

COMPUTATIONS IN DETERMINING A FINANCIAL PROXY WHICH
OPTIMIZES DE-TRENDED STOCHASTIC ASSET PRICES UNDER FIXED-MIX
PORTFOLIO STRATEGIES

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Abstract

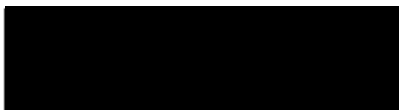
The performance of portfolios of a fixed-rate asset and a risky asset of major companies in a South African market index the FTSE/JSE with strategies which rebalances fixed proportions of wealth in every rebalancing period is analysed in a long term. Recent findings in portfolio management theory by Dempster, Evstigneev and Schenk-Hoppé (2010, 2008, 2007, 2003) and by Browne (1988) note optimality of fixed-mix portfolios which assert fast exponential growth in stationary markets.

A quantitative analysis is performed to analyse quantifiable measures in order to optimize the application of self-financing constant rebalanced portfolio strategies that contribute to the financial engineered prospects suggested by Dempster *et al.* (2010) for fixed-mix portfolios. The comparative performance of fixed-mix portfolios with a proxy strategy and without proxy strategy relative to a buy and hold strategy shows the superiority of fixed-mix portfolios in generic market conditions. The research extends the utilization of constant rebalanced self-financing portfolio investment strategies by assessing the market price of risk under the mean-variance model of Markowitz (1952). Effective implementation tactics of the strategy are examined by focusing on the market risk and the financial risk.

The frequent reversals and trending of stochastic asset prices in the financial market are analysed to adjust the market price of risk by considering tradable financial securities to determine the financial proxy of de-trending. The proxy hypothesis which evaluates the stationary financial condition in a fixed-mix portfolio is validated by an option-based myopic strategy using a lookback straddle option. A myopic strategy is a strategy which considers a single period ahead, Fabozzi, Forcardi and Kolm (2006). The realised growth under a financial proxy is found to have a linear strategic asset allocation with a low degree of concavity relative to a buy and hold performance in the market risk of self-financing portfolio strategies.

Declaration

I, Siyabonga Goodwill Chule, declare that the work contained in this thesis is my own original work, and that I have not previously submitted it to any university for a degree. Prior discussions of this proposed work were conducted in the talk during the 53rd SAMS conference at the University of Pretoria in November 2010. The results of the thesis were presented during the Institutional Research day on the 27th of November 2013.



Signature

01/09/2014

Date

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Contents

Abstract	ii
Declaration	iii
Acknowledgements	iv
Contents	v
List of figures	xii
List of Charts	xiv
List of Tables	xv
List of Appendices	xvii
Chapter 1: INTRODUCTION	1
1.1 Background.....	1
1.2 Market risk.....	3
1.3 Portfolio insurance.....	5
1.4 The financial proxy.....	5
1.5 Performed computations.....	6
1.6 Asset Liability portfolios.....	8
1.7 Research Problem Statement.....	10
1.8 Objectives.....	11
1.8.1 Main Objective.....	11
1.8.2 Supporting objectives.....	12
1.9 Research design.....	13
1.9.1 Research type.....	13
1.9.2 Data.....	13
1.9.3 Sample selection.....	14
1.9.4 Time series span.....	14
1.10 Structure of the thesis.....	15

1.11 Conclusion.....	16
Chapter 2: LITERATURE REVIEW.....	18
2.1 Introduction.....	18
2.2 Trading strategies.....	18
2.3 Active portfolio management.....	19
2.4 The quantification of risk-adjusted returns on a mean-variance model.....	22
2.5 Dynamic trading strategies.....	24
2.5.1 Lookback Call and lookback Put options.....	24
2.5.2 Lookback Straddle.....	25
2.6 Dynamic asset allocation portfolios.....	25
2.7 Optimization of dynamic portfolio strategies.....	27
2.8 Conclusion.....	28
Chapter 3: METHODOLOGY.....	30
3.1 Introduction.....	30
3.2 Financial risk.....	31
3.3 The application of fixed-mix strategies.....	33
3.4 Background of the designed strategy.....	34
3.4.1 A Quantitative Analytic Example.....	35
3.5 Risk and returns of asset prices.....	39
3.6 De-trending and the financial proxy.....	39
3.7 Data and Methodology.....	40
3.7.1 Major South African companies.....	41
3.7.2 Data collection method and validity.....	42

3.8 Test of stationarity.....	42
3.9 Performance measurement.....	43
3.9.1 Quantitative analysis of fixed-mix strategies performance.....	44
3.9.2 Unrealized potential of stochastic processes.....	44
3.9.3 Quantification of anticipated returns.....	45
3.9.4 Financial engineered formulation.....	47
3.9.5 The validity of a proposed hedging strategy.....	48
3.10 Summary.....	48
3.11 Conclusion.....	49
 Chapter 4: THE TRADING MODEL AND EMPIRICAL FINDING.....	 50
4.1 Introduction.....	50
4.2 The trading model	51
4.2.1 The Buy and Hold portfolio strategy.....	52
4.2.2 The Fixed-mix portfolio strategy... ..	52
4.3 Prime rates.....	52
4.4 Asset prices and total returns of a fixed-rate asset and risky assets of selected companies.....	54
4.4.1 Asset prices and total returns of a fixed-rate asset.....	54
4.4.2 Asset prices and total returns of a risky asset of ABSA Group Limited.....	55
4.4.3 Asset prices and total returns of a risky of Anglo American Platinum Limited.....	58
4.4.4 Asset prices and total returns of a risky asset of Sasol Limited.....	60
4.4.5 Asset prices and total returns of a risky asset of Standard Bank.....	62
4.4.6 Asset prices and total returns of a risky asset of AFGRI Limited.....	64

4.4.7 Asset prices and total returns of a risky asset of Anglo Gold Ashanti Limited.....	66
4.5 Quantification of anticipated portfolio growth.....	69
4.5.1 The model of asset prices and total returns of ABSA Group Limited.....	70
4.5.2 The model of asset prices and total returns of Standard Bank limited.....	70
4.5.3 The model of asset prices and total returns of Sasol Limited.....	71
4.5.4 The model of asset prices and total returns of Anglo American Platinum Limited.....	71
4.5.5 The model of asset prices and total returns of Anglo Gold Ashanti Limited.....	72
4.5.6 The model of asset prices and total returns of AFGRI Limited.....	73
4.6 The volatility in a fixed-mix portfolio.....	73
4.7 Financial engineered formulation approach.....	75
4.8 Summary.....	77
4.9 Conclusion.....	79
 Chapter 5: THE COMPUTATIONAL ANALYSIS.....	80
5.1 Introduction.....	80
5.2 Performance of portfolio strategies.....	81
5.2.1 Performance: Fixed-rate asset and ABSA Group Limited risky asset.....	83
5.2.2 Performance: Fixed-rate asset and AFGRI Limited risky asset.....	86
5.2.3 Performance: Fixed-rate asset and Anglo Gold Ashanti Limited risky asset..	89
5.2.4 Performance: Fixed-rate asset and Anglo American Platinum Limited risky asset.....	92
5.2.5 Performance: Fixed-rate asset and Sasol Limited risky asset.....	95
5.2.6 Performance: Fixed-rate asset and Standard Bank risky asset.....	98
5.3 Comparative performances relative to market conditions.....	101

5.3.1 Difference performance of portfolios of a risky asset of	
ABSA Group Limited relative to prime rates conditions.....	102
5.3.2 Difference performance of portfolios of a risky asset of	
AFGRI Limited relative to prime rates conditions.....	103
5.3.3 Difference performance of portfolios of a risky asset of	
Anglo Ashanti Limited relative to prime rates conditions.....	105
5.3.4 Difference performance of portfolios of a risky asset of	
Anglo American Platinum Limited relative to prime rates conditions.....	106
5.3.5 Difference performance of portfolios of a risky asset of	
Sasol Limited relative to prime rates conditions.....	108
5.3.6 Difference performance of portfolios of a risky asset of	
Standard Bank Limited relative to prime rates conditions.....	109
5.4 Summary of the computations.....	110
5.5 Conclusion.....	112
 Chapter 6: THE ANALYSIS OF A FINANCIAL PROXY.....	 114
6.1 Introduction.....	114
6.2 Quantitative analysis of performance of portfolio systems.....	115
6.2.1 Excess performance of a fixed-mix portfolio strategy.....	116
6.3 The unrealised potential of stochastic asset prices under fixed-mix strategies..	122
6.4 Quantification of portfolio growth of a fixed-mixed strategy.....	124
6.4.1 Quantification of a fixed-mixed strategy in a portfolio	
with 50% of a risky asset of ABSA Group Limited.....	124
6.4.2 Quantification of a fixed-mixed strategy in a portfolio	
with 50% of a risky asset of AFGRI Limited.....	127

6.4.3 Quantification of a fixed-mixed strategy in a portfolio	
with 50% of a risky asset of Anglo Gold Ashanti Limited.....	129
6.4.4 Quantification of a fixed-mixed strategy in a portfolio	
with 50% of a risky asset of Anglo American Platinum Limited.....	131
6.4.5 Quantification of a fixed-mixed strategy in a portfolio	
with 50% of a risky asset of Sasol Limited.....	134
6.4.6 Quantification of a fixed-mixed strategy in a portfolio	
with 50% of a risky asset of Standard Bank Limited.....	136
6.5 Quantification of excess value of risk in a portfolio.....	139
6.5.1 Quantification of excess value of risk in a portfolio	
of a risky asset of ABSA Group Limited.....	140
6.5.2 Quantification of excess value of risk in a portfolio	
of a risky asset of AFGRI Limited.....	141
6.5.3 Quantification of excess value of risk in a portfolio	
of a risky asset of Anglo Gold Ashanti Limited.....	142
6.5.4 Quantification of excess value of risk in a portfolio	
of a risky asset of Anglo American Platinum Limited.....	144
6.5.5 Quantification of excess value of risk in a portfolio	
of a risky asset of Sasol Limited.....	146
6.5.6 Quantification of excess value of risk in a portfolio	
of a risky asset of Standard Bank.....	147
6.6 Quantification of risk-adjusted portfolio growth of a fixed-mixed strategy.....	149
6.7 Time effective portfolio strategies that render a financial proxy.....	150
6.7.1 Performance of a proxy strategy in a portfolio of a risky	
asset of ABSA Group Limited.....	151

6.7.2 Performance of a proxy strategy in a portfolio	
of a risky asset of AFGRI Limited.....	155
6.7.3 Performance of a proxy strategy in a portfolio	
of a risky asset of Anglo Gold Ashanti Limited.....	157
6.7.4 Performance of a proxy strategy in a portfolio	
of a risky asset of Anglo American Platinum Limited.....	158
6.7.5 Performance of a proxy strategy in a portfolio	
of a risky asset of Sasol Limited.....	160
6.7.6 Performance of a proxy strategy in a portfolio	
of a risky asset of Standard Bank Limited.....	162
6.8 Summary of the interpretation	163
6.9 Conclusion.....	164
 Chapter 7: CONCLUSION.....	 165
7.1 Introduction.....	165
7.2 Computations of a financial proxy.....	167
7.3 Practical matters.....	170
7.4 The focused short comings of fixed-mix strategies.....	171
7.5 Recommendations.....	172
References.....	173

List of Figures

Figure 4.3.1: Down trending and up trending prime rates.....	53
Figure 4.4.1: Fixed-rate asset prices in down trending prime rates, up trending prime rates and in a constant rate.....	55
Figure 4.4.2.a: Prices of a risky asset of ABSA Group Limited.....	56
Figure 4.4.2.b: The total returns of a risky asset of ABSA Group Limited and a moving average of 5 periods.....	57
Figure 4.4.3.a: Prices of a risky of Anglo American platinum Limited.....	58
Figure 4.4.3.b: The total returns of daily prices of a risky asset of Anglo American Platinum Limited and a moving average of 5 periods.....	59
Figure 4.4.4.a: Prices of a risky asset of Sasol Limited.....	60
Figure 4.4.4.b: The total returns of daily prices of a risky asset of Sasol Limited and moving average of 5 periods.....	61
Figure 4.4.5.a: Prices of a risky asset of Standard Bank Limited.....	62
Figure 4.4.5.b: The total returns of daily prices of a risky asset of Standard Bank Limited and a moving average of 5 period.....	63
Figure 4.4.6.a: Prices of a risky asset of AFGRI Limited.....	64
Figure 4.4.6.b: The total returns of daily prices of a risky asset of AFGRI Limited and a moving average of 5 periods.....	65
Figure 4.4.7.a: Prices of a risky asset of Anglo Gold Ashanti Limited.....	66
Figure 4.4.7.b: The total returns of daily prices of a risky asset of Anglo Gold Ashanti Limited and a moving average of 5 periods.....	67
Figure 5.2.1: The performance of portfolios of a risky asset of ABSA Group Limited.....	83
Figure 5.2.2: The performance of portfolios of a risky asset of AFGRI Limited.....	86
Figure 5.2.3: The performance of portfolios of a risky asset of Anglo Gold Ashanti Limited.....	90
Figure 5.2.4: The performance of portfolios of a risky asset of Anglo American Platinum.....	92
Figure 5.2.5: The performance of portfolios of a risky asset of Sasol Limited.....	95
Figure 5.2.6: The performance of portfolios of a risky asset of Standard Bank.....	98
Figure 6.2.1: The excess ratios of a fixed-mix portfolio strategy of a risky asset of ABSA Group Limited.....	117
Figure 6.2.2: The excess ratios of a fixed-mix portfolio strategy of a risky asset of AFGRI Limited.....	118
Figure 6.2.3: The excess ratios of a fixed-mix portfolio strategy of a risky asset of Anglo Gold Ashanti Limited.....	119
Figure 6.2.4: The excess ratios of a fixed-mix portfolio strategy of a risky asset of Anglo American Platinum.....	120
Figure 6.2.5: The excess ratios of a fixed-mix portfolio strategy of a risky asset of Sasol Limited.....	120
Figure 6.2.6: The excess ratios of a fixed-mix portfolio strategy of a risky asset of Standard Bank Limited.....	121
Figure 6.4.1.a: The spread of trades of a fixed-rate asset in a fixed-mix portfolio of a risky asset of ABSA Group Limited.....	125
Figure 6.4.1.b: The spread of trades of a risky asset in a fixed-mix portfolio of a	

risky asset of ABSA Group Limited.....	126
Figure 6.4.2.a: The spread of trades of a fixed-rate asset in a fixed-mix portfolio of a risky asset of AFGRI Limited.....	127
Figure 6.4.2.b: The spread of trades of a risky asset in a fixed-mix portfolio of a risky asset of AFGRI Limited.....	128
Figure 6.4.3.a: The spread of trades of a fixed-rate asset in a fixed-mix portfolio of a risky asset of Anglo Gold Ashanti Limited.....	129
Figure 6.4.3.b: The spread of trades of a risky asset in a fixed-mix portfolio of a risky asset of Anglo Gold Ashanti Limited.....	130
Figure 6.4.4.a: The spread of trades of a fixed-rate asset in a fixed-mix portfolio of a risky asset of Anglo American Platinum Limited.....	132
Figure 6.4.2.b: The spread of trades of a risky asset in a fixed-mix portfolio of a risky asset of Anglo American Platinum Limited.....	133
Figure 6.4.5.a: The spread of trades of a fixed-rate asset in a fixed-mix portfolio of a risky asset of Sasol Limited.....	134
Figure 6.4.5.b: The spread of trades of a risky asset in a fixed-mix portfolio of a risky asset of Sasol Limited.....	135
Figure 6.4.6.a: The spread of trades of a fixed-rate asset in a fixed-mix portfolio of a risky asset of Standard Bank Limited.....	136
Figure 6.4.6.b: The spread of trades of a risky asset in a fixed-mix portfolio of a risky asset of Standard Bank Limited.....	137
Figure 6.5.1: The excess value of risk of portfolio strategies in a portfolio of a risky asset of ABSA Group Limited.....	140
Figure 6.5.2: The excess value of risk of portfolio strategies of a portfolio of a risky asset of AFGRI Limited.....	141
Figure 6.5.3: The excess value of risk of portfolio strategies of a portfolio of a risky asset of Anglo Gold Ashanti Limited.....	143
Figure 6.5.4: The excess value of risk of portfolio strategies of a portfolio of a risky asset of Anglo American Platinum Limited.....	145
Figure 6.5.5: The excess value of risk of portfolio strategies of a portfolio of a risky asset of Sasol Limited.....	146
Figure 6.5.6: The excess value of risk of portfolio strategies of a portfolio of a risky asset of Standard Bank Limited.....	147
Figure 6.6.1: The payoff timeline of a lookback straddle option at maturity.....	149
Figure 6.7.1: Performance of a proxy strategy of a risky asset of ABSA Group Limited.....	151
Figure 6.7.2: Performance of a proxy strategy of a risky asset of AFGRI Limited.....	155
Figure 6.7.3: Performance of a proxy strategy of a risky asset of Anglo Gold Ashanti Limited.....	157
Figure 6.7.4: Performance of a proxy strategy of a risky asset of Anglo American Platinum Limited.....	159
Figure 6.7.5: Performance of a proxy strategy of a risky asset of Sasol Limited..	160
Figure 6.7.6: Performance of a proxy strategy of a risky asset of Standard Bank..	162

List of Charts

Chart A.2.1.1: The ARIMA models of differenced relative price ratio.....	180
Chart A.2.1.1: Growth rates of a proxy fixed-mix portfolio of a risky asset of ABSA Group Limited.....	181
Chart A.2.1.2: Growth rates of a proxy fixed-mix portfolio of a risky asset of AFGRI Limited.....	181
Chart A.2.1.3: Growth rates of a proxy fixed-mix portfolio of a risky asset of Sasol Limited.....	182
Chart A.2.1.4: Growth rates of a proxy fixed-mix portfolio of a risky asset of Anglo Gold Ashanti Limited.....	182
Chart A.2.1.5: Growth rates of a proxy fixed-mix portfolio of a risky asset of Anglo American Platinum.....	183
Chart A.2.1.6: Growth rates of a proxy fixed-mix portfolio of a risky asset of Standard Bank.....	183
Chart A.4.1.1: Relative price ratios of a portfolio of a risky asset of ABSA Group Limited.....	188
Chart A.4.1.2: Relative price ratios of a portfolio of a risky asset of AFGRI Limited.....	188
Chart A.4.1.3: Relative price ratios of a portfolio of a risky asset of Anglo Ashanti Limited.....	189
Chart A.4.1.4: Relative price ratios of a portfolio of a risky asset of Anglo American Platinum Limited.....	189
Chart A.4.1.5: Relative price ratios of a portfolio of a risky asset of Sasol Limited.....	190
Chart A.4.1.6: Relative price ratios of a portfolio of a risky asset of Standard Bank.....	190
Chart A.4.1.7: Log returns of daily prices of a risky asset of ABSA Group Limited.....	191
Chart A.4.1.8: Log returns of daily prices of a risky asset of AFGRI Limited.....	191
Chart A.4.1.9: Log returns daily prices of a risky asset of Anglo Gold Ashanti Limited.....	192
Chart A.4.1.10: Log returns daily prices of a risky asset of Anglo American Platinum.....	192
Chart A.4.1.11: Log returns of daily prices of a risky asset of Sasol Limited.....	193
Chart A.4.1.12: Log returns of daily prices of a risky asset of Standard Bank.....	193
Chart A.4.1.13: Growth rates of a risky asset of ABSA Group Limited.....	194
Chart A.4.1.14: Growth rates of a risky asset of AFGRI Limited.....	194
Chart A.4.1.15: Growth rates of a risky asset of Anglo Gold Ashanti Limited.....	195
Chart A.4.1.16: Growth rates of a risky asset of Anglo American Platinum.....	195
Chart A.4.1.17: Growth rates of a risky asset of Sasol Limited.....	196
Chart A.4.1.18: Growth rates of a risky asset of Standard Bank Limited.....	196
Chart A.5.1.1: Value process of a fixed-mix strategy portfolio of a risky asset of ABSA Group Limited.....	198
Chart A.5.1.2: Value process of a fixed-mix strategy and a Buy and Hold strategy in	

a portfolio of a risky asset of AFGRI Limited.....	199
Chart A.5.1.3: Value process of a fixed-mix strategy portfolio of a risky asset of Anglo Gold Ashanti Limited.....	200
Chart A.5.1.4: Value process of a fixed-mix strategy portfolio of a risky asset of Anglo American Platinum.....	201
Chart A.5.1.5: Value process of a fixed-mix strategy portfolio of a risky asset of Sasol Limited.....	202
Chart A.5.1.6: Value process of a fixed-mix strategy portfolio of a risky asset of Standard Bank Limited.....	203
Chart A.6.1.1: Frequency distribution of discrete total returns of a fixed-mix portfolio strategy of a risky asset of ABSA Group Limited.....	216
Chart A.6.1.2: Frequency distribution of discrete total returns of a fixed-mix portfolio strategy of a risky asset of AFGRI Limited.....	216
Chart A.6.1.3: Frequency distribution of discrete total returns of a fixed-mix portfolio strategy of a risky asset of Anglo Gold Ashanti Limited.....	217
Chart A.6.1.4: Frequency distribution of discrete total returns of a fixed-mix portfolio strategy of a risky asset of Anglo American Platinum.....	217
Chart A.6.1.5: Frequency distribution of discrete total returns of a fixed-mix portfolio strategy of a risky asset of Sasol Limited.....	218
Chart A.6.1.6: Frequency distribution of discrete total returns of a fixed-mix portfolio strategy of a risky asset of Standard Bank Limited.....	218

List of Tables

Table 3.4.a: Analytic tabular of a strategy with 50 percent allocations in a two-asset portfolio in a down-reverting full cycle market scenario.....	37
Table 3.4.b: Analytic tabular of a strategy with 50 percent allocations in a two-asset portfolio in an up-reverting full cycle market scenario.....	37
Table A.5.2.13: Difference performance of a fixed-mix portfolio strategy and a buy and hold portfolio strategy of a risky asset of ABSA Group Limited and a fixed-rate asset in up trending and down trending prime rates.....	103
Table A.5.2.14: Difference performance of a fixed-mix portfolio strategy and a buy and hold portfolio strategy of a risky asset of AFGRI Limited and a fixed-rate asset in up trending and down trending prime rates.....	104
Table A.5.2.15: Difference performance of a fixed-mix portfolio strategy and a buy and hold portfolio strategy of a risky asset of Anglo Gold Ashanti Limited and a fixed-rate asset in up trending and down trending prime rates.....	105
Table A.5.2.16: Difference performance of a fixed-mix portfolio strategy and a buy and hold portfolio strategy of a risky asset of Anglo American Platinum Limited and a fixed rate asset in up trending and down trending prime rates.....	107
Table A.5.2.17: Difference performance of a fixed-mix portfolio strategy and a buy and hold portfolio strategy of a risky asset of Sasol Limited and a fixed-rate asset in up trending and down trending prime rates.....	108
Table A.5.2.18: Difference performance of a fixed-mix portfolio strategy and a buy and hold portfolio strategy of a risky asset of Standard Bank Limited and a fixed-rate asset in up trending and down trending prime rates.....	109

Table A.2.2.1: Performance of portfolios of a risky asset of ABSA Group Limited.....	184
Table A.2.2.2: Performance of portfolios of a risky asset of AFGRI Limited.....	184
Table A.2.2.3: Performance of portfolios of a risky asset of Anglo Gold Ashanti Limited.....	184
Table A.2.2.4: Performance of portfolios of a risky asset of Anglo American Platinum.....	185
Table A.2.2.5: Performance of portfolios of a risky asset of Sasol Limited.....	185
Table A.2.2.6: Performance of portfolios of a risky asset of Standard Bank.....	185
Table A.4.2.1: Descriptive statistics of daily log prices of risky assets.....	197
Table A.4.2.2: Descriptive statistics of daily total returns of a fixed-rate asset and of risky assets.....	197
Table A.5.2.1: Descriptive statistics of discrete total returns of portfolio strategies of a risky asset of ABSA Group Limited in up trending prime rates.....	204
Table A.5.2.2: Descriptive statistics of discrete total returns of portfolio strategies of a risky asset of ABSA Group Limited in down trending prime rates.....	205
Table A.5.2.3: Descriptive statistics of discrete total returns of portfolio strategies of a risky asset of AFGRI Limited in up trending prime rates.....	206
Table A.5.2.4: Descriptive statistics of discrete total returns of portfolio strategies of a risky asset of AFGRI Limited in down trending prime rates.....	207
Table A.5.2.5: Descriptive statistics of discrete total returns of portfolio strategies of a risky asset of Anglo Gold Ashanti Limited in up trending prime rates.....	208
Table A.5.2.6: Descriptive statistics of discrete total returns of portfolio strategies of a risky asset of Anglo Gold Ashanti Limited in down trending prime rates.....	209
Table A.5.2.7: Descriptive statistics of discrete total returns of portfolio strategies of a risky asset of Anglo American Platinum Limited in up trending prime rates.....	210
Table A.5.2.8: Descriptive statistics of discrete total returns of portfolio strategies of a risky asset of Anglo American Platinum Limited in down trending prime rates..	211
Table A.5.2.9: Descriptive statistics of discrete total returns of portfolio strategies of a risky asset of Sasol Limited in up trending prime rates.....	212
Table A.5.2.10: Descriptive statistics of discrete total returns of portfolio strategies of a risky asset of Sasol Limited in down trending prime rates.....	213
Table A.5.2.11: Descriptive statistics of discrete total returns of portfolio strategies of a risky asset of Standard Bank in up trending prime rates.....	214
Table A.5.2.12: Descriptive statistics of discrete total returns of portfolio strategies of a risky asset of Standard Bank in down trending prime rates.....	215

List of Appendices

Appendix 1: The Models.....	175
1. The Market model	175
2. The Trading model.....	176
3. The Merton model of asset prices.....	178
4. The Capital asset pricing model.....	178
5. The Markowitz's mean-variance model.....	179
6. The Sharpe ratio.....	179
7. The ARIMA model.....	180
Appendix 2: Charts and Tables of performances.....	180
Appendix 3: Test program.....	186
Appendix 4: Charts and Tables of preliminaries.....	188
Appendix 5: Charts and Tables of value processes.....	198
Appendix 6: Frequency distribution of log total returns.....	216
Appendix 7: Price formulas of lookback options.....	219

Chapter 1

Introduction

1.1 Background

The uncertainty of a return in an investment exposes a premium of the risk undertaken in active portfolio management. The premium of the risk of the investment is optimized by the choices of the asset manager as described by a utility of the investment (Fabozzi, Forcardi and Kolm, 2006). The utility of the investment is based on the central moments and higher moments of returns on the investment. The central moments of the distribution of returns which are the expectation and the variance for the uncertainty of the investment and the higher moments the skewness and kurtosis for the tail-risk are used in the computations to analyse the utility of the investment. The moments set trade-offs for active portfolio management for backtesting the tactical strategies by examining the extent of the performance of portfolios. The performance is examined to optimize the performance of portfolios under the risk-return trades-offs on the historical asset prices for reliable outcomes on real run applications. The second moment in terms of the volatility of asset prices which is the standard deviation of returns is radical for ensuring that the trade of asset prices in not null on fixed-mix portfolios as considered by Dempster, Evstigneev and Schenk-Hoppé (2007) analytically.

The volatility in active portfolio management is considered profoundly and contrary to the modern portfolio theory wisdom where the risk is considered diversified for its reduction. In these considerations, the risk settings from a stochastic contributing factor is viewed in terms of completing the assumptions of the non-degeneracy condition of prices ratios and as well as of the stationarity condition in active tactical portfolio management. The financial risk in attaining the proxy of the systematic fixed-mix portfolio trading model is raised and considered.

Dempster, Evstigneev and Schenk-Hoppé (2003, 2007, 2008, 2010) and Evstigneev and Schenk-Hoppé (2002) provide analytical results and computational examples to prove the long-term effect of volatility in active portfolio management. The risk of asset prices as a characteristic of uncertainty of the expected return on the investment is considered at minimum in the selection of efficient portfolios (Markowitz, 1952) to ensure less variability in the expected rate of financial growth. Dempster *et al.* (2007) regard the volatility as a reassuring factor in the financial growth. Furthermore, according to Dempster *et al.* (2003) the stationary markets where asset prices fluctuate under a distribution of a mean and variance which do not depend on time in fixed-mix portfolios yield exponential growth. To encounter the financial risk of a non-stationary condition over the market risk in fixed-mix portfolios a risk-minimization strategy using quadratic hedging motivates for the use of the approach which minimizes the risk of gains in discrete periods. Quadratics hedging is a hedging technique in an incomplete market, a market model where there are sources of uncertainty that cannot be tracked and the risk minimization strategy is called a locally risk-minimization delta hedge proposed by Schweizer (1991) (as cited by Poulsen *et al.*, 2009). The minimization strategy minimizes the variance of the cost process of non-self-financing hedges. The cost process is defined as the difference of the accumulated gains or losses to the value of the current portfolio. The envisioned financial risk is reduced for effective rebalancing gains by trading the portfolio of a risk-free asset and a risky asset to ensure optimal gains in volatile trending market conditions.

Dynamic asset allocation strategies, called fixed-mix portfolio strategies, are analyzed comparatively to a benchmark strategy called a buy and hold strategy in a South African financial market. The strategies provide a self-financing investment system which executes tactical trading decisions of assets without additional amount and consumption in a portfolio. The stationarity condition of asset prices according to Dempster *et al.* (2007) generally does not hold in financial markets and the evidence of stationarity is found in the currency market.

The practice of actively managing risk is found to be effective in the consideration of co-movement of market risks by James, Kasikov and Edwards (2012). The view of

market risk in terms of anticipated returns relatively to the connectedness of market asset prices is determined to provide excess diversification returns under fixed-mix portfolios. The connectedness of markets is referred to by James (2012) as the contagion effect which is a degree of correlation of markets greater than expected. The fixed-mix strategies are used for a dynamic portfolio management model of defined contribution schemes for liabilities of guaranteed returns according to Dempster, Germano, Madova, Rietbergen, Sandrini, Scrowston and Zhang (2007). The results of the computations show that fixed-mix strategies as benchmarks of dynamic stochastic programming of optimal solutions have low computational costs and less complexity. This shows that the fixed-mix decision rules are effective for the active management of portfolios. The consideration of managing risk-adjusted returns where the excess returns are due to rebalancing gains sets trade-offs of the active strategy in the market risk.

1.2 Market risk

In this subsection the market risk is discussed by focusing on the modelling of a stationarity form that is required to hold. The international perspective on the empirical performance of fixed-mix portfolios is also discussed. The condition of a stationarity form is derived by de-trending. De-trending assert a financial condition of a stationary form which yields fast exponential growth with fixed-mix portfolios. A resulting market condition by de-trending is a trend reverting condition such that with the modelling of the stated market risk on the trending market conditions the fixed-mix portfolio rebalancing is effective and efficient in comparison with the fixed-mix portfolio strategies which do not involve the proxy settings on a trading asset.

The mean-variance analysis of Markowitz provides a framework to analyze the market price of risk in rendering the necessary condition of stationarity. In the trending markets the benchmark strategy is considered as a single period strategy that yields gains over multi period investment models. The mean-variance analysis of Markowitz (1956) analyzes the expected returns of portfolios where portfolios of increased volatility with expected returns below the benchmark return, which are considered as

inefficient portfolios, are considered to improve the performance of fixed-mix portfolios. The improved performance by the utility of the tail risk has a potential of generating positive portfolio growth on the excess returns from the skeweness. The tail risk of returns is assessed by the skeweness which is the higher moment of the empirical distribution of returns of the investment asset, Fabozzi *et al.* (2006). In the modelling of the market risk for advanced consideration of portfolios of higher volatility which are considered inefficient, a probable prospect of the utility of risk is adapted. Dempster *et al.* (2007) analytically showed that volatile inefficient portfolios managed with fixed-mix strategies have a potential to yield excess growth of the portfolio wealth. The Markowitz perspective regards the volatility of asset prices as a draw-down for financial growth. In the research of dynamic asset allocation by Dempster, Evstigneev and Schenk-Hoppé (2003, 2007, 2008, 2010) the market risk in fixed-mix portfolios is analytically determined to be effective in the financial growth of portfolio wealth.

Mulvey *et al.* (2007) found that the equal weighted S&P (Standard and Poor) index perform higher with high volatility than the capital weighted index. Mulvey *et al.* (2007) show that overlay fixed-mix strategies which assign positions or securities in the futures markets are more beneficial at a lower risk and less costly than leveraged strategies in improving risk-adjusted returns. The empirical performance of fixed-mix portfolio rules show lesser excess returns on the international benchmark levels of growth of the returns.

The overlay strategies show effectiveness in handling risk-adjusted multi-period transactions with appropriate increased allocations which time the levels of a portfolio over allocations. The empirical results show the performance of fixed-mix portfolios on the international perspective to be above the performance of riskless assets and assert a performance improvement in further considering highly volatile market indices. Leveraging strategies and multi-period transactions are acquired to structure risk-increased transactions and the stationarity form of a gains process.

1.3 Portfolio insurance

The concept of portfolio insurance, which originates in Rubinstein and Leland (1981), and is considered by Yao (2012) as a technology of hedging the market risks is used to enhance the performance of an investment portfolio in a market of a fixed-rate asset and a risky asset. A robust approach is suggested in this research to implement fixed-mix portfolios to counter market gains, which are strategic returns on the market, in generic markets. The proposed robust approach of implementing fixed-mix strategy deals with counter-effective implementation tactics. The implementation of the fixed-mix strategy attains the condition of stationarity in a form of non-zero gains process composed of a stationary returns process in the considered market. A strategic transaction evaluates the market price of risk by effecting efficiency on the mean-variance framework. The expected financial growth is examined by analyzing the feasibility of replicating stationary tradable securities using portfolio insurance strategies. To attain a financial proxy a dynamic option is considered to insure the market risk by using risk-free assets and risky assets. The suggested approach utilizes straddle options which ensure effective gains in discrete trading periods in the events of high volatility. The straddle option retains non-negative returns for changes in volatile trending markets. This supports inefficient portfolios in single trading periods to yield returns exceeding a fixed-mix portfolio of risk free and risky assets. A trend following strategy of a lookback straddle option introduced in Fung and Hsieh (1997) (as cited by Darius, Ilhna, Mulvey, Simsek and Sircar, 2002) is used as a proxy for hedge funds to capture long volatility positions which improves the gains in multi period investments.

1.4 The financial proxy

The proxy hypothesis sets a condition required by a fixed-mix strategy to ensure a stationary financial condition in a trading portfolio. The risk-minimization strategy which reduces the variance of a cost process of non-self-financing hedges under a stochastic volatility model in Poulsen *et al.* (2009) is considered as a motivating approach to encounter the financial risk of rebalancing for self-financing portfolios in volatile trending markets with frequent reversals. Poulsen *et al.* (2009) derive an ex-

explicit formula for locally risk-minimization delta-hedges where the hedge is decomposed as the sum of the normal-delta hedge and the volatility/correlation-risk term. In this regard over the co-movements and the volatility of asset prices returns the structured returns are retained efficiently from the associated costs and gains of rebalancing which increases the hedging term of a risky asset for effective gains. The empirical analysis of fixed-mix portfolios in the currency market with de-trended exchange rates in Chule (2007) had shown a de-trending action to assert the necessary condition of exceptional exponential growth in trending markets. The financial proxy is a defined financial transaction which is expected to attain the condition of stationarity and to insure the expected financial growth. The research done here carries on the highlighted need of financial engineering suggested in Dempster *et al.* (2010) by analysing the feasibility and the analytical approach on how to realise or attain the expected financial growth in generic market conditions. The option-based portfolio insurance strategy is used to determine the financial proxy which encounters the financial risk of fixed-mix portfolios by de-trending stochastic asset prices over discrete rebalancing periods.

1.5 Performed computations

The implementation of the fixed-mix strategy requires maintaining the condition of stationarity in generic market conditions. The generic market conditions are market conditions with variations of prices under the equilibrium price level, over the equilibrium price and about the equilibrium price level with reference to the prevailing trend. To yield the expected result particularly in the trending market a quantitative analysis is performed to contribute to a financial engineered prospect highlighted in Dempster *et al.* (2010). The performed computations of a fixed-mix portfolio of a fixed-rate asset and a risky asset in the South African financial market, the Johannesburg Stock Exchange, show the empirical result of lesser excess returns of constant rebalanced portfolios over buy and hold portfolios (see subsection 5.2). Indicating a buy and hold strategy as a counter benchmarking strategy in trending market conditions. The result shows the significance of a strategy to fixed-mix portfolios in markets which the stationarity conditions are not found to hold.

To encounter this holdback condition the enhanced risk-adjusted returns in active portfolio management are analysed by considering market risk measures of asset prices. Fixed-mix strategies serve as effective hedging strategies which correct the investment policy of a strategy. The major problem or challenge in this research is to appraise the unrealised potential of stochastic asset prices under fixed-mix strategies. The financial engineering prospect is the market risk modelling which upholds the potential of the financial market to attain increased risk-adjusted returns on tactical self-styled market conditions with active portfolio management. The realised performance on fixed-mix portfolios with a proxy strategy is improved to a linearly strategic allocation which attains optimality with the leverage effect to outperform the performances of normal application of fixed-mix strategies and of buy and hold portfolios.

In the performed computations (see Appendix 2 Tables 2.2) the performance of fixed-mix strategy, as measured by a Sharpe ratio, underperforms the benchmark return of a fixed-rate asset for all allocations ranging from -200 percent to 200 percent in a fixed-rate asset in trending markets. The buy and hold portfolio strategy is found to outperform the benchmark return in trending markets. The market price of risk of the fixed-mix strategy is defined as the ratio of excess performance of a strategy to a performance of a fixed-rate asset and the riskiness of the strategy per definition of the market price of risk by Yao (2012) (see model 6 the Sharpe ratio at Appendix 1) . The market risk compensated by the buy and hold strategy is identified by high Sharpe ratios in trending market conditions. The fixed-mix strategy is determined to be a less risky strategy in trending markets and is known to outperform trend-following strategies in a market with stationary conditions Dempster *et al.* (2010). The fixed-mix strategy is found to have less downside risk and high market price of risk. The outperforming fixed-mix strategies in a market with frequent reversals are completely-mixed portfolios which are characterized by positive weights. The realized annual returns of fixed-mix portfolio strategies for optimal allocations are found to range from 7.19 percent to 11.47 percent in the performed computations (see Appendix 2.2 tables).

The results of fixed-mix portfolio strategies under the proxy hypothesis which is validated by a lookback straddle option (see growth rates in Appendix Charts 2.1) are found to grow at a consistent rate higher than the rate of fixed-mix portfolios and exceeding the performance of buy and hold strategies in the considered investment period. The performance of fixed-mix portfolios under a proxy is found to overcome the limitations of ill-timed transactions where the innovations render a stationary financial condition of positive expected gains per rebalancing. The proxy fixed-mix portfolio strategies with less restriction which uses excessive leveraging of a risky position in a portfolio are found to improve the performance of fixed-mix portfolio strategies in generic market conditions. Under a proxy hypothesis the fixed-mix portfolios show a growing potential. The performed computations of a proxy strategy are found optimal for trend-following strategies and with less performance concavity comparable to mean-reverting strategies.

1.6 Asset Liability portfolios

The assessments of marking-to-market transactions for asset/liability portfolios, which provide the performance of transactions at high frequency to effectively manage the portfolio strategic transactions, is determined to optimize the value of the market under fixed-mix active portfolios over the feasible ensured market risk. The determination of the optimized value of the market results from the active management of multi-period transactions which suffices to utilize the benchmark asset for effective transactions gains on the reversion of a stochastic condition has to attain a state price process as devised by Yao (2012) in the consideration of a market price of risk.

The performance of fixed-mix portfolios is considered to be maximized in trending markets for restricted transactions, which are allocation constraint, as devised by the general portfolio selection model (GPSM) in Markowitz (2013). The model involves leveraging in a portfolio, where the utility of a benchmark asset is considered sufficient for an analytic transaction to attain efficiency. In this regard, the Stochastic Margin Call Model (SMCM) of Markowitz which is a procedure that considers the

probability of the margin call, the variance and the expected return of the ruin in a single period is assumed to give analytical basis of the financial proxy. The analytical basis particularly considers the minimizing of the variance of the rewards of the transaction and the expectation of ruining the realized rebalancing gains. The GPSM model by (Markowitz, 2013) extends to a leveraged aversion problem which considers the risk component of leveraging. The proposed solution by Markowitz for the GPSM model of a leveraged aversion problem is a Stochastic Margin Call Model which determines the possibility of mark-to-market for efficient portfolios using procedures of analytic, numeric and Monte Carlo Methods. The analysis of the efficiency of portfolio with margin calls for marking- to -market transactions is considered to assert the possibilities of retaining optimal gains with an analytical computational effort.

The analytical solution by Kung, Wong and Wu (2013) of a stochastic control of an empirical portfolio strategy shows that fixed-mix portfolios provide solutions to a variety of problems in asset management under the Markowitz's framework of mean and variance. Kung *et al.* (2013) use a stochastic control to determine optimal weights of three assets in a portfolio with a leveraging strategy which shorts a risk-free asset of a short rate quoted from Treasury bill rate, a long position in a (Standard and Poor) S & P 500 index and a long position in a US (United States) Government bond which show that more risk-averse investors allocate more percentages in a short position. This indicates optimal control for a leveraged portfolio strategy with high expectations of equity performance and bond reliability.

The challenges in implementing the expected results are to derive the property of stationary asset prices which is attained by a de-trending action in the market considered. To test the condition of stationarity a unit root test of Dickey and Fuller from Brooks (2005) is carried out to examine the difference-stationarity condition. The trend-stationary condition is examined by performing a convergence analysis of a moving average of total returns of risky assets as identified by Dempster *et al.* (2007) which is found almost within the interval with bounds of 5 percent total return for each discrete period.

To enhance the performance of fixed-mix portfolios the basis of financial engineering is set using the concept of portfolio insurance to hedge the expected performance. The significance of the suggested approach which enhances the performance of fixed-mix strategies is subjected to a model capable of raising returns from a portfolio with most risk. Such appraisal indicates the significance of studying the performance of these strategies. This signifies that the Markowitz procedure of selecting efficient portfolios is relaxed when fixed-mix strategies are applied given that such procedures select portfolios of least volatility.

1.7 Research Problem Statement

Computations are performed in determining a financial proxy which optimizes detrended stochastic asset prices in order to attain the condition of stationarity in a fixed-mix portfolio. The condition of stationarity is a typical setting for fixed-mix portfolios which is considered not to hold for financial asset prices in general. However, the evidence of stationarity is witnessed on the empirical financial market returns, in the empirical computations in the currency market by Chule (2007) and on the currency market as analysed in Dempster *et al.* (2003) and (2007). The empirical analysis examines a strategic tactic which optimizes a performance of dynamic asset allocation portfolio strategies. The multi period investment model which is a portfolio strategy is analysed comparatively to a benchmark strategy, the buy and hold strategy. The buy and hold strategy is a single period trend-following strategy which effectively yields portfolio growth in trending markets. The trend stationary condition is determined to ensure the feasibility of the financial proxy.

To examine the problem of stationarity two types of stationarity are considered. These are the trend-stationary and the difference-stationary condition. In the trend-stationary condition according to Brooks (2005) stationarity is attained along the trend where the resulting stationary process has the trend excess drift which is considered zero over a long span of high frequency observations, and non-zero for short spans of frequent observations depicting variations of a trend. The contrarian strategies with leveraged allocations in a less risky asset are favourable for a fixed-mix

strategy to generate returns from selling appreciating assets of a large proportion in a portfolio. The trend-stationary condition in a risky asset increases the number of assets in a portfolio with leveraged funds where gains are realised on trades from frequent reversals about the trend of increasing prices. The difference-stationary condition is attained by lag differencing which is a short span frequent observation of prices where differencing de-trends the trending prices at high frequency.

The resulting stationary condition is required by a fixed-mix portfolio strategy with balanced allocations where the buying and selling trades are frequent about the average. According to Dempster *et al.* (2007) the volatility of asset prices in financial markets contributes positively in the financial growth. This identifies a constructive prospect in employing fixed-mix portfolios where the modelling of asset returns with active strategic transactions prompts a financial growth potential in markets with large frequent reversals. A major problem or challenge in this research is to assess the unrealized potential of stochastic asset prices under fixed-mix strategies. The analysis focuses on establishing the impact of volatility and to design a trading strategy that efficiently utilizes the presence of volatility. The designed portfolio has to be conditional to a mean-reverting random process or trend stationary process which is a primary setting for fixed-mix strategies to yield fast exponential growth of returns.

1.8 Objectives

1.8.1 Main Objective

The main objective is to determine the financial proxy which optimises de-trended stochastic asset prices under fixed-mix portfolio strategies. The purpose of this research is to suggest an extended implementation tactic in the utilization of constant rebalanced self-financing portfolio investment strategies. In Dempster *et al.* (2007), constant rebalanced portfolios are noted to yield fast exponential growth in volatile stationary markets. The research aims to induce a financial condition of stationarity

in a fixed-mix portfolio by determining a financial proxy which sets the basis to optimize on the application of self-financing constant rebalanced portfolio strategies.

1.8.2 Supporting objectives

- The first supporting objective in determining a financial proxy which optimises the application of de-trended stationary tradable securities for fixed-mix portfolio strategies is to perform a quantitative analysis of arbitrary constant rebalanced self-financing portfolio investment strategies.
- The second objective is to assess the unrealised potential of stochastic asset prices under fixed-mix strategies in generic markets particularly in trending market conditions where de-trending is required to attain a stationary form of traded prices.
- The third objective is to quantify the stochastic asset prices into a portfolio to yield positive expected results.
- The fourth objective is to establish a financial engineered approach that sets a basis to yield fast exponential growth induced by volatility on the application of self-financing constant rebalanced portfolio strategies.
- The fifth objective is to model risk-adjusted returns in order to quantify the expected performance.
- The sixth objective is to construct time effective portfolios that are favourable to portfolio objectives and also linked to financial engineered instruments that render an appropriate financial transaction.

A reinforced construction over partial co-movements in the effect of resonance noted by Dempster *et al.* (2008) to explain the phenomena of financial growth in fixed-mix portfolios can provide valuable potential for prices of assets which are dominant on the stochastic contributing factor to adapt portfolios from an inefficient frontier tactically to active adapt frontier. The active adapt frontier are portfolios of increased risk with economic sufficient tactical measure to adapt the efficient frontier. The asset allocation challenges determine the prospects of asset management for appropriate utilization of the market risk where the financial risk is the only major drawdown. I

quote “The portfolio manager of today needs to achieve positive returns with the least achievable risk - how on earth is he or she meant to do this? It is not easy, but forewarned is forearmed” (James, Kasikov and Edwards, 2012).

1.9 Research design

1.9.1 Research type

The research contributes to the existing knowledge by performing a quantitative analysis of fixed-mix strategies and trend-following strategies. The analysis examines the performance of two asset portfolios with leveraged positions and unleveraged positions. The two asset portfolios are composed of a fixed-rate asset and a risky asset with market conditions comprised of trending conditions with frequent reversals and market conditions reverting to a stable equilibrium condition. To determine the necessary condition the two asset types are used in the analysis to study the performance and to suggest a portfolio strategy which renders the required condition. The analysis is led by the two asset portfolio to effectively consider the benchmarking effect and the stochastic structure without the computational complexity of tradable multi assets where the contributing stochastic factor is of one risky security.

