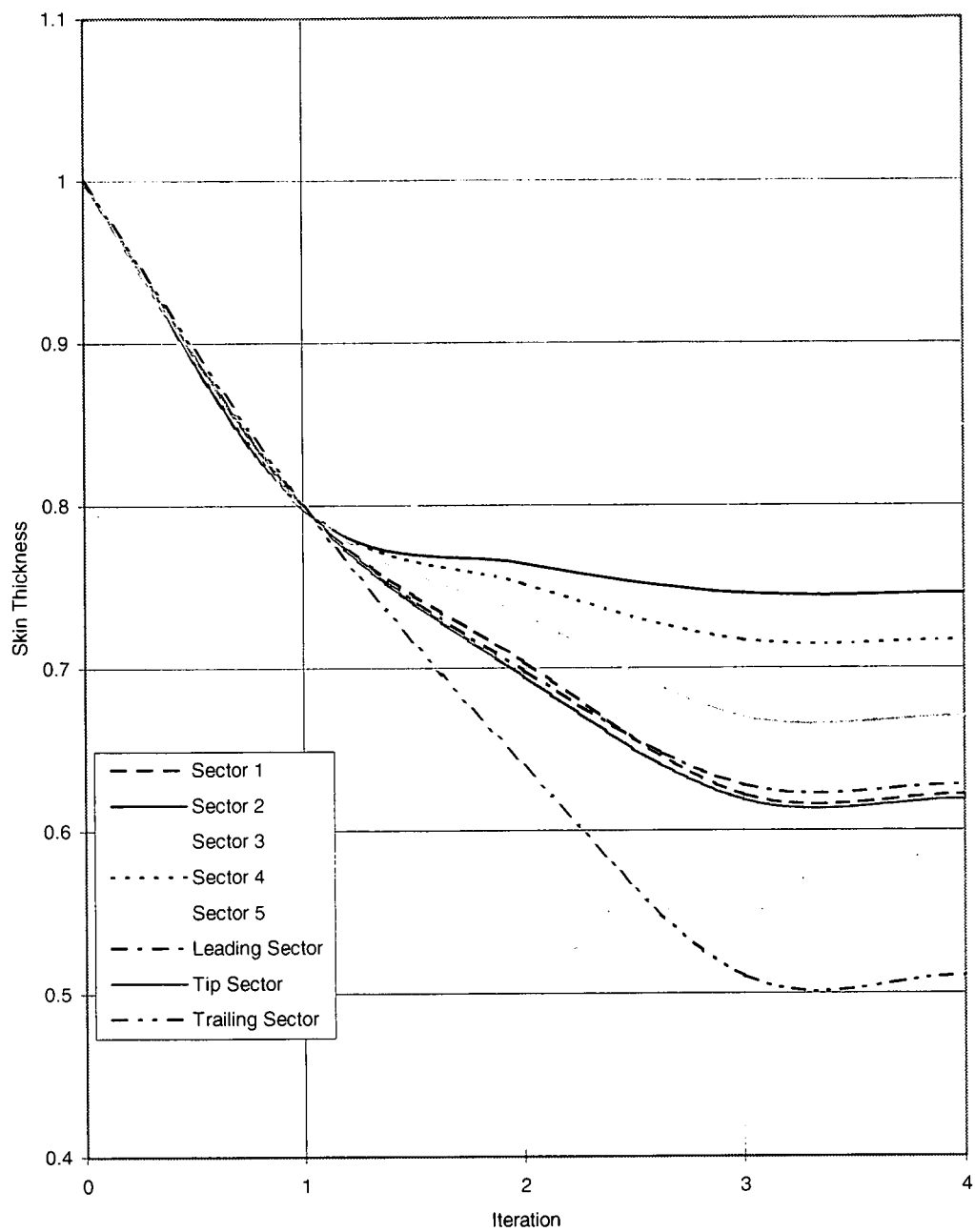


Figure 4.26 Iteration history for skin thickness: CP = 25%.