1.9.2 Data

The data of daily prices of risky assets and annual prime rates are retrieved from the MacGregor BFA web site (<http://www.macgregorbfa.com>). Daily prices of major companies in South Africa listed in the FTSE/JSE Top 40 index are used in the analysis. The index is a measure of the performance of top 40 companies listed in the Johannesburg Stock Exchange issued by a Financial Times Securities Exchange in the United Kingdom (Pike and Neale, 2006). The annual prime rates are used to quote the short rate of a fixed-rate asset. The quantitative analysis uses high frequency data to analyse the performance of trading strategies and to examine the robustness of the enhanced approach. To draw a conclusive evaluation a convenient sample of six risky assets is used to perform computations to assess the robustness of the two-asset approach in generic financial market conditions particularly in trending market con-

ditions and frequent reversal market conditions. The two-asset portfolios are composed of a fixed-rate asset with a short rate quoted as the effective prime rate and a risky-asset of a company in the FTSE/JSE Top 40 index.

1.9.3 Sample selection

The computations of trend-following strategies and contrarian strategies use asset prices of six major companies. Risky asset prices of the selected companies listed in the JSE with asset stock codes are ABSA Group Limited (JSE:ASA) and Standard Bank Limited (JSE:SBK) in the financial sector, Sasol limited (JSE:SOL) in the oil and energy sector, Anglo American Platinum Limited (JSE:AMS) and Anglo Gold Ashanti Limited (JSE:ANG) in the Gold mining sector and as well as AFGRI Limited (JSE:AFR) in the Food and fishery sector not in the Top 40. The time series of the companies are selected conveniently with trending market conditions and reversals market conditions. The selected asset prices are used to construct two-asset portfolios where their performances are analysed quantitatively. The prime rates are used for each risky asset to construct a portfolio where the performance of a portfolio is considered in two conditions of the prime rates. These are, the normal drawn condition which is a down-trending prime rates and secondly the reversed condition which is the up-trending prime rates. The third condition of the prime rates is a constant rate which determines the basic level of a fixed-rate asset to benchmark the performance of portfolios.

1.9.4 Time series span

The sample size of the considered time series is 3736 of daily observations of prices of risky assets over a span of 15 years. The span provides the long term quantitative performance.

1.10 Structure of the thesis

The first chapter introduces the integral matters of the focused problem and as well as some perspectives on the international empirical performance of fixed-mix rules. The theoretical analysis of fixed-mix portfolios appraises active portfolio management strategies to enhance the performance of an investment portfolio in volatile stationary markets. The global perspective of empirical performance of fixed-mix portfolios and the index funds performance in the United State market is noted for benchmark performance analysis. The market risk and portfolio insurance of fixed-mix portfolios are discussed and are understood to be the essential driver and tool, respectively, for the theoretically anticipated financial growth. The main research question is elaborated which is to determine an active portfolio management proxy that renders to the required financial condition by considering stationary tradable securities.

The second chapter reviews the literature and discusses the theoretical findings of active portfolio strategies. The modern portfolio theory of Markowitz is analysed to assess the risk-adjusted returns and suggests an approach which retains excess returns by inducing a required condition of fixed-mix portfolios. The mean-variance model of efficient portfolios is utilised to evaluate the excess growth enhanced by active implementation of fixed-mix strategies in volatile markets. The capital asset pricing model sets feasible measures for a financial engineered prospect to ensure the expected financial growth of fixed-mix portfolio strategies.

In chapter three the proxy hypothesis is postulated and the suggestions of a tactical approach in implementing the fixed-mix strategy are discussed. The riskless term-asset of a fixed-rate is considered in the two market conditions, the up-trending prime rates and the down-trending prime rates. The six risky assets of ABSA Group Limited, Standard Bank Limited, Sasol Limited, Anglo American Platinum Limited, Anglo Gold Ashanti Limited and AFGRI Limited, are used to perform the computations of the performances of two asset-type portfolios with a buy and hold strategy and a fixed-mix strategy. The use of a financial derivative instrument to attain the

required condition of stationarity which enhances the portfolio performance is suggested.

The fourth chapter shows the data analysis of a fixed-rate asset and risky assets for the selected companies. The Integrated Autoregressive moving average models of prices and of total returns of risky assets and of relative price ratios are determined.

The fifth chapter performs the computations of portfolio strategies that is, the buy and hold strategy and the fixed-mix portfolio strategy. The difference of performance of portfolio strategies relative to market conditions of a fixed-rate asset is computed.

The sixth chapter analyses and interprets the performed computations. The computations of a fixed-mix portfolio under a proxy hypothesis are performed. The recommendations and the limitations of the research are highlighted.

1.11 Conclusion

The quantitative analysis examines the performance of an active portfolio management strategy which is the fixed-mix strategy in comparison to the benchmark strategy the buy and hold strategy. The global perspective of empirical performance of fixed-mix portfolios indicates a superiority of equal weighted portfolios for long term performance. The market risk associated with volatility of asset prices which is a drawdown of the investment prospects is determined to yield positive returns under active portfolio management with option-based portfolio insurance strategies.

In the analysis of quantitative performance of fixed-mix strategies the effective implementation of constant rebalanced portfolios is suggested to induce a robust approach which yields the expected fast exponential growth. The robust approach induces growth from risk-adjusted returns by considering excess returns of a passive strategy for leveraged positions. The derivatives are used to hedge the risk of rebalancing costs and to retain a trend-following return in a non-stationary market condition. The concepts of portfolio insurance suggested by Rubinstein and Leland

(1981) to hedge the market risk of selling assets in portfolios is considered to derive the trend-stationary condition in trending market conditions and the difference-stationary condition in market conditions which are not trending. Derivative instruments quantify the required condition by establishing a financial engineered approach that sets basis to optimize on the application of self-financing constant rebalanced portfolio strategies. The multi period active portfolio management with fixed-mix portfolio strategies in generic market conditions renders the required condition of stationarity by constructing an appropriate financial engineered transaction.

The determined financial proxy over arbitrary performances of fixed-mix portfolio strategies and buy and hold strategies in generic markets is found optimal in trend-following strategies. The performance ratios of the proxy strategy has less degree of concavity over the buy and hold portfolios where the performance is dominated by strategies with leveraged positions of risky assets. In the considered market conditions which are up-trending, down-trending and stationary the actively managed portfolios yield excess returns with fixed-mix strategies benchmarked over a passive strategy the buy and hold strategy under the proxy hypothesis. The trend-following strategy the buy and hold strategy yield excess growth for leveraging positions in trending market conditions over fixed-mix strategies. The next chapter reviews the literature of active investment portfolio strategies.

Chapter 2

Literature Review

2.1 Introduction

The quantitative analysis of portfolio strategies is examined by implementing an active portfolio strategy of fixed-mix portfolios. The analysis of active portfolio strategies relative to a buy and hold strategy suggests effective implementation tactics which attain the required market conditions of the considered portfolio strategies. The global perspective of empirical performance of fixed-mix portfolios shows the effectiveness of equal weighted portfolios which are prone to rebalancing gains in volatile market conditions. The market risk encountered by fixed-mix strategies is determined to have financial risk particularly in trending market conditions. The portfolio insurance strategy is employed to effectively encounter the problem of the market risk in trending conditions by a financial proxy that holds a stationary condition in fixed-mix portfolios.

The analysis examines the effective methodology which is a portfolio trading strategy that replicates tradable stationary processes which are required by fixed-mix portfolio strategies to yield the fast exponential growth of returns. The main query of determining the financial proxy is analysed by viewing the literature of active portfolio management and performing a computational analysis of fixed-mix portfolio strategies to examine a financial engineered procedure which hedges the expected fast exponential growth. The analysis is performed on the data which is selected conveniently for trending and reversal market conditions.

2.2 Trading strategies

Two types of trading mechanisms which are considered in the analysis are the buy and hold portfolio strategy and the constant rebalanced portfolio strategy. Perold and Sharpe (1995) describe the trend-following strategy called the buy and hold strategy as a static type trading mechanism, which performs well in conditions of trending

markets and is characterised by a linear performance curve. The active trading strategy called the fixed-mix strategy is a strategy that allocates fixed-weights of portfolio wealth in each asset in a portfolio which are continuously or discretely re-adjusted to actual weights over the progress of a volatile market by selling or buying a specific determined number assets. The fixed-mix strategy performs well in stationary market conditions with frequent reversal and is characterised by a concave performance curve. The performance of constant rebalanced portfolios in continuous time models is presented by Browne (1998) who showed that fixed-mix portfolios have optimality properties and are sufficient for liability objectives.

The major focus is to analyse the quantitative performance of the fixed-mix strategy and to design a suggestive approach which optimally implement dynamic constant rebalanced portfolio strategies. A trading strategy is a portfolio strategy which determines the timing of the buy and sell of the assets. The financial derivative instruments are contract transactions which are tailored in a financial market for certain specific demands of trading assets.

2.3 Active portfolio management

The practice of active portfolio management involves the main basic fundamental principles which guide the expected level of wealth by timing of appropriate buy and sell transactions dependent on a mean-variance analysis of Markowitz (1952). The mean-variance model assesses the returns and risks of asset prices in a single trading period which select portfolios of least volatility as efficient portfolios. Efficient portfolios are a robust selection of dynamic portfolios in a mean-variance model and are described by expected returns above the benchmark return proportional to the increasing variance of the returns. The mean-variance model sets a framework to assess the market price of risk by considering risk and returns of portfolio strategies involving leveraged positions under a General Portfolio Selection Model (GPSM) presented in Markowitz (2013). The GPSM of Markowitz is theoretically considered in the analysis of efficient strategic transactions by assessing an analytical procedure

in a Stochastic Margin Call Model (SMCM) to encounter sufficiently the optimal transit of the strategic gains in a transaction period. In the findings of active portfolio management in Dempster *et al.* (2007) the volatility is analysed and found to be effective in the financial growth for inefficient portfolios. The fixed-mix portfolio of highly volatile risky assets yields portfolio growth strictly above the growth of individual assets in a portfolio. These considerations provide the extent of evaluating the price of market risk on the basis of benchmark utility and efficient utility of the market risk by raising the financial risk of the active lucrative strategy.

The focus of this analysis is on determining effective strategic tools in the implementation of fixed-mix portfolio strategies. The risk-minimization strategy proposed by Schweizer (1991) (as cited by Poulsen *et al.*, 2009) which minimizes the variance of a cost process in a portfolio of a risk-free asset and a risky asset is assumed for the structural design of the proxy transaction. The cost of a trading strategy is defined in Poulsen *et al.* (2009) as the difference in the value of the portfolio and the accumulation of the value process from the initial portfolio up to the current time. The strategy is called a risk-minimization delta hedge, which minimizes the variance of the cost of hedging the sold option. Poulsen *et al.* (2009) derive an explicit hedging formula for the locally risk-minimization delta-hedge for a class of stochastic volatility models. The explicit formula shows that the hedge can be decomposed into a sum of delta-hedges and a volatility/correlation-risk term. The locally risk-minimization indicates the increased positions in a portfolio to reduce the risk of the exposure where the increased position is dependent on the volatility and the correlation-risk of the underlying assets. The financial risk which is imposed by non-stationary market conditions in fixed-mix portfolios is reduced to a cost process with high expectations of gains in volatile trending market conditions where hedges on rebalancing are effective on option gains. This tactic reduces the risk of rebalancing gains due to the dynamics of risky asset prices. Globally the strategy attains a stationarity form resulting from strategic hedges which control the stochastic risk subject to the financial risk of the gains on rebalancing.

Dempster *et al.* (2010) underpin growing wealth under fixed-mix dynamic strategies to stationary market conditions with a volatile non-degeneracy process such that the strategy yields unbounded exponential growth. The non-degeneracy process is a relative price process which is considered to be non-constant in the assumption of volatility. The proposed strategic advancement in the implementation of fixed-mix portfolio strategies is developed by utilizing financial derivative instruments to hedge the projected portfolio growth. James, Kasikov and Edwards (2012) strategic risk management view in the drawdown of an asset is to trade off other assets not particularly an underperforming asset which is a counter strategic tactic for self-financing strategies such as overlay strategies and leveraged asset positions. The strategic risk management hedges the down risk of an asset and increases the expected gains of an asset.

The aim of the analysis is to optimize the performance of fixed-mix portfolio strategies by enhancing the required condition of stationarity. A multi period strategy is assessed with a benchmark strategy, the buy and hold strategy, in a long term. According to Dempster *et al.* (2007) constant rebalanced portfolio strategies yield exponential growth of wealth in volatile stationary markets under self-financing trading strategies. Evstigneev and Schenk-Hoppé (2002) point out that in stationary markets asset prices in general provide no financial growth, whereas under constant proportions such strategies yield fast exponential growth of wealth. Furthermore in Dempster *et al.* (2007) the condition of stationarity remains lucrative under self-financing fixed-mix strategies. Moreover, given that any self-financing constant proportions investment portfolio strategy yields a strictly positive exponential growth rate in volatile stationary markets in which the ratios of stochastic asset prices are characterized by a non-degenerate process. The derivations or the replications of the stationarity and the non-degeneracy in the financial market are a subtle in the implementations of fixed-mix portfolios. A financial proxy is required to attain the condition of stationarity in a portfolio in generic markets.

Brooks (2005) defines a stationary process as a process characterised by a distribution with statistical measures which are not dependent on the time factor of the pro-

cess. In determining stationary tradable securities which render the financial proxy three types of stationary conditions are considered. The first type is the trend-stationary condition where stationarity is attained along a trend. The second type of stationarity is the difference-stationary condition which attains stationarity by lagging and the third type is the covariance-stationary condition which determines stationarity on the co-variability and on the lag process. To evaluate the condition of stationarity, Dempster *et al.* (2007), used a convergence analysis to determine the asymptotic convergence of the average of the process such that a convergent average process is determined to be stationary.

The technology of portfolio insurance introduced by Rubinstein (1981) sets a hedging tool to encounter the financial risk by using financial derivative instruments. The theoretical results provided by Evstigneev and Schenk-Hoppé (2002) and the technology of portfolio insurance suggest a financial innovation on the settings of a mean-variance model. The main findings from the viewed literature to obtain a proposed strategic tool in the implementation of fixed-mix strategies are stationarity of financial markets and volatility of asset prices. To analyse the required financial proxy which renders stationary tradable securities the technology of portfolio insurance is considered by Yao (2012) in the optimization of the portfolio insurance model where a synthetic put option is used to time selling trades in a portfolio of riskless asset and risky asset. A proposed approach in implementing fixed-mix portfolio strategies requires a financial engineered approach which considers the underpinnings of growth of wealth by utilizing well-defined tools in the financial markets to attain projected outcomes.

2.4 The quantification of risk-adjusted returns on a mean-variance model

The mean-variance model serves as a basic fundamental model or a foundation to study optimization of a multi period trading model and extends the implementation of fixed-mix portfolios. The mean-variance model optimizes single period expected portfolio returns by reducing the risk of the portfolio. In this framework of Marko-

witz inefficient portfolios, which have expected returns below the benchmark return, are justified to enhance the performance on the basis of the volatility and the attained stationarity. The enhanced modelling of the market risk in the application of fixed-mix portfolios reduces the increased risk by benchmark utility with efficient attainability of the rebalancing gains over the assessment of the market price of risk of an inefficient frontier. The analysis by Bai, Liu and Wong (2009) on the Markowitz mean-variance analysis of self-financing portfolios highlights the importance of the mean-variance model to set feasible benchmarks with the objective of attaining expected portfolio performance and to resolve the estimation problem of the efficient portfolios from the theoretical counterparts. Moreover, the equal weighted portfolios are found to be more effective than portfolios selected with a Markowitz Mean-Variance criterion since they are less prone to estimation problems (see Bai *et al.*, 2009 and Fabozzi *et al.*, 2004). Bai *et al.* (2009) resolve the problem of over estimation of traditional returns of a self-financing portfolio which evaluates the theoretical value with sample mean and covariance matrix by a bootstrapping technique which develops new estimates that correct the over prediction problem and the accuracy estimates. This indicates the estimation procedure to be effective in managing model differentials and the efficiency of dynamic strategies with the emphasis on the accuracy of over prediction.

The optimization procedures mentioned in Bai *et al.* (2009) form a tactical approach in active portfolio management. The first optimizing criterion is to maximize the return subject to a given level of risk. The second is to minimize risk subject to a level of expected return. The main tactical objective resulting from the mean-variance analysis is to optimize the return subject to an increased level of risk which is a tactic on low costs. Secondly, to subjectively minimize the financial risk given the level of expected strategic option returns in volatile market conditions.

The capital-asset pricing model which is introduced by Sharpe and Lintner (1965) is a central model in a setting of a required rate of return of an asset under the trade-offs of risk-taking in the financial market considered. The model defines the required rate

of return of a risky asset as the risk-free rate of return plus the risk premium of the risky asset on the market risk in the market portfolio. Assets of low betas associated with high volatility are required for fixed-mix portfolios to enhance rebalancing gains as compared to high beta assets with low volatility and a high degree of co-movements of asset returns with the market returns. The management of risk levels in a portfolio is noted in James, Kasikov and Edwards (2012) to provide financial linkages which are associated with investor behaviours and opinions over the draw-downs of co-movement of asset returns at a correlation degree greater than expected.

2.5 Dynamic trading strategies

The derivative instruments are used in active portfolio management to provide the portfolio insurance and the tactical asset allocation. The put option sells depreciating assets in down-trending market conditions and the call option buys appreciating assets in up-trending market condition at predetermined prices. The derivative instruments are specifically used to retain a trend-following return in non-stationary market conditions and to reduce the rebalancing cost of the strategy. The non-stationary market conditions are characterised by non-stable reverting conditions as considered in trending conditions.

2.5.1 Lookback Call and lookback Put options

Darius, Ilhna, Mulvey, Simsek and Sircar (2002) define a lookback option as an agreement to trade a risky asset at a path-dependent price determined by a maximum value or a minimum value of an asset price over the duration of an option. A lookback call option is the right to buy an asset at a minimum value of the asset price over the duration of an option. The lookback put is the right to sell an asset at a maximum value of the asset price over the duration of an option. The price formula of a lookback call and of a lookback put is shown in appendix 7.

2.5.2 Lookback Straddle

The combination of options provides a strategic portfolio of puts and calls on the same asset. Daruis *et al.* (2002) define a lookback straddle as a combination of a lookback call and a lookback put option with the same strike price and expiration time. A lookback straddle is defined as the difference between the maximum value and the minimum value of a stock price taken over a period of an option. A straddle purchase has a combination that involves buying both put and a call option. A straddle purchase has a substantial payoff if the asset price is not close to a strike price where both depreciation and appreciation in asset prices ensure a strictly increase in the payoff. The reverse strategy of a straddle purchase is a straddle write which involves selling of both put and call options. The payoff of a straddle write is ensured when the asset price increases or decreases to a price close to a strike price. The lookback options are defined to retain the optimal intrinsic differential as a payoff in generic volatile market conditions associated with trending conditions of reversals from the trend. The lookback call and lookback put options retain optimal payoffs in volatile trending markets. Daruis *et al.* (2002) consider the lookback straddle as a derivative which captures the dominant trend. The lookback straddle is economically sufficient comparable to a less costly vanilla straddle because it captures the reversal of stochastic prices along a trend by returning better financial gains on the investment.

2.6 Dynamic asset allocation portfolios

Dynamic asset allocation portfolio strategies that are employed in active portfolio management are studied in Dempster, Evstigneev and Schenk-Hoppé (2003, 2007, 2008, 2010) and Evstigneev and Schenk-Hoppé (2002). Constant proportions or fixed-mix portfolio strategies are dynamic self-financing trading strategies which assign fixed-ratios of portfolio wealth in each asset in a portfolio without addition of cash throughout the life of the investment. The dynamic portfolio rebalances the weights of each asset to fixed-actual ratios.

According to Qian (2012) fixed-mix portfolios are multi-period active portfolio management strategies that add incremental value under certain exceptions as referred to by (Willenbrock, 2011) and others as the diversification return. The analytical attribute of the diversification return, by Qian (2012), is expressed as the excess of the weighted sum of individual variances and the variance of the fixed-weighted portfolio. A distinctive argument in the analysis of the underlying contribution effect of the rebalancing gains and the diversification returns considers the two similar. Qian (2012) shows that the diversification return is always positive for long-only unleveraged portfolios rebalancing to mean-reverting strategies for multi-asset class portfolios. For leveraged portfolios it is shown that rebalancing devalues the diversification return.

Dynamic replication of a synthetic put option with a riskless asset and a risky asset is used by Yao (2012) to construct optimal strategies of portfolio insurance where their optimality is dependent on the market price of risk. The trend-following portfolio strategy in (Daruis *et al.*, 2002) uses a lookback straddle to enhance multi period gains of risk-adjusted returns in the presence of volatility. The long straddle, purchased straddle and the lookback straddle are instruments that set feasibility of a portfolio strategy to ensure strictly increasing payoffs on progressively trending asset (*increasing* or *decreasing*) prices (*closer* or *apart*) from a strike price in generic markets over multi-periods.

It is shown analytical in Dempster *et al.* (2007) that the performance of fixed-mix portfolio strategies in volatile stationary markets is superior to the performance of individual assets in a portfolio. Volatile stationary stochastic asset prices are a major requirement in the implementation of fixed-mix portfolio strategies in order to achieve the approved results by Evstigneev and Schenk-Hoppé (2002). The market conditions which are considered in this analysis are trending and mean-reverting market conditions which are frequently in generic settings of financial markets.

2.7 Optimization of dynamic portfolio strategies

The non-anticipating algorithm of portfolio selection by Cover (1991) which is updated timely over the market asymptotically outperforms the best performing stock. The exponential rate of growth of the non-anticipating algorithm is asymptotically equal to an exponential rate of growth of an optimal constant rebalanced portfolio. According to Dempster *et al.* (2007), analytically and theoretically, the effectiveness of constant rebalanced portfolio strategies has the underpinnings of portfolio growth bounded on stationarity and volatility of stochastic portfolio asset prices such that for implementation purposes the focus is on dynamic enhancement of market volatility and stationarity. The optimization of the performance of the portfolio of investment assets is founded on the principle of maximizing the logarithmic growth rate of a portfolio which is known as the Kelly Rule, Kelly (1956). The optimization of the performance of the portfolio is determined by setting the appropriate weightings to accept the risk such that the growth rate of the portfolio is maximised.

The empirical analysis of fixed-mix portfolios in the currency market is carried in Chule (2007). It is shown that de-trended exchange rates enhance the performance of fixed-mix portfolios particularly for a strategy with 50 percent allocations in both the risk-free asset and the risky asset which attains the optimal performance. The carried empirical analysis used the exchange rates of the United States dollar, the Euro currency with a British pound base currency. The de-trending action of exchange rates raised a problem to determine a financial transaction corresponding to de-trending. The implementation of a fixed-mix portfolio strategy is prone to rebalancing costs which limit the performance of the fixed-mix strategy.

A Leland's framework of rebalancing under transaction costs, see (Albanese and Tompaidis, 2008), assign a no-transaction interval which asserts optimal trading. If transaction costs are proportional to the amount traded, the optimal rule of rebalancing is characterised by a no-transaction interval about the actual portfolio. The optimal strategy is to rebalance to the nearest boundary of the interval. The major limitations of the constant rebalanced portfolio strategies are transaction costs and the un-

realised potential of stochastic processes as a result of discrete rebalancing. Hence, to compensate these limitations, risk-adjusted well-timed transactions are proposed. The main query is to determine a financial proxy which attains the expected unbounded exponential growth under generic market conditions. This is achieved by considering an efficient strategic structure to effectively outpace the chance of rebalancing gain, reduce the risk and retain the optimal financial effort of stochastic.

The theory of fixed-mix portfolio strategies has laid expectations of yielding financial growth strictly above benchmark levels of the portfolio Dempster *et al.* (2007). The expected growth with fixed-mix portfolios outlays further scrutiny of passively managed portfolios. The actively managed portfolios with fixed-mix strategies provide a lucrative prospect in the analysis of inefficient portfolios as classified in the mean-variance model of Markowitz. The portfolio selection procedure of Markowitz sets a basis for the assumptions of stationarity in application of fixed-mix portfolio strategies. The theoretical findings in the application of fixed-mix portfolio strategies, in Dempster *et al.* (2010), highlight an active portfolio management benefits. The stochastic control is used by Kung, Wong and Wu (2013) to derive optimal solutions of an investment problem which characterises a securities market with dynamics and stochastic nature of asset prices. The stochastic control provides solutions which are implemented by fixed-mix portfolios to realize optimum performance results over the stochastic nature of the market.

2.8 Conclusion

The proposed methodology is structured to attain excess growth under normal techniques of rebalancing. The methodology is pioneered by a capital asset pricing model which quantifies the market price of risk in order to set risk-return trade-offs for a financial proxy which attains a necessary condition of stationarity. According to Qian (2012) trend-following portfolio strategies and mean-reverting portfolio strategies attributed to diversification return in the long term which implies that trend-following strategies are prone to yield more returns on leveraging. Mean-reverting strategies generate more value along a long-term average. Hence, active portfolio

management of fixed-mix portfolio strategies is required when adjusting excess returns in dynamic asset allocation portfolios. The potential of financial engineering which is raised in Dempster *et al.* (2010) is determined feasible under a trend-following strategy which de-trends stochastic asset prices. The suggested strategic tactic as a trend-following strategy ensures optimal utility of portfolio growth under active portfolio management particularly for fixed-mix portfolios with leveraged positions. The reviewed literature of active portfolio management with fixed-mix strategies provides the feasibility to quantify risk-adjusted returns under the framework of Markowitz mean-variance model with option-based portfolio insurance strategies.

The following chapter discusses the design of the research. The main research question to determining a de-trending action and a financial proxy are constructed. The sample selection and data collection methods as well as the validity of the method are explained. The analysis of fixed-mix strategies performance and the quantification of risk-adjusted returns are discussed. The financial engineered strategy and its validation by a derivative instrument is elaborated.

Chapter 3

METHODOLOGY

3.1 Introduction

This chapter outlines and discuss the implementations of the research objectives. The main objective of this quantitative analysis is to determining the financial proxy which optimizes the de-trended stochastic asset prices with fixed-mix portfolio strategies. The data and the methodology of the analysis are elaborated in the following subsections. The supporting objectives are discussed. The aims of this chapter are to design an analytical argument and to analyse a quantitative approach in attaining the main objective of a financial proxy.

The subsection 3.2 discusses the financial risk encountered in the application of fixed-mix portfolio strategies. The first objective which is to perform a quantitative analysis of the performance of arbitrary constant rebalanced self-financing strategies is discussed in subsection 3.3. The background design of a fixed-mix strategy and an analytic quantitative example of a fixed-mix strategy are discussed in subsection 3.4. The second objective is to assess the unrealized potential of stochastic processes under fixed-mix strategies. The third objective is to model a strategy which quantifies stochastic asset prices as products of total returns and linked into a portfolio to achieve positive expected results. In the modelling of a strategy the aim is to optimize in each single period under the mean-variance framework of Markowitz's. The risk and returns of asset prices for a fixed-mix portfolio strategy are analysed with a mean-variance model in subsection 3.5. The analysis and the implementation of de-trending and the financial proxy are discussed in the subsection 3.6. The data and methodology of a quantitative analysis is elaborated in subsection 3.7. The test of stationarity of the logarithm of asset prices and of returns is discussed in subsection 3.8. The performance measurements in the quantitative analysis are the market price of risk of the portfolio strategies, the unrealised potential of stochastic asset prices and the quantification of risk-adjusted returns are discussed in subsection 3.9.

The performance of portfolio strategies is interpolated over the performances of 6 leveraged portfolios with short positions in the fixed-rate asset or in the risky asset. The interpolation of completely-mixed portfolios is performed over 3 unleveraged portfolios with long positions in the fixed-rate asset and in a risky asset. The asset allocation of computed portfolios ranges from -200 percent to 200 percent allocations in a fixed-rate asset. Arbitrary portfolios are interpolated between the two computed portfolios to analyse the performance curves of fixed-mix portfolios and buy and hold portfolios.

The computations are performed to examine the performance of actively managed portfolios. The active portfolio management strategy called a fixed-mix strategy is implemented using the daily asset prices over a period of 15 years. The analysis examines the tactic of implementation, to evaluate the cost management effect during the short-term period of no rebalancing or the discrete period of rebalancing. The risk and returns of asset prices are computed to design a subroutine strategy that renders the required market condition and replicates the expected performance. The literature of active portfolio management particularly with fixed-mix strategies in Dempster *et al.* (2010) suggests a financial engineered prospect in this regard to further analyse the stationarity in fixed-mix portfolios which yield unbounded fast exponential growth of returns.

The analysis shows the application of fixed-mix strategies versus the buy and hold strategy. The strategy is simulated in market conditions of up-trending and market conditions with frequent reversal. The most important aspect of the analysis is to make a strategic appraisal by evaluating the market risk which is computed as a measure of the price of market risk.

3.2 Financial risk

The passive strategy which is the buy and hold strategy retains return differentials along the span of the investment. Hence such strategies efficiently perform well on trending markets. The market risk in a passive strategy is not actively managed and

portfolio risk levels are trend-following determined. The strategy has a financial risk of outperforming the trend over the market risk. According to Dempster *et al.* (2007) the dynamic investment strategy called the fixed-mix strategy utilizes the presence of volatility to generate growth which is not considered by a buy and hold strategy. Furthermore, the relative price ratios assert the assumption of volatility under fixed-mix portfolios such that the non-constancy of the ratios throughout the investment time horizon shows the presence of volatility in a two-asset portfolio. The risk-adjusted returns on a benchmark return, the return of a fixed-rate asset, in a trending market show an uncounted financial risk of a strategy where fixed-mix strategies underperform (see (John *et al.*, 2003) and Yoa (2012) the Sharpe ratio performance measure) for the evaluation of the returns on actively encountered risk.

Moreover, the financial risk which arises in determining the appropriate transaction which renders stationary tradable securities for fixed-mix portfolio strategies to yield expected results is not considered. In addition, the financial risk which is present in financial stochastic price processes as well as in trading systems is not treated by traditional investment management systems namely the buy and hold strategy and the fixed-mix strategy on asset prices. The encountered risk of rebalancing gains in trending markets is minimized by ensuring strictly positive gains in discrete periods. The designed strategy evaluates a geometric drift into a self-financing fixed-mix portfolio by stationary tradable securities to yield optimal performance under the considered market risk. To evaluate the market price of risk in two-asset portfolios the excess value of risk of a portfolio is computed to analyse the premium of a trend-following strategy. The excess value of risk of a portfolio is computed over the realised performance as a return of a fixed-rate asset and the excess return of a buy and hold strategy to a fixed-rate asset which varies with a position in the market risk. The excess value of risk in the realised performance shows the financial risk of a fixed-mix portfolio in trending conditions over a benchmark return of a fixed-rate asset.

The fixed-mix strategy requires hedging which reduces the loss on rebalancing gains where frequent trading is prone to transaction costs. To rebalance fixed-mix portfolios under proportional transaction costs a Leland's framework rebalancing rule is

used. The rebalancing of a fixed-mix portfolio is described by a no-transaction interval specified by a period of a contingent claim to maturity. The appropriate rebalancing in a fixed-mix portfolio of a tradable stationary asset which encounters the financial risk is a deterministic time hedging motivated in Albanese and Tompaidis (2008). The rebalancing of the portfolio is performed at optimal discrete periods which are set to a period of five discrete trading periods. The maturity period of contingent claims in fixed-mix portfolios is assigned to weekly rebalancing.

3.3 The application of fixed-mix strategies

The application of fixed-mix strategies in stationary markets has proved to be superior to the buy and hold strategy. This evidence arises from recent theory and numerical applications by Dempster, Evstigneev and Schenk-Hoppé (2003, 2007, 2008, 2010), and (Evstigneev and Schenk-Hoppé, 2002). Fixed-mix strategies, also known as constant proportions strategies, are dynamic strategies which discretely or set a threshold to readjust actual proportions of a portfolio throughout the investment period. The strategy is designed such that returns on market fluctuations and market reversals are retained and reinvested. The system dynamics of a fixed-mix trading strategy are analysed particularly in the South African financial market. The focus is on realizing the impact of volatility and to design a portfolio system that efficiently utilizes the presence of volatility.

The designed portfolio system has to be conditional to stationarity, which is primarily for fixed-mix strategies to realize fast exponential growth of returns. The interest of the research is to conduct a lucrative assessment of portfolio risk by focusing on the appraised suggestions of further financial engineering in Dempster *et al.* (2010). The computational analysis is performed using portfolios with long positions and short positions. The short positions are computed to determine the leveraged effect of the strategy in trending markets. Financial derivative instruments are used to handle the market risk appropriately for fixed-mix portfolios to retain the market reversal effect.

3.4 Background of the designed strategy

This subsection discusses the design of the fixed-mix strategy and provides a quantitative example of a strategy. The optimal performance of portfolios is considered in the context of optimizing the growth rate of wealth as the Kelly rule criterion in Kelly (1956). The Kelly rule criterion determines the critical portfolio weights which optimize the logarithmic growth rate of wealth. In the performed computations the optimal performance is evaluated in terms of the portfolio's total return. The condition of stationarity which is required for fixed-mix strategies to attain fast exponential growth of wealth sets further consideration of a stochastic asset price process to control the stochastic process of returns of asset prices.

The advancement of fixed-mix portfolios is attained by deriving the stationarity condition in the market considered. The aim of the research is to examine the performance of a fixed-mix strategy in a long run and to assess potentials of further financial engineering. Given that the market is not normally stationary, particularly the equity market, according to Dempster *et al.* (2007) the evidence of stationary market is usually found in the currency market. In the analytical results of long term performance of fixed-mix strategies Dempster *et al.* (2007) had shown that volatility is a driving factor of financial growth. The condition of stationarity of relative asset prices ratios is considered by Dempster *et al.* (2003) as a "driving force" of financial growth. The stationarity of relative asset prices ratios is tested using the unit root test. The results of the stationarity test are shown in the appendix in chart A.2.1.1.

The major aim of the research firstly is to implement dynamic fixed-mix portfolio strategies under sufficient conditions assumed in Dempster *et al.* (2007). Secondly, is to contribute to a financial engineered prospect in the modelling of optimal performance of fixed-mix portfolios in generic market conditions. The optimal performance of fixed-mix portfolios is evaluated over portfolios ranging from short positions in a fixed-rate asset to leveraged positions in a fixed-rate asset. The wealth of the portfolio is optimized for fixed-mix portfolios of a tradable stationary asset and a fixed-rate asset under the Merton's model of asset prices. The drift parameter and the

volatility parameter of tradable stationary asset prices under a Merton's model lay the core underpinnings of wealth creation when considered under the Markowitz mean-variance analysis. The stationary asset optimizes the drift parameter and reduces the volatility parameter to attain a geometric drift of a trending risky asset. To address this challenge we adopt a technology of portfolio insurance (Rubinstein and Leland, 1981) which precautions substantial loss and gains of wealth by hedging and shifting risk-adjusted returns using a financial derivative instrument which is called a lookback straddle.

The designed strategy completes the considered market by setting a sufficient condition to hedge risk-adjusted returns. The risk-adjusted returns are replicated by a combination of exotic options. Exotic options are options with payoffs defined implicitly on the realised prices of the underlying asset. To optimize the dynamic portfolio requires a complete market which is derived by controlling the stochastic process to attain a stationary tradable process of asset prices to hold a stationary financial condition in a fixed-mix portfolio. A fundamental concept of asset pricing sets fundamentals in attaining the sufficient condition of stationarity over the market risk of a trending risky asset.

3.4.1 A Quantitative Analytic Example

An exemplary market scenario of a fixed-mix portfolios strategy is analysed in an analytic form. The example shows the analytics of the strategy by considering a full-cycle of a market scenario. A full-cycle of a market scenario is defined as a combination of down-reverting and up-reverting market scenarios. The down-reverting price scenario is a stochastic change of a price of a risky asset by a positive logarithm of total return. The up-reverting price scenario is a stochastic change of a risky asset by a negative logarithm of total return. Holding to a full-cycle of a market scenario is null. No procedural counter-active effect to a stochastic financial market condition. However, the active management of a risky asset is quantitatively shown in the analytic table to retain the gain in a stochastic market situation.

An example of a market scenario is presented in a two-asset financial model of a risk-free asset and a risky asset. A vector of asset prices is denoted by, S , and the prices of assets at time, t , are given by the following 2-dimensional vector,

$$S_t = (S_t^0, S_t^1), \text{ for } t \leq T. \quad (\text{a})$$

The number of assets in a portfolio is denoted by a 2-dimensional vector, φ , given by the following,

$$\varphi_t = (\varphi_t^0, \varphi_t^1), \text{ for } t \leq T. \quad (\text{b})$$

The fixed actual weights of the portfolio wealth are denoted by a 2-dimensional vector, α , given by,

$$\alpha_t = (\alpha_t^0, \alpha_t^1), \text{ for } t = 0. \quad (\text{c})$$

The self-financing portfolio strategy is given by a value process setting for the next period of positioning of assets as a product of 2-dimensional vectors in equation (a) and (b) with a stochastic control vector process of equation (c) in the following,

$$V_t(\alpha) = \varphi_t(\alpha) \cdot S_t = \varphi_{t+1}^0(\alpha_0^0) \cdot S_t + \varphi_{t+1}^1(\alpha_0^1) \cdot S_t^1 = \varphi_{t+1}(\alpha) \cdot S_t, \text{ for } t \leq T. \quad (\text{d})$$

The analytics of three portfolio strategies are analysed in a full-cycle market scenario by the following strategies $V(0.5)$, $V(0.25)$ and $V(0.75)$. The analytics of $V(0.5)$, a portfolio value strategy, are shown in table 3.4.a and 3.4.b. The portfolio value strategies $V(0.25)$ and $V(0.75)$ are only analysed and not shown. A financial market vector of asset is assumed to be constant over a risk-free asset with a price value of 1 and varies by a variation factor of 2 to indicate the stochastic change for an increased price and decreased price of a risky asset.

The down-reverting and the up-reverting full-cycle market scenarios of a $V(0.5)$ strategy perform similar with a total return of 26.56 percent as shown in table 3.4.a and 3.4.b. However, the up-reverting market scenario has a higher two period mov-

ing average of a value process at a total factor of 1.3047 (see Table 3.4.b) comparable to a total factor of 1.1406 (see Table 3.4.a) in the down-reverting market scenario. The up-reverting full-cycle market scenario trades at discount by increasing the number of risky assets and realizes gains at the premium of the market.

An analytic table of a strategy with 50 percent allocations in a risk-free asset and in a risky asset is shown in table 4.3.a and table 4.3.b for two market scenarios of down-reverting and up-reverting respectively.

		φ_t^0	φ_t^1	S_t^0	S_t^1	$Trade S_t^0$	$Trade S_t^1$	V_t	$\bar{V}_{t,2}$
	0	0.5000	2.0000	1	0.2500	0.5000	2.0000	1.0000	1.0000
	1	0.7500	1.5000	1	0.5000	0.2500	-0.5000	1.5000	1.2500
	2	0.5625	2.2500	1	0.2500	-0.1875	0.7500	1.1250	1.1875
	3	0.4219	3.3750	1	0.1250	-0.1406	1.1250	0.8438	1.0156
	4	0.6328	2.5313	1	0.2500	0.2109	-0.8438	1.2656	1.1406

Table 3.4.a: Analytic table of a strategy with 50 percent allocations in a two-asset portfolio in a down-reverting full cycle market scenario.

The table shows the time of a trade in column 1, the number of a risk-free asset in a portfolio in column 2, the number off a risky asset in column 3, the price of a risk-free asset in column 4, the price of a risky asset in column 5, the number of traded risk-free assets in column 6 and the number of traded risky assets at the time of a trade in column 7. The last two columns 8 and 9 respectively are value processes and a two period moving average of the value process of a strategy.

		φ_t^0	φ_t^1	S_t^0	S_t^1	$Trade S_t^0$	$Trade S_t^1$	V_t	$\bar{V}_{t,2}$
	0	0.5000	2.0000	1	0.2500	0.5000	2.0000	1.0000	1.0000
	1	0.3750	3.0000	1	0.1250	-0.1250	1.0000	0.7500	0.8750
	2	0.5625	2.2500	1	0.2500	0.1875	-0.7500	1.1250	1.0000
	3	0.8438	1.6875	1	0.5000	0.2813	-0.5625	1.6875	1.3438
	4	0.6328	2.5313	1	0.2500	-0.2109	0.8438	1.2656	1.3047

Table 3.4.b: Analytic table of a strategy with 50 percent allocations in a two-asset portfolio in an up-reverting full cycle market scenario.

The variations of a full-cycle in a portfolio strategy indicate the variability of the value where a lesser two period moving average shows the low amplitude of the value process. Moreover, the increase in the variability has generative outcomes over the up-reverting market scenarios. The quantitative analysis shows the value of a hedging principle where the trading of assets has the benefit of covering the timing of the market over frequent trades. The holdings of a portfolio benefit by an increase of 0.1325 risk-free assets (see column 2 and 3 at the forth trade) and an increase of 0.5313 risky assets over both market scenarios.

The superiority of $V(0.5)$, is shown on the performances of $V(0.25)$ and $V(0.75)$ strategies. Both the strategic allocations of 25 percent and 75 percent in a risk-free asset realize a lesser performance than the $V(0.5)$ portfolio strategy. The realized performance is an increase of a return by 16.63 percent for both strategies in all considered market scenarios. Furthermore, the increasing potential of a value process is shown to be on an up-reverting market scenario with a two period value process of a total factor of 1.3149 for a $V(0.25)$ strategy at a relative factor of 1.22 to a down-reverting market scenario. The strategy, $V(0.75)$, of a two-period moving average total factor of 1.1939 in the up-reverting market scenario is relatively higher to the down-reverting market scenario at a relative factor of 1.071. The benchmark enhanced strategies of decreased risk are shown to have less performance dynamic as indicated by a lesser total factor and a reduced relative factor comparable in relative market scenarios.

The analytic quantitative analysis of a two- asset portfolio strategy indicates a frequency and the quantum of trades in timing the market of risky assets. The superiority of the strategy is shown by a balanced-strategic strategy of enhanced riskiness and gained stochastic effect indicating a dual generative factor a portfolio strategy.

3.5 Risk and returns of asset prices

The asset prices risk and returns are analysed to be of strategic valuable in the financial growth of an active portfolio. The presence of volatility is a profound result ac-

cording to Dempster *et al.* (2007) which sets a trade-off between the returns and risk that mainly arises from market fluctuations and market reversals. The diversification of asset prices results in a reduction of the volatility of a portfolio of assets such that it is always below the highest volatility of asset prices. Evstigneev and Schenk-Hoppé (2002) consider the stochastic process of returns of asset prices to form a stationary process. The evolution process of stationary total returns of asset prices is a requirement for theoretical results in Dempster *et al.* (2007) for fixed-mix portfolio strategies to yield exponential growth of returns in a long-run.

The mean-variance portfolio management approach by Markowitz (1952) sets the trade-offs of risks and returns in a two dimensional framework of expected return and variance. The efficient frontier is presented by a set of portfolios with a required expected rate of return above a rate of return of a fixed-rate asset where the volatility of the portfolios are most diversified over a fixed-rate asset and a risky asset. The analytical results in Dempster *et al.* (2007) consider the volatility of asset prices as a contributor to the financial growth which leads to the consideration of the inefficient frontier of risky assets, portfolios of high volatility, to assert positive risk-adjusted returns when subjected to effective financial engineering.

3.6 De-trending and the financial proxy

Stationarity of asset prices is a necessary condition of fixed-mix portfolios to yield expected unbounded exponential growth of returns in a long run. The performed empirical analysis of fixed-mix portfolios in Chule (2007) with de-trended exchange rates raised a problem in determining a financial transaction. The de-trended exchange rates were derived as excess rates over a released trend. The realised growth showed fixed-mix portfolios to perform optimally in the rendered condition of de-trending for portfolios with equal weighted allocations in each asset. Hence the evolution of asset prices is regarded as a combination of total returns where total returns represent a stationary stochastic process (Dempster *et al.*, 2007). Moreover, to attain the stationarity condition for stochastic asset prices, de-trending is used to trending asset prices to retain optimal return over trending stochastic asset prices. The attained

prices process is a tradable stationary asset prices process generated by a stochastic-stationary process of risky asset prices determined by the payoffs of the derivative instrument called the lookback straddle. The theoretical result of fixed-mix portfolio strategies in Dempster *et al.* (2007) proves the optimality of fixed-mixed strategies in stationary market over a long run, which leads to a problem of adjusting the stochastic prices process to a stationary tradable process. The problem arising is to consider the dynamical trading system to induce tradable stationary processes and to efficiently trade the market risk.

The derivative instruments are used in a dynamic trading strategy for leverage effects and gearing in accumulating rebalancing gains and in ensuring gains by collateralizing a stationary financial condition. The de-trending action embeds a financial proxy which is required to yields fast exponential growth. The problem analysed involves determining a transaction that is linked to de-trending which is required for fixed-mix strategies. The lookback straddle option is used for a strategy which de-trends asset prices in trending markets. The lookback straddle ensures the optimal trade of buying at a lower realized price and selling at a higher realized price which eliminates the financial risk of transactions and to gear the gains over rebalancing.

3.7 Data and Methodology

The problem which is being analysed is a problem of determining a transaction or an extended implementation of a fixed-mix strategy which fits the requirement of stationarity. This particular problem is addressed by performing a quantitative analysis to examine the performance of fixed-mix investment strategies in a frictionless market where the transaction costs are not considered. The price formulas of lookback options and a straddle option are shown in the appendix 7. The cost of a lookback straddle is assumed to be the initial price of the underlying risky asset at the beginning of the contract to realize the effect of a consistence financial condition by ignoring the cost impact in the considered frictionless market. The ignored cost impact is considered to have less effect under the assumption of minimum cost in the results of financial growth resulting from volatility by Dempster *et al.* (2007). The complexity

of the problem requires analytical recommendations for a financial engineering construction. In this research we perform a quantitative analysis of fixed-mix strategies of discrete processes of time series in the South African financial market.

3.7.1 Major South African companies

The six major companies in the JSE financial market are used in the analysis to examine the robustness of the approach. The type of research sample is a convenient sample of tradable financial securities with volatile trending market conditions and reversal market conditions. The prime rates are selected for a risk-free asset to correspond to the six selected financial time series. The risky asset prices of selected companies are constituents of the FTSE/JSE index namely; ABSA Group Limited, Standard Bank Limited in the financial sector, Sasol Limited in the Oil and energy sector, Anglo American Platinum Limited, Anglo Gold Ashanti Limited in the Gold mining sector and the company in the Food and fishery sector, AFGRI Limited are used to perform the analysis. The share prices are selected randomly and used in the analysis as initialised asset prices. The conveniently selected data of asset prices are required to have trending fluctuation conditions. The presence of volatility is determined by the non-constancy condition of the price relative ratios (see Dempster *et al.* (2007) the assumption of non-degeneracy). The selected data shows the market condition of up-trending and frequent reversals. The selected data of ABSA Group Limited, Standard Bank Limited, Sasol Limited and Anglo American Platinum Limited show the presence of an up-trending market condition and the selected data of asset prices of Anglo Gold Ashanti Limited and AFGRI Limited show market conditions of frequent reversals. The sample size of the selected JSE Financial markets time series is 15 years which sets to provide the evidence of robustness. However, a longer period is recommended.

3.7.2 Data collection method and validity

The performed analysis focuses on every tradable financial stochastic process. The targeted population is the FTSE/JSE Top 40 index which is a representative of 40

major companies in South Africa. The data is retrieved from the McGregor BFA web site (<http://www.mcgregorbfa.com>) from the price data report of the Share history. The data used for analysis is collected from the selected companies as daily closing asset prices. The analysis is performed without considering transaction costs. The analysed strategy, the fixed-mix strategy and the benchmark strategy, are used to assess the potential of risks and returns in the frictionless financial market model.

The analysis conducted is reliable given the scope of the research which focuses on the application of fixed-mix investment strategies and their optimal application. To utilize sufficiently the full effect of volatility in trading strategies, lucrative volatility-reduction strategies are examined. The implementation of the analysis is performed in a spreadsheet. The statistical analysis of the performed computations is carried on the IBM SPSS Statistics. A financial proxy that corresponds to an action which results to de-trending its feasibility and reliability is analysed.

3.8 Test of stationarity

The theory and application of fixed-mix strategies in Dempster, Evstigneev and Schenk- Hoppé (2003, 2007, 2008, 2010), Evstigneev and Schenk-Hoppé (2002) show that the implementation of constant rebalanced portfolio strategies has shortcomings for the optimal performance of the strategy. The required assumption of stationarity and non-constancy of the price ratio and as well as the appropriate transactions which yield the expected results prompts further inspection as it sets a financial engineering prospect. The test of stationarity condition of asset prices is carried by following a testing procedure in Brooks (2005) which models asset prices and returns of risky asset prices as Integrated Autoregressive Moving average and test the order of integration of the models using the Augmented-Dickey and Fuller test of unit root. The stationarity condition is proved by a model integrated of order zero. The total returns are computed as ratios of daily closing price to daily opening prices.

3.9 Performance measurement

The most important aspect of the analysis is to make an appraisal evaluation of the market risk which is computed as a measure of the price of market risk, the Sharpe ratio which is defined in John *et al.* (2003) and in Yoa (2012), as a ratio of the excess return of the strategy and the benchmark return to the riskiness of the strategy. The benchmark measure in the computation of a Sharpe ratio is a return of a fixed-rate asset. The risk of the strategy is the volatility of the portfolio strategy. The Sharpe ratio formula and the computed ratios of annual performances are shown in the appendix in model 5 and the ratios are shown in Tables 2.2.

To achieve the main objective of determining the financial proxy requires consideration of the following supporting objectives. The first objective is to perform a quantitative analysis of the performance of arbitrary constant rebalanced self-financing strategies. The performance of portfolio strategies is interpolated over the performances of 6 leveraged portfolios with short positions in the fixed-rate asset or in the risky asset. The interpolation of completely-mixed portfolios is performed over 3 unleveraged portfolios with long positions in the fixed-rate asset and in a risky asset. The asset allocation of computed portfolios ranges from -200 percent to 200 percent allocations in a fixed-rate asset. Arbitrary portfolios are interpolated between the two computed portfolios to analyse the performance curves of fixed-mix portfolios and buy and hold portfolios.

The analysis shows the application of fixed-mix strategies versus the buy and hold strategy. The strategy is simulated in market conditions of up-trending and market conditions with frequent reversal.

3.9.1 Quantitative analysis of fixed-mix strategies performance

The main question is to determine a financial proxy which optimises de-trended stochastic processes under fixed-mix portfolio strategies. The Merton's model considered in Dempster *et al.* (2007) represents the dynamics of a continuous distribution

of asset prices returns which is given by a differential equation. The differential equation represents a deterministic change and a stochastic change of a rate of return of asset price from the considered instantaneous time which is represented by an expected rate of return of an asset plus the volatility of asset prices and a randomly drawn standard normal distribution outcome. In the application of fixed-mix strategies the requirement of stationary markets is a condition which does not normally hold in financial markets (Dempster *et al.*, 2003).

The implementation of fixed-mix strategies under transaction costs provides dynamics which time the effectiveness of rebalancing. The implementation of the strategy using the discrete type of rebalancing is performed with the aim of evaluating the performance of the strategy in trending market conditions and in markets with reversals about a linear trend. The performance of a fixed-mix strategy is analysed by total performance of arbitrary portfolios versus the performance of arbitrary portfolios of buy and hold strategy. The annual rates of returns are computed to determine the respective performance of the strategies annually. The benchmarked performance of portfolio strategies to a fixed-rate asset is analysed by a Sharpe ratio which computes the market price of risk compensated by the strategy.

3.9.2 Unrealized potential of stochastic asset prices

The second objective is performed to assess the unrealized potential of stochastic processes under fixed-mix strategies. Considering the fixed-mix strategy as a composite strategy of a buy and hold strategy, to assess the unrealized potential of fixed-mix strategies the time of rebalancing is determined by deterministic time hedging for appropriate transactions which are expected to generate rebalancing gains. The unrealized potential is imbedded in the effectiveness of time rebalancing and in determining an appropriate transaction that de-trends stochastic asset prices. The analysis of the price of risk shows the expected gains for a strategy which fully considers the trend market risk. The price of risk is computed in the settings of a capital asset pricing model as the total expected return of the position in the considered risk. The total expected return of the position is the benchmark return, which is the return of a

fixed-rate asset, plus the market premium of the position of fixed-mix portfolio in the market risk.

The question of the analysis aims to attain a stationary process which holds by retaining a geometric return over a trending risky asset in the long-run. To eliminate a trend in the context of a portfolio of assets a stationary financial condition is attained by an appropriate financial engineering approach. The financial engineered approach determines a transaction that renders stationarity by a de-trending action which uses financial innovation. The theoretical results of Dempster *et al.* (2007) for constant rebalanced portfolios asserts an exponential yield of returns under the condition of non-constancy of price relative ratios given that the cost of rebalancing the portfolio is significantly small. Under proportional transaction costs the optimal rebalancing strategy for fixed-mix portfolios of tradable stationary asset prices is described by a deterministic time hedging which performs portfolio rebalancing at discrete times.

3.9.3 Quantification of risk-adjusted returns

The third objective is to model a strategy which quantifies stochastic asset prices as products of total returns and linked into a portfolio to achieve positive expected results. In the modelling of a strategy the aim is to optimize in each single period under the mean-variance framework of Markowitz's. The risk-adjusted returns are retained discretely by a tradable stationary price process. The fixed-mix portfolio is rebalanced by deterministic time hedging. The measure of the market price of risk (see Yoa 2012) is the ratio of excess returns of a strategy to a benchmark return which is a deterministic return over the standard deviation of discrete total returns of a strategy. The risk-adjusted returns are determined by positive market price of risk such that the common financial risk arises in the fluctuation risk or the reversal of the market increases or decreases the market price of risk compensated by the strategy. The market risk of the strategy is determined by a measure of market price of risk with a benchmark return equal to a risky return which varies with the market conditions. The assumption of volatility which is the non-constancy condition of relative price

ratios holds under the theory of fixed-mix strategies which prompts the action of rebalancing (Dempster, Evstigneev and Schenk-Hoppé, 2007).

The risk-adjusted returns compensate the market risk and positively encounter the financial risk of fixed-mix portfolios prone to transaction costs and the unrealised stationarity. A simple moving average of total returns is used to time the average convergence to evaluate the stationarity process and to determine the effectiveness of the innovative strategy (Dempster *et al.*, 2010). The risk-adjusted returns prompt a financial insurance strategy which hedges a geometric drift of a trending risky asset. The reliability of the approach is ensured by a well-defined technique which uses a financial innovative strategy to ensure optimal transaction in a fixed-mix portfolio. The limitations in retaining the risk-adjusted returns are due to the insignificance of the excess returns which are determined by the absence of volatile fluctuations or market reversals.

The tactic of hedging the risk-adjusted performance of a trending risky asset provides a market where the market risk is captured by an appropriate strategy. The suggested approach has the limitations of not considering the real trading costs of a financial derivative by assuming the cost of a transaction to be the initial price of the underlying risky asset at the time of initializing the contract. The performed analysis focuses on the impact of the payoff of the transaction. Hence, the problem to capture anticipated return occurs when the strategy underperforms the benchmark as shown in the market price of risk with negative values. To quantify risk-adjusted returns a major question that arises is to replicate mean-reverting tradable securities which are defined by a financial proxy to encounter frequent fluctuations and marker reversals.

3.9.4 Financial engineered formulation

The major aim is to establish a financial engineered approach that sets a basis to optimize on the application of self-financing constant rebalanced portfolio strategies. The approach required to derive the necessary condition of optimal application of portfolio theory of fixed-mix portfolios in a two-asset portfolio is proposed. The dy-

dynamic replication strategy replicates the return of a risky asset by a lookback straddle option which ensures an optimal price differential of a risky asset. The optimal differential of a risky asset is replicated by a difference of the maximum price and the minimum price of the underlying risky asset over the duration of an option. The strategy ensures optimal transactions in market risk.

The Merton's model of a stochastic distribution of an asset price quantifies the differential return of a portfolio as a normally distributed random outcome with a mean equal to a sum of the drift rates of the asset prices, the expected rate of return, and a standard deviation of asset's rate of return times the square root of the instantaneous time differential. The innovative strategy of dynamic portfolio insurance is used in hedging the financial risk under frequent fluctuations by minimizing the risk of returns and of risk of rebalancing costs over the market risk and to increase the rate of the expected returns. The strategy ensures a geometric drift of a tradable stationary price process in a trending risky asset.

A tactical solution to the objective is to set a financial innovative foundation which renders a financial proxy. The financial strategy is derived by using a combination of lookback options the lookback put option and a lookback call option. The combination of the options is considered by Darius *et al.* (2012) to be an effective trend-following strategy for hedge funds. The lookback call option is an agreed financial transaction to buy an asset at a specified date at a price determined by the minimum price of an asset over the duration of an option. The lookback put option is an agreed financial transaction to sell an asset at a price determined by the maximum price of an asset over the duration of an option.

The performance of fixed-mix strategy is studied to analyse the financial innovation in achieving the projected goal of optimizing the application of fixed-mix strategies. The limitation of the financial engineering firstly is the cost of the strategy which is compensated only in volatile trending market conditions where in market without volatile conditions the returns are not retained. The reliability of this approach ensures the efficient application of fixed-mix strategies to enhance the portfolio per-

formance by using appropriate financial instruments which render the assumed condition of stationarity.

3.9.5 The validity of a proposed hedging strategy

The derivative instrument is used to actively manage fixed-mix portfolio strategies to hedge the financial risk of determining the financial proxy that renders the condition of stationarity. The financial proxy is validated by a lookback straddle option. For analysis purposes the cost of the derivative instrument is considered to be the current price of the underlying risky asset at the time of writing the option. The considered maturity period of the option contract is five discrete periods.

3.10 Summary

Portfolio rebalancing strategies discretely rebalance portfolios by dynamic mechanisms of asset allocation which manage risky assets in a portfolio. Dempster *et al.* (2010) consider constant rebalanced portfolio strategies that yield fast exponential growth of returns in volatile stationary markets and a potential of financial engineering of fixed-mix portfolios. A de-trending action which is raised in Dempster *et al.* (2007) and carried in Chule (2007) is considered to transform a trending asset prices to a stationary process which raise a query to determine a financial proxy that de-trend stochastic asset prices. To attain tradable stationary processes that determine a financial proxy a lookback straddle option is used in fixed-mix portfolios to realise fast exponential growth of wealth.

The optimization of the dynamic portfolio model in a mean-variance framework set a justifiable evaluation of risk-adjusted returns. The mean-variance model reduces the risk of a portfolio by acquiring a required expected return. The capital-asset-pricing model set necessary fundamentals to quantify the market risk and to trade-off the financial risk of tradable stationary securities. These considerations provide a directive approach in determining a valid financial proxy. The selected convenient sample of trending market conditions and mean-reverting market conditions is considered to

evaluate the robustness of the proposed approach. The tactics of retaining risk-adjusted returns in frequent trading by timing transactions which retain excess returns over a trending asset eliminate the limitations of transaction costs for discretely rebalanced portfolios.

3.11 Conclusion

The quantitative analysis proposes a design of a sub-strategy which is a portfolio that dynamically replicates tradable stationary processes. The tradable stationary processes attain the required market condition of stationarity to yield fast exponential growth. The validity and the effectiveness of the sub-strategy to hedge the risk-adjusted return is provided by an option which hedges the market risk of a risky asset. The tradable stationary process attains discrete returns such that the stochastic process forms a geometric drift.

The analytic example of a fixed-mix strategy indicates the formation of a generative process which enhances the rebalancing gains in a portfolio. The formation of a generative process is shown and analysed under the objective of this research to be attained by a financial proxy using a lookback straddle derivative. The functionality of a straddle derivative option is extrinsic by enhancing the riskiness of market gains which is a strategic protective functional to the dual generative factor of fixed-mix portfolio. However, the dual generative forming process of the structure of the portfolio is a gain-stationary valued game process of a risky asset which value and trade the uncertainty which is effectively-null to the intrinsic generative process of fixed-mix portfolios because of a protective strategy to the potential of the market. The fixed-mix portfolios of tradable stationary processes consider the market risk and the financial risk which is not considered in absolute application of fixed-mix portfolios of risky assets.

The findings of the empirical asset prices and total returns are presented and their conditions are discussed. The volatility of a portfolio and the formulation of a financial engineered approach are further discussed in the following chapter.

Chapter 4

THE TRADING MODEL AND EMPIRICAL FINDINGS

4.1 Introduction

The trading model and its counterpart strategy are discussed as well as the empirical findings of asset prices of a fixed-rate asset and a risky asset. The financial risk encountered by stationary tradable securities sets a basis for optimizing the application of fixed mix portfolios in generic market conditions. The designed strategy of fixed-mix portfolios hedges the time effective evolution of asset prices process by further analysing the return structure of a price process of a risky asset. The aim of the chapter is to analyse the empirical of assets prices and to assess the quantification of returns of a financial engineered strategic approach.

The strategic action of de-trending determines the financial proxy required to set a sufficient condition of fixed-mix portfolios to perform optimally particularly in trending conditions. The analysis is performed on a conveniently selected data set with trending and reversal market conditions. The unrealised potential of fixed-mix portfolios is determined by a financial proxy. The test of stationarity is performed on logarithms of asset prices, logarithms of total returns of asset prices and relative price ratios of assets in a portfolio. The quantitative analysis and the performance measure of trading portfolios are performed in the following chapter. The validity and formulation of the approach which quantifies the risk-adjusted returns is discussed and further analysed in chapter six.

The design of the quantitative analysis asserts an effective financial engineering approach that renders to the required stationary condition which yields the expected performance. The quantitative findings and the suggested financial-engineered approach is carried out in the following section. This chapter studies preliminary settings of fixed-mix portfolios and the empirical findings to formulate an approach in meeting the objectives of the research. The computational analysis of fixed-mix portfolios is carried out relatively to a buy and hold portfolio strategy.

Fixed-mix portfolio strategies are characterised by pre-determined portfolio weights. The weights of asset prices to the wealth of the portfolio generate an evolution of proportions which are discretely adjusted to fixed-portfolio weights.

This chapter examines the findings of the performance of portfolio strategies and the suggested methods which enhance the performance of a fixed-mix portfolio strategy. The objectives of the research are: to do a quantitative analysis of the performance of constant rebalanced self-financing portfolio investment strategies; to assess the unrealised potential of stochastic processes under fixed-mix strategies; to establish a financial engineered optimisation on the application of self-financing constant rebalanced portfolio strategies; to model risk – adjusted return differentials in order to quantify the level of anticipated performance; to construct time-effective portfolios that are favourable to portfolio objectives and are also linked to financial engineered instruments that acquire an appropriate financial transaction. The computations of trading strategies are performed without considering trading costs.

4.2 The Trading model

The model considered in the analysis is a continuous time Merton's model of asset prices (Dempster *et al.*, 2007) which describes the current rate of return of asset prices. The asset-price model describes the rate of change of asset prices in a discrete period by a deterministic component and a random component. The deterministic component is a drift rate of an asset which is the current average rate of return of asset prices. The random component is a multiple of a volatility parameter factored by an identically and independently distributed random shock with a mean of zero and a variance of instantaneous time differential. The prices of a risky asset are presented as logarithm of prices. To encounter the normality aspects of asset prices the analysis of asset prices is focused on the logarithm return process of asset prices which are found to be distributed normal, identically and independently, hence by virtue of the Merton's model of asset prices. The Merton's model of asset prices is shown in the appendix in model 3.

4.2.1 The Buy and Hold portfolio strategy

The model of a financial market where two assets are traded is used to perform computations of a portfolio management strategy. The buy and hold portfolio strategy allocates a fixed number of assets in each asset in a portfolio specified by a pre-determined ratio throughout the investment period of the strategy which assigns long and short positions of assets in a financial market. The initial allocated number of assets in each asset in a portfolio is held throughout the investment period. The buy and hold portfolio strategy is considered a single period model. The final value of the strategy is determined by the initial portfolio and the final price vector. The total return of the strategy is determined by the ratio of the final value to an initial value of the portfolio. The trading model of a buy and hold strategy is shown in the appendix in model 2.1.

4.2.2 The Fixed-mix portfolio strategy

The vector of asset prices determines the price processes of financial assets, at each trading time, where for each time the fixed-mix portfolio strategy rebalances trades by tactically allocating capital among constituent assets in a portfolio, as determined by constant investment proportions. The fixed-mix portfolio strategy discretely determines the number of assets held in a portfolio to each traded financial asset. The final value of the fixed-mix portfolio strategy is determined by the final portfolio and the final price vector. The total return of the strategy is determined by the ratio of the final value to an initial value. The trading model of a fixed-mix strategy is shown in the appendix in model 2.2.

4.3 Prime rates

In the computations the prime rates are selected over a span of 15 years with 3736 daily observations from 1 September 1994 to 1 September 2009. The prime rates are shown in figure 4.3.1 below. The prime rates are from 18.5 percent to 10 percent with a range of 15.5 over a span of 15 years. The average of the prime rates is 15.35 percent. The minimum prime rate is 10 percent and the maximum rate is 25.5 per-

cent. The prime rates are considered down-trending with a total underperformance of 54 percent over a period of 15 years.

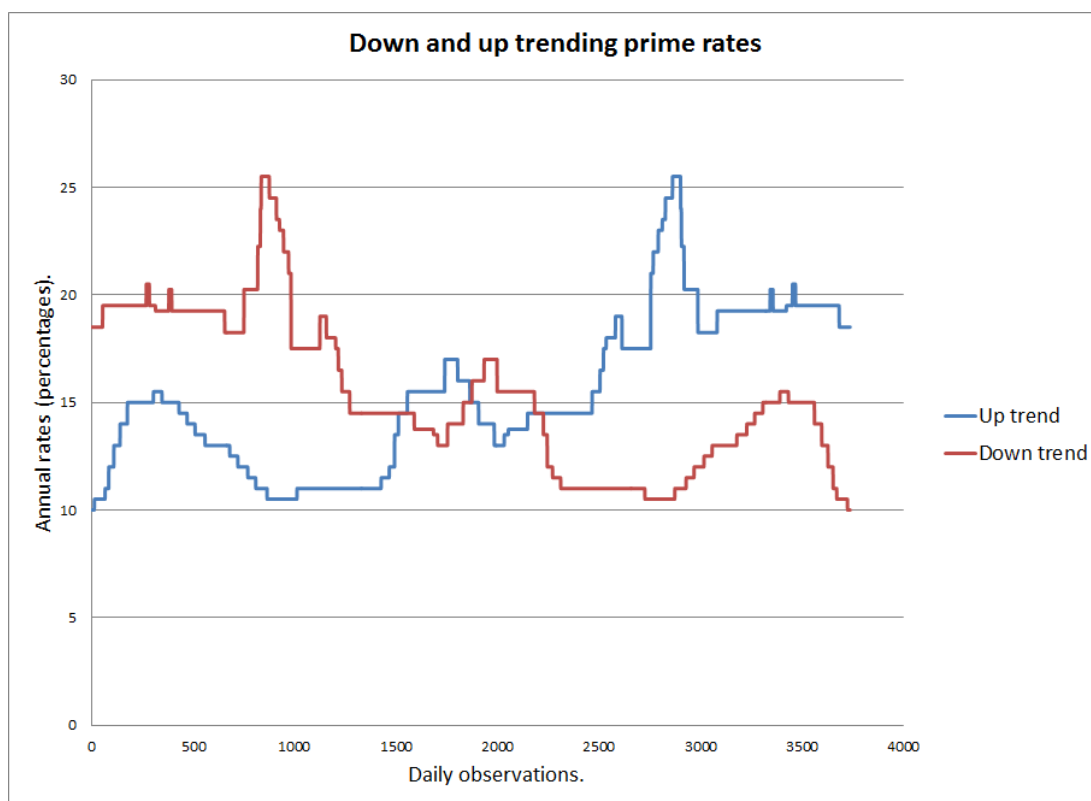


Figure 4.3.1: The prime rates and the reversed-prime rates.

The variations of the prime rates are indicated by the standard deviation of 3.49. The annual and the daily standard deviation rates are 0.90 percent and 0.06 percent respectively. The distribution of the prime rates is found to be positively skewed which indicates a high frequency of rates to be above the mean rate of 13.35 percent and a negative kurtosis statistic. This shows large dispersion of low frequent rates from the average rate.

For simulation purposes the prime rates are reversed to prime rates from 10 percent to a rate of 18.5 percent which are used in the computations to consider the scenario of an up-trending prime rate over the period of 15 years.

4.4 Asset prices and total returns of a fixed-rate asset and risky assets of selected companies

The time series presented are of a span of 15 years. The asset prices are shown as natural logarithms of initialised asset prices. The price data report of ABSA Group Limited presents 3736 daily closing asset prices and the price data report of AFGRI Limited presents 3187 daily closing asset prices. The price data report of Anglo Gold Ashanti Limited presents 2987 daily closing asset prices and the price data report of Anglo American Platinum Limited presents 3736 daily closing asset prices. The price data report of Sasol Limited presents 3736 daily closing asset prices. Finally the price data report of Standard Bank Limited presents 3736 daily asset prices. The prices of risky assets are presented as initialised prices which are weighted by an initial price. The uncertainty of asset prices is defined as the standard deviation of asset total returns.

4.4.1 Asset prices and total returns of a fixed-rate asset

The fixed-rate asset is considered for three prime rates conditions, the up-trending prime rates, the down-trending prime rates and the constant rate. The effective rate of the fixed-rate asset in the down-trending prime rates is from the initial rate of 0.0007 to a rate of 0.0004 in 255 trading days in one year. The up-trending prime rates are from 0.0004 to 0.0007 and the constant rate is constant at a rate of 0.0004 over 3736 discrete observations. The fixed-rate asset earns a short rate over a discrete trading period which is quoted as a current prime rate. The standard deviation of log prices of a fixed-rate asset is 0.636. The annual and daily standard deviation rates are 16.42 percent and 3.97 percent respectively.

The accumulating asset prices of a fixed-rate asset in three conditions of the prime rates are shown in the figure 4.4.1 in units of 100 percent. The fixed-rate asset prices in down-trending prime rates are shown to have small curvature and higher prices than in up-trending prime rates because of the effectiveness of the initial high rates in down-trending prime rates.

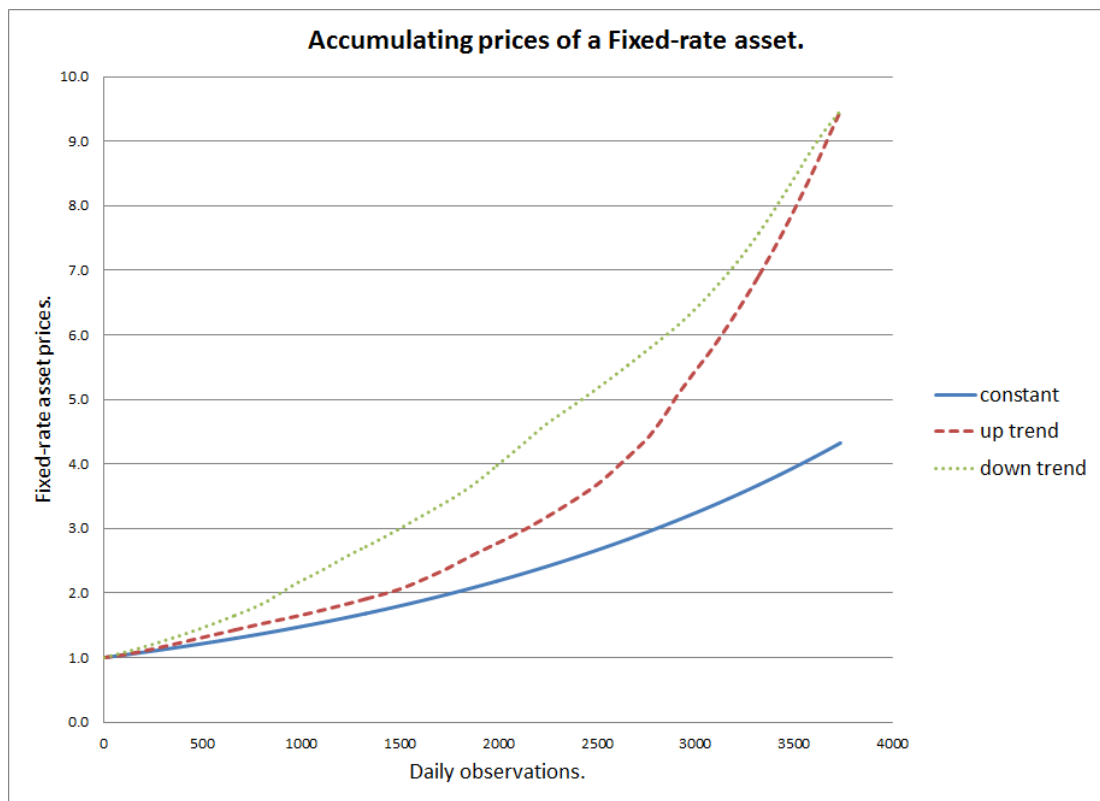


Figure 4.4.1: The fixed-rate asset prices of a one day short rate.

The average of total returns of a fixed-rate asset is 0.0006 and the return of an initialised price over 3736 discrete trading periods is 946.64 percent. The fixed-rate asset is set as a benchmark for the performance of the portfolio.

4.4.2 Asset prices and total returns of a risky asset of ABSA Group Limited

The trend analysis shows the linear trend of the logarithm of asset prices to revert along a trend approximately over a thousand daily observations in a population of 3736 data. The descriptive statistics of logarithm of prices of risky asset are shown in the appendix in table A.4.2.1. The logarithm of initialised asset prices ranges from a minimum value of -.0488 to a maximum value 2.6479 with a statistic range of 2.6967. The average value of logarithm of initialised asset prices is 1.3976 and the standard deviation is 0.6932.

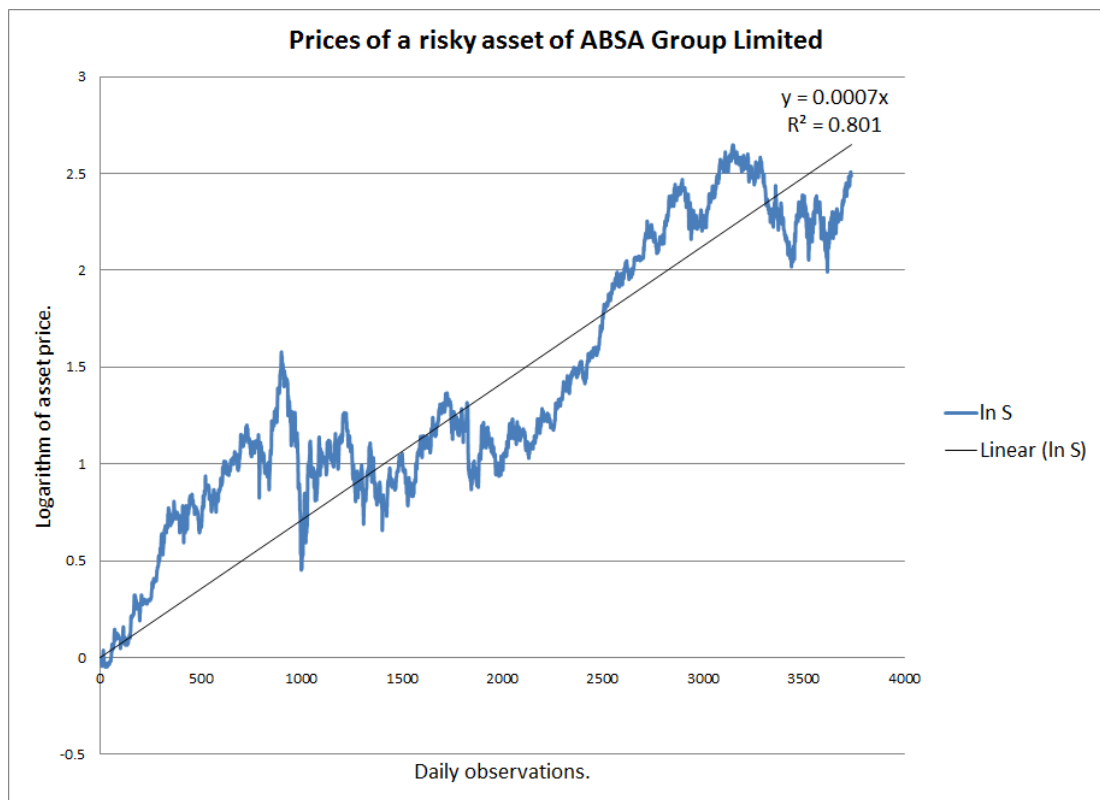


Figure 4.4.2.a: Natural logarithm of initialised daily prices of a risky asset of ABSA Group Limited.

The annual and the daily standard deviation rates are 17.90 percent and 4.34 percent respectively. The rates are computed on the empirical parameters (see Appendix 1: The market model).

The skewness of asset prices is positive which shows asset prices to be more frequently below the average value of 1.3976. The asset prices are found to have fatter tails which is indicated by a negative kurtosis statistic which shows prices of large dispersion from the mean to be distributed at a small frequency. The descriptive statistics of total returns of asset prices are shown in the appendix in table A.4.2.2. The total returns of logarithm of initialised asset prices ranges from a minimum value of -0.1796 to a maximum value of 0.1371 with a statistic range of 0.3167. The mean of total returns of logarithm of initialised prices of a risky asset is 0.0007 which is similar to an estimated linear rate of logarithm of initialised asset prices indicating the

drift of asset prices. The standard deviation of total returns is 0.0230. The annual and the daily standard deviation rates are 0.59 percent and 0.14 percent respectively. The total returns are negatively skewed which indicates more frequent total returns to be above the long run rate of 0.0007. The total returns are found to have thinner tails which are shown by a positive kurtosis statistic which indicates more frequent total returns to average at a long run rate.

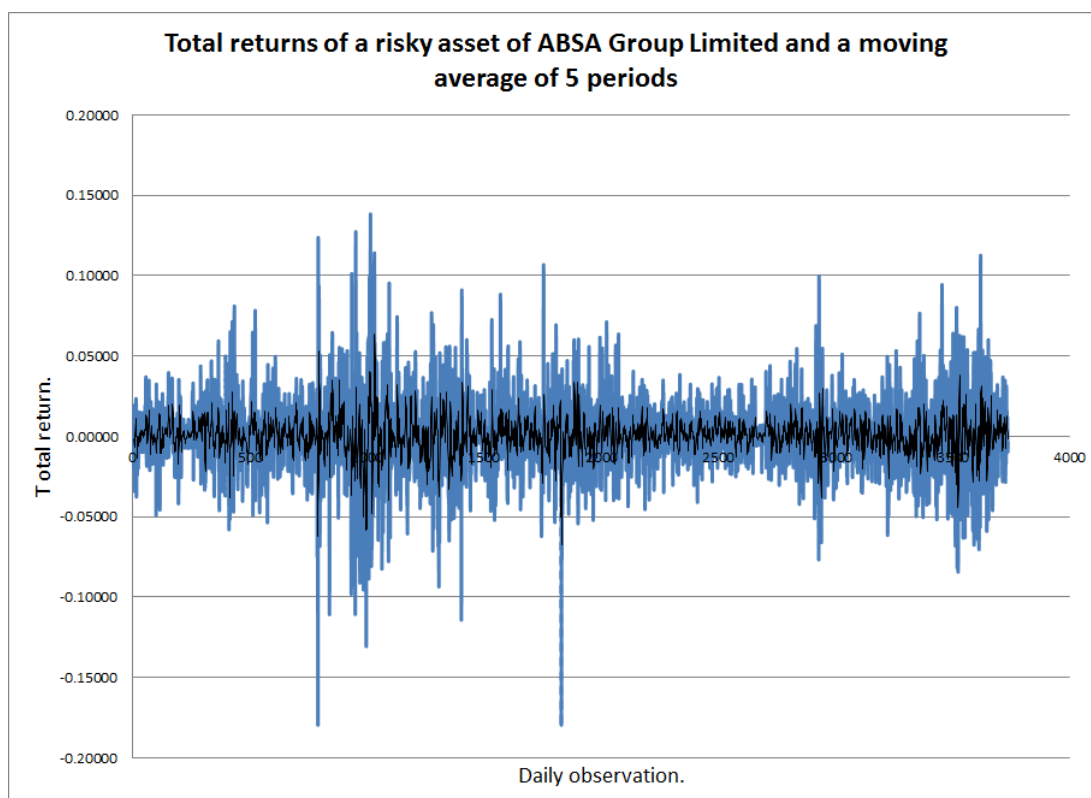


Figure 4.4.2.b: The total returns of daily asset log prices of ABSA Group Limited and a moving average of 5 periods.

The total return of an initial price of a risky asset over a period of 3736 daily observations is 1203 percent. The basis point is computed as a percentage of 0.01. The total returns of the Absa Group Limited natural logarithms of initialised asset prices are shown in figure 4.4.2.b.

The growth rates of a risky asset of the ABSA Group Limited over the span of 3736 discrete periods are shown in the appendix in chart A.4.1.13. The rate in the long run

is strictly above zero. The condition of non-degeneracy of asset prices in a portfolio is shown in chart A.4.1.1 where the price ratios are non-constant and vary with variations of a risky asset. The total returns are found to average at a value of 7 basis points and frequently distributed above the average which is found similar to an estimated long run of asset log prices.

4.4.3 Asset prices and total returns of a risky of Anglo American Platinum Limited

The prices of a risky asset of Anglo American Platinum Limited are shown in figure 4.4.3.a.

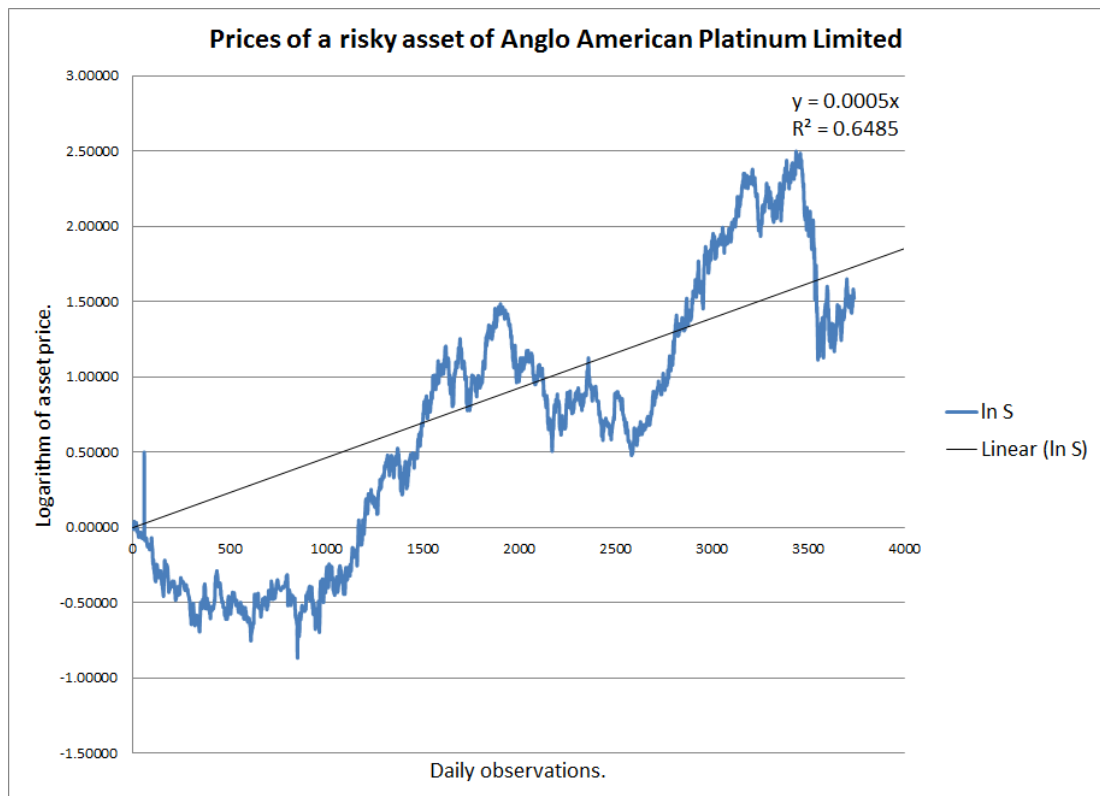


Figure 4.4.3.a: Natural logarithms of initialised daily prices of a risky of Anglo American Platinum Limited.

The logarithm of initialised asset prices of Anglo American Platinum Limited is ranging from a minimum value of -0.8655 to a maximum value of 2.4987 with an

average value of 0.6855 over 3736 daily observations. The estimated linear curve is shown with a linear rate of 0.0005. The distribution of logarithm of initialised asset price is positively skewed where frequent logarithms of initialised prices are below the long run rate. The standard deviation of logarithm of initialised asset prices is 0.9017. The annual and the daily standard deviation rates are 23.28 percent and 5.65 percent respectively.

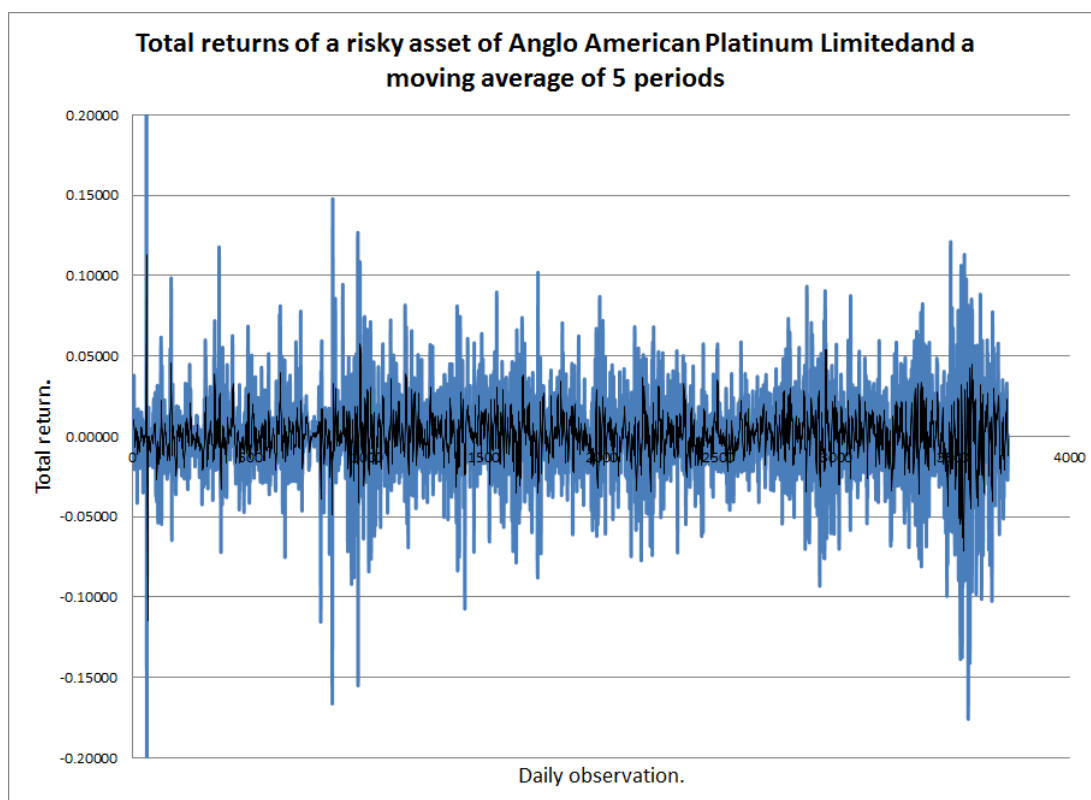


Figure 4.4.3.b: The total returns of daily prices of a risky asset of Anglo American Platinum Limited and a moving average of 5 periods.

The total returns of Anglo American Platinum Limited initialised asset prices range from a minimum value of -0.5601 to a maximum value of 0.5579 with an average value of 0.0004 which is slightly less different to an estimated long run rate of log prices. The total returns are distributed with a negatively skewed distribution where more frequent total returns are above the average value of 0.0004. More frequent total returns are close to an average value with less difference at a standard deviation of 0.0302. The annual and the

daily standard deviation rates are 0.78 percent and 0.19 percent respectively. The total return of an initial price of a risky asset over a period of 3736 daily observations is 529 percent. The excess of an estimated long run rate of log prices and the average of total returns is 1 basis point. The portfolios are found to perform efficiently under an active portfolio management strategy given that the portfolio is inefficient. The growth rates of log prices are shown in chart A.4.1.16 with a non-zero rate at the long run. The price ratios in a portfolio are shown in chart A.4.1.4 with a non-constancy condition.

4.4.4 Asset prices and total returns of a risky asset of Sasol Limited

The time series of Sasol limited is shown with a linear trend estimated at a linear rate of 0.0006 over 3736 daily observations.

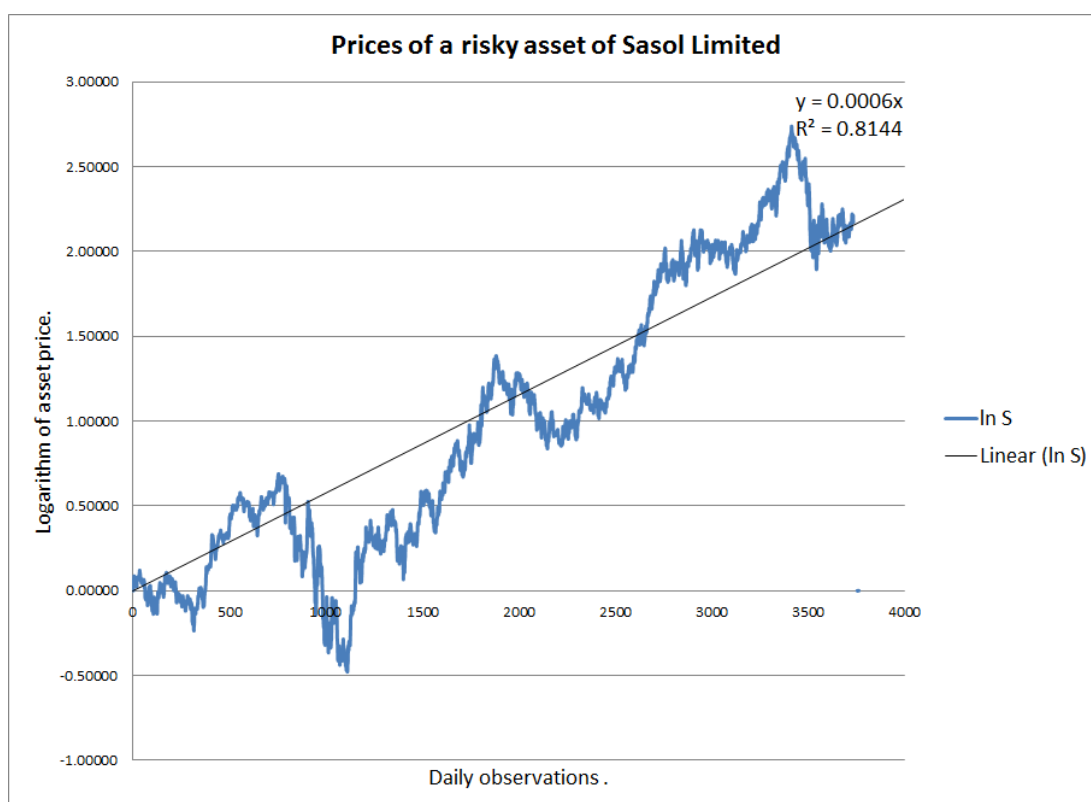


Figure 4.4.4.a: Natural logarithms of initialised daily prices of a risky asset of Sasol Limited.

The total return of an initial price of a risky asset over a period of 3736 daily observations is 878 percent. The logarithm of initialised asset prices of Sasol Limited ranges from a minimum value of -0.4788 to a maximum value of 2.7382 with an average value of 1.0046. The distribution is positively skewed where more frequent prices are below the average value of 1.0046. There is less difference in the frequent distribution of asset price with a standard deviation of 0.8241. The annual and the daily standard deviation rates are 21.28 percent and 5.16 percent respectively.

The total returns of logarithm of initialised asset prices are from a minimum value of -0.11735 to a maximum value of 0.1429 with an average value of 0.0006 common to an estimated long run rate of the logarithm of prices. The standard deviation is 0.0249 with a negatively skewed distribution where more frequent total returns are above the average value of 0.0006.