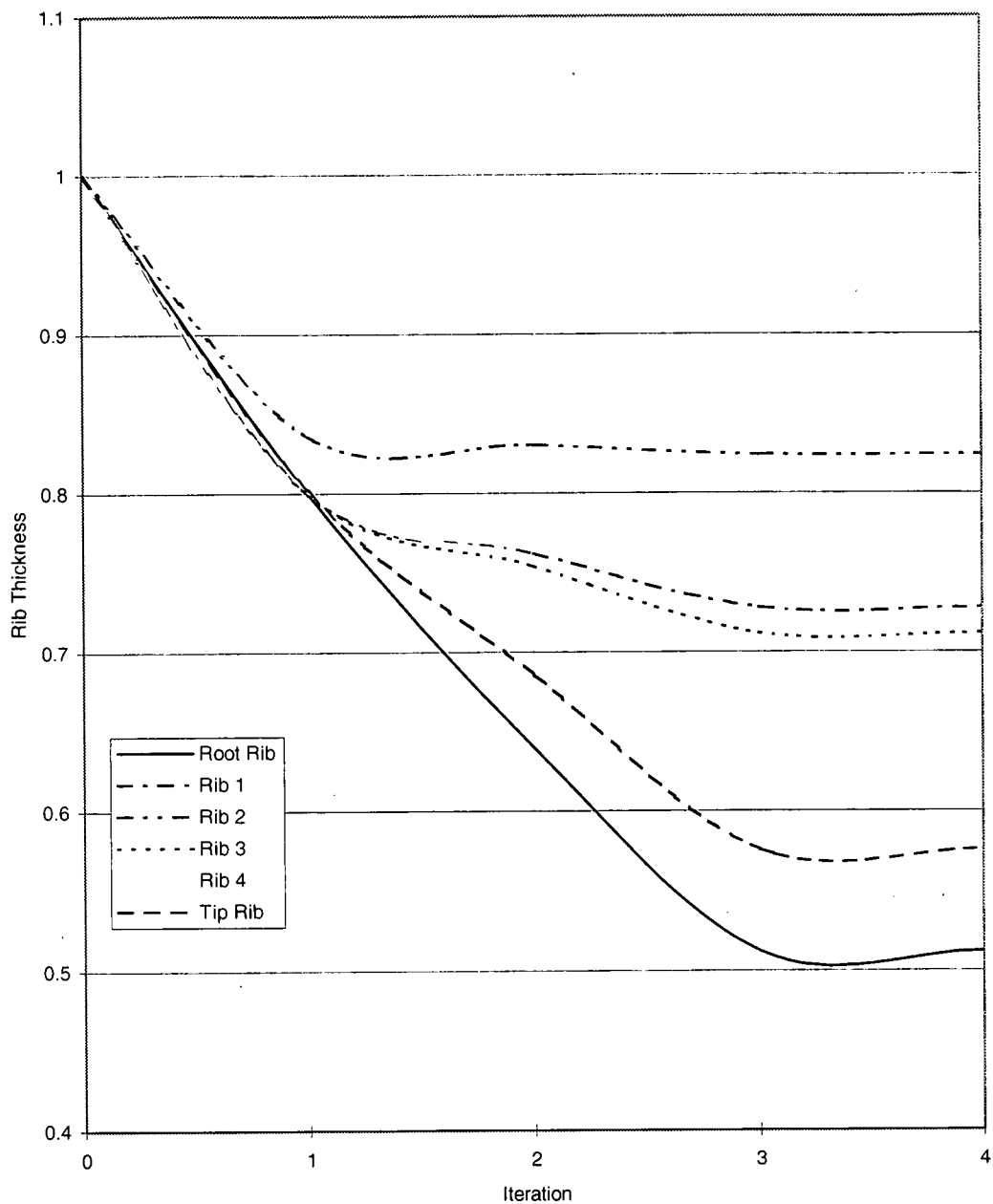


Figure 4.27 Iteration history for rib thickness: CP = 25%.

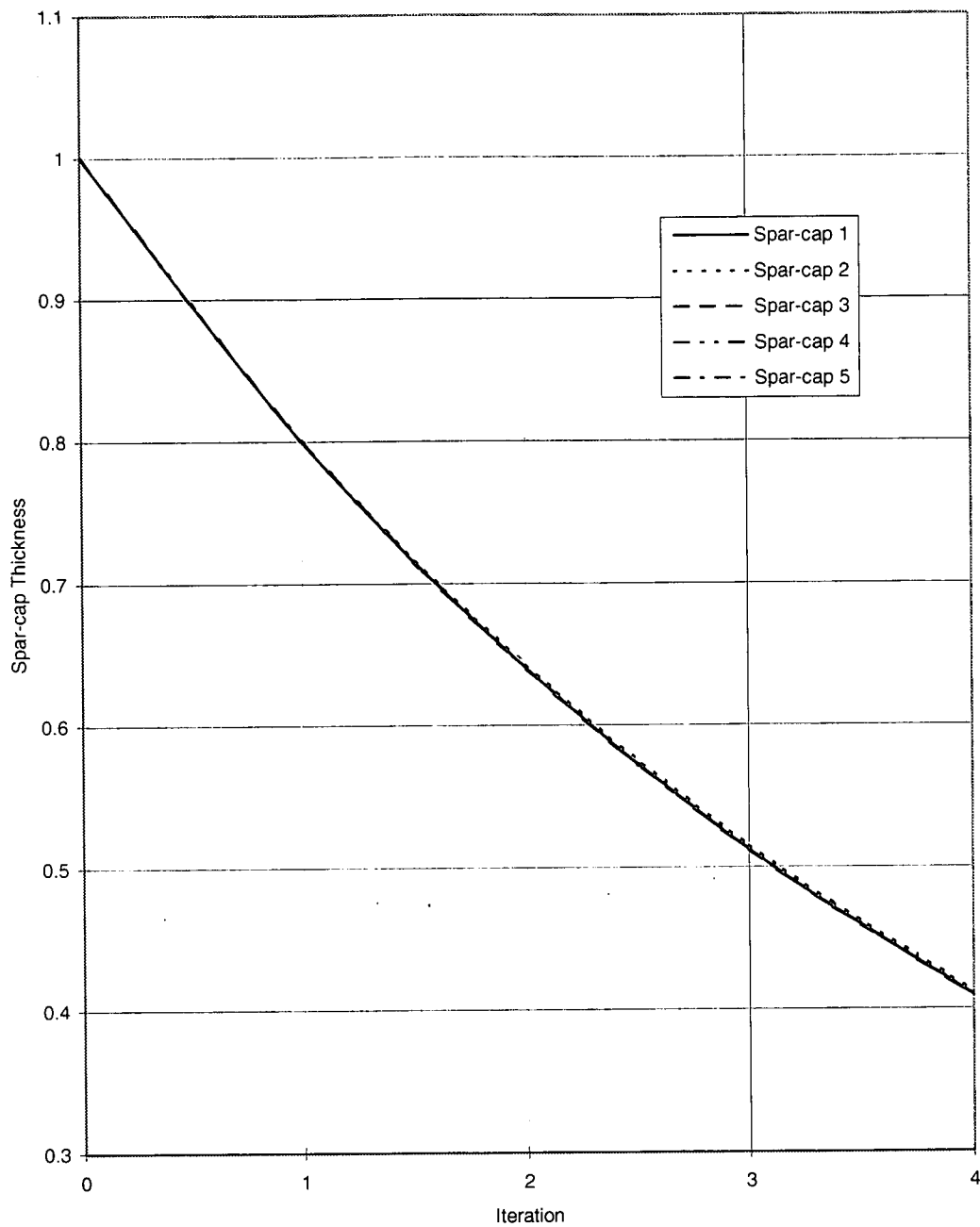


Figure 4.28 Iteration history for spar-cap thickness: CP = 25%.