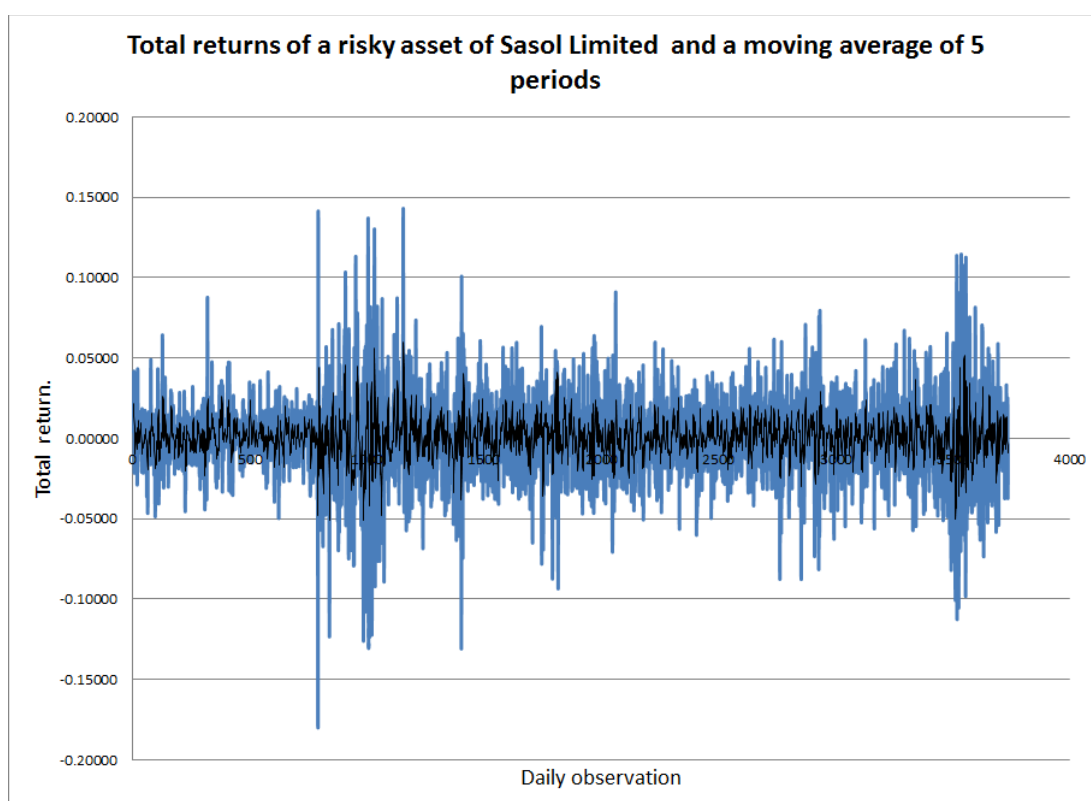


Figure 4.4.4.b: The total returns of daily prices of a risky asset of Sasol Limited and a moving average of 5 periods.

The annual and the daily standard deviation rates are 0.64 percent and 0.16 percent respectively. The excess kurtosis is 1.053 which shows a high frequency of total returns to be distributed closely to an average of 0.0006. The estimated long run rate of logarithm of prices is observed to be similar to the average of daily total returns at a value of 0.0006. The growth rates of prices are shown in the appendix in chart A.4.1.17 with a non-zero rate in the long run. The non-constancy condition of a portfolio of a risky asset of Sasol Limited is shown in the chart A.4.1.5.

4.4.5 Asset prices and total returns of a risky asset of Standard Bank Limited

The logarithm of initialised asset prices of Standard Bank Limited depicts a market trending with an estimated linear rate of 0.0006 over 3736 daily observations as shown in figure 4.4.5.a.

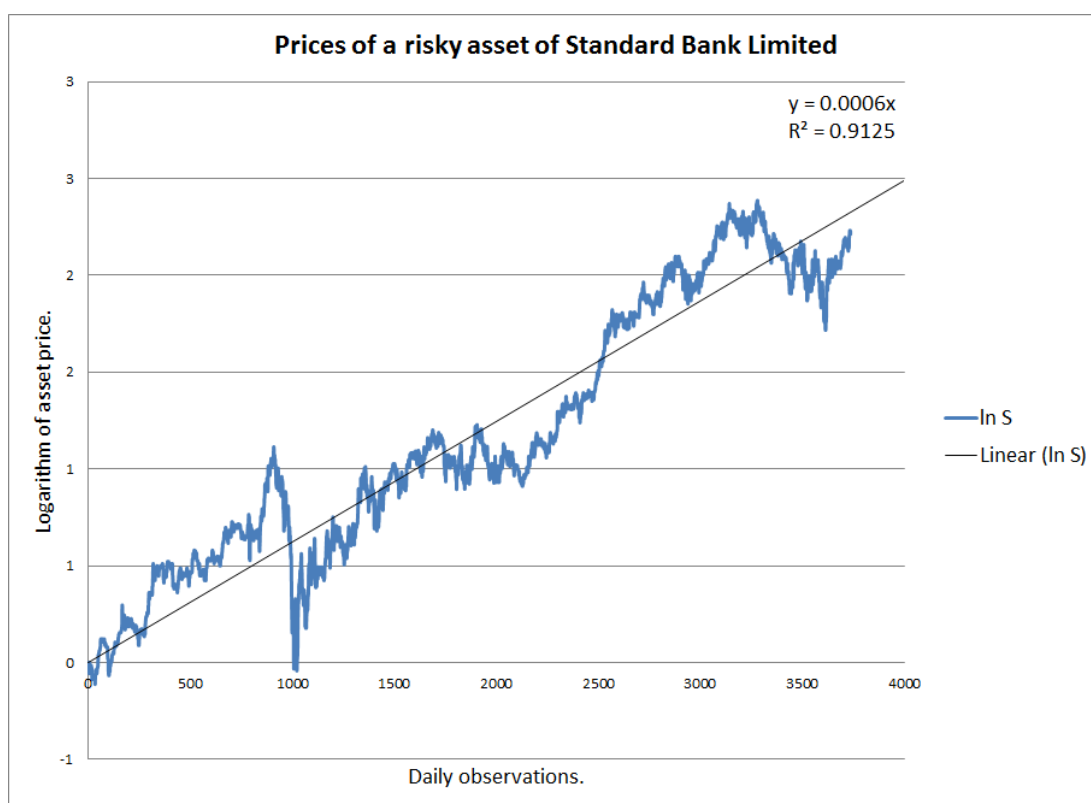


Figure 4.4.5.a: Natural logarithms of initialised daily prices of a risky asset of Standard Bank Limited.

The total return of asset prices over the observation period is 914 percent. The logarithm of initialised asset prices range from a minimum value of -0.1126 to a maximum value of 2.3868 with an average value of 1.1617. The distribution of logarithm of initialised asset price is positively skewed with more frequent asset prices below an average value of 1.1617 and less frequent prices over fatter tails of the distribution. The standard deviation of log prices is 0.6757. The annual and the daily standard deviation rates are 17.45 percent and 4.23 percent respectively.

The total returns of initialised asset prices of Standard Bank Limited range from a minimum value of -0.1863 to a maximum value 0.1379 with an average value of 0.0007. The total returns are distributed with a negatively skewed distribution where more frequent total returns are above the average return of 0.0007 and more distributed closely to an average return at a standard deviation of 0.0226. The annual and the daily standard deviation rates are 0.58 percent and 0.14 percent respectively.

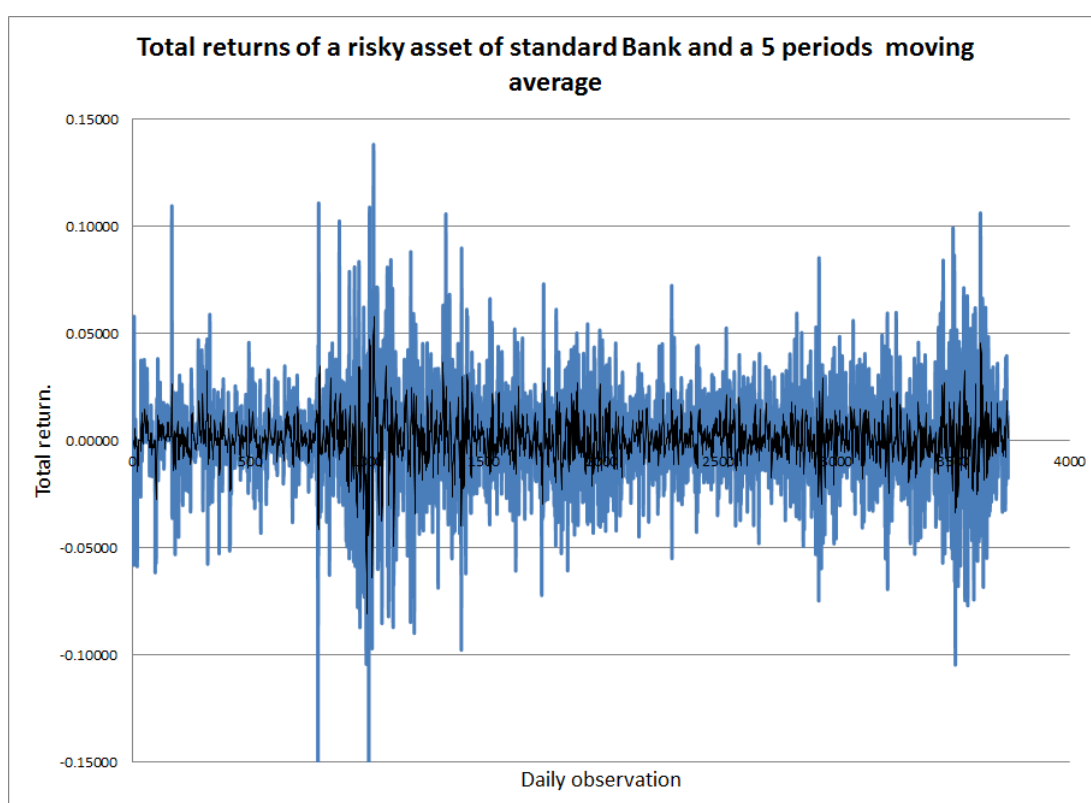


Figure 4.4.5.b: The total returns of daily prices of a risky asset of Standard Bank Limited and a moving average of 5 periods.

The efficient portfolios of a risky asset of Standard Bank Limited and a fixed-rate asset are determined to perform above the long run total return of 0.0007 under active portfolio strategies. This is in line with the Merton model (see Dempster *et al.*, 2007).

4.4.6 Asset prices and total returns of a risky asset of AFGRI Limited

The asset prices are daily closing prices of a risky asset of AFGRI Limited over a span of 15 years and are shown in figure 4.4.6.a as logarithm of prices. The data has 3187 daily observations of AFGRI Limited risky asset prices.

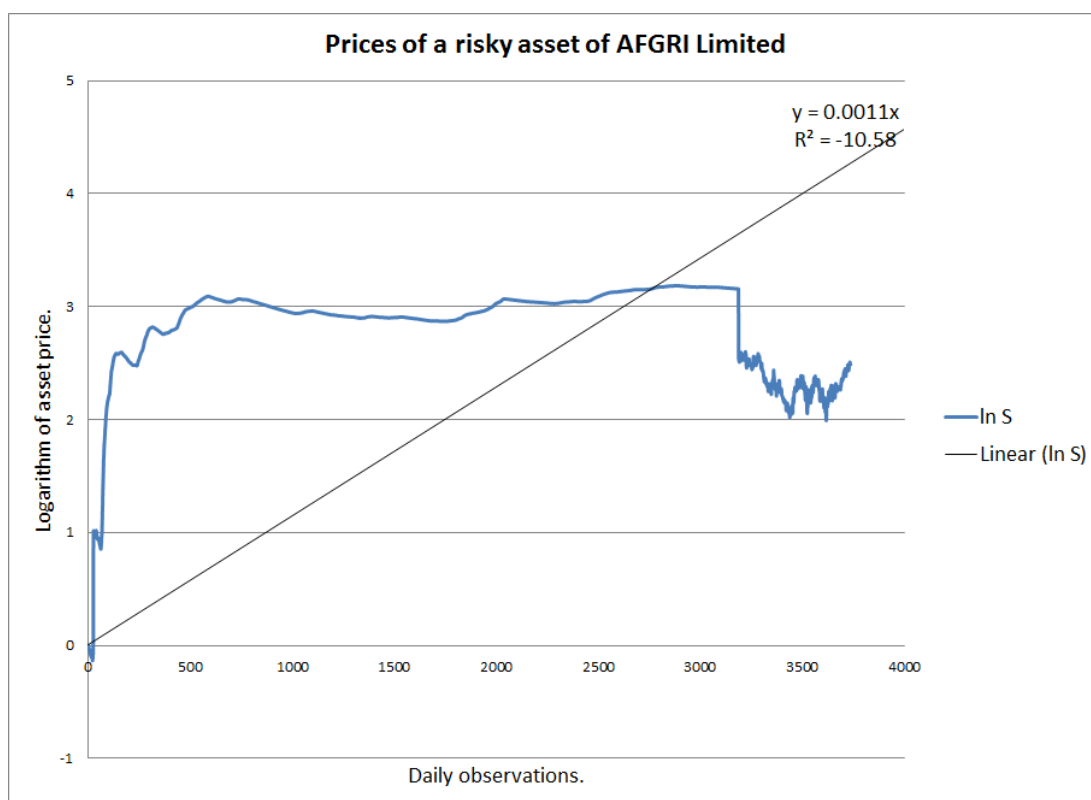


Figure 4.4.6.a: Natural logarithms of initialised daily prices of a risky asset of AFGRI Limited.

The trend analysis shows a growth of AFGRI Limited logarithm of initialised asset prices with an estimated linear rate of 0.0007. The logarithm of asset prices ranges from a minimum value of -0.1335 to a maximum value of 3.1842 with a standard de-

viation of 0.4292 and an average of 2.8310. The annual and the daily standard deviation rates are 11.08 percent and 2.69 percent respectively. The logarithm of initialised asset prices has a negatively skewed distribution where frequent returns are below the average of 2.8310. The positive kurtosis statistic shows the distribution to have more frequent total returns close to an average value.

The total returns of AFGRI limited asset prices are shown in figure 4.4.6.b. The total returns range from a minimum value of -0.6187 to a maximum value of 1.0953 with a standard deviation of 0.0235 and an average value of 0.0007. The annual and the daily rates of the volatility of a risky asset of AFGRI Limited are 0.61 percent and 0.15 percent respectively. The total returns are negatively skewed where more frequent returns are below the average value of 0.0007 and more frequent total returns are close to an average value. The total return of asset prices over a period of 3187 daily observations is 1203 percent.

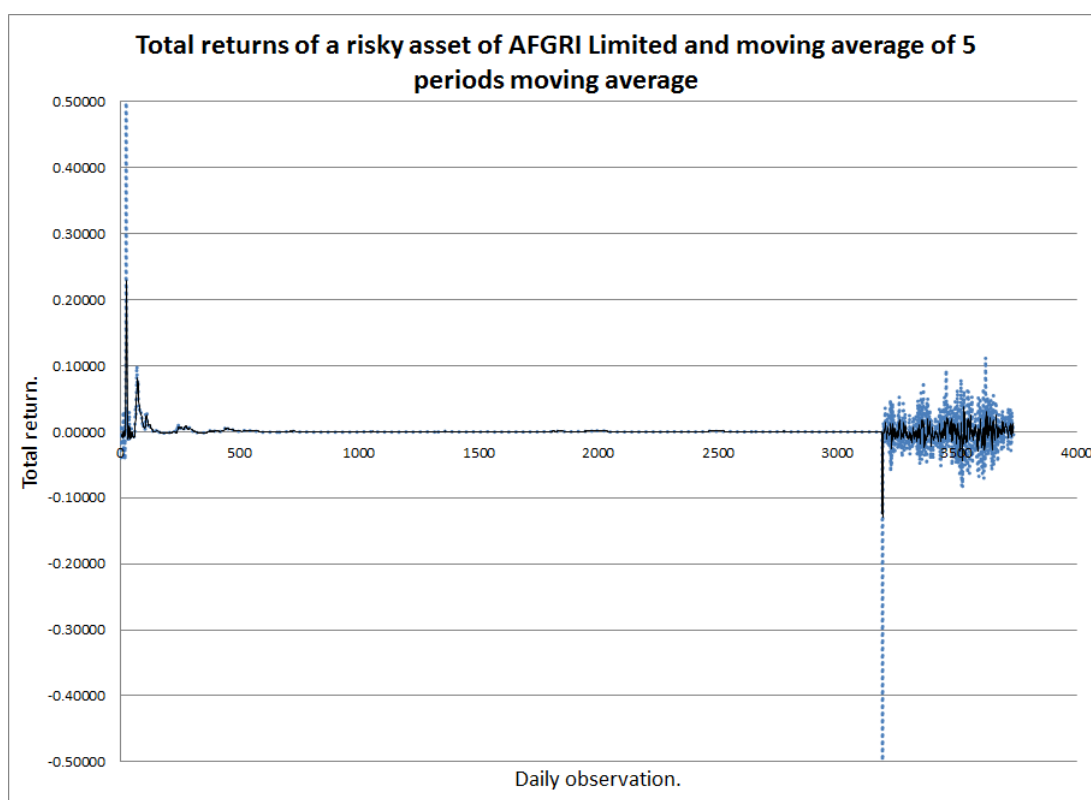


Figure 4.4.6.b: The total returns of daily prices of a risky asset of AFGRI Limited and a moving average of 5 periods.

The efficient portfolios are determined to outperform the long run rate of total returns of 0.0007 under active portfolio strategies. In line with the fixed-mix portfolio strategy in terms of the drift rate, the rate of a fixed-rate asset plus the premium of the strategy (see a Two-fund model of Merton in Mulvey *et al.*, 2007). The long-run rate of total returns of a risky asset specifies the estimated linear trend.

4.4.7 Asset prices and total returns of a risky asset of Anglo Gold Ashanti Limited

The logarithm of initialised daily asset prices of Anglo gold Ashanti Limited are shown to revert along a trend with an estimated linear rate of -0.00004 as depicted in figure 4.4.7.a.

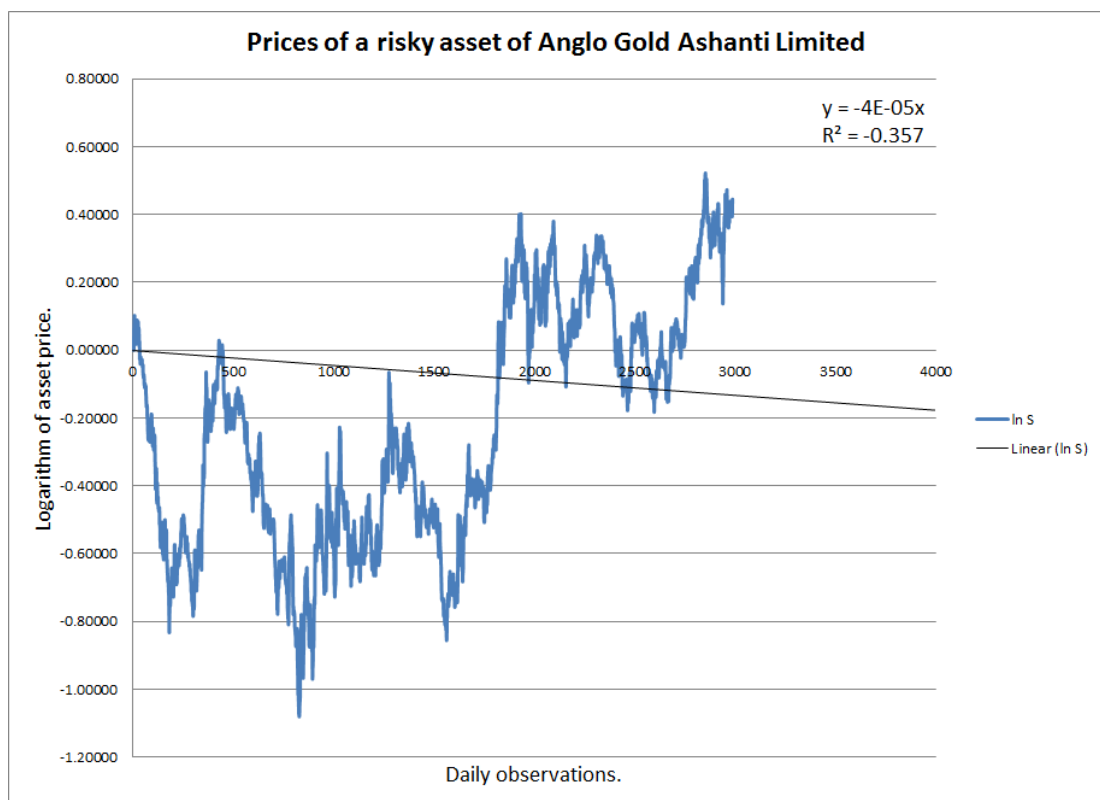


Figure 4.4.7.a: Natural logarithms of initialised daily prices of a risky asset of Anglo Gold Ashanti Limited.

The logarithm of initialised asset prices ranges from a minimum value of -0.0800 to a maximum value of 0.5215 with an average value of -0.2238. The standard deviation of the logarithm of initialised asset prices is 0.3553 where more frequent logarithm of initialised asset prices are below the average value of -0.2238. The annual and the daily rates of the standard deviation of a risky asset of Anglo Gold Ashanti Limited are 9.17 percent and 2.23 percent respectively. The prices frequently underperform the initial price of a risky asset.

The total returns of Anglo Gold Ashanti Limited are shown in figure 4.4.7.b and range from a minimum value of -0.1053 to a maximum value of 0.1494 with an average value of 0.0001. The total returns are positively skewed with a standard deviation of 0.0259 where more frequent total returns are below the average value of 0.0001. The annual and daily rates of the standard deviation of the total returns of a risky asset of Anglo Gold Ashanti Limited are 0.67 percent and 0.16 percent respectively.

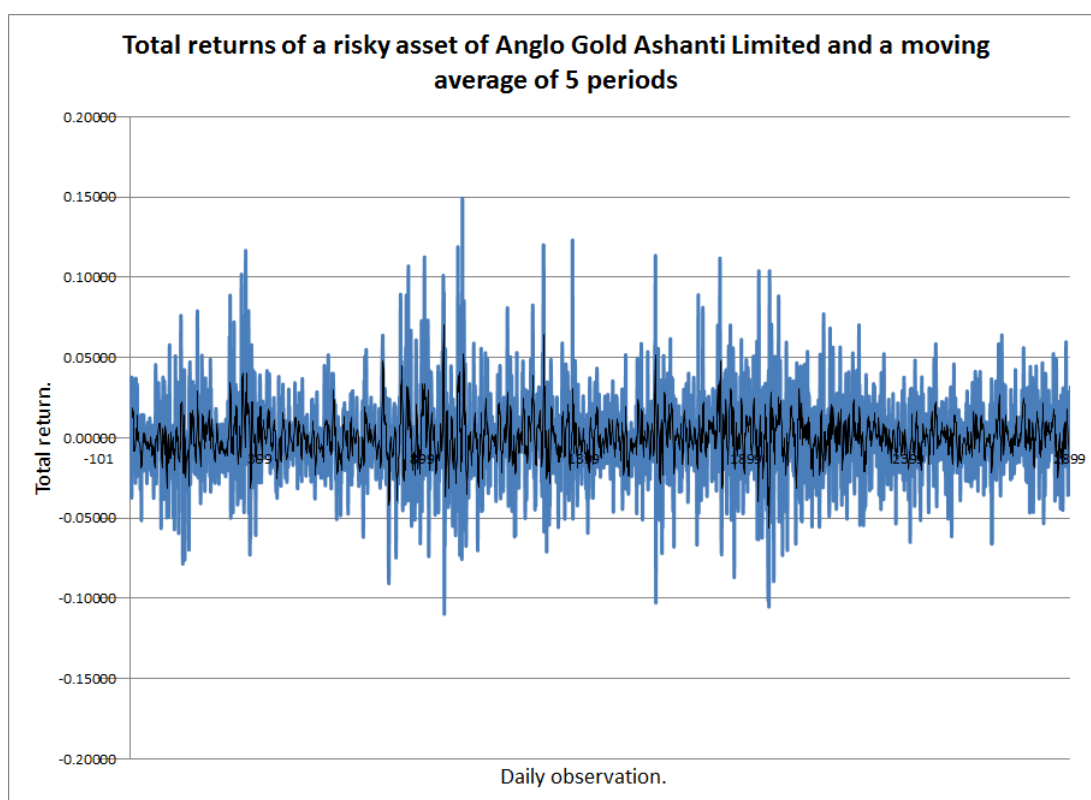


Figure 4.4.7.b: The total returns of daily prices of a risky asset of Anglo Gold Ashanti Limited and a moving average of 5 periods.

The total returns are distributed with a slightly less different frequency where large deviations from the mean are frequently common. The total return of asset prices over a period of 2987 daily observations is 147 percent. The portfolios of a risky asset of Anglo Gold Ashanti Limited and a fixed-rate asset are determined as efficient over a benchmark return.

The frequent reversals are required by fixed-mix portfolio strategies to yield exponential growth of returns. The quantification of total returns shows frequent reversals which appear above and below the average line. This shows discrete total returns of logarithm of initialised asset prices which average to an estimated long run rate of asset prices. The quantification of total returns sets the feasibility of a risk-adjusted strategy under a prospect of financial engineering. The significance of the quantification of returns is determined by the deviations of total returns from the average line. The modelling of risk - adjusted returns is validated by the significance of total returns in order to determine a risk-adjusted strategy. For modelling purposes a financial engineered strategy is determined to reward an estimated return process which is strictly greater than the absolute total returns process. The anticipated or estimated total returns process is determined by time effective portfolios that are favourable to portfolio objectives such that the estimated risk adjusted returns are modelled by a financial engineered instrument.

The expected gains or unrealized potential of stochastic asset prices under fixed-mix portfolio strategies are due to a rebalancing technique which adjusts portfolio proportions discretely. The expected gains arise between rebalancing periods which are analytically characterised by total returns between rebalancing periods as determined by Qian (2012) as the excess of volatility in a portfolio and a portfolio strategy. The evolution of total returns process induces a stationary processes which triggers rebalancing at rebalancing periods or after a specified deviation of portfolio proportions.

4.5 Quantification of anticipated portfolio growth

The financial growth which results from the contribution of volatility in the financial market according to Dempster *et al.* (2007) widens the scope of portfolio selections where highly volatile assets are expected to yield financial growth. The result of fast exponential growth by Dempster *et al.* (2010) under the condition of prices with a trend underlines the efficient trades of a fixed-mix portfolio such that the growth rate of a fixed-mix portfolio is strictly greater than the growth rate of a trending index.

According to Dempster *et al.* (2007) in financial applications and empirically the condition of stationarity is commonly noted in asset prices returns and as well as in the currency market. The empirical analysis of risky asset prices of the selected companies' shows the evidence of trend-stationarity which is shown by an estimated linear graph with a non-zero linear rate in trend-stationarity and an Integrated Autoregressive Moving Average of order zero. The logarithm of initialised asset prices are modelled by an ARIMA model which is a combination of a Moving average of order q and an Integrated Autoregressive model of order p . According to (Brook , 2002) the Moving average of order p represents the current asset price with an estimated drift and p independently and identically distributed disturbance terms and the Autoregressive model represent the current price with an estimated drift and p previous weighted prices and the current disturbance term. To induce stationarity in the ARIMA (p, d, q) model differencing is performed up to d order. The models of relative asset prices are found stationary with an Autoregressive integrated moving average model of order zero showing no presence of a unit root which characterises a stationary model implying a stationary condition in a portfolio. The charts of one lag differenced models of relative asset ratios in up-trending prime rates and in down-trending prime rates are shown in the appendix in chart A.2.1.1.

4.5.1 The model of asset prices and total returns of ABSA Group Limited

The logarithms of initialised asset prices of ABSA Group Limited shown in figure 4.4.2.a are found to revert along a trend of an estimated linear rate of 0.0007. The long run exponential growth rate of asset prices is 0.0007 which is common with a linear rate of the trend. The prices are positively skewed with high frequency of asset prices below the average.

The ARIMA model of natural logarithms of initialised asset prices is ARIMA (2, 1, 8) with a stationary R – squared statistic of 0.018 and R – squared statistic of 0.99 which are model fit statistics. The model of total returns is ARIMA (2, 0, 8) with a stationary R-squared and R-squared statistics of a value of 0.018. Graphically the log returns are noted to be distributed normally with a high frequency of prices above the average as shown in chart A.4.1.7. The log returns are found to be stationary and with a converging asymptotic average of the moving averages of 5 periods. The 5 periods moving averages of total returns range almost within the interval of -0.05 to 0.05 total returns as shown in figure 4.4.2.b which according to the results of fast exponential growth the asymptotic average or the long run mean of moving averages is required to converge at a constant value.

4.5.2 The model of asset prices and total returns of Standard Bank Limited

The logarithms of initialised asset prices of Standard Bank Limited are found to trend and revert along a linear trend with an estimated linear rate of 0.0006 as shown in figure 4.4.5.a. The long run exponential growth rate is 0.0006. The prices are found to have a high frequency of prices below the average as indicated by a negative skewness statistic shown in the Table 4.2.1. The model of logarithms of initialised asset prices is a ARIMA (2, 1, 1) with a stationary R-squared statistic of 0.08 and R-squared statistic of 0.99. The model of total returns is ARIMA (2, 0, 1) with a stationary statistic R-squared and R-squared statistics of a value of 0.08. The log returns graphically are distributed normally with high frequency of returns above average as

shown in chart 4.1.12. The total returns are found to be stationary where a 5 periods moving average ranges almost within the interval of -0.05 to 0.05 of the total returns indicated in figure 4.4.5.b. The model of relative asset prices integrated of order one into a unit root model for up-trending prime rates and down-trending prime rates are determined as an ARIMA (0, 0, 0). The charts of differenced models are shown in the appendix in chart A.2.1.3 and chart A.2.1.4 indicating a stationary condition in a portfolio.

4.5.3 The model of asset prices and total returns of Sasol Limited

The logarithm of initialised asset prices of Sasol Limited trends at an estimated linear rate of 0.0006 as shown in figure 4.4.4.a. The long run growth rate is determined to be 0.0006. The prices are frequently distributed below the average as indicated by a negative skewness statistic shown in Table 4.2.1. The ARIMA model of logarithm of initialised asset prices is ARIMA (0, 1, 3) with a stationary R-squared statistic of 0.009 and R-squared statistic of 0.9. The logarithms of returns graphically are distributed normally with high frequency of prices above the average as shown in chart 4.1.11. The model of total returns is ARIMA (0, 0, 3) with a stationary R-squared statistic and R-squared statistic of a value of 0.009 indicating a stationary process. The total returns are stationary with a 5 period moving averages almost within the interval of -0.05 to 0.05 of the total returns as shown in figure 4.4.4.b. The model of relative asset prices integrated of order one in a unit root model for up-trending prime rates and down-trending prime rates are determined as an ARIMA (0, 0, 0) and the chart of differenced models are shown in the appendix in chart A.2.1.5 and chart A.2.1.6. The non-degeneracy condition of a portfolio of a risky asset of Sasol Limited is shown in chart 4.1.5 indicating a volatile portfolio.

4.5.4 The model of asset prices and total returns of Anglo American Platinum Limited

The logarithms of initialised asset prices of Anglo American Platinum Limited trends and reverts along a trend with a linear rate of 0.0005 as shown in figure 4.4.3.a. The exponential growth rate of prices observed within 500 observations varies in the in-

interval of -0.0033 to 0.01 and gradually increases to a long run rate of 0.00056 as indicated by chart 4.1.16. The prices are frequently distributed below the average as indicated by a negative skewness statistic in Table 4.2.1. The model of logarithms of initialised asset prices of Anglo American Platinum Limited is ARIMA (1, 1, 15) with a stationary statistic of 0.021 and R-squared statistic of 0.99. The logarithms returns graphically are distributed normally with a high frequency of prices above the average as shown in chart 4.1.10. The model of total returns is ARIMA (1, 0, 15) with a stationary R-squared and R-squared statistics of 0.021 indicating a stationary process of returns. The 5 periods moving averages range almost within the interval of -0.05 to 0.05 total returns as shown in figure 4.4.3.b. The model of relative asset prices integrated of order one in a unit root model for up-trending prime rates and down-trending prime rates are determined by an ARIMA (0, 0, 0) and the charts of differenced models are shown in the appendix in chart A.2.1.7 and chart A.2.1.8. Indicating a stationary condition in a portfolio and a non-degeneracy condition indicating a volatile portfolio is shown in chart A.4.1.4.

4.5.5 The model of asset prices and total returns of Anglo Gold Ashanti Limited

The logarithm of initialised asset prices of Anglo Gold Ashanti Limited is found to revert along a trend with a linear rate of -0.00004 as shown in figure 4.4.7.a. The logarithm of initialised asset prices grow at a negative exponential rate which ranges within the interval of -0.005 to 0.04 for prices observed up to 500 observations and gradually increases in a long run at a rate of 0.00014 as indicated in chart A.4.1.15. The prices are distributed frequently above the average as indicated by a positive skewness statistic in Table 4.2.1. The model of logarithms of initialised asset prices is ARIMA (0, 1, 1) with a stationary R-squared statistic of 0.006 and R-squared statistic of 0.995.

The logarithms of returns graphically are distributed normally with a high frequency of returns above the average zero as shown in chart A.4.1.9. The model of total returns is ARIMA (0, 0, 1) with stationary R-squared and R-square statistics of 0.006.

The total returns are stationary with a 5 periods moving averages almost within the interval of -0.05 to 0.05 total returns as indicated in figure 4.4.7.b. The model of relative asset prices integrated of order one in a unit root model for up-trending prime rates and down-trending prime rates are determined as an ARIMA (0, 0, 0) and the chart of differenced models are shown in the appendix in chart A.2.1.9 and chart A.2.1.10. The non-degeneracy condition of a portfolio of a risky asset of Anglo Gold Ashanti Limited indicating a volatile portfolio is shown in chart A.4.1.3.

4.5.6 The model of asset prices and total returns of AFGRI Limited

The logarithm of initialised asset prices of AFGRI Limited trends at an estimated linear rate of 0.01 as shown in figure 4.4.6.a. The exponential growth rate of a risky asset gradually decreases from 0.042 to 0.0006 indicating a stable growth of a risky asset as shown in chart A.4.1.14. The prices are negatively skewed with a high frequency of prices above the average as indicated in by a skewness statistic in Table 4.2.1.

The model of logarithms of initialised asset prices is ARIMA (1, 1, 1) with a stationary R-squared statistic of 0.03 and R-squared statistic of 0.97. The total returns are positively skewed with a high frequency of prices below the average as shown in chart A.4.1.8 and indicated by a positive skewness statistic in Table 4.2.2. The model of total returns is ARIMA (1, 0, 1) with a stationary R-squared and R-squared statistics of 0.003. The log returns are found to range within the interval from -2 to 2 where frequent total returns show significant small change.

4.6 The volatility in a fixed-mix portfolio

The variations of discrete total returns which are measured by a standard deviation of the value process of the portfolio strategy are shown to be reduced for fixed-mix portfolios (see Tables in appendix 5.2). The risk of buy and hold portfolios increases linearly with an increasing risk of leveraged positions of a risky asset. The risk of fixed-mix portfolios increases concavely with an increasing risk of a risky asset with short positions in a fixed-rate asset. The variations of discrete total returns of buy and

hold portfolios of a risky asset of ABSA Group Limited in down-trending prime rates increases as the portfolio risk increases with a larger factor of almost 1.3 unit rates of the standard deviation estimated by a trend. The reduced variations of discrete total returns of fixed-mix portfolios increase at a fraction of a unit rate which is a trend estimation rate of 0.3 units. The annual rates of the standard deviation of log prices of the selected companies ranges from a rate of 9.17 percent for a risky asset of Anglo Gold Ashanti Limited to a value of 23.28 percent for a risky asset of Anglo American Platinum Limited indicating increased fluctuations of risky assets from large frequent reversals. The annual volatility of asset prices ranges from a rate 0.69 percent for a risky asset of Standard Bank to a rate of 0.78 percent for a risky asset of Anglo American Platinum Limited showing low degree of variability of annual total returns of risky assets. The annual rates of the standard deviation of log prices of a fixed-rate asset in up-trending prime rates and in down-trending prime rates are 16.43 percent and 16.42 respectively indicating a significant variation in the growth of a fixed-rate asset. The volatility of a fixed-rate asset in up-trending and down-trending prime rates is 0.0001 indicating an upside of 1 basis point in a fixed-rate asset position.

The evidence of non-degeneracy of asset prices as the assumption of a volatile portfolio in a fixed-mix portfolio of a risky asset of ABSA Group Limited and a fixed-rate asset is noted by the non-constancy of the ratios of relative prices from a ratio of 1 up to a ratio of 14. The fixed-rate asset yields a value of 1 in the short rate quoted from the prime rates over a period of 255 discrete trading periods. The relative price ratios of assets in up-trending prime rates and the down-trending prime rates have a small significant relative change ranging from 1 with an estimated linear rate of $2E-07$ as shown in the appendix in chart A.4.1.1. The relative change in the up-trending prime rates and the down-trending prime rates is significant when considering the effective short rate of 0.0004 in the up-trending prime rate and 0.0007 in the down-trending prime rates.

The ratio of relative prices in a fixed-mix portfolio of a risky asset of AFGRI Limited and a fixed-rate asset ranges from a ratio of 0.6 to a ratio of 1.54 as indicated in chart

A.4.1.2. The ratios of relative prices of assets in a fixed-mix portfolio of a risky asset of Anglo Gold Ashanti Limited are from a ratio of 0.34 to a ratio of 1.69 as indicated in chart A.4.1.3. The ratio of relative prices of assets in a fixed-mix portfolio of a risky asset of Anglo American Platinum Limited and a fixed-rate asset ranges from a ratio of 0.42 to a ratio of 12.17 as indicated in chart A.4.1.4. The ratio of relative prices in a fixed-mix portfolio of a risky asset of Sasol Limited and a fixed-rate asset ranges from a ratio of 0.62 to a ratio of 15.46 as shown in chart A.4.1.5. The ratio of relative prices in a fixed-mix portfolio of a risky asset of Standard Bank and a fixed-rate asset ranges from a minimum ratio of 0.88 to a maximum ratio of 10.88 as indicated in chart A.4.1.6.

4.7 Financial engineered formulation approach

The expectation of excess total returns under active portfolio management strategies is considerable in the presence of high volatility as shown by Qian (2012) in the diversification return. The Markowitz's (1952) model of optimization under a mean-variance model considers the efficient frontier as a set of portfolios with higher expectation of returns and minimum volatilities. In this regard inefficient portfolios under an active fixed-mix portfolio strategy are lucrative in yielding financial growth. The mean-variance model provides a solution in determining excess total returns which optimizes the total return over a deterministic time hedging as considered by Albanese and Tompaidis (2008) under the limitation of trades in high transaction costs. The mean-variance model of Markowitz with a maximum return and minimum volatility objectives is shown in the appendix in model 5.

To assess the potential of stochastic asset prices under the Markowitz's framework, the inefficient frontier of portfolios of higher volatility are expected to induce financial growth from volatile stochastic asset process. Under the formulation of a managed-risk reduction strategy which renders the financial condition of stationarity in a portfolio. Volatility reduction sets a strategy to reduce portfolio volatility by enhancing strategic appraisal of market of risk as defined by the Sharpe ratios as a measure of expected excess returns over the riskiness of the strategy.

The use of financial derivatives in hedging portfolio risk by use of portfolio insurance strategies in the optimization of dynamic portfolio insurance model by (Yao, 2012) dynamically hedge and transform the risk to yield financial growth. To derive a sufficient condition of fixed-mix portfolio strategies a transformation strategy which renders stationarity is attained by deriving a trend – stationary condition in the trending market with a lookback financial derivative instrument.

The required conditions are the volatility of asset prices and the stationarity of asset prices which generate a financial growth. The most fundamental shortcomings of the strategy are transaction costs since the rebalancing strategy requires frequent trading and as noted in Dempster *et al.* (2010) under balanced fixed-mix strategies a common barrier of excess returns are ineffective trades and diversification returns which is not optimal. To adjust to the shortcoming of transaction costs an approach by (Albanese and Tompaidis, 2008) suggests an interval of no transaction. The interval of no transaction sets a deterministic time hedge which rebalances at the end of the interval. A second shortcoming is the condition of stationarity in a fixed-mix portfolio to yield expected fast exponential growth of returns. To adjust to this limitation the evolution of asset prices which is regarded as a combination of total returns, where total returns normally represent a stationary stochastic processes (Dempster *et al.*, 2007) synthetically derives a tradable stationary security by use of a financial derivative instrument.

Under such consideration, to attain the condition of stationarity, de-trending is used to eliminate a linear trend in a stochastic process by employing a lookback straddle, a derivative instrument which ensures stationary returns with a payoff defined by a difference of a maximum and a minimum value of the asset price over the duration of a financial derivative. The lookback straddle is used as a proxy for hedge funds in trend following strategies to retain option-like returns in Darius *et al.* (2002). According to (Evstigneev and Schenk-Hoppé, 2002) the theory of fixed-mix strategies proves the optimality of such strategies in stationary market and the problem of adjusting or shifting the stochastic system to a stationary form is resolved by guiding

the drifts of asset prices and proportions of assets. This shows active management of portfolio strategies to be effective in managing risk and generating financial growth. The excess return of rebalancing which is defined by Qian (2012) as the diversification return is optimized. Qian (2012) shows that rebalancing leveraged portfolios generate a negative diversification return where the diversification return is approximated by the excess volatility of a portfolio strategy. The volatility reduction in a portfolio is determined to increase the rebalancing gains by an appropriate strategic asset allocation.

Under the limitations of fixed-mix portfolio strategies which are trading costs and stationarity of asset prices the stochastic processes of products of total returns are linked into a portfolio to achieve positive expected results. In this regard a sub-strategy is constructed such that under a no-trade region or over the intervals of rebalancing the expectation of market price of risk is not zero, that the present risk is manageable.

The strategy opts to design a routine which optimizes in each single period by constructing a portfolio of two securities the riskless assets and the financial derivative instrument. The strategy is expected to act as a risk management tool during rebalancing. Financial risk arises in fluctuation risk which is the non-degeneracy assumption of the theory of fixed-mix strategies which prompt the action of rebalancing, (Dempster *et al.*, 2010). The sub-strategy is constructed such that it establishes a financial engineered approach which sets a basis to utilize the effect of market price of risk efficiently in the application of self-financing constant rebalanced portfolio strategies by using a synthetic financial derivative instrument in the portfolio of a riskless asset and a risky asset.

4.8 Summary

The logarithms of prices of a price report of risky assets of the six selected companies are found to have up-trending market conditions and mean-reverting market conditions about the zero average. The risky assets with up-trending market condi-

tions are a risky asset of ABSA Group Limited, a risky asset of logarithm Anglo American Platinum Limited, a risky asset of Sasol Limited and a risky asset of Standard Bank. The market condition of mean-reverting about the zero average is found on a risky asset of Anglo Gold Ashanti Limited. The risky asset of AFGRI Limited is determined to have an up-trending market condition where frequent reversals are about a trend with an estimated linear rate of zero. The portfolios of a fixed-rate asset and a risky asset of ABSA Group Limited, Sasol Limited, Standard Bank Limited and AFGRI Limited are determined efficient under a mean-variance model over a benchmark return. The portfolios of a risky asset of Anglo Gold Ashanti Limited and Anglo American Platinum Limited are determined inefficient.

The financial proxy which adjusts to expected market conditions is a sub-strategy under fixed-mix strategies that attains returns above the riskless performance of a strategy over a no-trade region. The sub-strategy is a composite of a riskless financial security, the risky asset and a financial derivative instrument such that the proportions of the strategy assert the expected market characteristics. In the mean-reverting market the sub-strategy ensures an equal weight of a riskless financial security and a risky asset in a portfolio over a period of no-rebalancing and in a trend-following market the sub-strategy attains proportions which adjust to a time-dependent linear drift which increases with time. The considered financial securities retain option returns are straddle options which are combinations of puts and calls options that ensure non-zero payoff in a volatile market condition. The lookback straddle option favours leveraging particularly in trending markets conditions because of the expected non - zero payoff in volatile market conditions (Darius *et al.*, 2002). The implementation of fixed-mix portfolio strategies in the consideration of limitations of constant rebalanced portfolio strategies which are transaction costs and stationarity suggests a sub-strategy which is a time-effective portfolio. The time-effective portfolio is considered to follow a multi period dynamic stochastic programming approach that is favourable to portfolio objectives and also linked to a financial engineered security that renders an appropriate financial transaction. The long-run rates of total returns of logarithm of prices of a risky asset are observed to be approximately identical to the estimated rate of a linear trend.

4.9 Conclusion

In this chapter the quantitative analysis examines the performance of each asset considered and analyses the required conditions for the performance of a fixed-mix portfolio strategy. The significance of risk-adjusted returns is analysed by studying a financial engineered approach which renders the required condition of stationarity in a portfolio. The computed effective short rates of fixed-rate asset prices ranges from 4 to 7 basis points in an up-trending prime rate. The models of asset prices are ARIMA models with the integrated order of 1 which are found not stationary. The models of total returns are ARIMA models with integrated order zero which are found stationary. The observation of the estimated linear trend which is found approximately similar to the long-run rate of total returns of risky assets determines the specification of a trending risky asset for optimization in the fixed-mix portfolio strategy.

The risk-adjusted returns are quantified in a portfolio strategy by using a strategy of portfolio insurance to replicates the expected growth. In the following chapter the computational analysis of a fixed-mix portfolio strategy and a buy and hold portfolio strategy are discussed. The computed example of a financial proxy which renders the required market condition is performed.

Chapter 5

COMPUTATIONAL ANALYSIS

5.1 Introduction

In this Chapter we discuss the computational analysis of portfolio strategies of the six selected companies' risky assets. The aim of the chapter is to analyse the performance of fixed-mix portfolios comparable to the performance of an inactive buy and hold portfolio strategy referential to a benchmark asset, the fixed-rate asset, of up-trending prime rates and down-trending prime rates.

The fixed-rate asset in up-trending prime rates and down-trending prime rates performs at a total return of 947.64 percent and 947.32 percent respectively. The risky return of prices of a risky asset of ABSA Group Limited over 3736 discrete trading days is 1202.86 percent with an annual volatility of 18 percent. The market conditions of a risky asset are up-trending outperforming the benchmark returns. The risky return of prices of a risky asset of AFGRI Limited is 1202.86 percent with an annual volatility of 11 percent. The market conditions of a risky asset trends to a stationary condition with less frequent reversals outperforming the benchmark returns.

The risky return of prices of a risky asset of Anglo Gold Ashanti Limited is 147.14 percent with an annual volatility of 9 percent. The market conditions of a risky asset have large reversals which frequently underperform the total performance of a risky asset and the benchmark returns. The risky return of prices of a risky asset of Anglo American Platinum is 529.17 percent with an annual volatility of 23 percent. The market conditions of a risky asset are up-trending with stationary reversals with a total performance which underperforms the benchmark returns. The risky return of prices of Sasol Limited is 878.35 percent with an annual volatility of 21 percent. The market conditions of a risky asset are reverting along an uptrend with stationary reversals and less underperforming the benchmark return. The risky return of prices of a risky asset of Standard Bank is 914.35 percent with an annual volatility of 17 percent. The market conditions of a risky asset revert along an uptrend with stationary

frequent reversals and lesser underperforming the benchmark returns. The moving averages of five periods of the total returns of risky assets are almost within the interval of 5 percent of total return which indicates a basic return of a tradable stationary process. The total returns and relative price ratios are found stationary by the ARIMA model.

The analysis of computations of fixed-mix portfolios and the benchmark strategy the buy and hold strategy is performed in a dynamic market model where trading costs and dividends rates are not considered. The analysis is performed in up-trending prime rates and in down-trending prime rates conditions for a fixed-rate asset. The performance of portfolio strategies is comparatively analysed for market conditions of up-trending and down-trending prime rates. The best performing fixed-mix strategies are tactically determined in the performed computations. The value process of fixed-mix portfolio strategies with 50 percent allocations in a fixed-rate asset is computed and shown in the appendix in charts A.5.1.