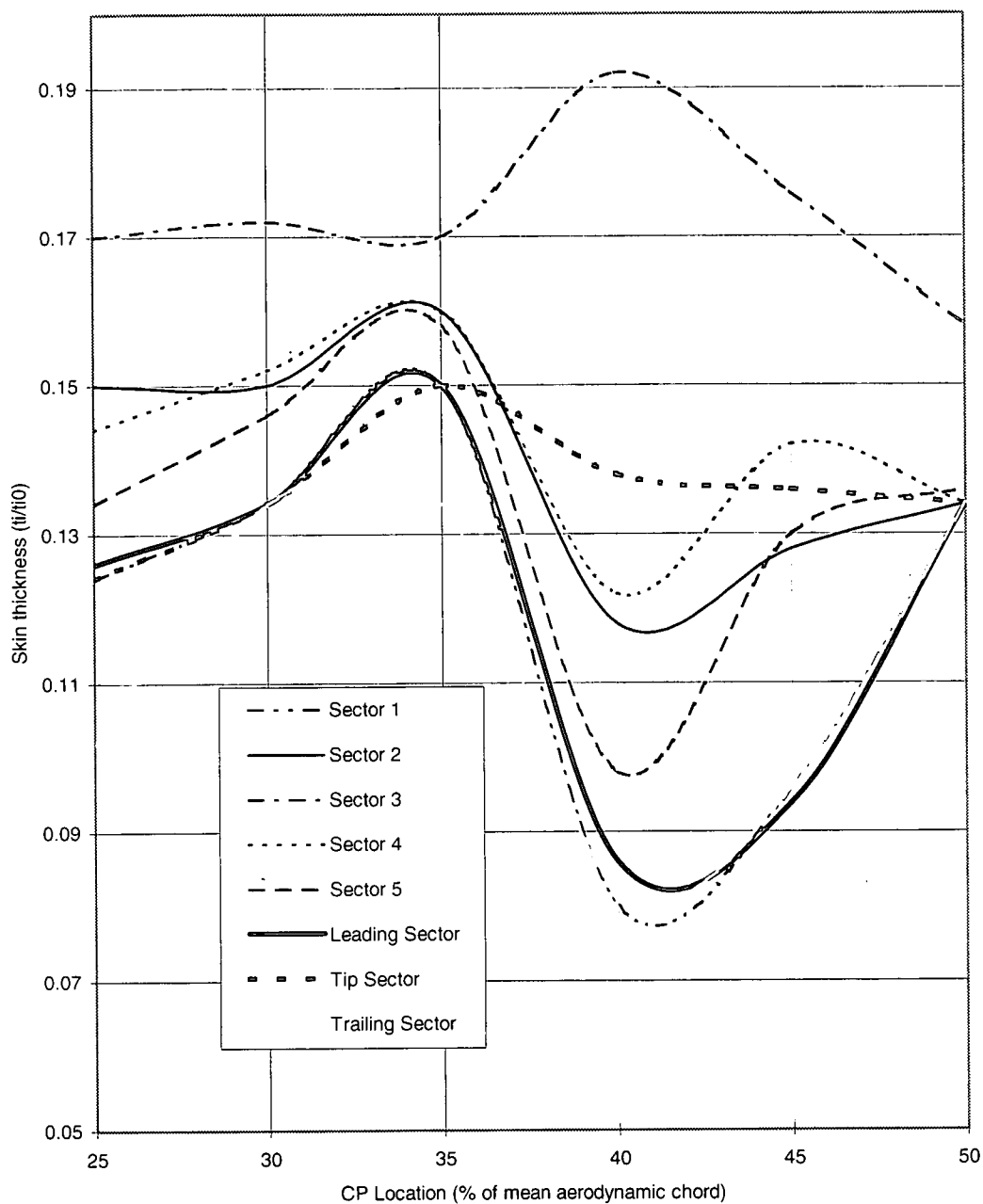


Figure 4.30 Optimal wing skin thicknesses CP = 25% to 50%.