The considered risky assets of ABSA Group Limited, Standard Bank, Sasol Limited, Anglo American Platinum Limited, Anglo Gold Ashanti Limited and AFGRI Limited are used to perform the analysis of computations of portfolio strategies. The short rate of a fixed-rate asset is quoted in two scenarios of the prime rates the up-trending prime rates scenario and the down-trending prime rates scenario. The analysis is carried over daily closing prices of risky assets in period of 15 years with 3736 observations.

5.2 Performance of portfolio strategies

The computations of the fixed-mix portfolios and the buy and hold portfolios of a fixed-rate asset and a risky asset are shown in the following figures. The portfolio performance system is composed of arbitrary portfolios which vary with allocations in a fixed-rate asset ranging from a short position of 200 percent to a position of 200 percent. The short position in a fixed-rate asset borrows a fixed proportion of fixed-rate assets to purchase risky assets in a portfolio. The short position in a risky asset

sells risky assets to purchase a fixed-proportion of fixed-rate assets in a portfolio. The computed portfolios are portfolios with unleveraged and leveraged positions in a fixed-rate asset or in a risky asset. The completely-mixed strategies are strategies which do not involve short positions in a portfolio. The completely-mixed portfolios of a fixed-mix strategy are 0FiM, 0.5FiM and FiM. For a buy and hold strategy the completely-mixed portfolios are 0BoH, 0.5BoH and BoH. The fixed-mix portfolios with leveraged positions are 1.5FiM, 2FiM, -0.5FiM, -FiM, -1.5FiM, -2FiM. The buy and hold portfolios with leveraged positions are 1.5BoH, 2BoH, -0.5BoH, -BoH, -1.5BoH, -2BoH.

The performance of buy and hold portfolios is shown by a line performance curve. The concave performance curve shows the performance of fixed-mix portfolios. The total return of a risky asset over an investment period is referred to as a risky return and the total return of a fixed-rate asset is referred to as the benchmark return. To analyse the realised performance in a portfolio a relative asset performance ratio is computed as a ratio of a risky return to a benchmark return. The relative asset performance ratio determines the price of risk of a portfolio when considering the market performance of a risky asset. The mean of total returns of a fixed-rate asset in up-trending prime rates and in down-trending prime rates is 0.0004 which determines the effective rate of a 1 day short rate in a fixed-rate asset. The basis point is computed as a percentage of 0.01 which is a relevant calibration. The benchmark line shown in the performance chart below depicts the benchmark return of a portfolio and a level of complete allocation of a fixed-rate asset in a portfolio. The performance of portfolios is shown in units of 100 percent. The line performance curve shows the performance of buy and hold portfolios and a concave performance curve shows the performance of fixed-mix portfolios over a period of 15 years.

5.2.1 Performance: Fixed-rate asset and ABSA Group Limited risky asset

The performance of portfolios of a fixed-mix strategy and a buy and hold strategy over 3736 daily price observations of a risky asset of ABSA Group Limited in up-trending prime rates is performed. The total performance of portfolio strategies is shown in figure 5.2.1. The performance of portfolio strategies is computed in terms of total return of a portfolio strategy over an investment period.

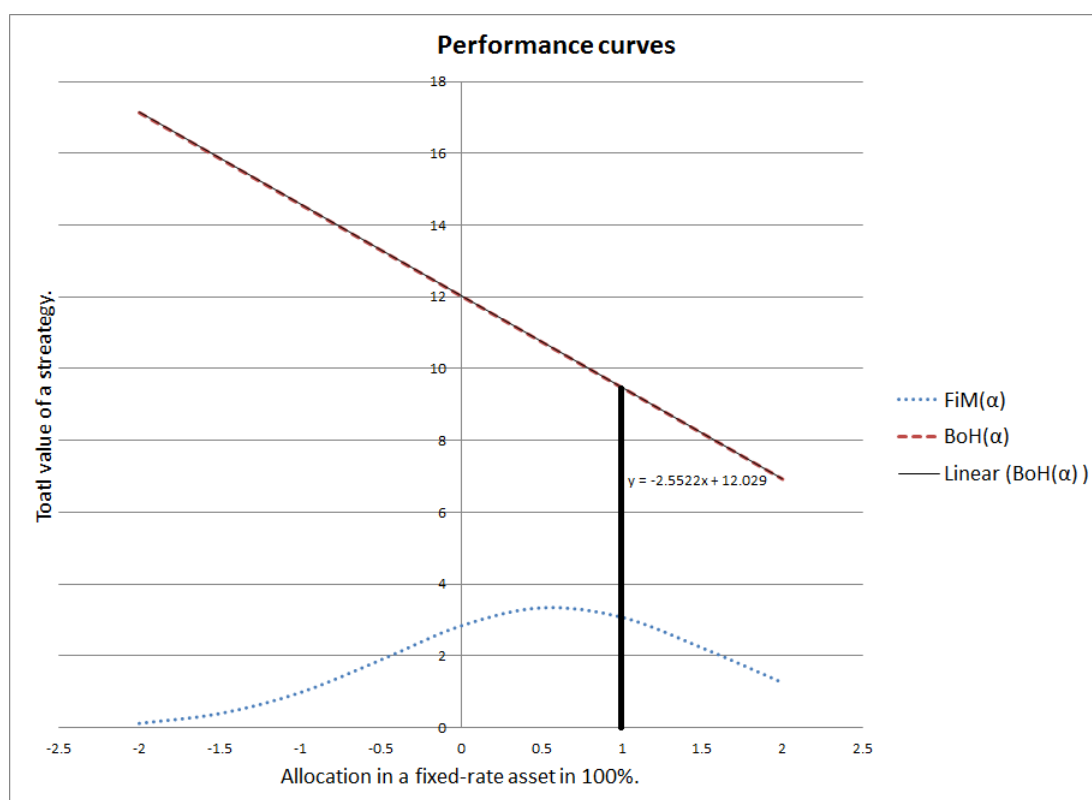


Figure 5.2.1: The performance of portfolios of a risky asset of ABSA Group Limited.

The total returns of portfolios of a buy and hold strategy in the duration of 15 years decrease at an estimated total return of 255 percent over increasing allocations in a fixed-rate asset. The total returns of buy and hold strategies are shown to be favourable to leveraged positions in a fixed-rate asset because of a risky asset which relatively outperforms the return of a fixed-rate asset. The total value return of fixed-mix portfolios and of buy and hold strategies in up trending prime rates and in down trending prime rates are shown in table 5.3.1 in the subsection 5.3.1. The relative as-

set performance ratio of a portfolio is 1.2693. The complete allocation in a risky asset yields a total return of 1203 percent with a buy and hold portfolio strategy. The absolute difference of a buy and hold portfolio strategy is a change of portfolios total returns over a change of 50 percent in asset allocation. The absolute difference of a buy and hold strategy is similar for all asset allocations from -200 percent to 200 percent in a fixed-rate asset at a value of 1.2761 basis points.

The fixed-mix portfolio with complete allocation in a risky asset yields a total return of 284 percent. The fixed-mix portfolios are optimal over completely-mixed strategies at a total return of 334 percent for a portfolio 0.5FiM. The value process of an optimal strategy is shown in the appendix in chart A.5.1.1. The value process increases at an estimated linear rate of 0.0004 from a line regression of a 10 periods moving average. The absolute differentials of fixed-mix portfolios are changes of total returns of portfolio strategies over a 50 percent change in asset allocation. The average of absolute differentials of fixed-mix strategies is 6630 basis points. The absolute differential shows the sensitivity of change in portfolio total return with respect to a change in asset allocations. The optimal allocation in a portfolio is a strategy with optimal total return in a portfolio which is signified by a smaller absolute differential.

The buy and hold portfolios with leveraged positions of a risky asset are found to outperform completely-mixed portfolios and portfolios with leveraged positions of a fixed-rate asset. The fixed-mix portfolios are optimal over completely-mixed strategies at an allocation of 50 percent in a fixed-rate asset at a minimum absolute differential of 2606 basis points. The optimal portfolio performance system of a risky asset of ABSA Group Limited is a leveraged trend-following strategy -2BoH and a completely-mixed mean-reverting strategy 0.5FiM.

The discrete total return is a total return of a strategy at discrete trading period. The descriptive statistics of discrete total returns of portfolio strategies of a risky asset of ABSA Group Limited in up-trending prime rates are shown in table A.5.2.1 in the appendix. The range of discrete total returns of a buy and hold portfolio 2BoH de-

creases from a total return of 2942.43 percent at a minimum of 83.19 percent. The range decreases to a total return of 1317.14 percent at a minimum of 95.24 percent for a portfolio 0BoH with complete allocation in a risky asset. This shows high variability of discrete total returns of strategies with increased allocations in a fixed-rate asset. The range of total returns decreases down to a total return of 933.96 percent for a portfolio 1.5BoH with a short position in a risky asset. The range for a portfolio 2BoH with short position in a risky asset is increased to a total return of 1202.57 percent at a minimum of -200.86 percent because of a leveraged trending risky asset. The standard deviations of total returns decrease from 7.097 for a portfolio -2BoH with short position in a fixed-rate asset to 2.117 for a portfolio 2BoH with short position in a risky asset. The standard deviations and means of total returns of portfolio strategies decrease with decreasing allocation in a risky asset. The discrete total returns are found to be positively skewed for all portfolio strategies which show a portfolio growth with less effect on total returns which are frequently below the average total return.

The range of total returns of fixed-mix portfolio strategies decreases from a total of 1156.38 percent for a portfolio 2FiM with short position in a risky asset at a minimum total return of 10.02 percent to a total return of 125.19 percent at a minimum of 40.37 percent for a portfolio 0FiM with complete allocation in a risky asset. The portfolios with increased allocations in a fixed-rate asset have high variability of discrete total returns. The means of total returns of fixed-mix strategies increases from 109.85 percent for a portfolio -2FiM with short positions in a fixed-rate asset to an optimal mean of 218.64 percent for a portfolio 0FiM with complete allocation in a risky asset. The means decrease to 85.06 percent for a portfolio 2FiM with a short position in a risky asset. The standard deviations of total returns decreases from 1.461 for a portfolio -2FiM with short position in a fixed-rate asset to 0.865 for a portfolio 0FiM with complete allocation in a risky asset. The standard deviations of fixed-mix portfolios decrease to 0.2188 for a portfolio 2FiM with short position in a risky asset. The discrete total returns are positively skewed indicating portfolio growth with less effective of discrete total returns below the average total return. The high effective impact of discrete total returns is found for -0.5FiM, 0FiM and 0.5FiM

portfolios as shown in the chart A.5.1.1 in the appendix.

The performance of portfolios of a risky asset and a fixed-rate asset in down-trending prime rates are shown in the appendix in chart A.5.1 to have similar performance curves with significant differences which are analysed by difference performance table shown in the appendix in table A.5.2. The performances of fixed-mix portfolios are found to have higher total returns in up-trending prime rates and the buy and hold portfolios are found to have higher total returns in down-trending prime rates.

5.2.2 Performance: Fixed-rate asset and AFGRI Limited risky asset

The performances of buy and hold strategies in up-trending prime rates of a risky asset of AFGRI Limited over 3736 daily price observations is performed and shown in figure 5.2.2.

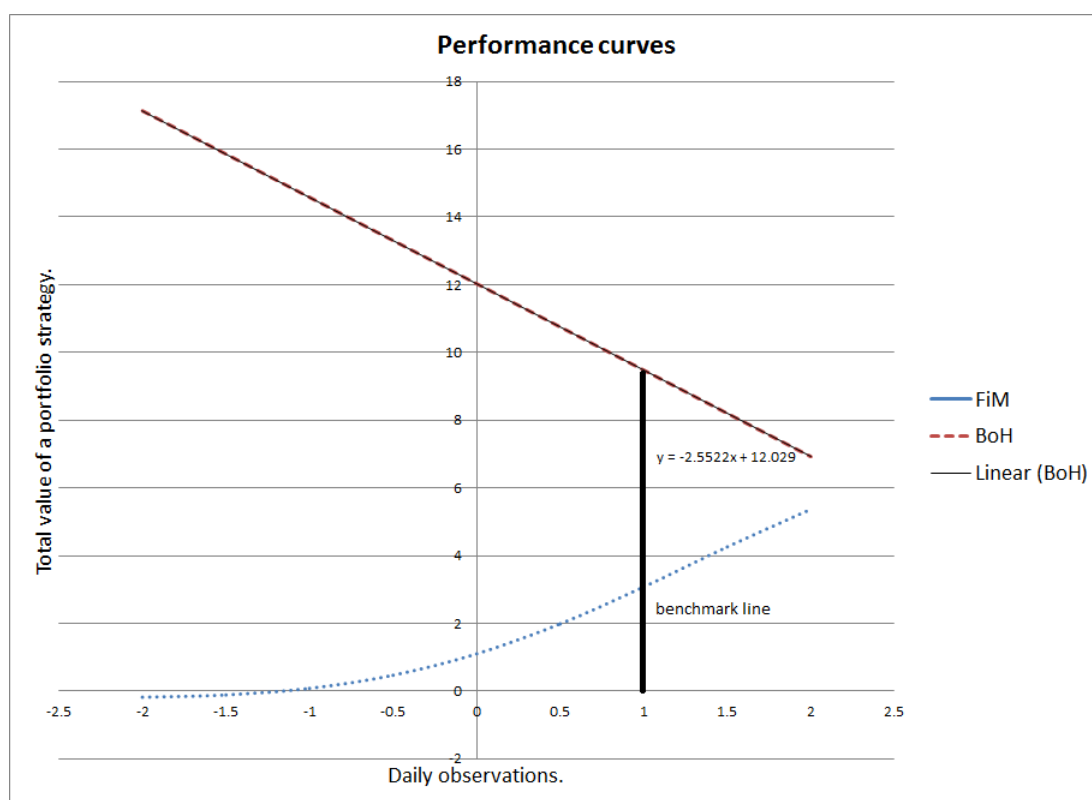


Figure 5.2.2: The performance of portfolios of a risky asset of AFGRI Limited.

The relative asset performance ratio of a portfolio is 1.2693. The performance of a buy and hold strategy which completely allocates in a risky asset attains a total return of 1203 percent. The total value return of fixed-mix portfolios and buy and hold portfolios in up trending prime rates and in down trending prime rates are shown in Table 5.3.2 in the section 5.3. The absolute difference of a buy and hold portfolio strategy is a total of 1.2761 basis points. The total value return of portfolio strategies decreases as allocations in a fixed-rate asset increase at an estimated linear rate of 255.22 percent total return. The fixed-mix portfolio which completely allocates in a risky asset yields a total value return of 110 percent. The performance of portfolios with unleveraged positions increases as the allocations in a fixed-rate asset increase up to 200 percent. Further allocations in a fixed-rate asset increase the total return of a portfolio with deviations of negative returns. The absolute differentials of total returns of fixed-mix portfolios in a change of 50 percent in asset allocation average to a value of 6933 basis points. The optimal total return of 537 percent is attained with the portfolio which allocates 200 percent in a fixed-rate asset where further allocations are determined to increase the value of a portfolio as absolute differentials are noted above the average. The value process of a strategy with 50 percent in a fixed-rate asset is shown in the appendix in chart A.5.1.2 with large dispersion of the value process about a 10 periods moving average.

The descriptive statistics of discrete total returns of portfolio strategies of a risky asset of AFGRI Limited in up-trending prime rates are shown in the table A.5.2.3 in the appendix. The ranges of total returns of buy and hold portfolio strategies decrease from a total return of 6259.74 percent at a minimum total return of 60.98 percent for a portfolio -2BoH with short position in a fixed-rate asset to a range of 847.64 percent at a minimum total return for a portfolio BoH with complete allocations in a fixed-rate asset. Further allocations in a fixed-rate asset increase the range from 1827.55 percent for a portfolio 1.5BoH at a minimum total return of -893.17 percent to a range of 2925.41 percent at a minimum total return of -1923.67 percent for a portfolio 2BoH with a short position in a risky asset.

The wide range of total returns indicates high levels of discrete total returns in a

buy and hold portfolio. The means of discrete total returns of buy and hold portfolio decrease from a mean of 4747.34 percent for a portfolio -2BoH with a short position in a fixed-rate asset to a mean of 337.79 percent for a portfolio BoH with complete allocations in a fixed-rate asset. Further allocations in a fixed-rate asset, the portfolios 1.5BoH and 2BoH with short positions in a risky asset are found to have discrete total returns which average to -397.13 percent and -1132.06 percent.

The standard deviations of total returns of a buy and hold strategy decrease from 16.609 for a portfolio -BoH with a short position in a fixed-rate asset to 2.249 for a portfolio BoH with complete allocation in a fixed-rate asset. Further allocations in a fixed-rate asset increases the riskiness of a portfolio, the portfolios 1.5BoH and 2BoH with short positions in a risky asset the standard deviations are 4.668 and 7.497 respectively. The portfolios with short positions the discrete total returns are distributed frequently above the average total return. Indicating an increased volatility of discrete total returns of portfolios with leveraged positions. The portfolios with completely-mixed positions are found to have discrete total returns which are frequently below the average total return. The discrete total returns of buy and hold portfolios are found to be distributed about the average total return at decreasing standard deviations for further allocations in a fixed-rate asset.

The range of total returns of a fixed-mix portfolio decreases from a range of 12518.95 percent at a minimum of -94.16 percent total returns to a range of 207.92 percent with a maximum total return of 307.92 percent for a portfolio FiM with complete allocation in a fixed-rate asset. The further allocations in a fixed-rate asset increase the range from 1827.55 percent for a portfolio 1.5FiM with short position in a risky asset to a range of 2925.41 percent for a portfolio 0FiM with complete allocations in a risky asset. The means of total returns of fixed-mix portfolios decrease from 3060.78 percent for a portfolio -2FiM with short position in a fixed-rate asset to a mean of 29.81 percent total return for a portfolio 2FiM with allocations in a fixed-rate asset. The standard deviations of fixed-mix portfolios decrease from 33.428 for a portfolio -2FiM with 200 short position in a risky asset to 0.574 for a portfolio FiM with complete allocations in a fixed-rate asset. Further leveraging a

fixed-rate asset position the portfolios 1.5FiM and 2FiM with short positions in a risky asset increases the standard deviations from 1.087 to 1.732. The discrete total returns of fixed-mix portfolios are positively skewed with total returns frequently distributed below the average total return of the portfolio. The discrete total returns of fixed-mix portfolios with 100 percent short positions of a risky asset have highly significant frequency of total returns about the average total return which decreases with increasing allocations in a fixed-rate asset because of the avoided risk.

The fixed-mix portfolio with short unleveraged positions are found to outperform the performance of fixed-mix portfolios. The optimal portfolio system of a risky asset of AFGRI Limited is -2BoH and 2FiM. The mean-reverting strategy of completely-mixed portfolios perform optimal in conditions with frequent reversals. The performance curves of portfolio strategies in down-trending prime rates are found similar with the performance curves in up-trending prime rates. The small significant performance differences are shown in the appendix in table A.5.2.13 to table A.5.2.18.

5.2.3 Performance: Fixed-rate asset and Anglo Gold Ashanti Limited risky asset

The performances of buy and hold strategies and of the fixed-mix strategies with 2989 daily observations of prices of a fixed-rate asset in up-trending prime rates and prices of a risky asset of Anglo Gold Ashanti Limited are shown in figure 5.2.3. The relative asset performance ratio of a portfolio is 0.2729 which shows an underperforming risky asset. The above performance of portfolio strategies shows the linear performance curve of buy and hold portfolios indicating an estimated linear rate of a total growth of 392 percent return over the allocations in a fixed-rate asset

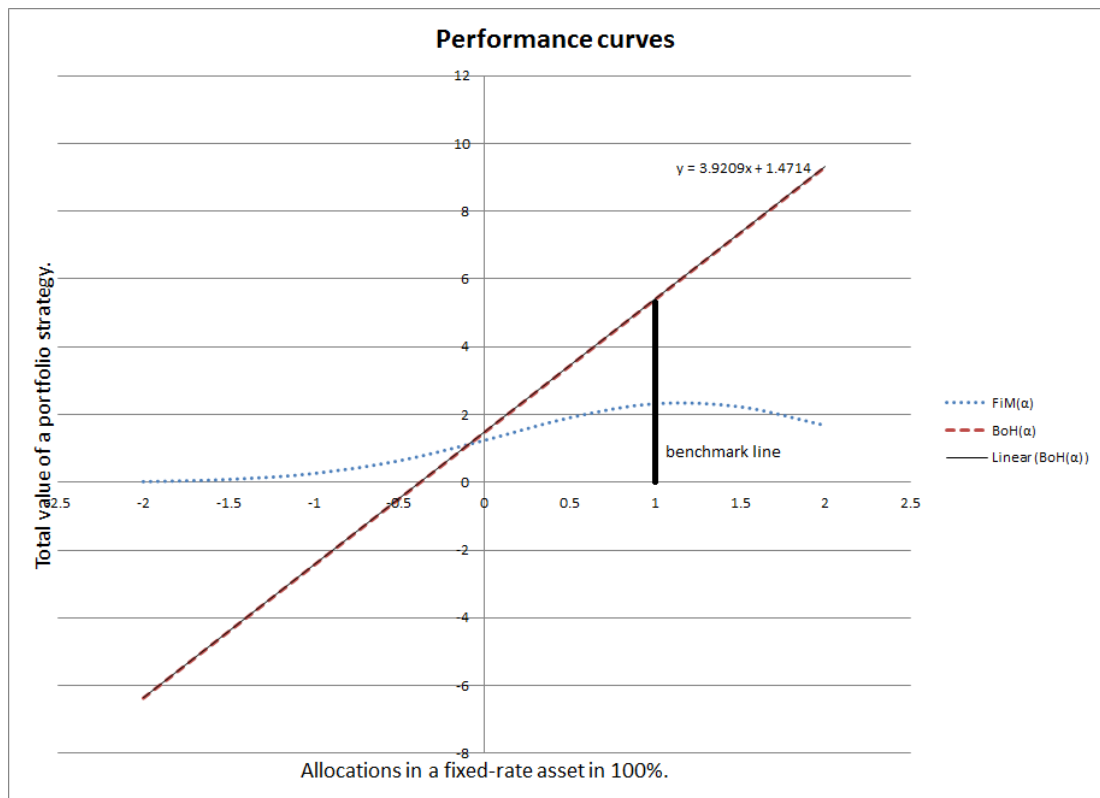


Figure 5.2.3: The performance of portfolios of a risky asset of Anglo Gold Ashanti Limited.

The buy and hold portfolio strategy with complete allocation in a risky asset attains a total value return of 147 percent. The value total returns of buy and hold portfolios with allocations in a fixed-rate asset ranging from -200 percent to 200 percent are shown in Table 5.3.3 in the section 5.3. The total return increases as the allocations in a fixed-rate asset increase. The absolute difference of a buy and hold strategy is a total of 1.9604 basis points over a change of 50 percent in asset allocation. The absolute differentials of value of fixed-mix portfolio strategies average to a value of 3684 basis points which is much above a differential of 943 basis points at an optimal allocation of 100 percent in a fixed-rate asset. The optimal value is 232 percent for a fixed-mix strategy which allocates 100 percent in a fixed-rate asset. Both strategies the buy and hold and the fixed-mix strategies are optimal for leveraged positions. The value process of a strategy with 50 percent in a fixed-rate asset is shown in the appendix in chart A.5.1.3 increasing at an estimated linear rate of 0.0002 from a line regression of a 10 periods moving average with less dispersion of the value process.

The descriptive statistics of discrete total returns of portfolio strategies of a risky asset of Anglo Gold Ashanti Limited in up-trending prime rates are shown in the table A.5.2.5 in the appendix. The ranges of total returns of buy and hold portfolio strategies decrease from a total return of 819.62 percent at a minimum total return of -687.71 percent for a portfolio -2BoH with short positions in a fixed-rate. The range decreases to a total return of 270.07 percent at a minimum total return of 75.89 percent for a completely-mixed portfolio 0.5BoH. The range of a portfolio BoH is at a total return of 439.23 percent. Further short positions in a risky asset have the range increasing to a total return of 640.63 percent for a portfolio 1.5BoH and to a total return of 842.58 percent for a portfolio 2BoH. The mean of discrete total returns of portfolios is at minimum for a portfolio -2BoH with a short position in a fixed-rate asset at a total return of -226.68 percent because of an underperforming risky asset with a relative asset performance ratio of 0.2729. The mean of discrete total returns of buy and hold strategies increases to a total return of 396.95 percent for a portfolio 2BoH with short positions in a risky asset. The standard deviations of buy and hold portfolio strategies decrease from 1.702 for a portfolio -2BoH with short positions in a fixed-rate asset to 0.303 for a portfolio 0BoH with complete allocations in a risky asset. The standard deviations of strategies increase to 2.063 for a portfolio 2BoH with a short position in a risky asset. The total returns of buy and hold portfolios from -2BoH to -BoH with short positions in a fixed-rate asset are negatively skewed which shows a high frequency of total returns above average total return. The strategies which increase the allocations of a fixed-rate asset have discrete total returns which are frequently below the average total return. The total returns of buy and hold strategies have small significant frequency with at a wider range from the average total return.

The range of total returns of fixed-mix portfolios decrease from a total return of 117.56 percent with a minimum total return of 0.86 percent for a strategy with 200 percent short position in a fixed-rate asset. The range decreases to 93.78 percent at a minimum total return of 46.84 percent for a portfolio 0FiM with complete allocations in a risky asset. For fixed-mix portfolio strategies the range increases to a total return of 166.62 percent with an increasing allocation in a fixed-rate asset at a minimum

total return of 0.90 percent. The means of discrete total returns increases from a total return of 0.14 percent for a portfolio -2FiM with short positions in a fixed-rate asset to a mean of 158.24 percent for a portfolio 1.5FiM with short positions in a risky asset. The discrete total returns of fixed-mix portfolios are distributed wider at a high frequency below the average total return. The strategies with short positions in a fixed-rate asset have discrete total returns less wide about average total return.

5.2.4 Performance: Fixed-rate asset and a risky asset of Anglo American Platinum Limited

The performances of a buy and hold strategy and a fixed-mix strategy over 3736 daily price observations of a risky asset of Anglo American Platinum Limited in up-trending prime rates is performed. The total performance of portfolio strategies is shown in figure 5.2.4. The total growth of prices of a risky asset is 529 percent.

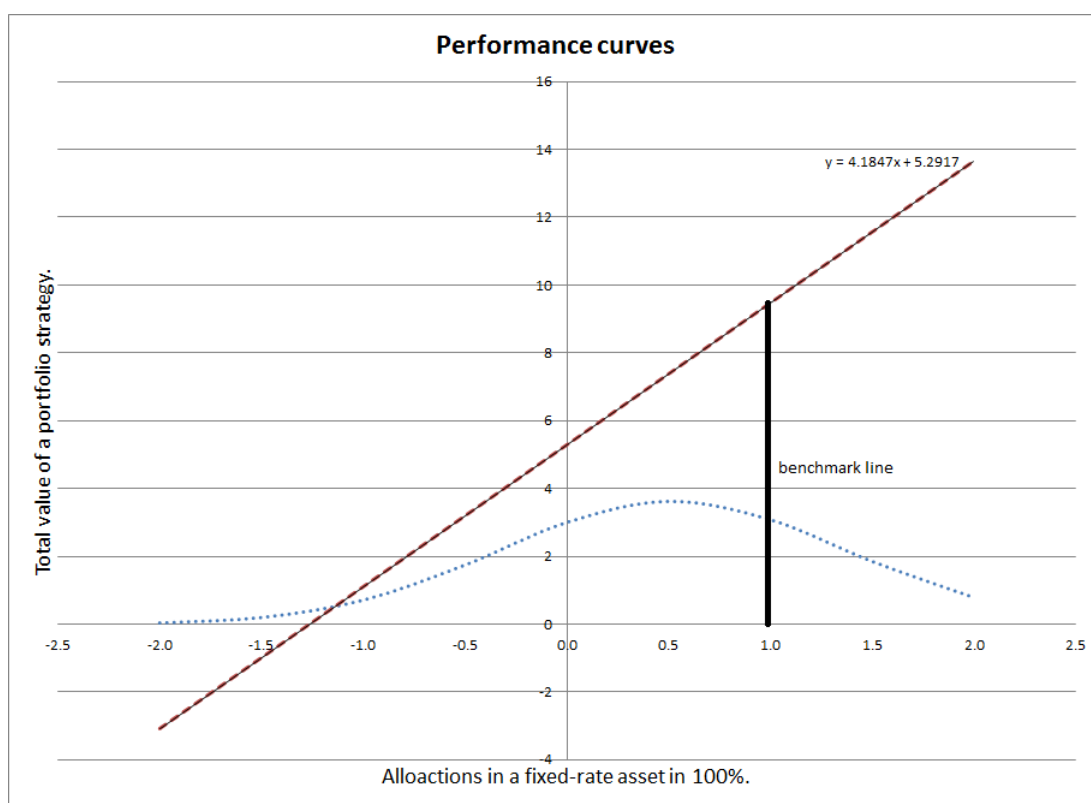


Figure 5.2.4: The performance of portfolios of a risky asset of Anglo American Platinum Limited.

The performance by total value returns of buy and hold portfolios is shown to increase at an estimated linear rate of 418.47 percent total return over the allocations in a fixed-rate asset.

The absolute difference of a buy and hold portfolio strategy over a 50 percent change in asset allocation is a total of 2.0924 basis points. The complete allocation of a risky asset in a fixed-mix portfolio yields a total value of 300 percent. The completely-mixed strategies of a fixed-mix portfolio attain an optimal performance of 362 percent at an allocation of 50 percent. The absolute differentials of a fixed-mix strategy average at a value of 8027 basis points. The narrower absolute differential of a value of 5420 basis points is for an optimal value of 362 percent.

The comparative performances of portfolio strategies in down-trending prime rates are shown by difference performance Table 5.3.4 in the section 5.3. The relative asset performance ratio of a portfolio is 0.5584 which shows a high performing asset to be a fixed-rate asset. The short unleveraged portfolios outperform trend-following strategies because of the underperforming risky asset. The mean-reverting strategy of fixed-mix portfolios is optimal for long unleveraged portfolios. The value process of a completely-mixed portfolio 0.5FiM is shown in the appendix in chart A.5.1.4 with large dispersion of the value process about a 10 periods moving average from 2000 discrete periods to 3000 discrete periods.

The descriptive statistics of discrete total returns of portfolio strategies of a risky asset of Anglo American Platinum Limited in up-trending prime rates are shown in table A.5.2.7 in the appendix. The range of buy and hold portfolios decreases from a total return of 2930.79 percent at a minimum of -776.06 percent for a portfolio -2BoH with short position in a fixed rate asset to a range of 847.64 percent for a portfolio BoH with complete allocations in a fixed-rate asset. The portfolios 1.5BoH and 2BoH with short positions in a risky asset their ranges are 1118.26 percent and 1407.55 percent respectively with minimums of 71.03 percent and 39.75 percent. The means of total returns of buy and hold strategies increases from a mean of 210.68 percent for a portfolio -2BoH with short positions in a fixed-rate asset to a

mean of 380.16 percent for a portfolio 2BoH with short positions in a risky asset. The standard deviations of buy and hold strategies decreases from 5.245 for a portfolio -2BoH with short positions in a fixed-rate asset to 2.249 for a portfolio BoH with complete allocations in a fixed-rate asset. For further short positions in a risky asset the standard deviation of portfolios 1.5BoH and 2BoH increases to 2.438 and 2.856 respectively. The total returns of buy and hold strategies are distributed frequently at a less wide frequency below the average total return.

The range of total returns of fixed-mix portfolios increases from a range of 407.05 percent at a minimum total return of 0.61 percent for a portfolio -2FiM with short position in a fixed-rate asset to a range of 457.08 percent for a portfolio with complete allocations in a risky asset at a minimum of 71.91 percent. The range of strategies with increasing allocations in a fixed-rate asset from 50 percent decreases from a total return of 341.62 percent at a minimum total return of 88.54 percent to a total return of 160.57 percent at a minimum of 15.38 for a portfolio 1.5FiM with short positions in a risky asset. The means of discrete total returns of fixed-mix portfolio strategies increases from 54.35 percent for a portfolio -2FiM with short positions in a fixed-rate asset to a mean of 213.59 percent for a portfolio FiM with complete allocations in a risky asset. The means of discrete total returns of fixed-mix portfolios decreases from a total return of 212.45 percent for a completely-mixed portfolio 0.5FiM to a total return of 66.81 percent for a portfolio 2FiM with short positions in a risky asset.

The standard deviations of discrete total returns of strategies increase from 0.587 for a portfolio -2FiM with short position in a fixed-rate asset to 109.13 for a portfolio FiM with complete allocations in risky asset. The standard deviations of discrete total returns of fixed-mix strategies decrease from 0.933 to 0.409 for a completely-mixed portfolio to a portfolio 2FiM with short position in a risky asset. The discrete total returns of portfolios are positively skewed with high frequency of total returns below the average total returns. The portfolios with leveraged positions in a fixed-rate asset have total returns which are distributed less wide about the average total return. The

strategies with increasing allocations in a fixed-rate asset have total returns with wider discrete total returns about the average total return.

5.2.5 Performance: Fixed-rate asset and Sasol Limited risky asset

The performance of portfolios of a risky asset of Sasol Limited in up-trending prime rates over 3736 daily price observations is performed. The total performance of portfolios strategies is shown in figure 5.2.5. The growth of prices of a risky asset is 878 percent.

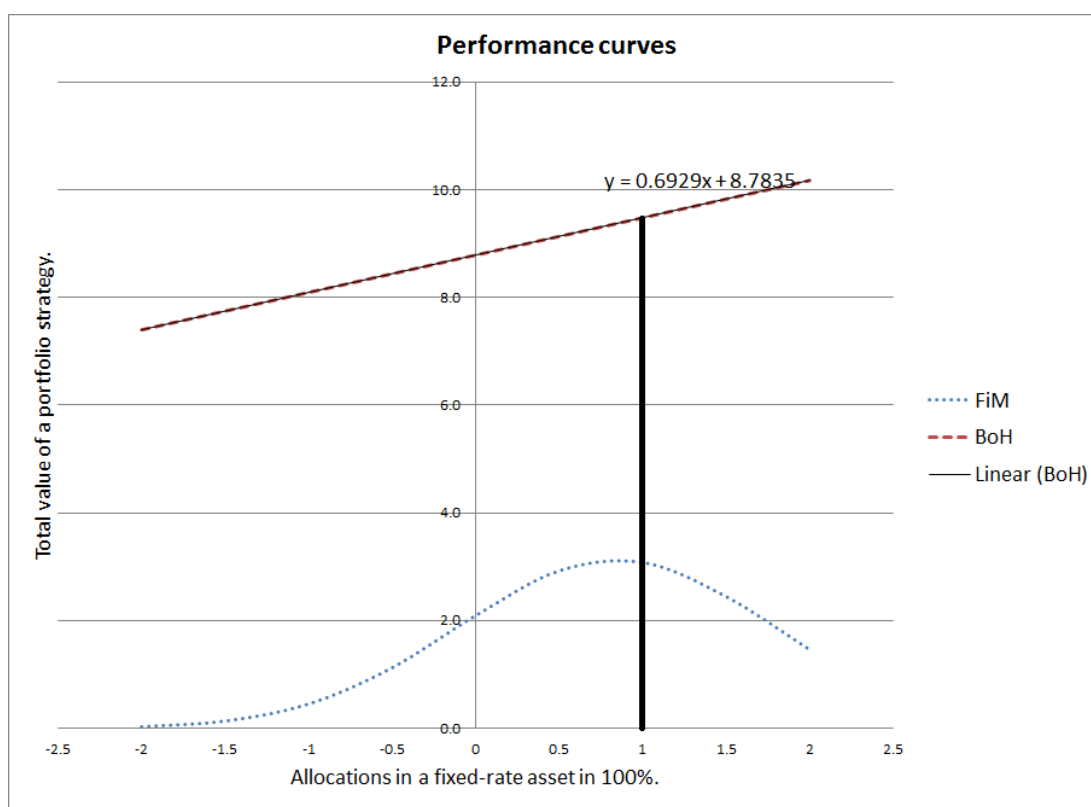


Figure 5.2.5: The performance of portfolios of a risky asset of Sasol Limited.

The relative asset performance ratio of a portfolio is 0.9269. The total values of a buy and hold portfolios increases at an estimated linear rate of 69.29 percent total return over the allocations in a fixed-rate asset. The value of buy and hold portfolio strategies increase at an absolute difference of 3464 basis points over a change of 50 percent in asset allocation.

The complete allocation in a risky asset in a fixed-mix portfolio attains a growth of 209.09 percent. The completely-mixed strategies of a fixed-mix portfolio increase up to an optimal total value of 307.92 percent for a portfolio FiM with complete allocations in a fixed-rate asset. The absolute differentials of fixed-mix strategies average to a value of 5835 basis points where the allocation with a minimum absolute differential of 1553 basis points for an optimal allocation.

The performance of portfolios of a fixed-rate asset in down-trending prime rates is shown by a difference performance Table 5.3.5 in the section 5.3. The relative asset performance ratio is higher than the ratio in up-trending prime rates at a ratio of 0.9272 which indicates a higher performance of a fixed-rate asset. The total values of a buy and hold portfolio grow over allocations from -200 percent to 200 percent in a fixed-rate asset increase at an estimated linear rate of 68.97 percent total return over the allocations in a fixed-rate asset. The fixed-mix portfolio 0.5FiM attains an optimal total value of 307.88 percent. The value process of a fixed-mix strategy with 50 percent in a fixed-rate asset is shown in the appendix in chart A.5.1.5 with large dispersion of the value from a 10 periods moving average from 2000 discrete periods to a closing period of the strategy. The trend-following strategy is optimal for short unleveraged portfolios. The mean-reverting strategy of a fixed-mix portfolio is optimal for long unleveraged portfolios. The growth of portfolio strategies is comparatively higher in up-trending prime rates where a fixed-rate asset outperforms in down-trending prime rates condition.

The descriptive statistics of discrete total returns of portfolio strategies of a risky asset of Sasol Limited in up-trending prime rates are shown in the table A.5.2.9 in the appendix. The range of total returns of a buy and hold portfolio -2BoH with short position in a fixed-rate asset decreases from a total return of 3314.16 percent at a minimum of -161.58 percent to a total return of 847.64 percent for a portfolio BoH with complete allocations in a fixed-rate asset. The ranges of discrete total returns of portfolios 1.5BoH and 2BoH with short positions in a risky asset are 899.68 percent and 1124.41 percent respectively with minimums of 0.956 and -0.608. The means of discrete total returns of buy and hold strategies decreases from a total return of

476.57 percent for a portfolio -BoH with short position in a fixed-rate asset to a mean of 291.53 percent for a portfolio 2BoH with short positions in a risky asset. The standard deviations of discrete total returns decrease from 5.839 to 1.901 for a portfolio -2BoH with short positions in a fixed-rate asset to a portfolio 2BoH with short positions in a risky asset. The discrete total returns are positively skewed with high frequency of discrete total returns less wide below the average total return. The value process of a completely fixed-mix portfolio 0.5FiM is shown in the appendix in chart A.5.1.3.

The range of discrete total returns of fixed-mix portfolio increases from a total return of 394.68 percent at a minimum total return of 1.83 percent for a portfolio -2FiM with short positions in a fixed-rate asset to a range of 855.59 percent for a portfolio 1.5FiM with short position in a fixed-rate asset at a minimum of 0.48 percent total returns. The range of discrete total returns of a portfolio 0FiM with complete allocations in a risky asset is 695.14 percent at a minimum of 0.71 total returns to a range of 164.68 percent for a portfolio 2FiM with short positions in a risky asset at a minimum of 0.36 percent total returns. The means of discrete total returns of a fixed-mix portfolios increases from a total return of 62.11 percent to an optimal total return of 197.64 percent for a strategy with 100 percent allocations in a risky asset. The means of strategies with increasing allocation in a fixed-rate asset from 50 percent increase from a total return of 196.57 percent to a total return of 103.91 percent for a strategy with 100 percent short position in a risky asset. The standard deviations of total returns increases from 0.66 for a fixed-mix portfolio -2FiM with short positions in a fixed-rate asset to 1.64 for a portfolio -0.5FiM with short positions in a fixed-rate asset. The standard deviations of discrete total returns of a portfolio FiM with complete allocations in a risky asset is 1.439 and decreases to 0.368 as short positions in a risky asset increases to 100 percent. The discrete total returns of fixed-mix portfolios are positively skewed with high frequency of total returns less below the average of total returns.

5.2.6 Performance: Fixed-rate asset and a risky asset of Standard Bank Limited risky asset

The performance of portfolios of a risky asset of Standard Bank in up-trending prime rates over 3736 daily price observations is performed. The total performance of portfolio strategies is depicted in the figure 5.2.6 and the total value return of portfolios is shown in Table 5.3.6 in the section 5.3. The buy and hold portfolios increase at an estimated rate of 33.29 percent total return over the allocations from -200 percent to 200 percent in a fixed-rate asset. The complete allocation in a risky asset in a buy and hold portfolio attains a total value of 914 percent. The absolute difference of portfolio strategies is a total of 1664 basis points. The complete allocation in a risky asset in a fixed-mix portfolio attains a total value of 426 percent which is an optimal total value over completely-mixed strategies. The absolute differentials of a fixed-mix portfolio over a 50 percent change in asset allocations average at a value of 8895 basis points which shows a significant impact of asset allocation in a portfolio.

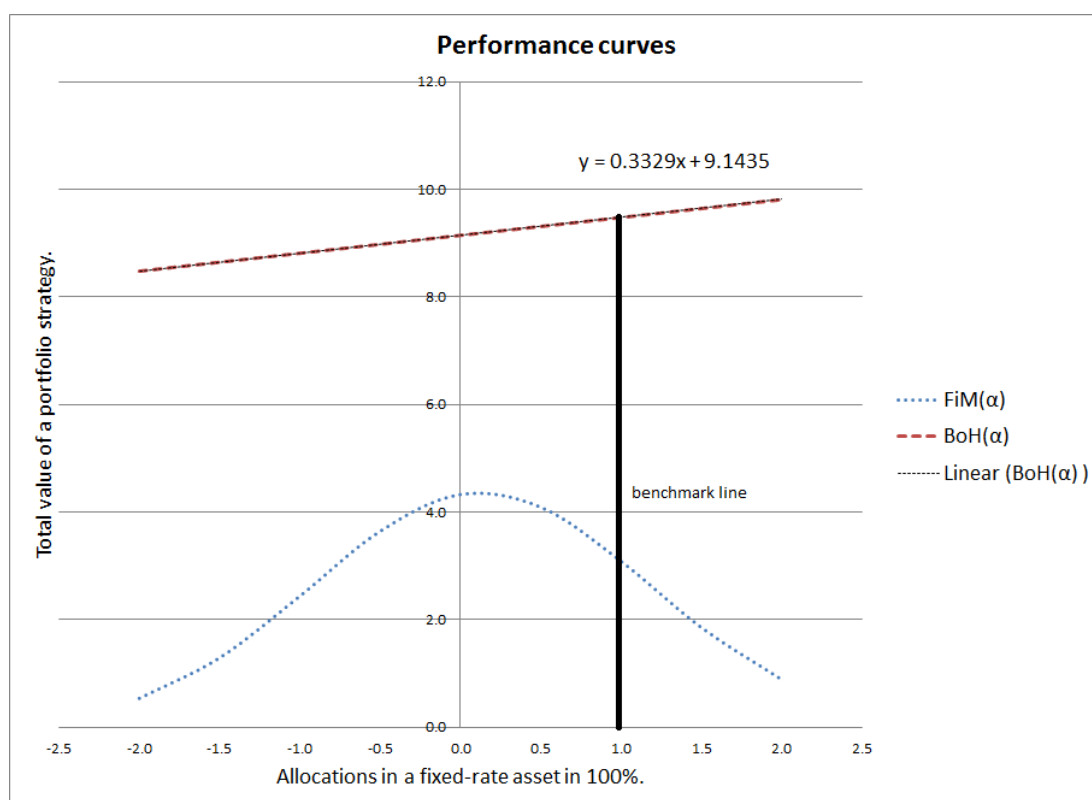


Figure 5.2.6: The performance of portfolios of a risky asset of Standard Bank Limited.

The range of total returns of buy and hold portfolio strategies decrease from a total return of 2062.53 percent at the minimum total return of 5.48 percent for a portfolio -2BoH with short position in a fixed-rate asset. The range of discrete total returns decreases to 843.43 percent at the minimum total return of 95.33 for a completely-mixed portfolio 0.5FiM. The range of a strategy with complete allocations in a fixed-rate asset is a total return of 847.64 percent. The range increases up to a total return of 1158.73 percent at a minimum of 14.44 percent total returns for a portfolio 2FiM with short position in a risky asset. The value process of a completely-mixed portfolio 0.5FiM is shown in the appendix in chart A.5.1.6 with large dispersion of the value process from a 10 period moving average.

The means of discrete total returns of buy and hold strategies decreases from a total return of 543.09 percent for a portfolio -2BoH with short positions in a fixed-rate asset up to a mean of 269.36 percent for a portfolio 2BoH with a short position in a risky asset. The standard deviations of the strategies decreases from 4.469 for a portfolio with 200 percent short position in a fixed-rate asset to 2.191 for a portfolio 1.5BoH with short positions in a risky asset and increase to 2.275 for a portfolio 2BoH with a short position in a risky asset. The discrete total returns of buy and hold strategies are positively skewed indicating a high frequency of total returns below the average total return. The discrete total returns of portfolios from -2BoH to 2BoH with increasing allocations in a fixed-rate asset are frequently below the average total return. The discrete total returns of strategies with short positions in a fixed-rate asset and in a risky asset are less wide about the average total return. For strategies with 50 percent short position in a fixed-rate asset to a strategy with 50 percent allocations in a fixed-rate asset are distributed wider about the average total return.

The descriptive statistics of discrete total returns of portfolio strategies of a risky asset of Standard bank in up-trending prime rates are shown in the table A.5.2.11 in the appendix. The range of total returns of fixed-mix portfolios is 890.63 percent at a minimum of 1.73 percent for a portfolio -2FiM with short positions in a fixed-rate asset. The range of discrete total returns decreases to 207.92 percent for a strategy with complete allocations in a risky asset. The range increases to a total return of

218.32 percent for a portfolio 1.5FiM with a short position in a risky asset and for a portfolio 2FiM with short position in a risky asset is 217.83 percent. The minimums of strategies with short positions in a risky asset are 71.57 and 32.46 percent respectively. The means of discrete total returns of fixed-mix strategies increases from 159.82 percent for a portfolio -2FiM with a short position in a fixed-rate asset to a mean of 231.11 percent for a portfolio 0.5FiM with short positions in a fixed-rate asset.