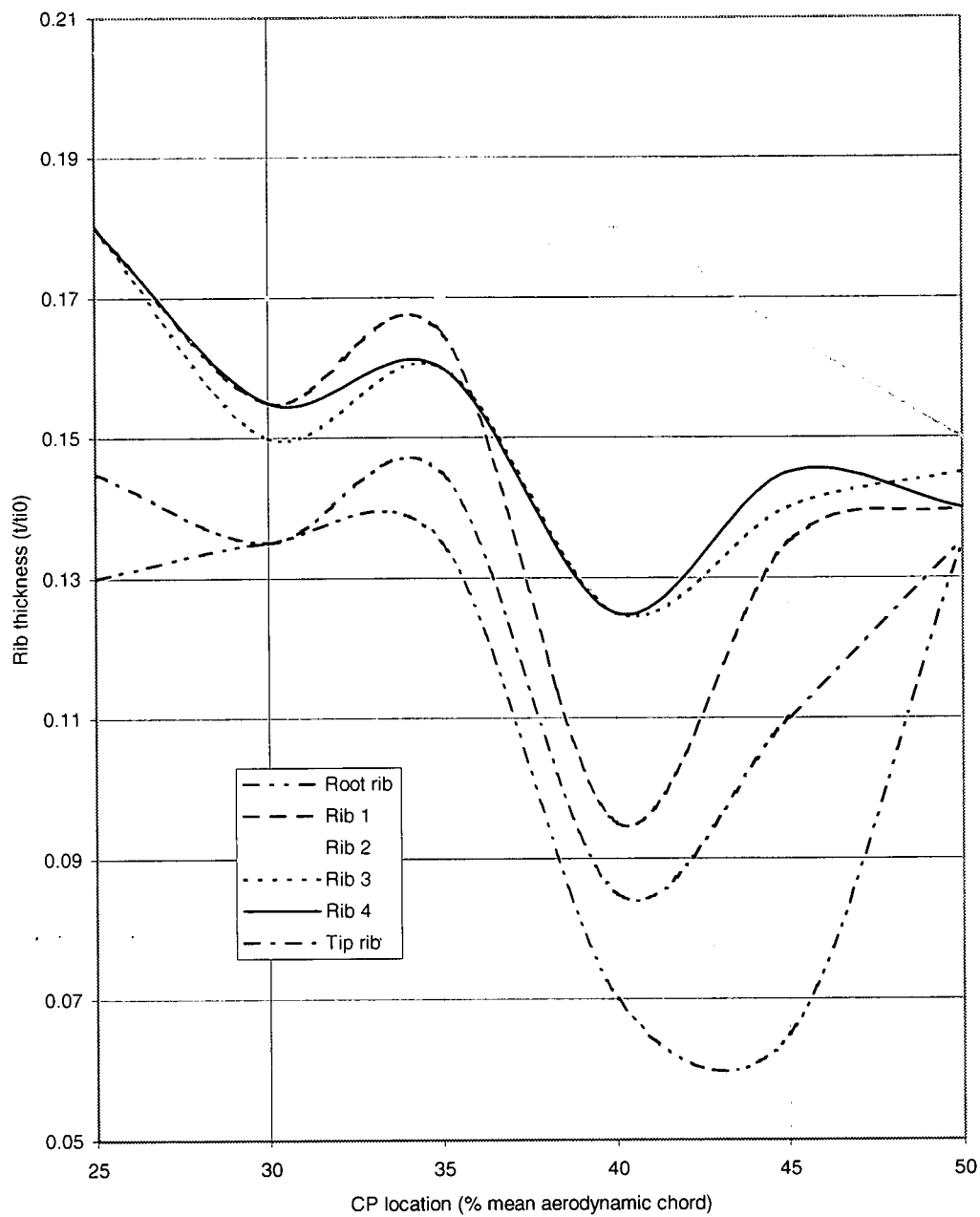


Figure 4.31 Rib/Spar thickness: CP = 25% to 50%.