The means of discrete total returns of portfolios 0.5FiM and FiM with complete and 50 percent allocations in a fixed-rate asset respectively are 223.86 percent and 203.12 percent. The means decreases to a total return of 114.11 percent for strategies which increases the allocations in a fixed-rate asset up to a strategy with 100 percent short position in a risky asset. The standard deviations of fixed-mix portfolios increase from 1.843 for a portfolio -2FiM with short position a in a fixed-rate asset to 2.124 for a portfolio 2FiM with short position in a fixed-rate asset. The standard deviations decrease from 1.905 for a portfolio 0.5FiM with short positions in a fixed-rate asset to 0.534 for a portfolio 1.5FiM with short positions in a risky asset. The discrete total returns of fixed-mix portfolios are distributed frequently below the average total return. For strategies with allocations in a fixed-rate asset exceeding from 100 percent and with a short position in a fixed-rate asset from 150 percent to 200 percent the total returns are less different from the average total return. The discrete total returns are wider about the average total return for portfolios from -FiM with short positions in a fixed-rate asset to a portfolio FiM with complete allocations in a fixed-rate asset.

The computed portfolio strategies of risky assets are found to yield portfolio growth under specific markets conditions the trending market condition and the mean-reverting market condition. The portfolios of a buy and hold strategy are optimal for portfolios with short positions in a fixed-rate asset and leveraged positions in a risky asset which are trend-following strategies. The portfolios of a risky asset of ABSA Group Limited and of AFGRI Limited are optimal for trend-following strategies with short positions in a fixed-rate asset. The underperforming buy and hold portfolios are

found to outperform fixed-mix portfolios because of trending risky assets. The buy and hold portfolios of Anglo Gold Ashanti Limited and of Anglo American Platinum Limited are found optimal for strategies with leveraged positions of a risky asset. Portfolios of Anglo Gold Ashanti with short position in a fixed-rate asset underperform with negative returns for leveraging exceeding 50 percent in a fixed-rate asset. The portfolios of Anglo American Platinum Limited underperform with negative returns for leveraging exceeding 100 percent in a fixed-rate asset. The portfolios of a risky asset of Sasol Limited and of Standard Bank are found to be optimal for portfolios with short positions in a risky asset. The underperforming short leveraged portfolios exceed the performance of fixed-mix portfolios which perform strictly above zero returns.

5.3 Comparative performances relative to market conditions

The performance of portfolio strategies indicates a growth value of each strategy with its level of performance for considered market risk. The fixed-rate asset total return is riskless over short periods of time as it is predetermined on the current prime rate with riskless expected performance in a short term. The quantitative analysis performance of portfolio strategies in relative trending market conditions is performed below and the table of performances are shown in the following tables.

The average logarithm price of a fixed-rate asset in down-trending prime rates is 1.2372 with a maximum logarithm price of 2.2485. Comparatively to up-trending prime rates the difference in the average of logarithm prices 2257 basis points lower. The difference of the maximum log price is 3 basis points higher. The mean and volatility of total returns of a fixed-rate asset in up-trending prime rates and in down-trending prime rates is 6 basis points and 1 basis point respectively. The relative asset performance ratio of a portfolio is computed as a ratio of a risky return to a benchmark return which determines the benchmark comparative performance level of a risky asset in a portfolio. The difference performance of buy and hold portfolio strategies is common for all respective portfolio strategies. The leveraged portfolios of

buy and hold strategies are favourable to down-trending prime rates and the unleveraged portfolios are favourable to up-trending prime rates. The difference performance of buy and hold strategies is symmetric for portfolios with leveraged positions and with unleveraged positions.

The performance and the difference performance of fixed-mix portfolios and buy and hold portfolios in the relative market conditions is shown in the tables for each selected company risky asset. The difference performance for a completely-mixed buy and hold portfolio strategy with 50 percent in a fixed-rate asset is 16 basis points. The difference performance of a strategy with complete allocations in a fixed-rate asset is 32 basis points. The difference performance of a strategy with a 50 percent and 100 percent short position in a risky asset is 47 basis points and 63 basis points respectively. The difference performance of unleveraged portfolios is symmetric to leveraged portfolios for buy and hold strategies which shows an increasing difference performance for short positions in a portfolio.

5.3.1 Difference performance of portfolios of a risky asset of ABSA Group Limited relative to prime rates conditions

The relative asset performance ratio of a portfolio of a fixed-rate asset in up-trending prime rates and a risky asset of ABSA Group Limited is 1.2693 and in down-trending prime rates is 1.2697. The fixed-rate asset performs lower in down-trending prime rates which results to a higher relative asset performance ratio. The buy and hold portfolio strategy performs comparatively higher in down-trending prime rates for short allocations in a fixed-rate asset. The difference performances of strategies relative to up-trending and down-trending prime rates are shown in the table 5.3.1. The difference performance for fixed-mix portfolios is 1 basis point to 2 basis points for leveraged allocation in a fixed-rate asset. For further allocations in a fixed-rate asset the difference performance increases from 1 basis point to 4 basis points.

Table 5.3.1: Difference performance of portfolio strategies of a risky asset of ABSA Group Limited and a fixed-rate asset in up-trending and down-trending prime rates.

α	Total return of Fixed-mix strategy:FiM(α) up trend	Total return of Buy and Hold strategy:BoH(α) up trend	Relative ratio of the strategies	Total return difference of Fixed-mix strategy in up trend and down trend	Total return difference of Buy and Hold strategy in up trend and down trend
-2.0	0.1217	17.1329	140.7790	0.0002	-0.0063
-1.5	0.3928	15.8568	40.3681	0.0003	-0.0047
-1.0	0.9768	14.5807	14.9275	0.0003	-0.0032
-0.5	1.8850	13.3047	7.0581	0.0001	-0.0016
0.0	2.8371	12.0286	4.2398	0.0000	0.0000
0.5	3.3398	10.7525	3.2195	0.0001	0.0016
1.0	3.0792	9.4764	3.0776	0.0004	0.0032
1.5	2.2228	8.2003	3.6892	0.0006	0.0047
2.0	1.2542	6.9242	5.5210	0.0004	0.0063

The optimal difference performances of 3 basis points and 6 basis points are realised for a strategy with 150 percent short position in a fixed-rate asset and for a strategy with 50 percent short position in a risky asset.

The descriptive statistics of discrete total returns of portfolio strategies of a risky asset of ABSA Group Limited in up-trending prime rates are shown in the table A.5.2.2 in the appendix. The ranges of discrete total returns of buy and hold portfolios and fixed-mix portfolios are lesser for strategies in down-trending prime rates. The minimum discrete total returns of portfolio strategies with short positions in a fixed-rate asset are lower in down-trending prime rates and higher for strategies with increasing positions in a fixed-rate asset. The means and standard deviations of discrete total returns are higher in up-trending prime rates for strategies with short positions in a fixed-rate asset and lower for strategies with increased allocations in a fixed-rate asset.

5.3.2 Difference performance of portfolios of a risky asset of AFGRI Limited relative to prime rates conditions

The relative asset performance of a portfolio of a risky asset of AFGRI Limited is 1.2693. The difference performance of a fixed-mix portfolio strategy is comparative-

ly high by 1 basis point for a strategy with 200 percent short position in a fixed-rate asset in up-trending prime rates. The difference performances of strategies relative to up-trending and down-trending prime rates are shown in the table 5.3.2. The difference performance is high by 1 basis point for strategies with decreasing short positions in a fixed-rate asset in down-trending prime rates to 3 and 2 basis points.

Table 5.3.2: Difference performance of portfolio strategies of a risky asset of AFGRI Limited and a fixed-rate asset in up-trending and down-trending prime rates.

α	Total return of Fixed-mix strategy:FiM(α) up trend	Total return of Buy and Hold strategy:BoH(α) up trend	Relative ratio of the strategies	Total return difference of Fixed-mix strategy in up trend and down trend	Total return difference of Buy and Hold strategy in up trend and down trend
-2.0	-0.1770	17.1329	-96.8199	0.0001	-0.0063
-1.5	-0.1122	15.8568	-141.2956	-0.0001	-0.0047
-1.0	0.0830	14.5807	175.6634	-0.0003	-0.0032
-0.5	0.4715	13.3047	28.2167	-0.0002	-0.0016
0.0	1.1025	12.0286	10.9102	0.0000	0.0000
0.5	1.9875	10.7525	5.4100	0.0003	0.0016
1.0	3.0792	9.4764	3.0776	0.0004	0.0032
1.5	4.2628	8.2003	1.9237	-0.0001	0.0047
2.0	5.3696	6.9242	1.2895	-0.0015	0.0063

For complete allocation in a fixed-rate asset the fixed-mix strategy is comparatively high with a performance difference of 4 basis points favourable to up-trending prime rates. The increasing allocations in a fixed-rate asset have a comparative performance difference from 1 basis point to 15 basis points favourable to down-trending prime rates.

The descriptive statistics of discrete total returns of portfolio strategies of a risky asset of AFGRI Limited in up-trending prime rates are shown in the table A.5.2.4 in the appendix. The ranges of discrete total returns of portfolios with short position in a fixed-rate asset are comparatively higher for up-trending prime rates and lower for increasing allocations in a fixed-rate asset. The minimum of discrete total returns are lower for strategies with short positions in a fixed-rate asset in down-trending prime rates and lower for strategies with increasing allocations in a fixed-rate asset. The

means of discrete total returns are higher for strategies with short positions in a fixed-rate asset and higher for strategies with increased allocations in a fixed-rate asset in down-trending prime rates. The standard deviations are increased for strategies with short positions in a fixed-rate asset in up-trending prime rates and lower for strategies with increased allocations in a fixed-rate asset.

5.3.3 Difference performance of portfolios of a risky asset of Anglo Ashanti Limited relative to prime rates conditions

The relative performance ratio in a portfolio of a fixed-rate asset in up-trending prime rates is 0.2729 and in down-trending prime rates is 0.2312. The comparative performance of a buy and hold portfolio strategy is favourable to up-trending prime rates for leveraged allocations in a fixed-rate asset. The difference performances of strategies relative to up-trending and down-trending prime rates are shown in the table 5.3.3. The difference performance increases with further leveraged allocations in a fixed-rate asset from 0.4859 to 1.9435 which is favourable in up-trending prime rates. The difference performance of a buy and hold portfolio strategy with completely-mixed allocations and allocations exceeding complete allocation in a fixed-rate asset is similar to a difference performance of leveraged allocations and is favourable to down-trending prime rates.

Table 5.3.3: Difference performance of portfolio strategies of a risky asset of Anglo Gold Ashanti Limited and a fixed-rate asset in up-trending and down-trending prime rates.

α	Total return of Fixed-mix strategy:FiM(α) up trend	Total return of Buy and Hold strategy:BoH(α) up trend	Relative ratio of the strategies	Total return difference of Fixed-mix strategy in up trend and down trend	Total return difference of Buy and Hold strategy in up trend and down trend
-2.0	0.0215	-6.3704	-295.7706	0.0033	1.9435
-1.5	0.0840	-4.4099	-52.4697	0.0099	1.4577
-1.0	0.2598	-2.4495	-9.4287	0.0207	0.9718
-0.5	0.6368	-0.4890	-0.7679	0.0258	0.4859
0.0	1.2381	1.4714	1.1884	0.0000	0.0000
0.5	1.9071	3.4318	1.7995	-0.0805	-0.4859
1.0	2.3223	5.3923	2.3219	-0.2002	-0.9718
1.5	2.2280	7.3527	3.3001	-0.2944	-1.4577
2.0	1.6761	9.3132	5.5565	-0.3020	-1.9435

The fixed-mix strategy is favourable in up-trending prime rates for leveraged allocations in a fixed-rate asset. The optimal difference performance is 0.258 for a strategy with 50 percent leveraging in a fixed-rate asset. The difference performance of fixed-mix portfolios increases with completely-mixed allocations and exceeding complete allocations in a fixed-rate asset which is favourable to a down-trending asset.

The descriptive statistics of discrete total returns of portfolio strategies of a risky asset of Anglo Ashanti Limited in up-trending prime rates are shown in table A.5.2.6 in the appendix. The difference of the range, of the minimums, of the means and of the standard deviations of discrete total returns for strategies with 200 percent and 150 percent short positions in a fixed-rate asset is similar for up-trending prime rates and down-trending prime rates. The range of discrete total returns is higher and increasing for strategies with increasing allocations in a fixed-rate asset in down-trending prime rates. The minimums, the means and the standard deviations of discrete total returns are lower for strategies with 100 percent and 50 percent short positions in a fixed-rate asset in down-trending prime rates. For strategies with increased allocations in a fixed-rate asset are lower in up-trending prime rates.

5.3.4 Difference performance of portfolios of a risky asset of Anglo American Platinum Limited relative to prime rates conditions

The relative asset performance for a portfolio of a fixed-rate asset in down-trending prime rates is 0.5586 and in up-trending prime rates is 0.5584. The difference performances of strategies relative to up-trending and down-trending prime rates are shown in the Table 5.3.4. The difference performance of a fixed-mix strategy is favourable in up-trending prime rates at a difference of 484 basis points to 12798 basis points for leveraged positions in a fixed-rate asset up to a complete allocation in a risky asset. The difference performance of completely-mixed allocations in a fixed-rate asset are favourable in up-trending prime rates for allocations of 50 percent and 100 percent in a fixed-rate asset at a value of 7698 basis points and 4 basis points respectively.

Table 5.3.4: Difference performance of portfolio strategies of a risky asset of Anglo American Platinum Limited and a fixed-rate asset in up-trending and down-trending prime rates.

α	Total return of Fixed-mix strategy:FiM(α) up trend	Total return of Buy and Hold strategy:BoH(α) up trend	Relative ratio of the strategies	Total return difference of Fixed-mix strategy in up trend and down trend	Total return difference of Buy and Hold strategy in up trend and down trend
-2.0	0.0375	-3.0778	-82.0915	0.0484	-0.0063
-1.5	0.1977	-0.9854	-4.9842	0.2124	-0.0047
-1.0	0.7078	1.1069	1.5638	0.6068	-0.0032
-0.5	1.7465	3.1993	1.8318	1.1199	-0.0016
0.0	3.0020	5.2917	1.7627	1.2798	0.0000
0.5	3.6212	7.3840	2.0391	0.7698	0.0016
1.0	3.0792	9.4764	3.0776	0.0004	0.0032
1.5	1.8491	11.5688	6.2563	-0.3899	0.0047
2.0	0.7838	13.6611	17.4302	-0.3294	0.0063

The difference performance of fixed-mix portfolios with short positions of a risky asset is favourable in down-trending prime rates for a 50 percent short position in a risky asset at a value of 3899 basis points. The difference performance of a 100 percent short position in a risky asset is 3294 percent favourable to down-trending prime rates.

The descriptive statistics of discrete total returns of portfolio strategies of a risky asset of Anglo American Platinum Limited in up-trending prime rates are shown in the table A.5.2.8 in the appendix. The range of discrete total returns of strategies with short positions in a fixed-rate asset is higher in up-trending prime rates and wider for strategies with increasing allocations in a fixed-rate asset in down-trending prime rates. The minimums, the means and the standard deviations of discrete total returns are lower for strategies with short positions in a fixed-rate asset in down-trending prime rates and higher for strategies with increasing allocations in a fixed-rate asset. The value process of a strategy with 50 percent in a fixed-rate asset is shown in the appendix in Chart A.5.1.4.

5.3.5 Difference performance of portfolios of a risky asset of Sasol Limited relative to prime rates conditions

The relative asset performance ratio of a portfolio of a fixed-rate asset in up-trending prime rates is 0.9269 and in down-trending prime rates is 0.9272. The difference performances of strategies relative to up-trending and down-trending prime rates are shown in the Table 5.3.5.

Table 5.3.5: Difference performance of portfolio strategies of a risky asset of Sasol Limited and a fixed-rate asset in up-trending and down-trending prime rates.

α	Total return of Fixed-mix strategy:FiM(α) up trend	Total return of Buy and Hold strategy:BoH(α) up trend	Relative ratio of the strategies	Total return difference of Fixed-mix strategy in up trend and down trend	Total return difference of Buy and Hold strategy in up trend and down trend
-2.0	0.0292	7.3977	253.7371	0.0001	-0.0063
-1.5	0.1339	7.7442	57.8520	0.0001	-0.0047
-1.0	0.4503	8.0906	17.9655	0.0002	-0.0032
-0.5	1.1228	8.4371	7.5146	0.0002	-0.0016
0.0	2.0909	8.7835	4.2008	0.0000	0.0000
0.5	2.9239	9.1299	3.1226	0.0000	0.0016
1.0	3.0792	9.4764	3.0776	0.0004	0.0032
1.5	2.4444	9.8228	4.0185	0.0009	0.0047
2.0	1.4612	10.1693	6.9597	0.0010	0.0063

The difference performance of fixed-mix portfolio strategies is from 1 basis point for leveraged portfolios to 10 basis points for unleveraged portfolios which show less effectiveness of a short rate for leveraged fixed-mix portfolios. The performance of fixed-mix portfolios is favourable in up-trending prime rates. Further allocations in a fixed-rate asset are favourable to up-trending prime rates up to a difference performance of 10 basis points.

The descriptive statistics of discrete total returns of portfolio strategies of a risky asset of Sasol Limited in up-trending prime rates are shown in the Table A.5.2.10 in the appendix. The range of discrete total returns is higher for buy and hold strategies in down-trending prime rates. For fixed-mix strategies the range is higher for strategies with short positions in a fixed-rate asset in up-trending prime rates and lower for

strategies with increasing allocations in a fixed-rate asset. The minimums of portfolios strategies are higher for strategies with short positions in a fixed-rate asset in up-trending prime rates and lower for increasing allocations in a fixed-rate asset. For fixed-mix portfolios the minimums are wider for strategies with increased allocations in down-trending prime rates. The means and standard deviations of discrete total returns are higher for portfolio strategies with short positions in a fixed-rate asset in up-trending prime rates and lower for strategies with increasing allocations in a fixed-rate asset. The standard deviations are higher for strategies with short positions in a fixed-rate asset in up-trending prime rates and lower for increasing allocations in a fixed-rate asset. The value processes of fixed-mix portfolios with 50 percent allocations in a fixed-rate asset are shown in the appendix. The values increase consistently with variations from market risk position in a portfolio.

5.3.6 Difference performance of portfolios of a risky asset of Standard Bank Limited relative to prime rates conditions

The relative asset performance of a portfolio of a fixed-rate asset in down-trending prime rates is 0.9649 and in up-trending prime rates is 0.9652. The difference performances of strategies relative to up-trending and down-trending prime rates are shown in the Table 5.3.6.

Table 5.3.6: Difference performance of portfolio strategies of a risky asset of Standard Bank and a fixed-rate asset in up-trending and down-trending prime rates.

α	Total return of Fixed-mix strategy:FiM(α) up trend	Total return of Buy and Hold strategy:BoH(α) up trend	Relative ratio of the strategies	Total return difference of Fixed-mix strategy in up trend and down trend	Total return difference of Buy and Hold strategy in up trend and down trend
-2.0	0.5359	8.4778	15.8182	-0.0007	-0.0063
-1.5	1.2896	8.6442	6.7029	-0.0012	-0.0047
-1.0	2.4382	8.8106	3.6135	-0.0014	-0.0032
-0.5	3.6451	8.9771	2.4628	-0.0009	-0.0016
0.0	4.3269	9.1435	2.1132	0.0000	0.0000
0.5	4.0887	9.3100	2.2770	0.0006	0.0016
1.0	3.0792	9.4764	3.0776	0.0004	0.0032
1.5	1.8480	9.6428	5.2180	0.0000	0.0047
2.0	0.8827	9.8093	11.1128	-0.0003	0.0063

The difference performance of 0.0837 for a fixed-mix portfolio strategy is optimal at a leveraged allocation of 50 percent in a fixed-rate asset. The leveraged allocations are favourable to down-trending prime rates. The fixed-mix strategy with complete allocations in a risky asset has a difference performance of 0.0653 favourable to down-trending prime rates.

The descriptive statistics of discrete total returns of portfolio strategies of a risky asset of Standard Bank in up-trending prime rates are shown in the table A.5.2.12 in the appendix. The range of discrete total returns of buys and hold portfolios strategies is higher for strategies with a fixed-rate asset in up-trending prime rates. The range of fixed-mix portfolios is higher for strategies with short positions in a fixed-rate asset in up-trending prime rates. The minimums of discrete total returns are lower for portfolio strategies with short positions in a fixed-rate asset in down-trending prime rates and lower for increasing positions in a fixed-rate asset in up-trending prime rates. The means and standard deviations of discrete total returns are higher for portfolio strategies with a fixed-rate asset in up-trending prime rates.

5.4 Summary of the computations

The performed computations of portfolios of a buy and hold portfolio strategy and fixed-mix portfolio strategy in a financial market model which do not consider trading costs are found to underperform the benchmark return and in some instances to outperform the benchmark return over a period of 15 years. The fixed-mix strategy is found to underperform the benchmark return and the risky return in all performed computation except the portfolio of a risky asset of Anglo Gold Ashanti Limited where the fixed-mix strategy outperform the risky return in a market condition of large frequent reversals about a zero rate trend. The performed computations of buy and hold portfolio strategies are optimal for portfolios with short positions in a fixed-rate asset and with increasing allocations in a fixed-rate asset. The complete difference of total performance of buy and hold portfolios is a diversified performance of assets which is computed as the excess of total performance of assets in a portfolio.

The computed portfolios of a risky asset of ABSA Group Limited and a fixed-rate asset are found to have total returns of buy and hold strategies to decrease in a straight line as allocations in a fixed-rate asset increase. The optimal performance of a fixed-mix strategy is a total of 333.97 percent with the allocation of 50 percent in a fixed-rate asset which outperforms the strategies in down-trending prime rates. The value of a fixed-mix portfolio is determined to increase with increasing span of investment. The complete difference of total performance of a buy and hold portfolios is 255 percent which is lower than the optimal total performance of fixed-mix portfolios which indicates a higher diversification return of rebalancing gains resulting from strategic asset allocation. The risky return of a risky asset of AFGRI Limited is a total of 1202.94 percent. The total returns of portfolios of a buy and hold strategy increase in a straight line over allocations in a fixed-rate asset. The total returns of fixed-mix portfolio strategy increase over completely-mixed allocations up to a total return of 536.96 percent at an allocation of 200 percent in a fixed-rate asset.

The benchmark return of a fixed-rate asset in a portfolio of a risky asset of Anglo Gold Ashanti Limited is a total of 539.23 percent over a span of 2987 discrete periods. The risky return is a total of 147.14 percent where the performance of buy and hold portfolio strategies increases as allocations in a fixed-rate asset increase. The optimal total return of a fixed-mix strategy is 232.23 percent at an allocation of 100 percent in a fixed-rate asset which outperforms the risky asset. The complete difference of total performance of buys and hold portfolios is 392.09 percent which is higher than the optimal performance of fixed-mix portfolios and dominated by the performance of a fixed-rate asset indicating less diversification return over the considered period where the risky asset performs less.

The risky return of a risky asset of Anglo American Platinum Limited is 529.17 percent over a span of 3736 discrete observations. The performance of a buy and hold portfolios increases as allocations in a fixed-rate asset increases where the performance of fixed-mx portfolios is optimal at a total performance of 362.12 percent with a strategy allocating 50 percent in a fixed-rate asset. The complete - difference of the total performances of buy and hold portfolios is 418.47 percent which indi-

cates a spread of diversification return of portfolio strategies. The optimal fixed-mix portfolio releases 86.53 percent of the spread of diversification returns.

The portfolio of a risky asset of Sasol Limited obtains an optimal total return of a fixed-mix strategy of 307.92 percent at the allocation of 50 percent in a fixed-rate asset. The complete difference of total performance of buy and hold portfolios is - 69.29 percent which is lower than the optimal performance of fixed-mix portfolios indicating rebalancing gains higher than the diversification return of buy and hold portfolios over a performing risky asset. The risky return of a risky asset of Standard Bank is a total of 914.35 percent where the fixed-mix strategy attains an optimal total return of 426.15 percent with a complete allocation in a risky asset.

The performed computations of portfolio strategies show a trend-following strategy the buy and hold portfolio strategy to dominate the performance of portfolio strategies over a trading risky asset. The fixed-mix strategy is found to perform lesser over a trend following strategy which is a less risky strategy in comparison with a buy and hold strategy. The performance of a fixed-rate asset in the considered conditions of the prime rates is found to outperform the risky return of risky assets of Anglo Gold Ashanti Limited, Anglo American Platinum Limited and Standard Bank and underperforms the risky return of AFGRI Limited and ABSA Group Limited.

5.5 Conclusion

The fixed-mix strategy underperforms the risky returns and the benchmark return particularly in trending market conditions. The performance of a fixed-mix strategy shows the strategy to be less risky than the buy and hold strategy over unanticipated market conditions. The growth of computed portfolios of a buy and hold strategy as estimated by a linear rate over allocations in a fixed-rate asset is determined positive for relative asset performance ratios below 1. The estimated linear rate of total value returns of the computed portfolios of a buy and hold strategy is negative where the relative asset performance is above 1. The value of fixed-mix portfolios increases consistently at different variations which result from fluctuation conditions of risky

asset prices. The fixed-mix portfolios are performing less relative to buy and hold portfolios in trending market conditions. In market conditions with frequent volatile reversals the fixed-mix portfolios outperform buy and hold strategies. The complete difference of the total performance of buy and hold strategies show the diversified total performance of assets over completely-mixed portfolios. The optimal total performances of fixed-mix portfolios are increased over diversification return of a portfolio.

The next chapter analyses the quantification of the financial condition attained by a de-trending strategy in a fixed-mix portfolio to yield fast exponential growth and to perform the computations of the best perform strategies in a fixed-mix portfolio.

Chapter 6

ANALYSIS OF A FINANCIAL PROXY

6.1 Introduction

The fixed-mix portfolio strategy outperforms the risky return rarely under the conditions of volatility and stationarity in a portfolio of a fixed-rate asset and a risky asset since the empirical of asset prices is dominated with trending market conditions. The realised performance of fixed-mix portfolios increases consistently over time. The excess return of a fixed-mix strategy over a benchmark return is determined negative particularly for trending market conditions where the performance is dominated by a trending risky asset. The difference in performance of portfolio strategies relative to market conditions of the prime rates has a substantial impact in the performance of portfolio strategies which are dominated by the performance of a fixed-rate asset. The strategies with short positions in a fixed-rate asset increase the performance in trending market conditions with leveraged positions of a risky asset. The chapter aims to analyse the excess value of risk and the excess performance of a portfolio system of fixed-mix portfolios and the inactive holding strategy. The second aim of the chapter is to quantify the unrealised potential of portfolio growth with risk-adjusted returns on a timeline payoff of a lookback straddle.

The unrealized growth of fixed-mix portfolios and risk-adjusted returns are quantified. The total excess performance of fixed-mix portfolios to risky return and benchmark return is analysed to evaluate the optimal rebalancing gains. The trades of fixed-mix portfolios with 50 percent allocations in a fixed-rate asset are determined. The sufficient condition of a financial proxy in the performed computations is attained by trend-following strategies.

The potential of excess returns of fixed-mix portfolio strategies is determined over trend-following strategies where there is less stationarity. The diversification return which is defined as the excess of weighted growth rates of individual assets and the growth rate of a fixed-mix portfolio in Qian (2012) and approximated to an excess of volatility of a fixed-mix portfolio is optimised by a portfolio strategy. The strategy is

a functional which reduces the downside risk of a risky asset to attain a stationary condition of returns. The analysis of the active investment strategy is performed. The financial proxy required to attain a stationary condition of total returns is analysed and the computations of a best performing portfolio strategy are performed.

The computations of active portfolio management strategies the fixed-mix portfolio strategy relative to the buy and hold strategy, as a benchmark strategy, were performed on daily prices. The daily prices are of risky assets and a fixed-rate asset of a short rate quoted as the effective rate of the current prime rates over a span of 15 years from 1 September 1994 to 1 September 2009. The discrete total returns of fixed-mix portfolios of risky assets of ABSA Group Limited, Anglo American Platinum Limited, Sasol Limited and Standard Bank Limited are found to have log returns which are frequently above their means. The charts and descriptive statistics of total returns of fixed-mix portfolio strategies are shown in the appendix in chart A.4.1.13. The empirical result shows the value of risk-adjusted returns particularly over completely-mixed allocations with total returns which are distributed at a lesser frequency than a normal distribution and wide excess returns from a drift of portfolio returns. The models of log prices are ARIMA model integrated of order 1. The log returns of initialised asset prices are found to be stationary with an estimated ARIMA model integrated of order 0.

The growth rates of fixed-mix strategies are to be strictly above the growth rates of individual assets in a portfolio strategy with 50 percent allocations in a fixed-rate asset and above the growth rate of the returns of the lookback derivative instrument. The growth rates of risky assets are shown in the appendix in chart A.4.1.13.

6.2 Quantitative analysis of performance of portfolio systems

The analysis of the computations performed considers a portfolio of two asset type which is a portfolio of a riskless asset and a risky financial security. The riskless asset used in the computations is a fixed-rate asset which returns a deterministic payoff of one in a discrete trading period with a short rate derived from the current annual

prime rates as a one day effective rate in 255 trading days in one year term. The price of a fixed-rate asset is a discounted payoff under the short rate. The prime rates over a span of 15 years are down-trending from a rate of 18.5 percent to a rate of 10 percent. For simulation purposes the prime rates are considered as up-trending from a rate of 10 percent to a rate of 18.5 percent. The computational analysis examines the performances of arbitrary portfolios of two assets under generic market conditions, particularly the up-trending market condition and the market condition with frequent fluctuations to analyse the required financial proxy.

The characteristics of a portfolio which trades in a fixed-rate asset and a risky asset is defined by a benchmark return which is a riskless returns of a fixed-rate asset and a risky return of a risky asset. The allocations in a portfolio strategy are described by proportions according to the objectives of the investment portfolio. The riskiness of a portfolio is determined by allocations in a riskless and a risky asset where the high risky portfolio invests more on a risky asset. The fixed-mix strategy rebalances the allocated proportions in a portfolio discretely in each trading period.

6.2.1 Excess performance of a fixed-mix portfolio strategy

The performance of a fixed-mix strategy relative to a benchmark return determines the excess performance of the strategy to a fixed-rate asset. The excess ratios of a strategy are shown in figure 6.2.1. The performance of the fixed-mix strategy relative to a risky return is determined by the excess performance of the strategy to a risky asset. The relative asset performance ratio strictly greater than 1 implies a small allocation in a fixed-rate asset in a conventionally best performing risky asset. The relative asset performance ratio of a portfolio of a risky asset of ABSA Group Limited is 1.26932. The total return of a fixed-mix strategy underperforms relatively to a benchmark return and a risky return.

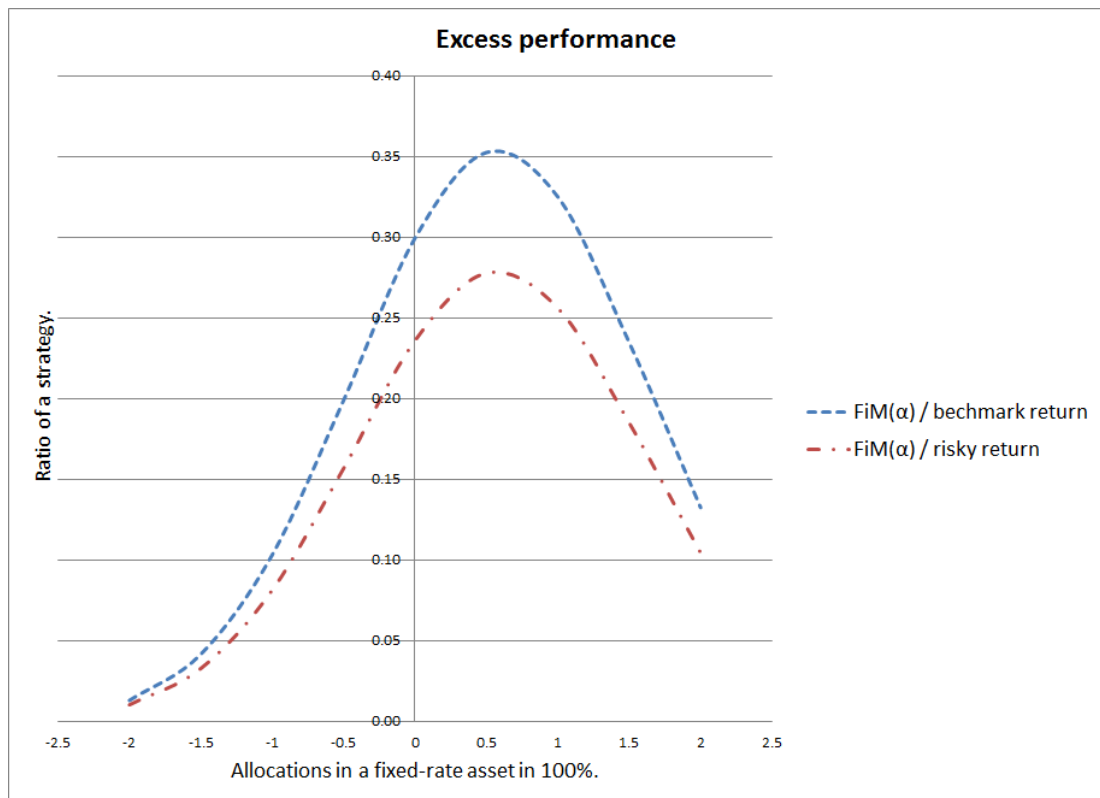


Figure 6.2.1: The excess ratios of a fixed-mix portfolio strategy of a risky asset of ABSA Group Limited.

The excess ratios of a fixed-mix strategy are shown to underperform the benchmark return and the risky return of a portfolio. The total return of a fixed-mix portfolio is only 35 percent of a benchmark return at an optimal allocation of 50 percent in a fixed-rate asset over completely-mixed- allocations. The risky excess ratio of a fixed-mix strategy is a total of 28 percent of a risky return at 50 percent allocation in a fixed-rate asset.

The relative asset performance ratio of a portfolio of a risky asset of AFGRI Limited is 1.2693. The risky asset outperforms the benchmark return.

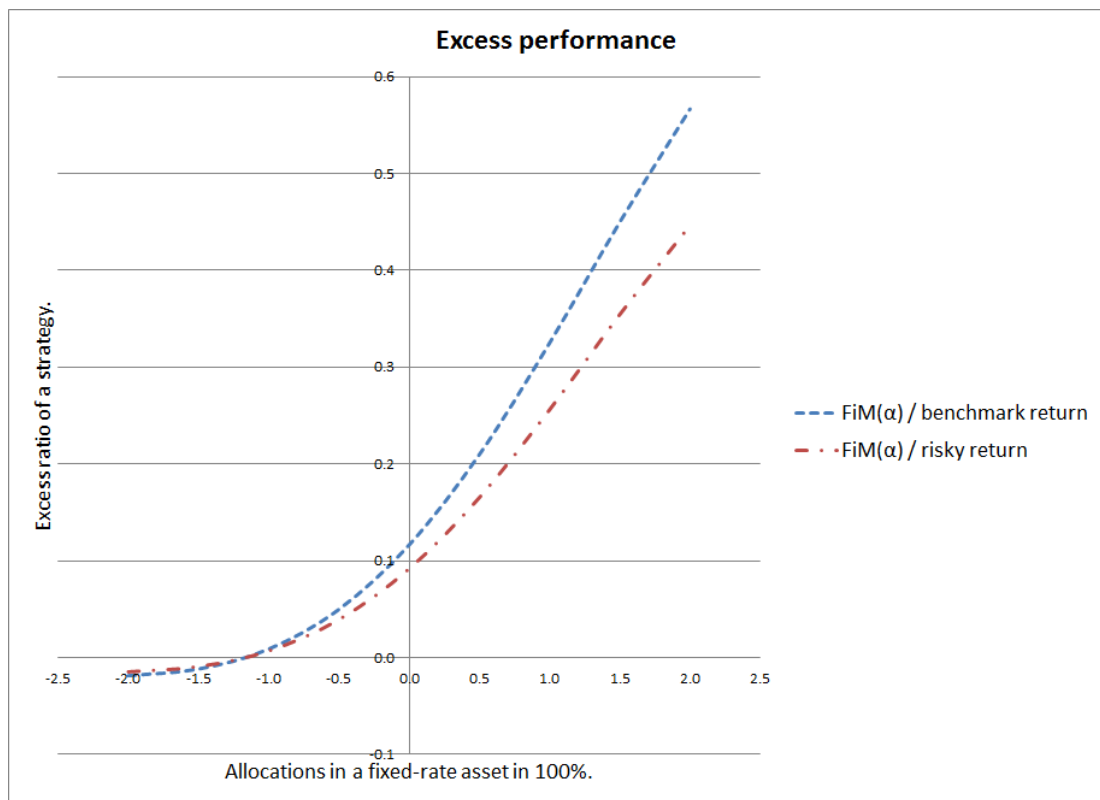


Figure 6.2.2: The excess ratios of a fixed-mix portfolio strategy of a risky asset of AFGRI Limited.

The excess ratios of a strategy over a benchmark return and a risky return are shown in figure 6.2.2 where excess returns increase with further allocations in a portfolio which shows a leverage effect into a fixed-mix strategy. The increasing combination of a risky return and a benchmark return increases the performance of a fixed-mix portfolio strategy. The fixed-mix portfolio shows an increasing excess return of a strategy as allocations in a fixed-rate asset increase. The excess ratios of a fixed-mix strategy are shown in figure 6.2.3.

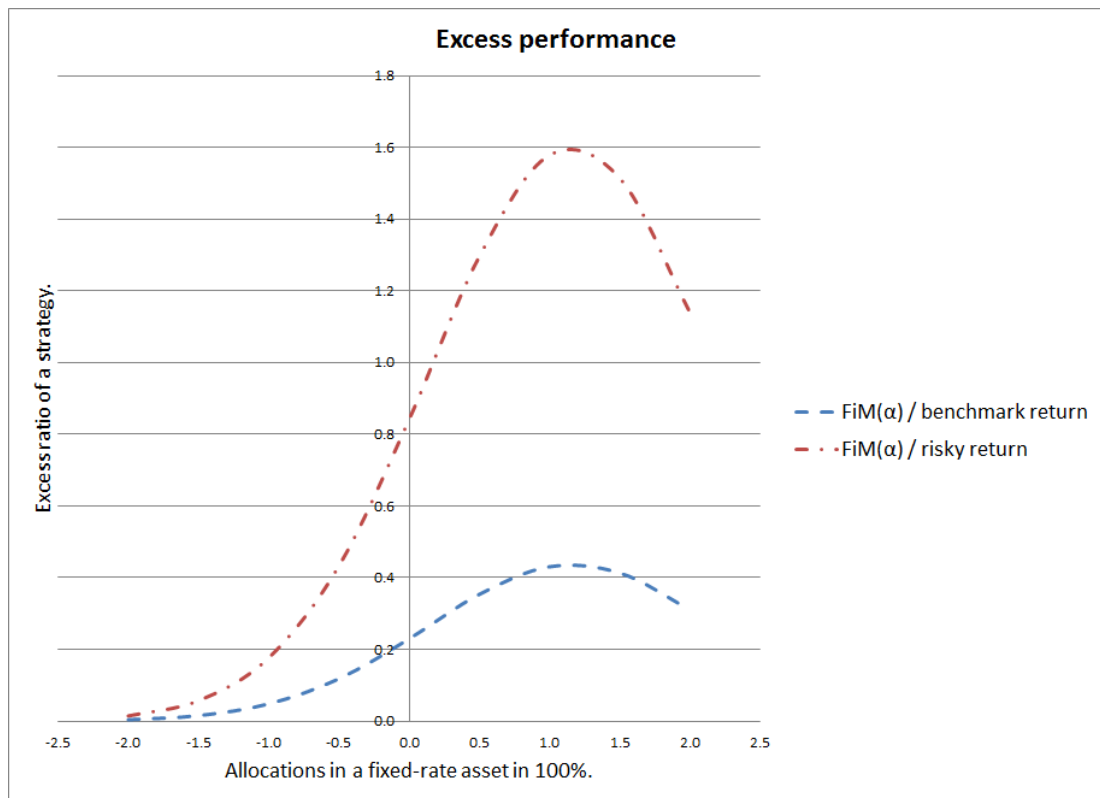


Figure 6.2.3: The excess ratios of a fixed-mix portfolio strategy of a risky asset of Anglo Gold Ashanti Limited.

The relative asset performance ratio of a portfolio of a risky asset of Anglo Gold Ashanti Limited is 0.2729 which shows a risky asset underperform the benchmark return. The optimal excess ratio of a fixed-mix strategy over a benchmark return is 43.07 percent and the excess ratio over a risky asset is total of 157.83 percent for completely-mixed allocations at 50 percent in a fixed-rate asset. The allocation of a fixed-mix strategy outperforms the risky asset. The frequent reversals of a risky asset and the relative asset performance which showing a high performing fixed-rate asset determine excess performance of a fixed-mix strategy.

The excess ratios of a fixed-mix strategy of a risky asset of Anglo Gold American Platinum Limited are shown below in figure 6.2.4. The relative asset performance ratio of a portfolio of a risky asset of Sasol Limited is 0.9269 which shows a fixed-rate asset to outperform the risky return as shown in figure 6.2.5.

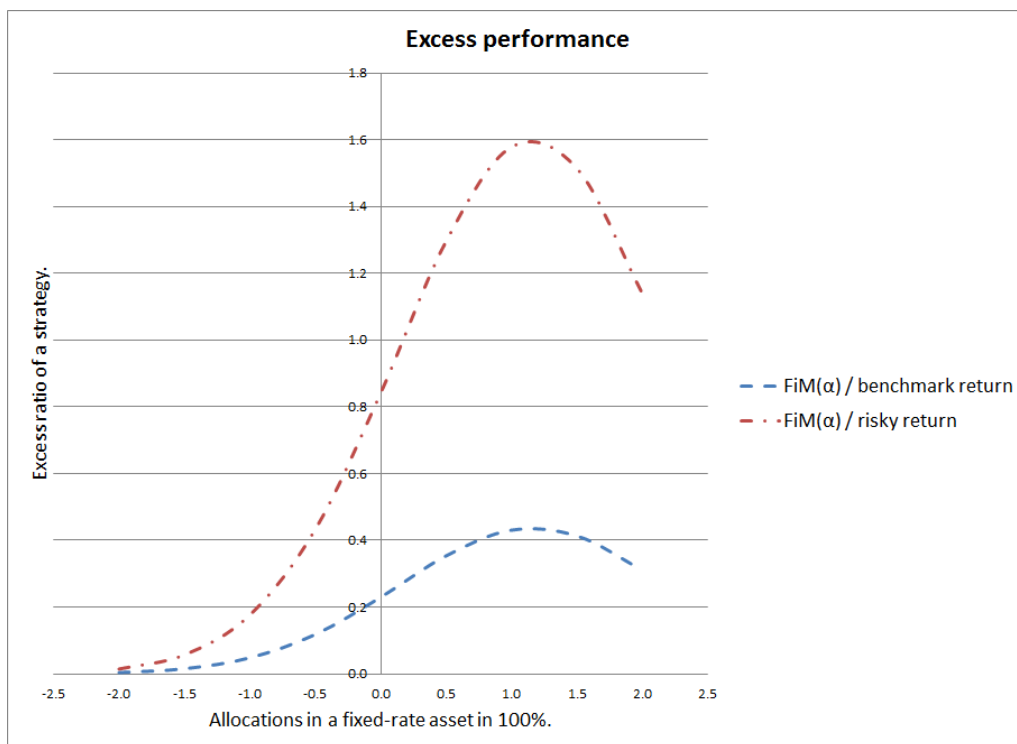


Figure 6.2.4: The excess ratios of a fixed-mix portfolio strategy of a risky asset of Anglo American Platinum Limited.

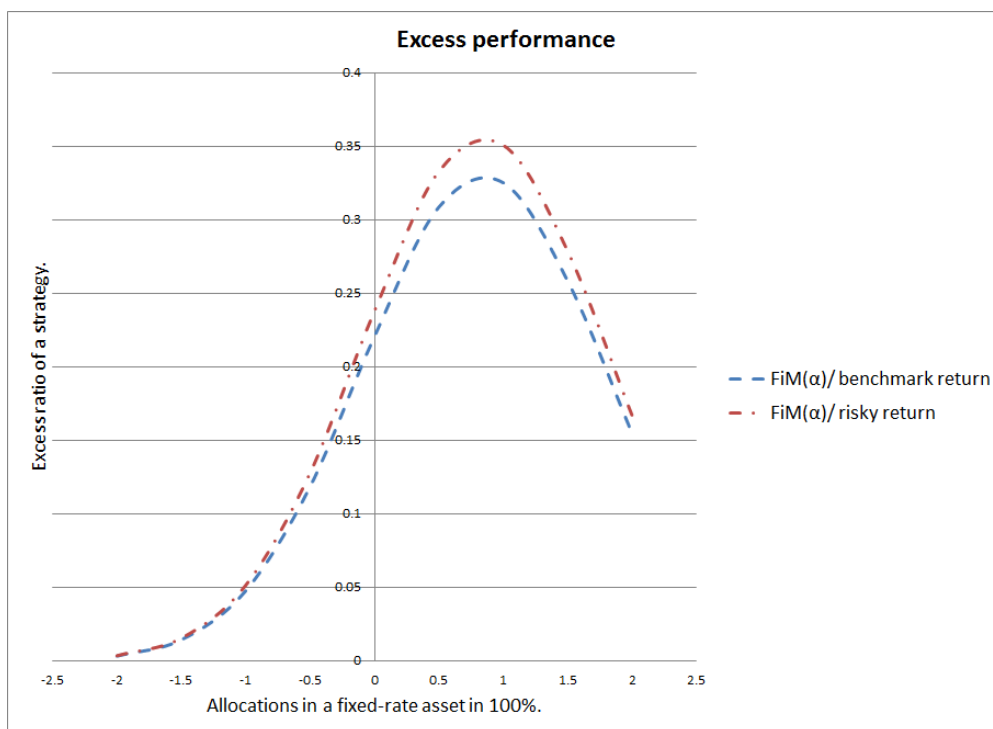


Figure 6.2.5: The excess ratios of a fixed-mix portfolio strategy of a risky asset of Sasol Limited.

The excess ratio of a fixed-mix strategy over a risky return is total 68.43 percent which under performs the risky asset. The excess ratio at the optimal allocation of 50 percent in a fixed-mix portfolio strategy is a total of 35.24 percent of the risky return. The excess ratio of a fixed-mix strategy over a benchmark return is a total of 32.50 percent. The excess ratios of a fixed-mix portfolio strategy are shown in figure 6.2.6.