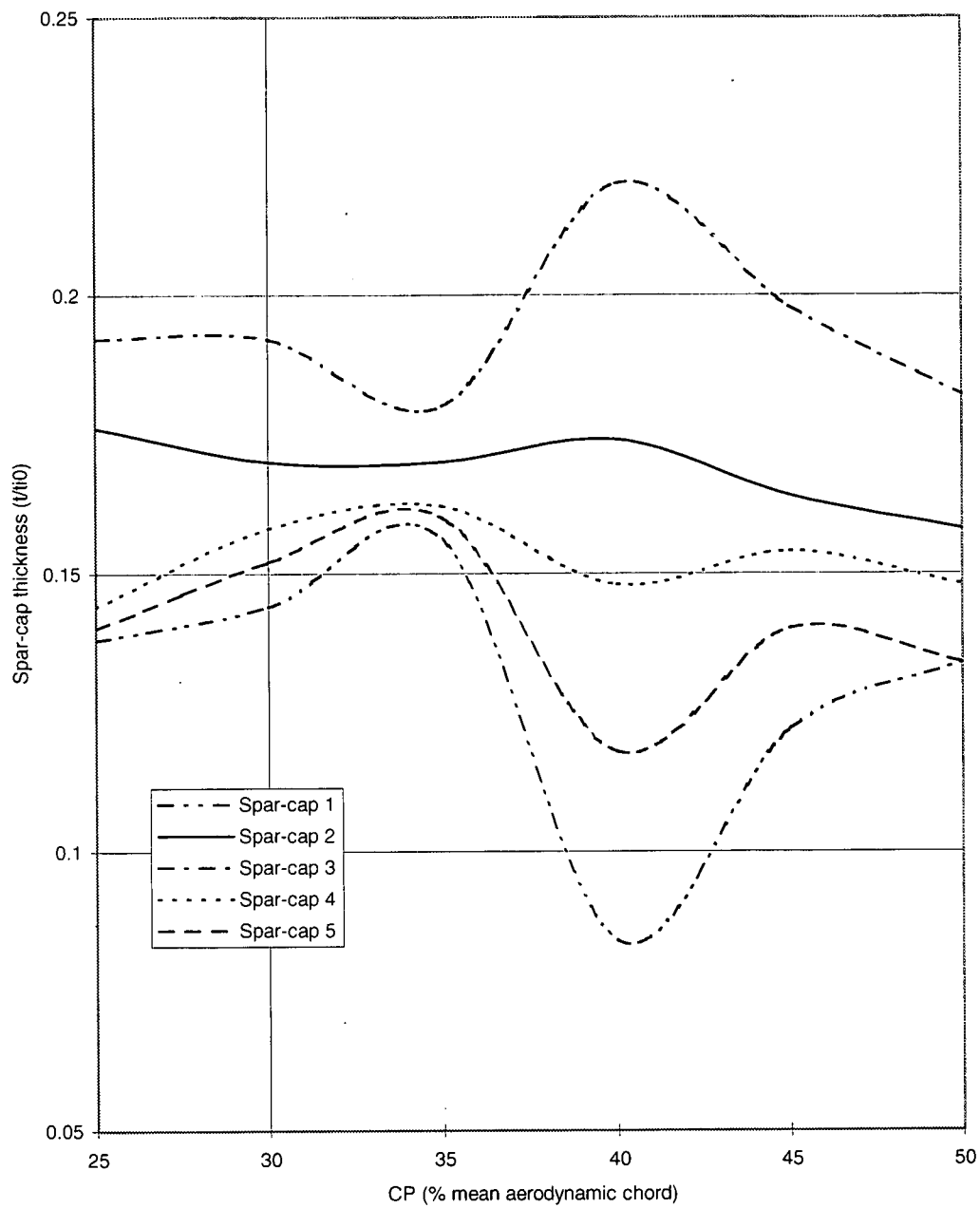


Figure 4.32 Spar-cap thickness: CP = 25% to 50%.