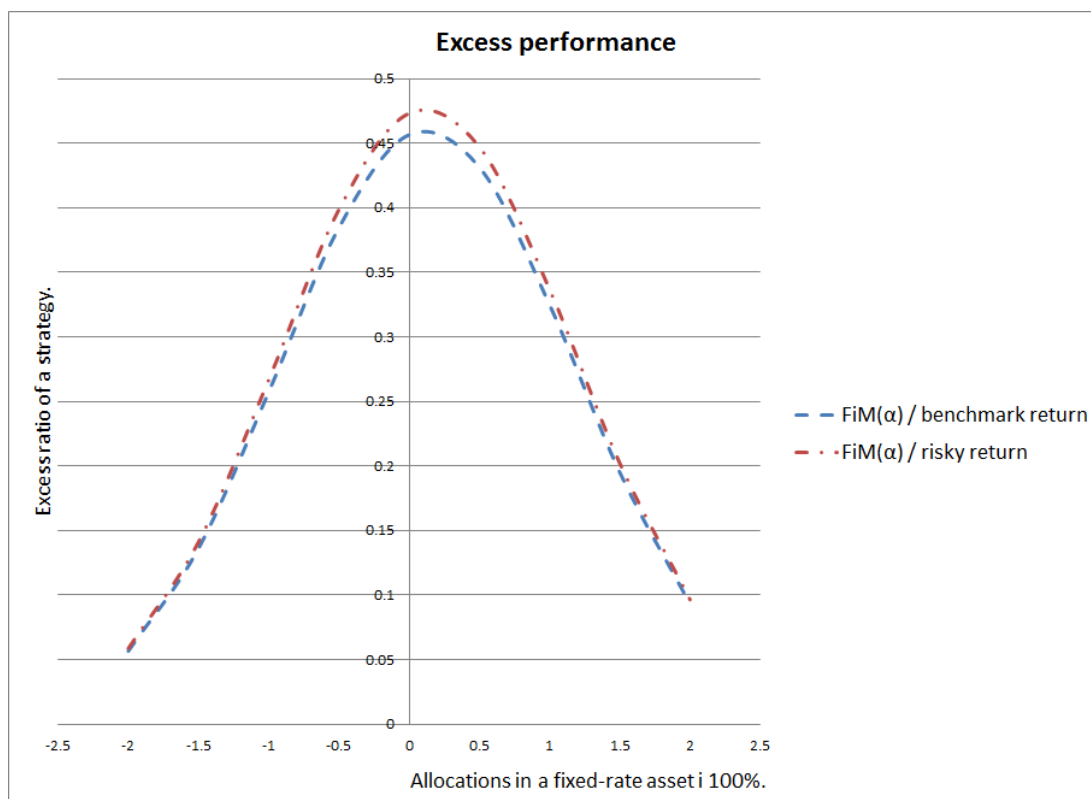


Figure 6.2.6: The excess ratios of a fixed-mix portfolio strategy of a risky asset of Standard Bank Limited.

The relative asset performance ratio of a portfolio of a risky asset of Standard Bank is a total of 96.49 percent. The fixed-rate asset outperforms the risky asset with a small difference. The excess ratio of a fixed-mix portfolio strategy at a complete allocation in a risky asset is total of 44.97 percent of a benchmark return. The strategy underperforms the fixed-rate asset. The excess ratio of a fixed-mix strategy is total of 46.61 percent over a risky return.

The performance of fixed-mix portfolios is relatively below the performances of constituents assets in a portfolio resulting from trending asset prices. The realised performance is strictly above the value of the initial portfolio. The down side risk of trending asset prices is realised positively by fixed-mix portfolios.

6.3 The unrealised potential of stochastic asset prices under fixed-mix strategies

According to Evstigneev and Schenk-Hoppé (2002) the application of the theory of fixed-mix strategies asserts a fast exponential growth of portfolio returns in a stationary market. The multi period accumulation of returns contributes to a fast exponential growth of wealth under the condition of stationarity of assets prices in a portfolio and the non-constancy of the relative price ratio. In the performed computations the buy and hold strategy which is a trend-following strategy is considered as a benchmark strategy with a single period return. The returns in a fixed-mix portfolio strategy are accumulated in multi-periods. In trending market conditions a fixed-mix portfolio strategy under performs the buy and hold strategy which return a single period return. The fixed-mix portfolio strategy outperforms a trend-following strategy in stationary asset prices. The computations show the performance of buy and hold strategies to be a lucrative risky strategy under leveraging since the trend is not deterministic. According to Dempster *et al.* (2007) the condition of trending asset prices is rendered by de-trending.

The significance of total returns and a tactical asset allocation of a moving average of asset prices to identify the asset trends formulate a strategic approach for multi-active managed portfolios. De-trending renders stationarity along a trend by timing the reversal of asset prices about a trend with a tactical asset allocation strategy. The optimal tactical asset allocation strategy under stationary prices is a 50 percent allocation (see Dempster *et al.*, 2007). The significant total returns in a trending market with frequent reversals are retained as stationary tradable securities. The multi period measure of market price of risk assert strictly positive gains in the presence of volatility under the assumption of non-degenerate of asset prices by Dempster *et al.*

(2010) such that the assets total returns are not constant throughout the trading periods. The rebalancing gains are induced by volatility over tactical asset allocation which asserts strictly positive gain in single trading periods.

A tradable stationary security endorses the expected result of exponential growth. The performance of a fixed-mix strategy with a tradable security is determined to strictly outperform a trend following strategy by a geometric return in a market with trending reversals. The excess growth of a fixed-mix strategy with a tradable security is dominated by a growth rate of a trend-following strategy as indicated in Dempster *et al.* (2010). The multi period contrarian strategy accumulates risk-adjusted returns over volatile market condition. The system performance of fixed-mix and buy and hold strategies shows a significance excess-growth between the strategies particularly for completely-mixed strategies and short positions in a risky asset. The fixed-mix portfolios of strategies with complete short positions in a fixed-rate asset are found to have high excess growth because of an increasing price of market risk which is rewarded accordingly by the portfolio strategy over risk-adjusted returns.

The excess growth which results from a trend-following strategy and a fixed-mix strategy is dependent upon the strategic allocation of assets. Optimal excess growth from the computations of portfolio strategies is attained at -200 percent allocations in a fixed-rate asset because of a leveraged effect on a trend-following strategy. The excess growth of the system performance set a framework to link the potential of stochastic asset prices with the deterministic growth under buy and hold strategies. In considering a multi period portfolio strategy a subroutine strategy is constructed to hedge and control the geometric return by effectively linking the excess growth between the trend-following and the contrarian strategies using the concept of portfolio insurance. The portfolio insurance strategy is used to replicate the excess growth between the fixed-mix strategy and buy and strategy by ensuring a premium allocation in considering a short-term trend. The short term strategy which is a single period subroutine is constructed to yield geometric returns which are strictly above the risky return of a trending asset.

The replication technique induces a routine strategy which efficiently identifies the effective allocations of asset prices. The risk-adjusted returns are hedged over each period in a multi period investment portfolio by using a derivative instrument which is a lookback straddle. The derivative returns the difference of the maximum price and the minimum price over the duration of the derivative. The main hypothesis of a financial proxy is to derive tradable stationary process with a geometric return strictly exceeding the risky return over a trending risky asset. The stationary tradable securities ensure the total returns to strictly exceed the short-term total returns over a no-trade region which empirically agrees with the result of fast exponential growth by Dempster *et al.* (2010).

6.4 Quantification of portfolio growth of a fixed-mixed strategy

The growth rates of risky assets are shown to grow at a rate above zero over a period of 15 years. The growth rates of a risky asset are shown in the appendix in chart A.4.1.14 to chart A.4.1.18. The fixed-mix strategy initially purchases a specified number of fixed-rate assets at a current short rate and a specified number of risky assets in the financial securities market. The specified amounts are described by the asset proportions as determined by portfolio objectives. The changes in asset prices deviates the pre-fixed assets proportions in a portfolio. The deviating proportions are re-adjusted discretely during rebalancing to fixed-proportions by assigning a discretely determined amount of asset to be held in a portfolio.

6.4.1 Quantification of a fixed-mixed strategy in a portfolio with 50% of a risky asset of ABSA Group Limited

In a portfolio of a risky asset of ABSA Group Limited the best performing strategy has 50 percent allocations in a fixed-rate asset. The transactions of a fixed-rate asset in a portfolio of a fixed-mix strategy with 50 percent allocations in a fixed-rate asset are shown in figure 6.4.1.a.

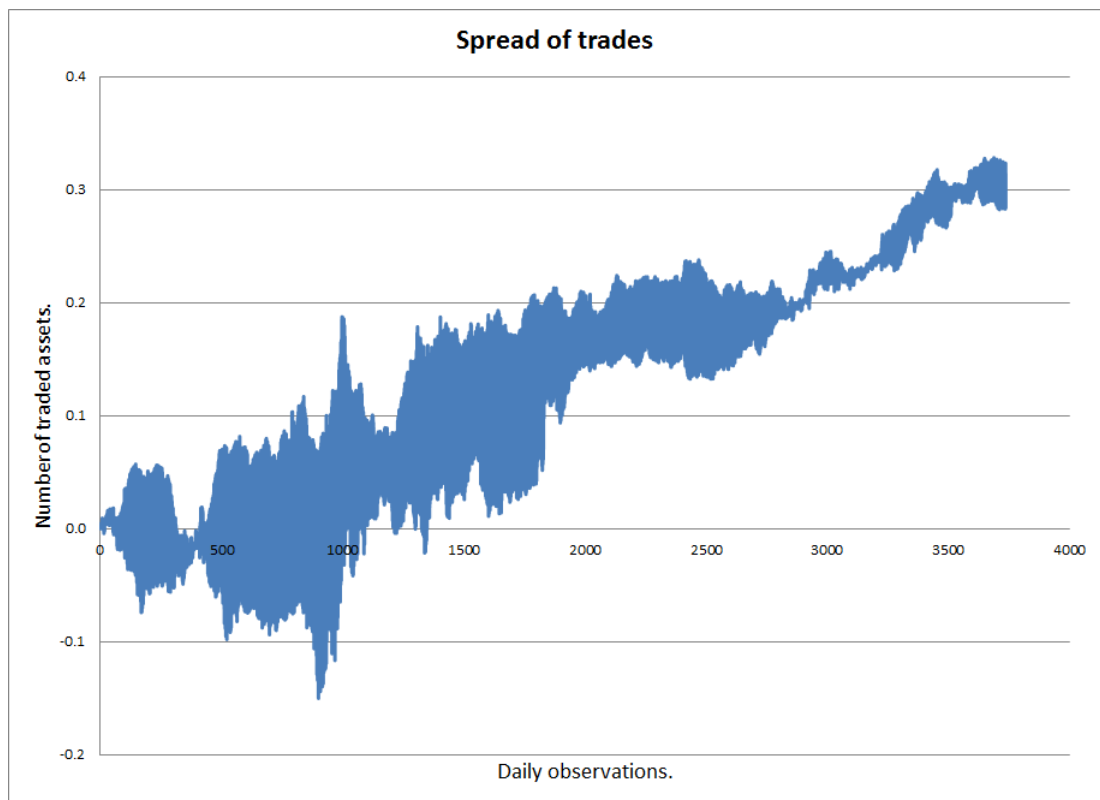


Figure 6.4.1.a: The spread of trades of a fixed-rate asset in a fixed-mix portfolio of a risky asset of ABSA Group Limited.

The numbers of fixed-rate assets shown in the figure 6.4.1.a are offset holdings from the initial fixed-mix portfolio. The spread of a trade is a discrete offset of the holdings of an asset which depicts the structure of transactions and differential holdings. The lesser spread shows the small immediate changes in holdings as indicated in the spread of trade of a fixed-rate asset at the beginning of the investment. The wider spread of trades indicates large changes in the holdings of an asset. The holdings of a fixed-rate asset initially they vary from the assigned holdings as decreasing and increasing up to below 500 observations. The spread increases wide with a variation of decreased holdings and increased holdings. The spread shifts to selling trades which increases as the position of a fixed-rate asset compounds. The spread of trades diminishes indicating the stable position of a fixed-rate which is led by selling transactions. The position of a fixed-rate asset closes with almost an offset of 60 percent which is

a differential in holdings of 0.3 units. The number of fixed-rate assets sold over the span of the investment increases throughout the investment.

The number of risky assets held in a fixed-mix portfolio is shown to decrease as the risky asset trends. The spread of trades of a fixed-rate asset shows a potential of increasing gains in a portfolio by employing an innovative tactical rebalancing method to hedge the trades over deterministic spread. The gains of a fixed-mix strategy in a risky asset are shown in figure 6.4.1.b as a spread of the transactions. The holdings of a risky asset decreases up to a total offset position of 0.35 units over observations below 1000. The trades immediately vary from a selling trade to a buying trade at the 1000th observation indicating a decrease in the price of a risky asset. The spread of selling trades diminishes throughout the investment to a total offset of about 0.35 units.

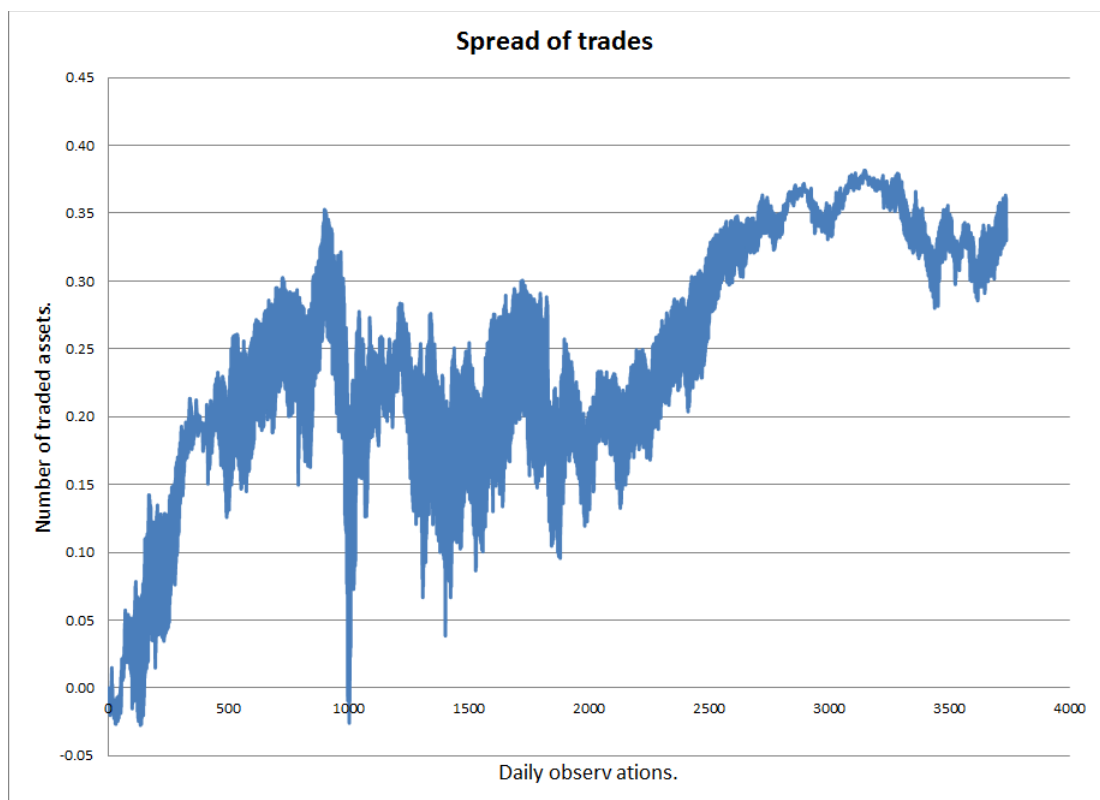


Figure 6.4.1.b: The spread of trades of a risky asset in a fixed-mix portfolio of a risky asset of ABSA Group Limited.

The discrete total returns of a fixed-mix strategy are found to range from a total return of 0.6 to a total return of 1.6 at an average of 1.03 and to frequently accumulate gains of a wider spread. The performing total returns range from 1 to a maximum return of 1.6 at a higher frequency of 150 over 3736 total returns. The frequency of total returns is shown in the appendix in chart A.6.1.1.

6.4.2 Quantification of a fixed-mixed strategy in a portfolio with 50% of a risky asset of AFGRI Limited

The fixed-mix strategy initially assigns selling trades of about 20 percent of the initial holdings of the fixed-rate assets which are coupled with frequent buying transactions of about 100 percent of the initial holdings. The holdings of a fixed-rate asset increase up to and below 200 percent of the initial holdings over the reversals of the asset price. Large spread of trades is a result of frequent fluctuations of asset prices.

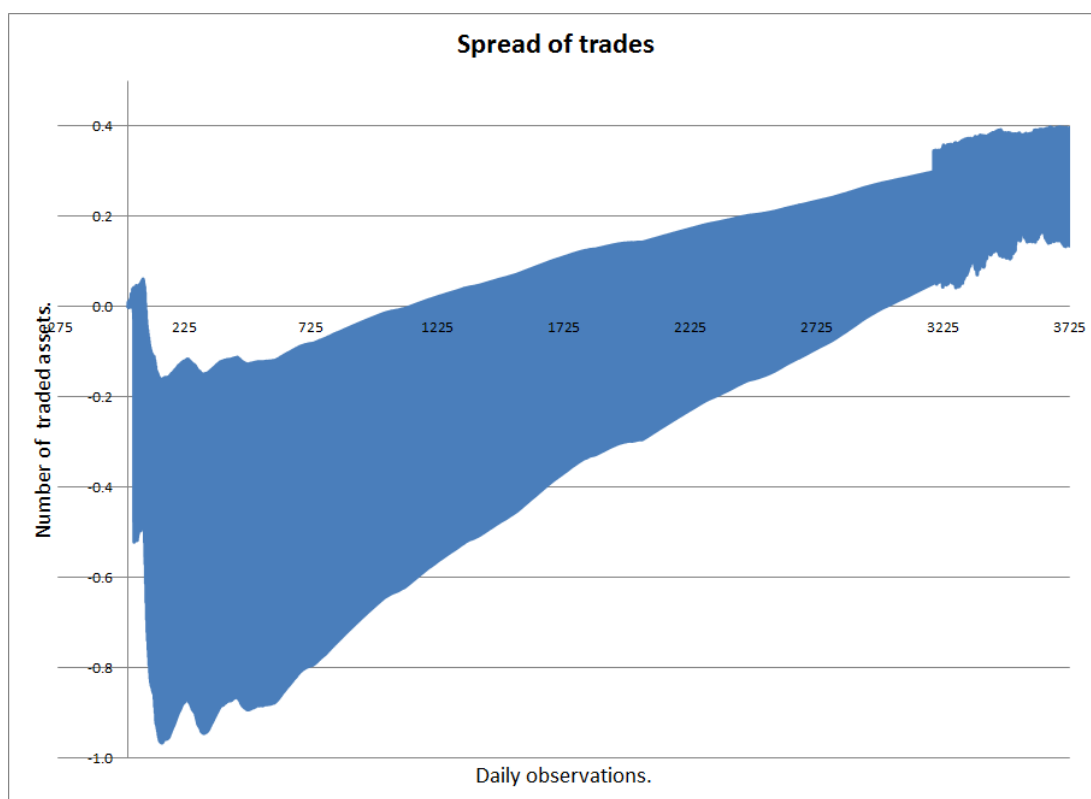


Figure 6.4.2.a: The spread of trades of a fixed-rate asset in a fixed-mix portfolio of a risky asset of AFGRI Limited.

The increased holdings of a fixed-rate asset decreases with variations of selling and diminishing buying trades up to 3000 observations. The selling trades are increase to 80 percent of the initial holdings with a spread of about 40 percent of the differentials of holdings. The selling trades are prompts by compounding and accumulation of rebalancing units of a fixed-rate asset. The spread of trades of a fixed-rate asset is shown in figure 6.4.2.a.

The spread of trade of a risky asset is shown in figure 6.4.2.b. The initial spread of trades of a risky asset increases up to less than 20 percent of the initial holdings signalled by purchases of a risky asset. The trades widely increase with selling transactions by a differential of holdings of about 60 percent. The selling trades increase up to 90 percent of the initial holdings as the prices of the risky asset increases to a stable form.

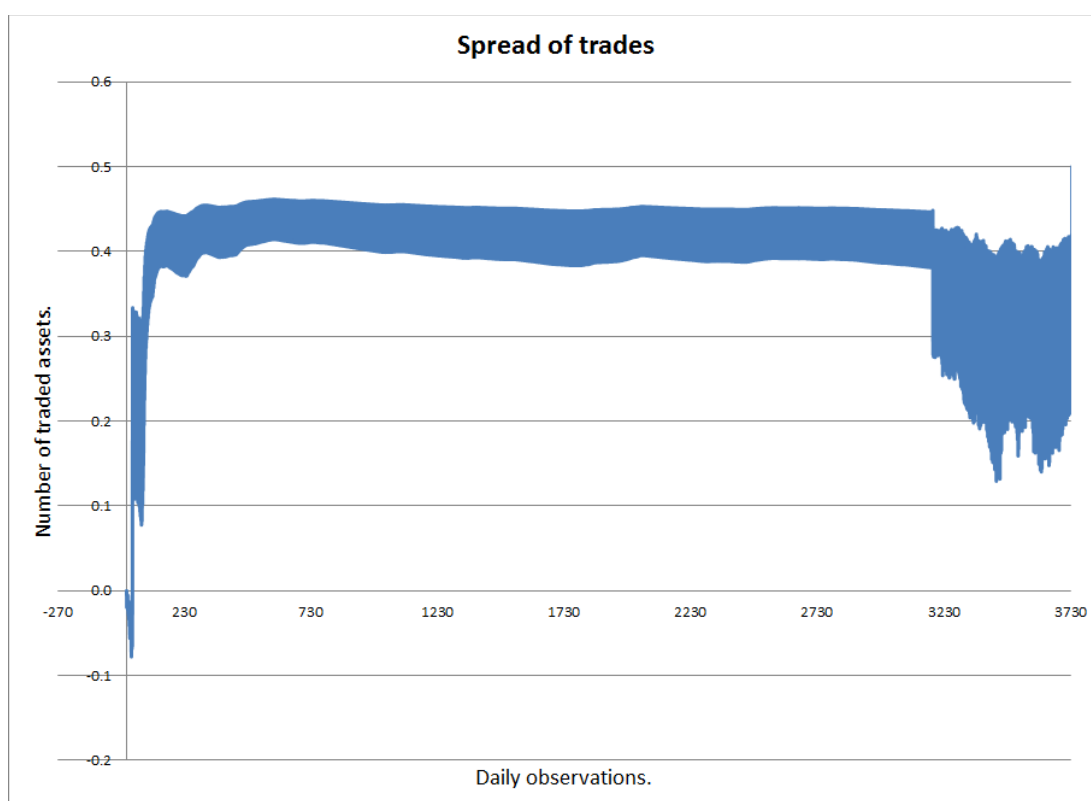


Figure 6.4.2.b: The spread of trades of a risky asset in a fixed-mix portfolio of a risky asset of AFGRI Limited.

The spread of trades uniformly vary with selling transactions up to 3000 observations. The variations in the price of a risky asset beyond 3000 observations drastically vary as indicated by a wider spread. The uniform spread of trades ranges by a difference closely approximately to 15 percent of the total differential of holdings of a risky asset which shows a stationary property on the returns of the trades. The total returns of a best performing strategy with 50 percent allocations in a risky asset of AFGRI Limited are shown to range from 28.17 percent to 351.51 percent with total returns below 50 percent and between 200 percent and 250 percent frequently attained at 1500 frequency over 3736 trading periods.

6.4.3 Quantification of a fixed-mixed strategy in a portfolio with 50% of a risky asset of Anglo Gold Ashanti Limited

The spread of trades of a fixed-rate asset in a fixed-mix strategy increases which shows immediate frequent trades of a fixed-rate asset.

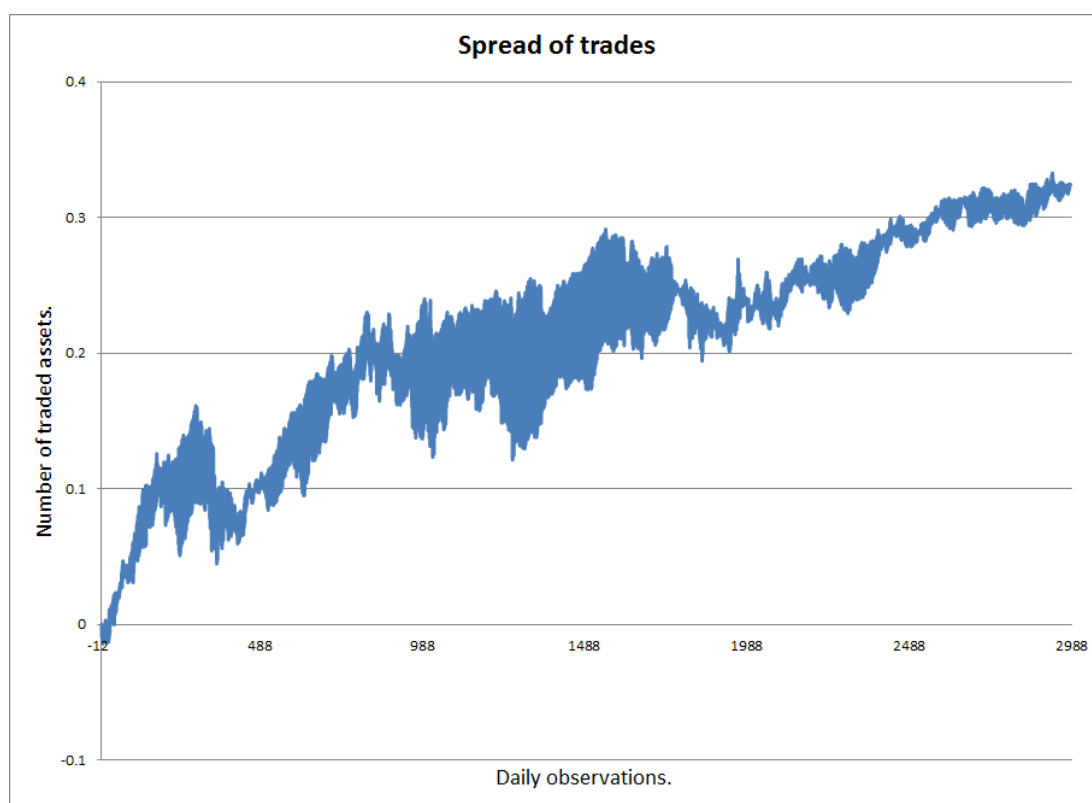


Figure 6.4.3.a: The spread of trades of a fixed-rate asset in a fixed-mix portfolio of a risky asset of Anglo Gold Ashanti Limited.

The spread of trades of a fixed-rate asset are shown in figure 6.4.3.a. The trades of a fixed-rate asset increase up to 40 percent of the total differential of holdings with a less wide spread indicating selling transactions with immediate small differentials of discrete holdings. The spread of trades of a fixed-rate asset widens indicating a large factor of differential change in units resulting from rebalancing gains and continuous compounding. The holdings of a fixed-rate asset decrease by a total differential of holdings of about 68 percent signalling a position dominated with selling transactions.

The risky asset in a fixed-mix strategy is shown to vary frequently by large variations because of frequent fluctuations of a risky asset. The spread of trades is dominated by purchases of a risky asset which increase the units in a portfolio. The spread of trades of a risky asset of Anglo Ashanti Limited is shown in figure 6.4.3.b

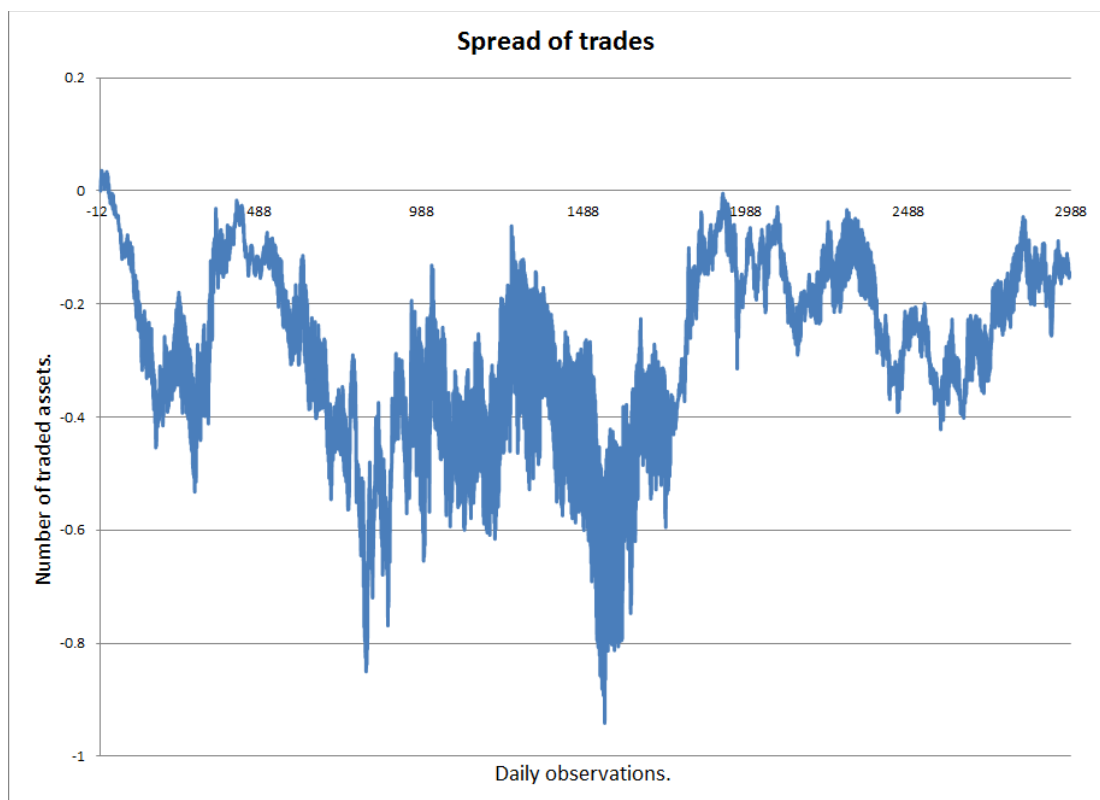


Figure 6.4.3.b: The spread of trades of a risky asset in a fixed-mix portfolio of a risky asset of Anglo Gold Ashanti Limited.

The large differentials of holdings of a risky asset in a portfolio are frequent over the observations between 1000 and 3000. The differential holdings of a risky asset are increased by almost about 200 percent of the initial holdings. The increased units of a risky asset decrease not below the initial holdings of a risky asset as prices increases. The portfolio closes with units of a risky asset above the initial holdings of 28 percent at a price of 147 percent total return. The total returns of a fixed-mix strategy with 50 percent allocations in a risky asset of Anglo Gold Ashanti Limited range from 68.28 percent to 146.64 percent where high frequent total returns are within a 95 percent confidence interval of 80 percent to 120 percent. Total returns above 140 percent are attained at a lower frequency. The growth rates of a fixed-mix strategy are above the growth rates of the index of the portfolio where the spread of trades of a risky asset are large as shown in figure 6.4.3.b above. The holdings of a risky asset are frequently exceeding the allocations where selling and buying of assets keep the holdings from increasing.

6.4.4 Quantification of a fixed-mixed strategy in a portfolio with 50% of a risky asset of Anglo American Platinum Limited

The spread of trades in a fixed-mix portfolio of a fixed-rate asset are shown to widen and dominated with buying transactions. The buying transaction is about 34 percent of the initial holdings which decreases with a wider spread over 500 observations of above 20 percent. The spread of trades diminishes and is dominated by selling transactions. Over the observations ranging from 1000 to 2000 the spread widens large as a result of gains in units which is indicated by increased units of a fixed-rate asset in the 1500th observation by a differential holding of about 20 percent. The selling transaction over the range of observations between 1000 and 2000 is limited below a differential of holdings of 40 percent. The spread of trades vary frequently over purchases and selling of a fixed-rate asset. The spread of trades decreases which shows the selling of fixed-rates assets in a portfolio to occur frequently at a minimum spread. The portfolio closes with a total differential of holdings of 60 percent with selling transactions valuing the portfolio with 0.2 units. The spread of trades of a fixed-rate asset is shown in figure 6.4.4.a.

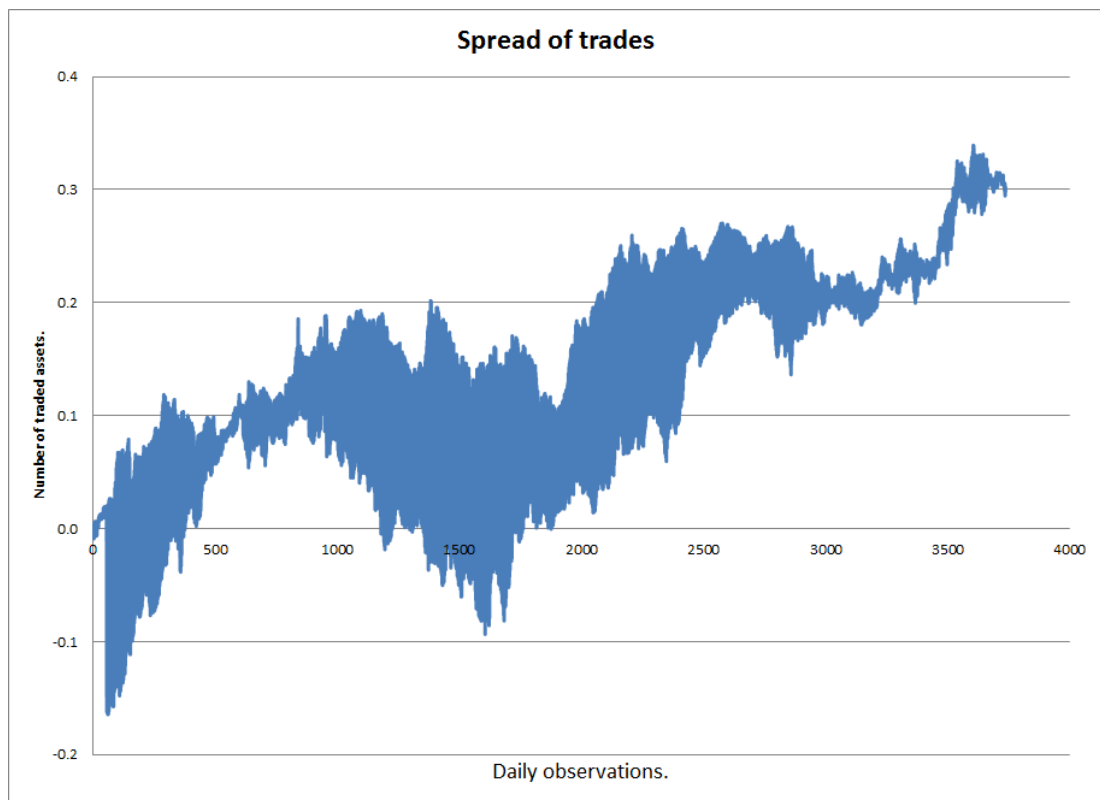


Figure 6.4.4.a: The spread of trades of a fixed-rate asset in a fixed-mix portfolio of a risky asset of Anglo American Platinum Limited.

The spread of trades of a risky asset widens with variations over 500 observations to a total differential of holdings of 128 percent. Between the 500 and 1000 observations the spread varies frequently with a limit buying transaction to a total differential of 170 percent. The buying trades decrease up to 1500 observations with decreasing units as the fixed-rate assets accumulates units in a compounding position in returns. The trades vary with selling and buying transactions within the threshold of 40 percent units traded. For observations exceeding 3000 the spread of selling the fixed-rate asset diminishes where the total differential of holdings do not exceed 70 percent of the units. Towards the closing of the portfolio beyond the 3500th observation the gained units of a fixed-rate asset are traded and 10 percent of the 0.5 units is purchased. The portfolio closes with less units at a total of 72 percent of the initial hold-

ings. The following figure 6.4.4.b shows the spread of trades of a risky asset of Anglo American Platinum Limited.

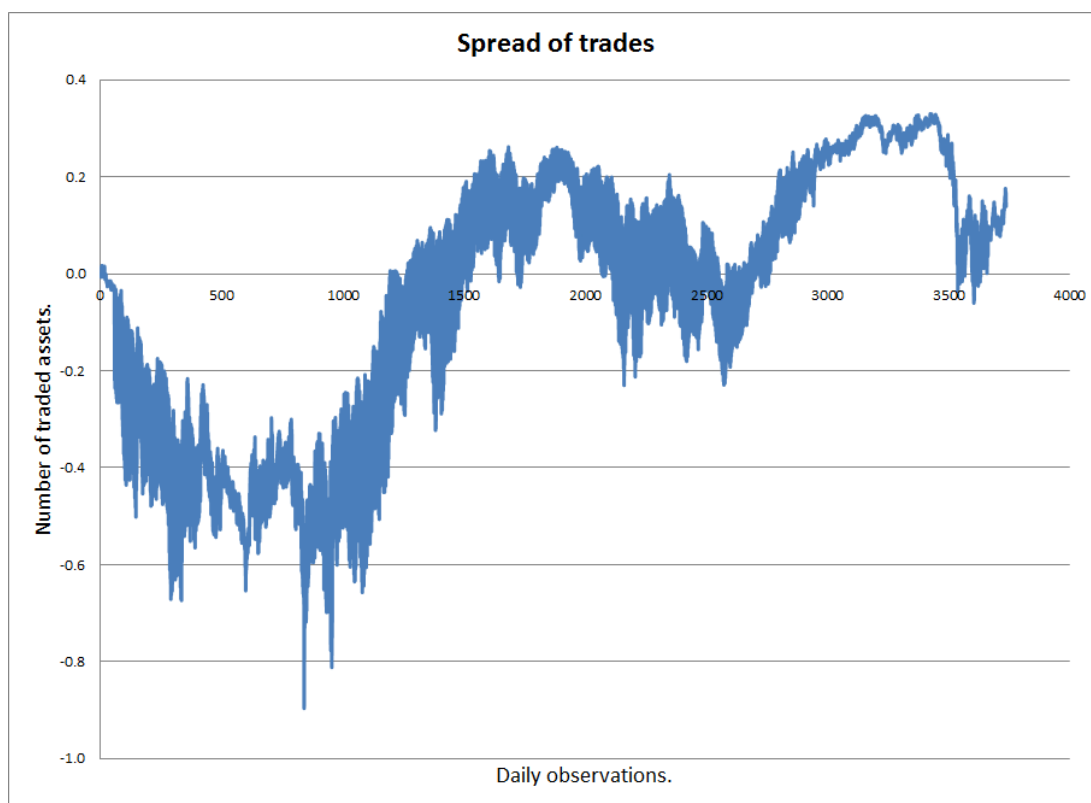


Figure 6.4.4.b: The spread of trades of a risky asset in a fixed-mix portfolio of a risky asset of Anglo American Platinum Limited.

The total returns of a fixed-mix strategy with 50 percent allocations in a risky asset of Anglo American are shown to range from 44.86 percent to 225.34 percent. The total returns of a higher frequency of 300 over 3736 total returns are from 44.86 percent to 100 percent total returns. From total returns of 100 percent to 225.34 percent the frequency ranges to 100. The frequency of total returns is shown in the appendix in chart A.6.1.4. The growth rates of a fixed-mix strategy index are below 0.00028.

6.4.5 Quantification of a fixed-mixed strategy in a portfolio with 50% of a risky asset of Sasol Limited

The spread of trades of a fixed-rate asset in a fixed-mix portfolio strategy is shown to widen over observations below 750 which shows a spread dominated with selling transactions up to a total differential of holdings of 24 percent.

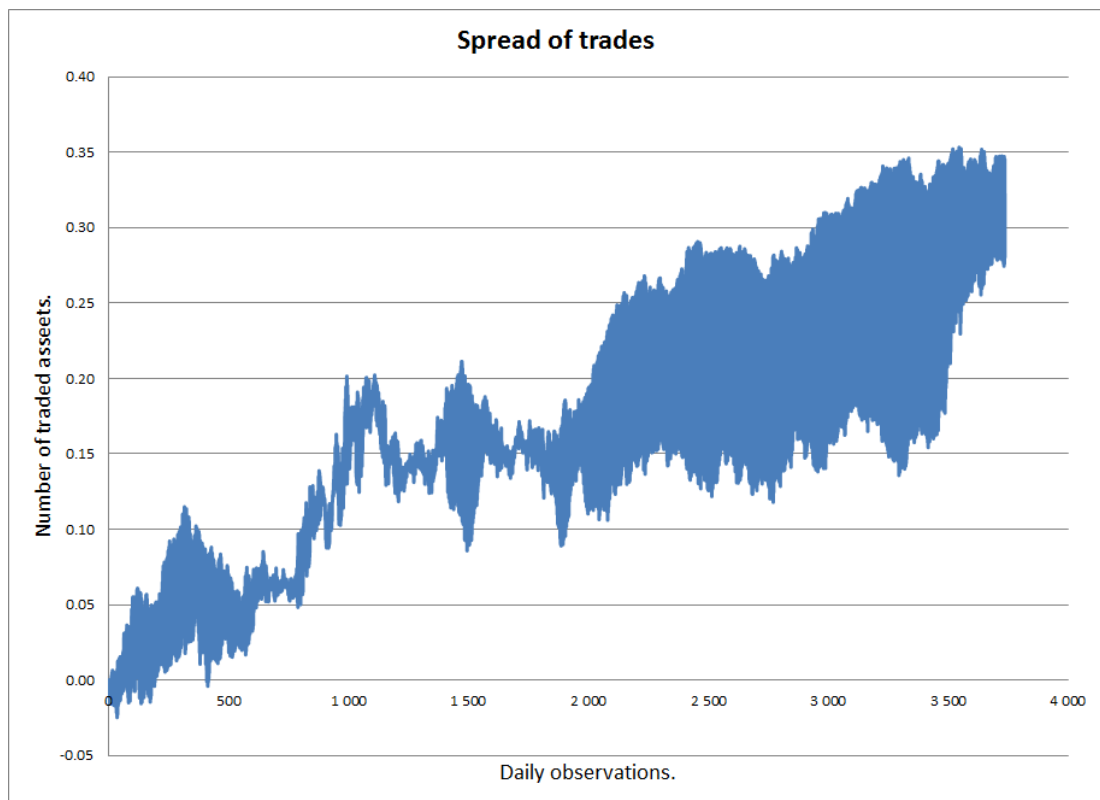


Figure 6.4.5.a: The spread of trades of a fixed-rate asset in a fixed-mix portfolio of a risky asset of Sasol Limited.

The trades increase with variations at a lesser spread over the limit of 40 percent total differential of holdings in the range of observations from 1000 to 2000. The trades of a fixed-rate asset are dominated with selling transactions resulting from a compounding position and gained units from a wider spread of transactions. The increased selling transactions close at a total differential of holdings of 70 percent. The spread of about 15 percent total differential of holding indicates variations in the gains of units of a fixed-rate asset.

The total differential of holdings of selling transactions of a fixed-rate asset is about 0.35 units. The portfolio closes with a total of 30 percent of the initial holdings of the fixed-rate asset in the portfolio. The spread of trades of a risky asset is shown figure 6.4.5.b dominated with selling transactions.

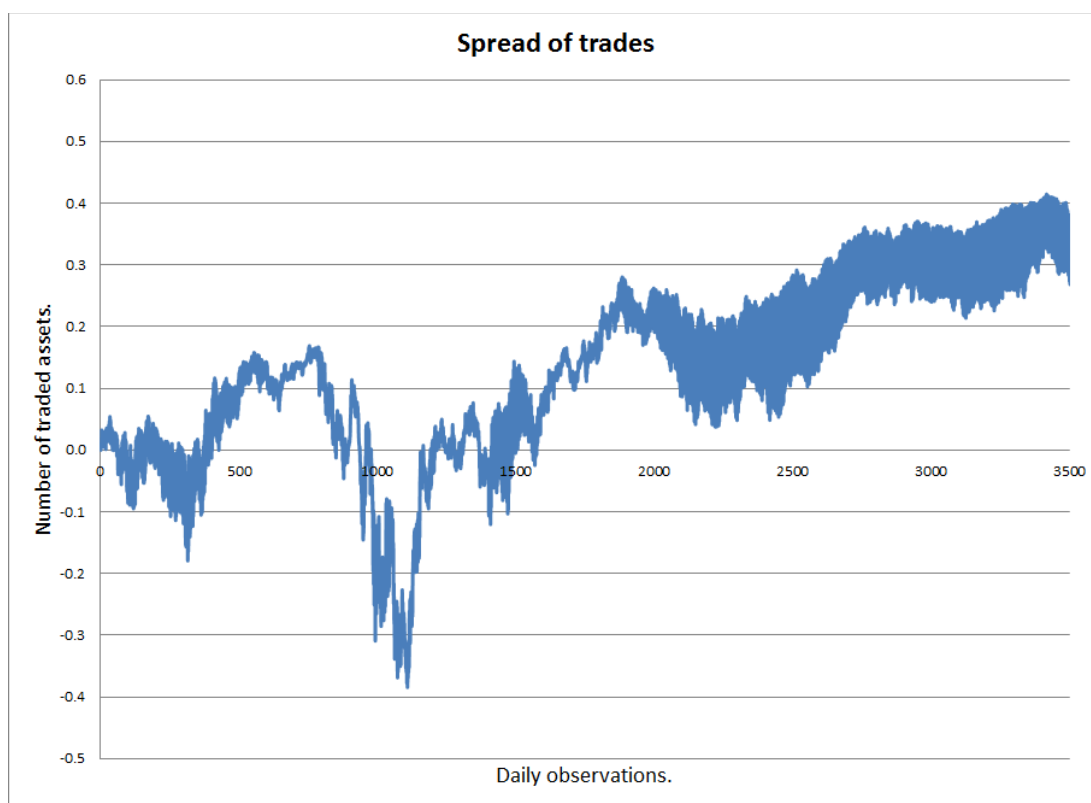


Figure 6.4.5.b: The spread of trades of a risky asset in a fixed-mix portfolio of a risky asset of Sasol Limited.

The holdings of a risky asset increases to a total differential of below 40 percent of the initial holdings at a varying spread. The spread varies with increasing selling transactions below a total differential of 40 percent. The trades vary with buying and selling transactions up to 1500 observations with rebalancing gains resulting from increased units of fixed-rate asset. The spread diminishes with decreasing units of a risky asset limited below a total differential of holdings of 60 percent up to 2500 observations. The spread widens resulting from increased prices of a risky asset to a total differential of 80 percent of the selling transactions. The portfolio closes with units of about 25 percent of the initial holdings. The value of the risky position is a

quarter of a total performance of 878 percent. The total returns of a best performing strategy with 50 percent allocations in a risky asset of Sasol Limited are shown to range from 43.19 percent to 229.42 percent with a volatility of 38.91 percent where total returns of a higher frequency are below the average of 106.68 percent. The value of a fixed-mix strategy with 50 percent allocations in a risky asset grows up to a rate of 1.6 which is dominated by the variations in the number of fixed-rate assets traded in a portfolio and grows strictly above the rate of the index.

6.4.6 Quantification of a fixed-mixed strategy in a portfolio with 50% of a risky asset of Standard Bank Limited

The spread of trades of a fixed-mix strategy with 50 percent allocations in a fixed-rate asset is shown in figure 6.4.6.a.

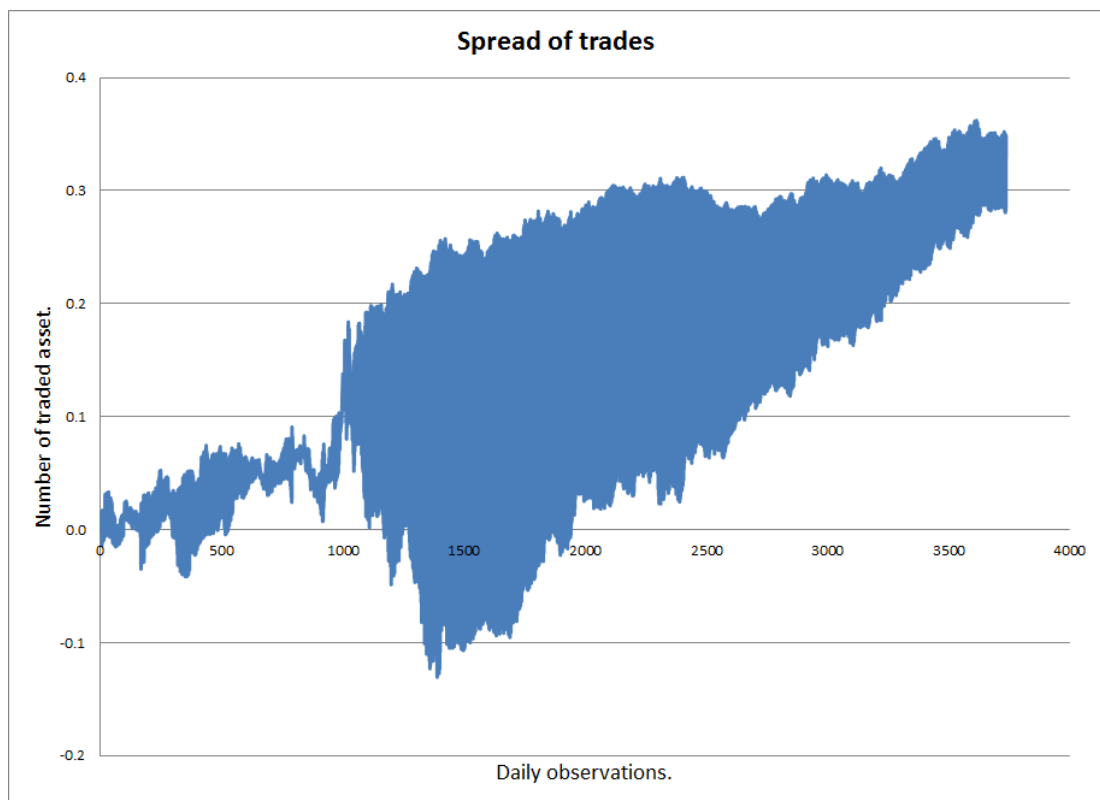


Figure 6.4.6.a: The spread of trades of a fixed-rate asset in a fixed-mix portfolio of a risky asset of Standard Bank Limited.

The figure shows the spread of trades of a fixed-rate asset in a portfolio of a risky asset of Standard Bank Limited. The trades of a fixed-rate asset are dominated with selling transactions at an increasing spread as the risky asset trends and the fixed-rate asset accumulates units. The increasing selling transactions widens the spread over 1000 observations coupled with buying transactions limited at a total differential of holdings of about 26 percent. The wide spread depicts gains of units resulting from rebalancing gains from a risky asset. The steep increasing below limits of the spread indicates selling transactions that are consistently increasing resulting from the gains in the variation of a risky asset. The portfolio closes with fixed-rate asset units of about 30 percent of the initial holdings showing a dominant up trend and the accumulation of rebalancing gains.

The spread of trades of a risky asset of Standard Bank Limited is shown in figure 6.5.6.b below.

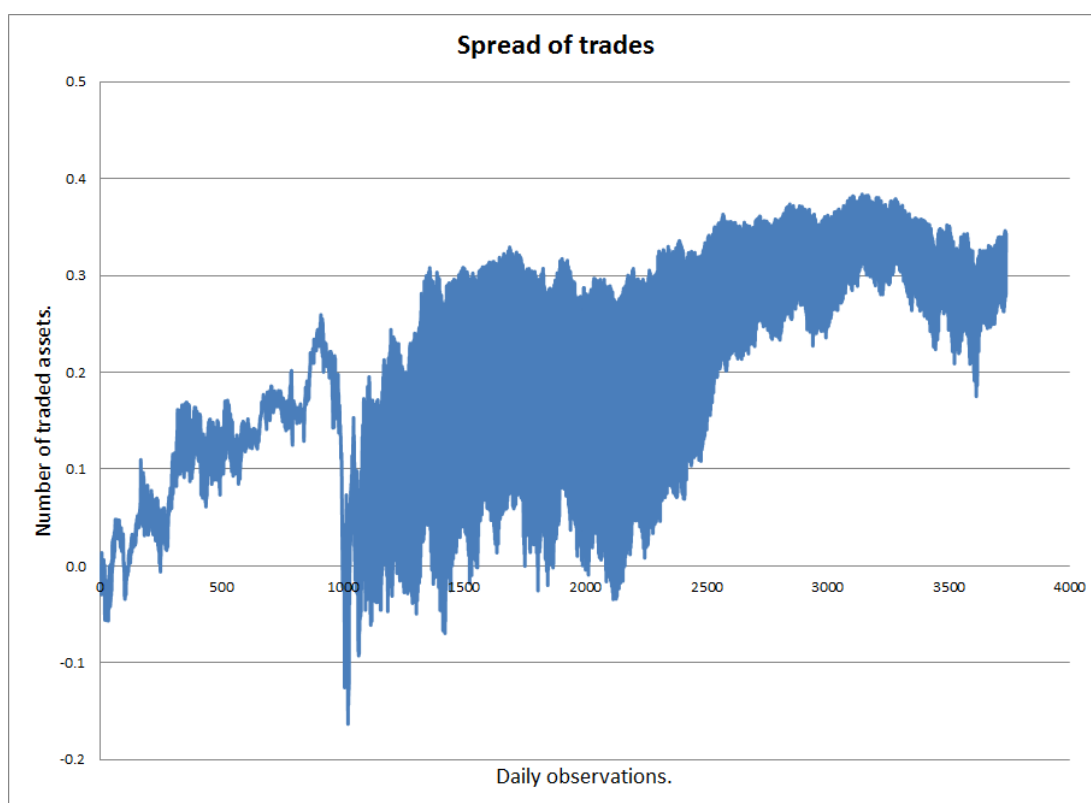


Figure 6.4.6.b: The spread of trades of a risky asset in a fixed-mix portfolio of a risky asset of Standard Bank Limited.

The spread of trades of the risky asset shows variations up to 1000 observations dominated by selling transactions which result from a trending risky asset. From the 1000th observation the spread of trades revert to buying transactions with a wider spread over selling transactions as the price of the risky asset suddenly changes with a reversal of about a total of the initial price..

The frequent fluctuations of a risky asset spread the selling transactions which vary with an above limit total differential of holdings of about 60 percent indicating increasing selling transactions to a total differential of holdings limited below 80 percent of the total differential of holdings. The spreads of trades of a risky asset diminish towards the closing of a portfolio. The portfolio closes with a 32 percent of the initial holdings of a risky asset

The quantification of portfolio growth of fixed-mix portfolios is shown to be volatile with large variations indicated by the spread of trades. The variations of trades of a fixed-rate asset in a fixed-mix portfolio are found to be dominated by selling transactions as the fixed-rate asset position accumulates and regains from a trending risky asset. The trades of a fixed-rate asset increase with less variation which showing an increasing momentum of a fixed-rate asset position in a portfolio as characterized by frequent selling and buying transactions of a fixed-rate asset. The trades of a risky asset vary frequently and widen as rebalancing transactions have large differentials in number of units resulting from the reversals of prices showing an increased volatility. The spread of trades in the performed computations indicated a diminishing number of units of assets in a portfolio which resulted from trending positions where accumulation gains and rebalancing gains are dominated by a unitary transaction with mainly selling instructions. The selling instructions are due to trending market conditions and the compounding of a fixed-rate asset. The value of excess risk in a portfolio determines the potential of the market risk and the unstructured transaction which impose a financial risk on the encountered market conditions. Further analysis examines the quantification of the gains and the financial risk involved over the market risk.

6.5 Quantification of excess value of risk in a portfolio

The excess value of risk is determined by the premium of a buy and hold strategy which is more risky than the fixed-mix strategy. The risk of the buy and hold strategy comes with the non-deterministic trend in a risky asset and a 67 percent chance not to do well from reversals and down-trending conditions. The capital asset pricing model is considered to set a basis of equilibrium for a risky asset in a portfolio as defined in the appendix in model 4. The expected return of a risky asset is the return of a benchmark and the premium of a strategy in the considered market.

The premium of a portfolio strategy is the excess return of the strategy to a benchmark return. The fixed-mix strategy is a discount strategy which is not compensated by a performing market where it loses units of assets. The strategy performs on the excess of underperforming and over performing market conditions where the equilibrium of trades is a binary transaction. The excess performance of the buy and hold strategy and the fixed-mix strategy is the difference in the performance of the strategies. The financial risk and the market risk of the strategies is coupled in the excess performance of the strategy. The coefficient of excess value of risk from the initialization of the investment up to terminal time, $v(0, \alpha) = \alpha \cdot v_0$, is computed as the ratio of the allocation in a fixed-rate asset and the relative asset performance ratio. The excess value of risk is computed as the benchmark value, λ , of an internal rate of returns, r_λ , and the coefficient of excess value of risk times the excess value return of a buy and hold strategy to a fixed-rate asset. The value of risk in a portfolio with an initial investment of x_0 accounts for the price of market risk as quantified by the excess value of the strategy given by,

$$r(t, \alpha) = \lambda + v(t, \alpha) \cdot (\text{BoH}(\alpha) - \lambda) \quad (\text{a})$$

where $\lambda = x_0(e^{r_\lambda(T-t)})$ for $t \leq T$, $x_0 > 0$ and $-2 \leq \alpha \leq 2$.

6.5.1 Quantification of excess value of risk in a portfolio of a risky asset of ABSA Group Limited

The excess value of a portfolio strategy computed by equation (a) is shown in figure 6.5.1 as the offset of the benchmark return. The increased risk and the avoided risk are at optimum in the value of 9.979 for 50 percent allocations in a fixed-rate asset. The avoided risk is not compensated for and the increased risk yields rewards and losses which in total eliminates the potential of risk. The strategies which allocate complete allocations in a risky asset and in a fixed-rate asset are symmetric at a benchmark value of 9.476 indicating an excess value on the increased allocations for the risky return. The further allocation of market risk from a short position of a fixed-rate asset offsets a 9.476 performance which indicates a strategy at excess.

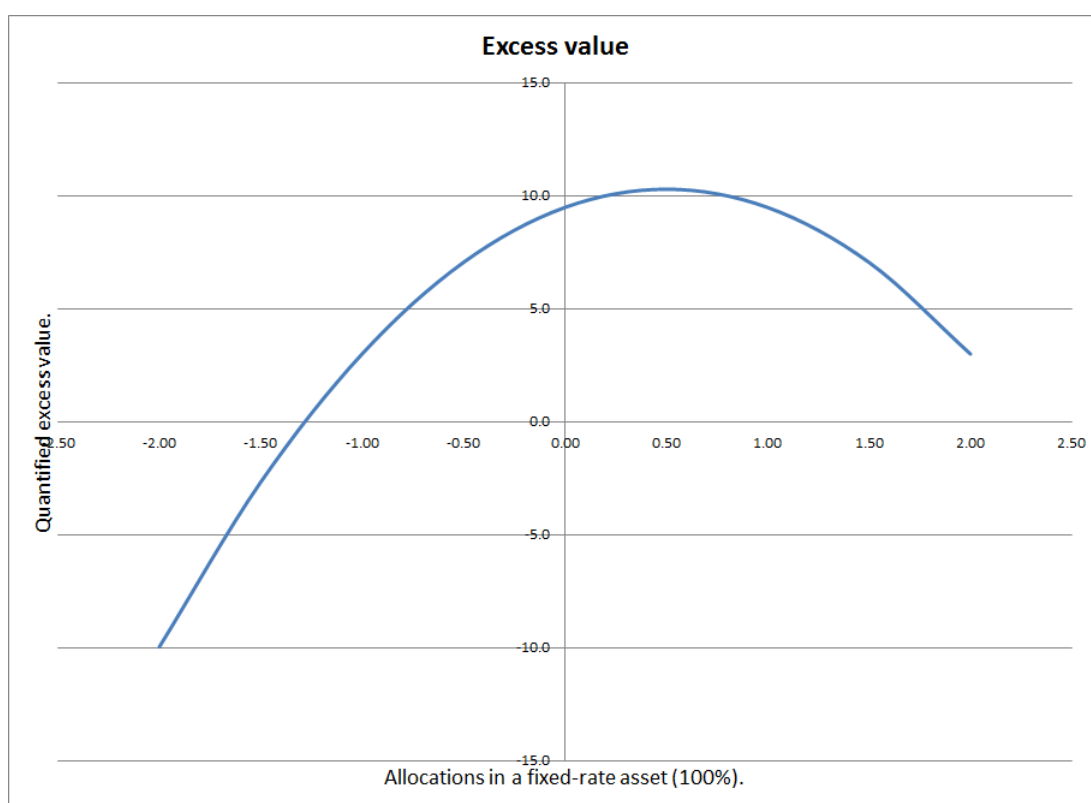


Figure 6.5.1: The excess value of risk of portfolio strategies of a risky asset of ABSA Group Limited.

The performance of a buy and hold strategy measured with a Sharpe ratio at a value of 0.0218 indicates an outperformance of a strategy with 200 percent short positions

in a fixed-rate asset exceeding the performance of a benchmark strategy. The Sharpe ratio decreases to a ratio of -0.0387 for strategies with increased allocations in a fixed-rate asset to a strategy with 100 percent short position in a risky asset which underperforms. The Sharpe ratios of fixed-mix strategies increase from a ratio of -0.7788 indicating an underperforming strategy with 200 percent short position in a fixed-rate asset to a ratio of -0.3467 for a strategy with 50 percent allocations in a fixed-rate asset indicating an increasing performance over the down side risk. The strategy with 100 percent allocations in a fixed-rate asset has an increased downside risk at a ratio of -0.5113 which decreases to -2.4149 for further short positions in a risky asset indicating an increasing downside risk for a fixed-mix strategy.

6.5.2 Quantification of excess value of risk in a portfolio of a risky asset of AFGRI Limited

The excess value of risk of a portfolio strategy of a risky asset of AFGRI Limited is shown in figure 6.5.2.

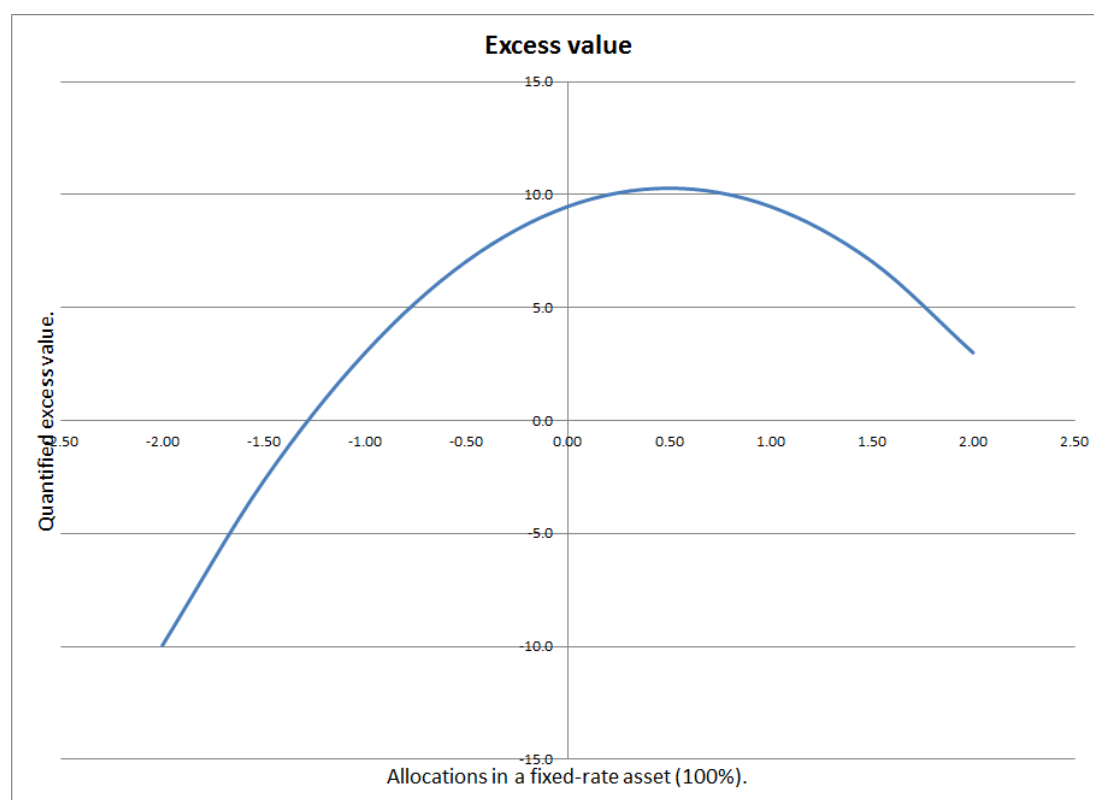


Figure 6.5.2: The excess value of risk of portfolio strategies of a risky asset of AFGRI Limited.

The portfolio strategies of a portfolio of a risky asset of AFGRI Limited are determined to have excess value of risk which is optimal over completely-mixed allocations. The excess value of risk of a portfolio strategy of a risky asset of AFGRI Limited is symmetric at a 50 percent allocation in the market risk at a value of 10.286. The offset value of risk of the portfolio decreases with an increasing market risk. The portfolio offset the benchmark for further allocations exceeding at short position of 150 percent in a fixed-rate asset.

The shown excess value of -9.961 for further allocations of market risk indicates a demand for a portfolio strategy to encounter the value of risk in the market considered. The added risk and avoided risk by short positions in the portfolio is 2.997. The buy and hold strategies outperform with increasing Sharpe ratios from 0.0088 to an optimal ratio of 0.0128 for a strategy with 50 percent allocations in a fixed-rate asset. The Sharpe ratios decreases from -0.0077 for allocations exceeding the benchmark to a ratio of -0.0104 for a strategy with a 100 percent short position in a risky asset showing increasing underperformance. The performance of a fixed-mix strategy with short positions of 200 percent and 150 percent in a fixed-rate asset is not measured for strategies underperforming with negative returns. The Sharpe ratios decreases from -0.1025 for a strategy with 100 percent short position in a fixed-rate asset to a ratio of -0.4851 for increasing allocations in a fixed-rate asset indicating an increasing performance at a downside risk. The ratios increase from -0.1821 for a strategy with 50 percent short position in a risky asset to a ratio of -0.0813 for a strategy with 100 percent short positions in a risky asset which shows an increasing performance of an underperforming strategy.

6.5.3 Quantification of excess value of risk in a portfolio of a risky asset of Anglo Gold Ashanti Limited

The excess value of risk of a portfolio of a risky asset of Anglo Gold Ashanti Limited is shown in figure 6.5.3. The excess value of risk is dominated by a fixed-rate asset position where short positions increase the market risk. The added risk and avoided risk which is not an offset of the strategy is 7.532 for both complete short

positions in a portfolio. The complete allocation of risk in a portfolio exposes a financial risk of 5.392 from a fixed-rate asset position. The value at equilibrium is 5.125 at 50 percent allocation in the marker risk. The excess value is increased up to 12.0 for further shorting in a fixed-rate asset. The increased position of a fixed-rate asset encounters the financial risk of a portfolio underperforming the benchmark where the excess value of risk increases. The excess value of risk is rewarded as the risky asset underperforms. The excess value shows the premium of a portfolio strategy over the downside risk of a trend-following strategy. The excess performance of strategies is increased from a complete allocation in a risky asset to increasing short positions in a risky asset.

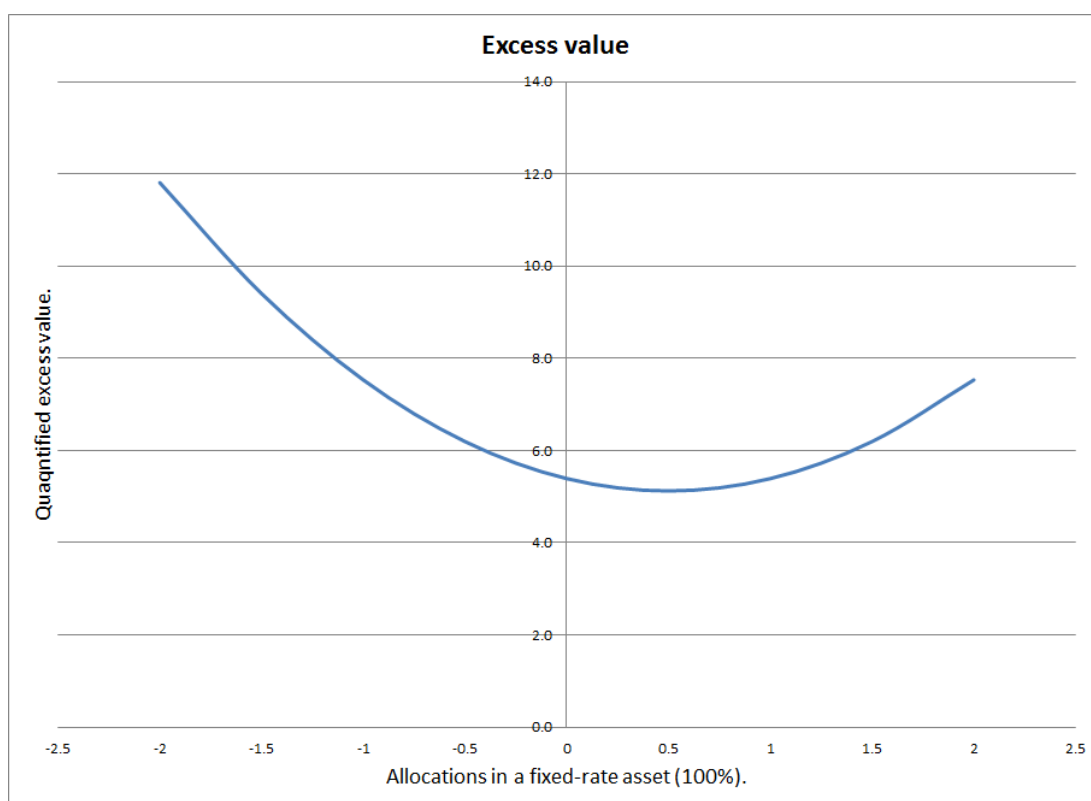


Figure 6.5.3: The excess value of risk of portfolio strategies of a risky asset of Anglo Gold Ashanti Limited.

Exceeding the benchmark allocation, the increasing premium indicates the potential of market risk for an active strategy. The added and avoided risk compensates equilibrium sets by the capital asset pricing model as shown in model 3 in appendix.

The performance of buy and hold strategies with 200 percent to 50 percent short positions underperforms with negative returns. The strategy with 100 percent allocations in a risky asset underperforms with a ratio of -6.9328 which further decreases to -8.6610 for a strategy with 50 percent allocations in a fixed-rate asset. The performance increases to a ratio of 10.5034 for a strategy with 100 percent short position in a risky asset. The Sharpe ratios of fixed-mix portfolios decrease from a ratio of -3.5238 for a strategy with 200 percent short position in a fixed-rate asset to a performance of -1.2248 for a strategy with 50 percent short position in a fixed-rate asset indicating an underperforming performance relative to a benchmark return of 947 percent. The ratio decreases from -7.8542 for a strategy with 100 percent allocations in a risky asset to a performance of -432.3442 for a strategy with 100 percent allocations in a fixed-rate asset indicating an increasing performance of strategy with a minimum risk. The performance decreases for strategies with 50 percent and 100 percent short position in a risky asset at Sharpe ratios of -28.9660 and -22.4597 respectively indicating underperforming strategies with minimum riskiness.

6.5.4 Quantification of excess value of risk in a portfolio of a risky asset of Anglo American Platinum Limited

The excess value of risk of portfolio strategies in a portfolio of a risky asset of Anglo American Platinum Limited is shown in figure 6.5.4. The excess value of risk of a portfolio is at equilibrium at 8.892 in the 50 percent allocation of the market risk. The increased market risk shows the excess value of risk of about 23.0 for short positions in a fixed-rate asset indicating the risk of short positions over the underperformance in the market risk. The excess value of risk of the added risk and avoided risk is 14.150.

The excess value of risk for further allocations in a fixed-rate position shows the avoided risk by short positions in a risky asset to reward the portfolio over underperformance of a risky position. The performance of buy and hold strategies with 200 percent and 150 percent short positions in a fixed-rate asset underperforms with negative returns. The performance increases from a Sharpe ratio of -0.1461 for an un-

derperforming strategy with 100 percent short position in a fixed-rate asset to a performance of a Sharpe ratio of 0.0335 for a strategy with 100 percent short position in a risky asset.

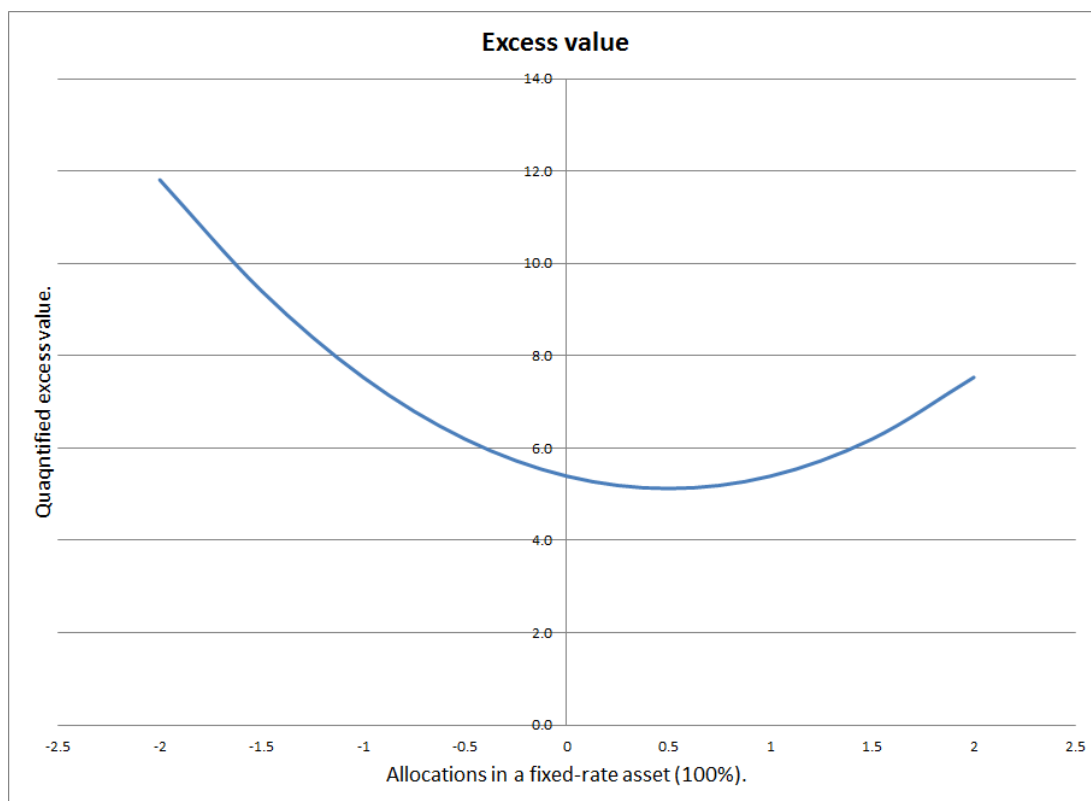


Figure 6.5.4: The excess value of risk of portfolio strategies of a risky asset of Anglo American Platinum Limited.

The underperforming fixed-mix strategy of 200 percent short position in a fixed-rate asset performs at a Sharpe ratio of -2.4620 increases the performance to a Sharpe ratio of -0.2695 for increasing allocations in a fixed-rate asset. For a strategy with 100 percent allocations in a fixed-rate asset the performance decreases from a Sharpe ratio of -0.5113 to a Sharpe ratio of -1.5905 indicating a decreasing performance of strategies with increasing allocation in a fixed-rate asset.

6.5.5 Quantification of excess value of risk in a portfolio of a risky asset of Sasol Limited

The excess value of risk of portfolio strategies of a risky asset of Sasol Limited is shown in the figure 6.5.5 with a portfolio performance dominated by a position of a fixed-rate asset. The excess value of risk at equilibrium is 9.316 over the market risk of 50 percent. The excess value of risk over the underperforming market of risk is at 13.300 for increased market risk in a short position of 200 percent of a fixed-rate asset. The added risk and avoided risk is in excess of 10.761 exceeding the value at equilibrium indicating the potential of market risk and the financial risk of the appropriate transactions.

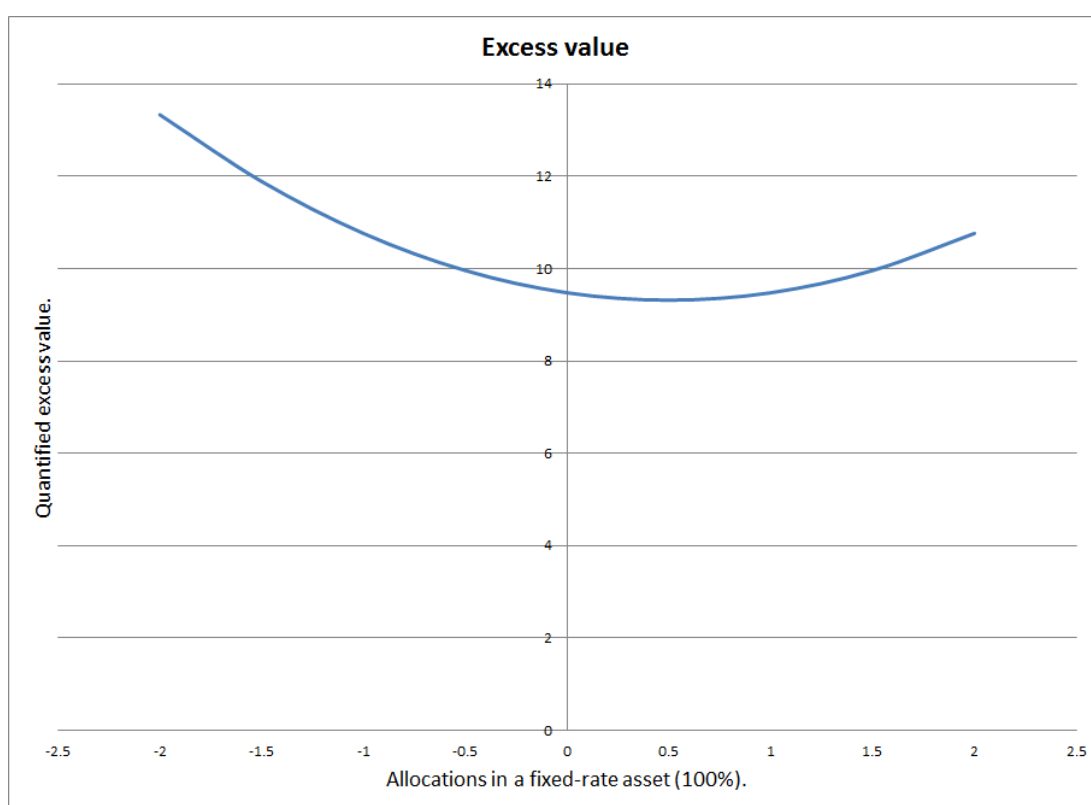


Figure 6.5.5: The excess value of risk of portfolio strategies of a risky asset of Sasol Limited.

The performance of a buy and hold strategy underperforms the benchmark at a Sharpe ratio of -0.0093 for a strategy with 150 percent short position in a fixed-rate asset to a Sharpe ratio of -0.0031 for a strategy with 50 percent allocations in a fixed-

rate asset. The performances outperform the benchmark to a Sharpe ratio of 0.0097 for a strategy with 100 percent short position in a risky asset. The Sharpe ratios of fixed-mix portfolios increases from a ratio of -1.7294 for a strategy with 150 short position in a fixed-rate asset which underperforms the benchmark to a ratio of -0.2192 for a strategy with 50 percent allocations in a fixed-rate asset. The Sharpe ratio decreases from -0.3051 to -1.2590 indicating a decreasing performance of strategies with short position in a risky asset.

6.5.6 Quantification of excess value of risk in a portfolio of a risky asset of Standard Bank Limited

The excess value of risk of portfolio strategies of a risky asset of Standard Bank is shown in figure 6.5.6 with a dominating performance of a fixed-rate asset position.

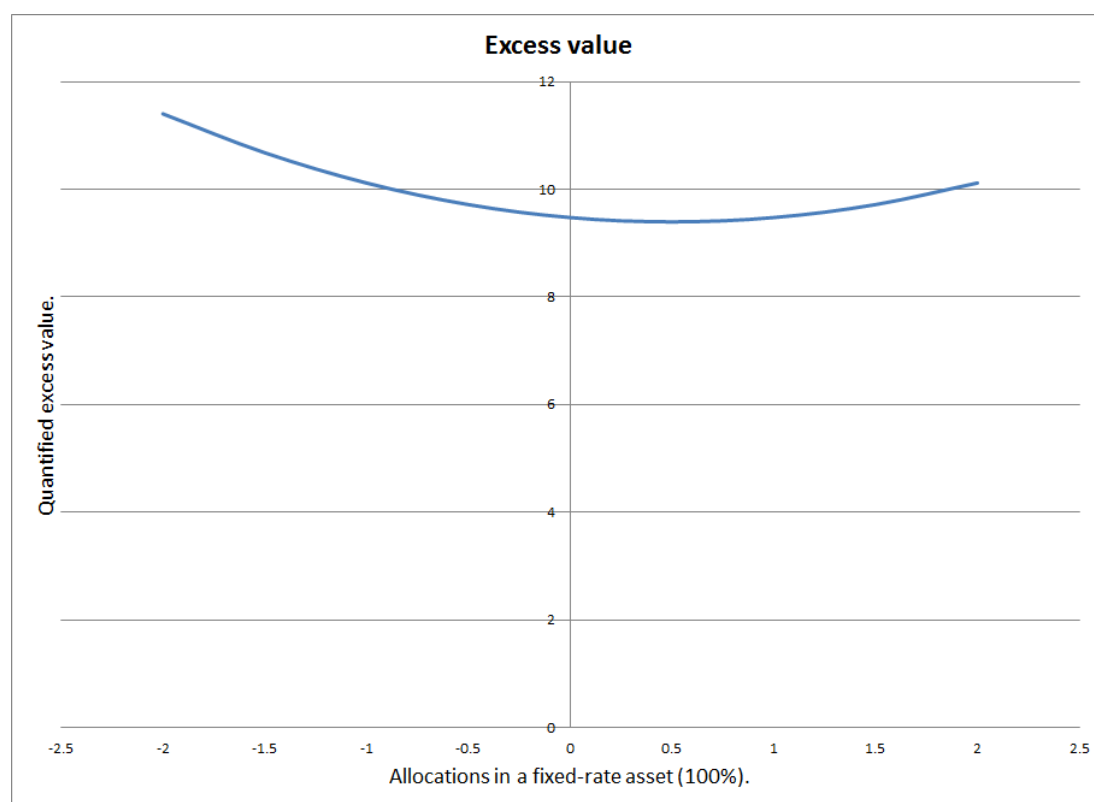


Figure 6.5.6: The excess value of risk of portfolio strategies of a risky asset of Standard Bank.

The excess value of risk at equilibrium is 9.396 below the performance of a fixed-rate asset. The increased market risk with short positions in a fixed-rate asset of 200 percent is in excess of 11.403 with a performance of a risky asset of 9.144. The added risk and avoided risk is at a value of 10.119 over short positions in a portfolio.

The underperforming performance of buy and hold portfolio strategies increases from a Sharpe ratios -0.0065 for a strategy with 200 percent short position in a fixed-rate asset to a ratio of -0.0019 for a strategy with 50 percent in a fixed-rate asset. The performance increases to a Sharpe ratio of 0.0040 outperforming the benchmark return at a strategy with 100 percent position in a risky asset.

The Sharpe ratios of fixed-mix portfolios are underperforming the benchmark strategy at a ratio of -19.7719 for a strategy with 200 percent short position in a fixed-rate asset. The Sharpe ratio increases to -16.9748 for a strategy with 100 percent short position in a fixed-rate asset as the market risk is reduced. The performance decreases from a Sharpe ratio of -18.8646 for a strategy with 50 percent short position in a risky asset to an underperforming performance of -67.6308 for a strategy with 50 percent short position in a risky asset as the market risk is transferred to a fixed-rate asset over a volatile market.

The excess value of risk of portfolio strategies shows the premium of a trend-following strategy which offsets the benchmark return. The offset premium shown above the benchmark return is for portfolio strategies which underperform over the market risk. The offset premium below the benchmark return shows the excess value of a portfolio strategy over a risky return. The computed excess values of portfolio strategies depicts the market price of risk captured along a trending risky asset and the downside market risk over frequent reversal along an estimated trend. The value of market risk over trending risky assets is valued as a single period transaction. The performance of a fixed-rate asset has excess value underperforming the risky position which impose a financial risk in short positions. The risky position performance is found to have excess value of risk over the market risk. The financial risk of risky

positions over trending market with short positions is realised by a transaction which considers the reversals of the risky asset for effective rebalancing gains.

6.6 Quantification of risk-adjusted portfolio growth of a fixed-mixed strategy

The computed excess value of risk is considered to induce risk-adjusted returns under volatile conditions of the market. The relative asset price ratios are determined to optimise the discrete performance over volatile conditions by adjusting the performance of a fixed-rate asset with a position of minimum risk and manageable risk. The hypothesis considered is termed a proxy hypothesis which ensures the stationary condition in a portfolio. The condition of stationarity is validated by the application of portfolio insurance strategy which renders the condition in a portfolio. The proxy hypothesis uses a derivative instrument called the lookback straddle to retain risk-adjusted returns in the underlying risky asset. The risk-adjusted returns are retained to mitigate the risk of a trend-following transaction. The strategic approach of using portfolio insurance hedges the strictly positive returns over trending trading periods.

The figure 6.6.1 illustrates a payoff timeline of a lookback option at maturity.

$$\begin{aligned}
 S_\tau - 1 &= \Delta S > 0, \text{ up trend} \\
 &= \Delta S < 0, \text{ down trend} \\
 &= \Delta S = \varepsilon_\tau = \sum_{t=0} \Delta S_t, \text{ reversal change}
 \end{aligned}$$

$$\min_{0 \leq t \leq \tau} \langle \text{-----} a \text{-----} S_t \text{-----} b \text{-----} \rangle \max_{0 \leq t \leq \tau}$$

$$(1 - \Delta S) \qquad \qquad \qquad (1 + \Delta S)$$

Figure 6.6.1: The payoff timeline of a lookback straddle option at maturity.

To ensure a positive return in a volatile trading period, for a trading period of length τ with an initial logarithm price $S_0 = 1$ the payoff timeline is shown over progressive changes of the risky asset.

The payoff of a lookback call option increases as the risky asset lesser performs which is denoted by a change size of a and the payoff of a lookback put option increases as the risky asset over performs which is denoted by a change size b . The combination of the options which buy at a minimum price and sell at a maximum price give a payoff $a + b$ which is at minimum if there is no change or if there is a small change in prices of a risky asset and optimal if there are large changes or reversals in prices of a risky asset.

The stationary tradable securities validate the proxy hypothesis where stationary risk-adjusted returns from the payoff of a lookback straddle option assert the stationarity condition derived from the underlying risky asset. The market risk is captured by an appropriate strategy which retains risk-adjusted returns. The financial engineered approach that sets a basis to optimize on the application of self-financing constant rebalanced portfolio strategies uses a portfolio insurance tactic to induce the stationarity condition in a portfolio. The lookback straddle option retains a return of an underlying asset over a difference of the maximum price and the minimum price in the duration of the derivative instrument. The retained returns over reversals of a risky asset are of a trend-following transaction and the reversal transaction. The excess value of risk of a portfolio is determined to enhance risk-adjusted returns and exceed the performances of portfolio strategies as the risky asset trends. The proxy hypothesis shows the value of a financial proxy which renders a financial condition of stationarity in a portfolio which retains returns over trending and reversal conditions.

6.7 Time effective portfolio strategies that render a financial proxy

The stationary condition of asset prices which is the assumption for fixed-mix portfolio strategies to yield unbounded exponential growth is derived by a subroutine strat-

egy over a constant rebalanced portfolio. The subroutine strategy replicates total returns which are strictly above the total returns of a no-trade region over a short-term rebalancing period. The tactic of replicating the risk-adjusted returns and the excess growth which results in the spread performance of the strategies is a tactic that uses portfolio insurance strategy. The strategic approach which replicates stationary tradable financial securities is a fixed-mix strategy on a portfolio of a fixed-rate asset and a lookback straddle option on an underlying risky asset. The designed strategy renders the stationarity condition on the portfolio by replicating a time-effective portfolio that is favourable to portfolio objectives.

6.7.1 Performance of a proxy strategy in a portfolio of a risky asset of ABSA Group Limited

The performance of a proxy strategy in a portfolio of a risky asset of ABSA Group Limited is shown in figure 6.7.1.

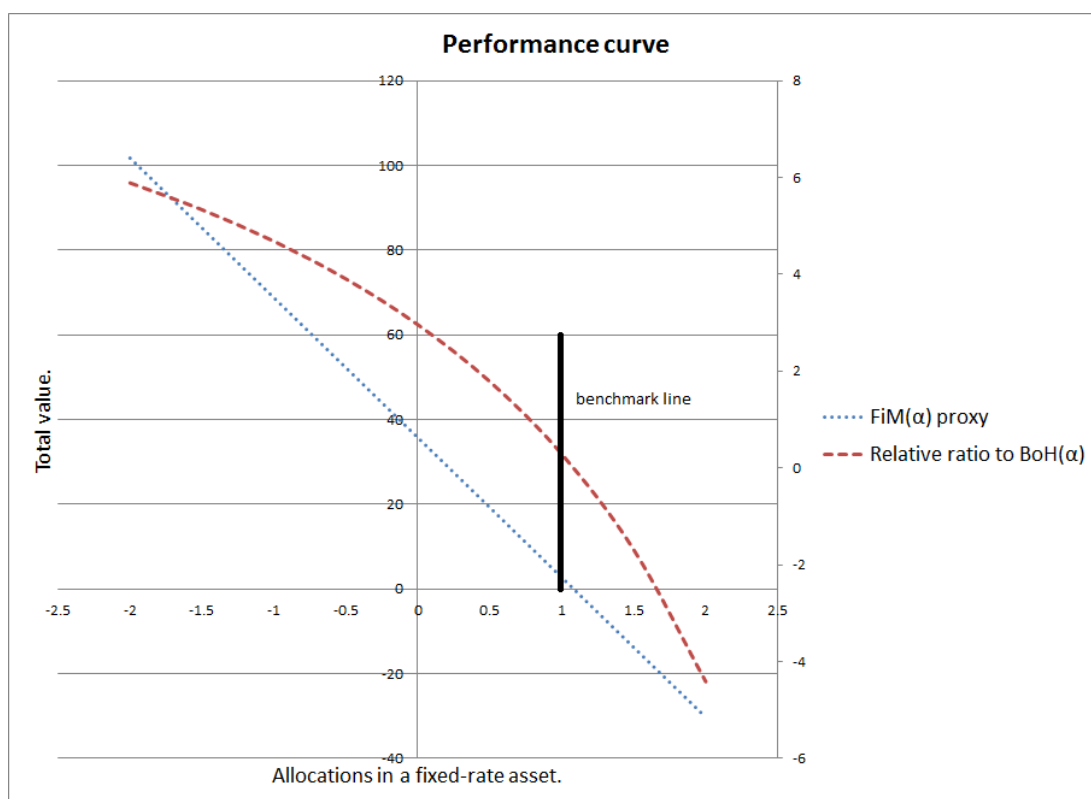


Figure 6.7.1: Performance of a proxy strategy of a risky asset of ABSA Group Limited.

The financial proxy is a financial transaction of a fixed-mix portfolio strategy which replicates the stationarity condition in a portfolio of a fixed-rate asset and a risky asset. The stationarity condition is replicated by a lookback straddle option which retains a return of a risky asset over a maximum price and a minimum price over a time of an option.

The performance of a proxy strategy of a fixed-mix portfolio strategy in a risky asset of ABSA Group Limited exceeds the buy and hold strategy at an increasing relativity factor. The relative ratio increases as the allocations in a fixed-rate asset decrease and range from -4.41 up to 5.89 as shown in figure 6.7.1 by a curve line. The short positions in the risky asset lead to an underperforming strategy for further allocations in a fixed-rate asset. The total value of a fixed-mix proxy strategy increases as allocations in a risky security increase. The strategy that allocates 50 percent in a fixed-rate asset outperforms the benchmark return and the risky return. The complete allocations in a risky security retain a total value of 3571.03 percent. The complete allocations in a fixed-rate asset return a total value of 269.68 percent which underperforms the benchmark. Allocations above completely-mixed strategies under perform with negative returns. The proxy strategy of a fixed-mix portfolio with 50 percent allocations in a fixed-rate asset grows at an exponential rate of 0.0007 which decreases gradually to a rate strictly above 0.0002. The growth rate of a proxy strategy with -200 percent is shown in the appendix in chart A.2.1.1 with a rate from 0.0036 which decreases gradually to a long run rate of 0.0003.

The annual performance of arbitrary constant rebalanced self-financing portfolio investment strategies and annual performances of the buy and hold strategy are determined. The annual return of a fixed-mix portfolio of a risky asset of ABSA Group Limited is optimal at a return of 8.23 percent with a strategy which allocates 50 percent in a fixed-rate asset and the annual return of a buy and hold strategy is 16.21 percent. The fixed-mix strategy underperforms the trend-following strategy in a trending risky asset. The annual returns of a buy and hold strategy increases from a return of 13.21 percent at the allocation of -200 percent in a fixed-rate asset to an allocation of 200 percent at a return of 19.32 percent. The annual return of a fixed-mix

proxy strategy is 6.77 percent at a complete allocation in a fixed-rate asset and increases to an annual return of 31.55 percent as allocations in a fixed-rate asset decrease to -200 percent. The performance of buy and hold strategies to a benchmark return which is a return of a fixed-rate asset is determined by a Sharpe ratio which shows the buy and hold strategy to outperform the benchmark return for strategies with leveraged allocations in a fixed-rate asset as shown by a positive ratio. The fixed-mix strategy underperforms the benchmark return over allocations from -200 percent to 200 percent in a fixed-rate asset on a high market risk. The large positive Sharpe ratios of fixed-mix proxy strategies show an increasing excess returns for leveraged allocations in a fixed-rate asset up to a 50 percent allocation in a fixed-rate asset. The larger negative Sharpe ratios for a strategy with 100 percent in a fixed-rate asset indicate an underperforming strategy not utilizing the market risk. The proxy strategies with short positions in a risky asset underperform with negative returns.

6.7.2 Performance of a proxy strategy in a portfolio of a risky asset of AFGRI Limited

The proxy strategy of a portfolio of risky asset of AFGRI Limited is found to grow optimally as allocations in a risky security increases.

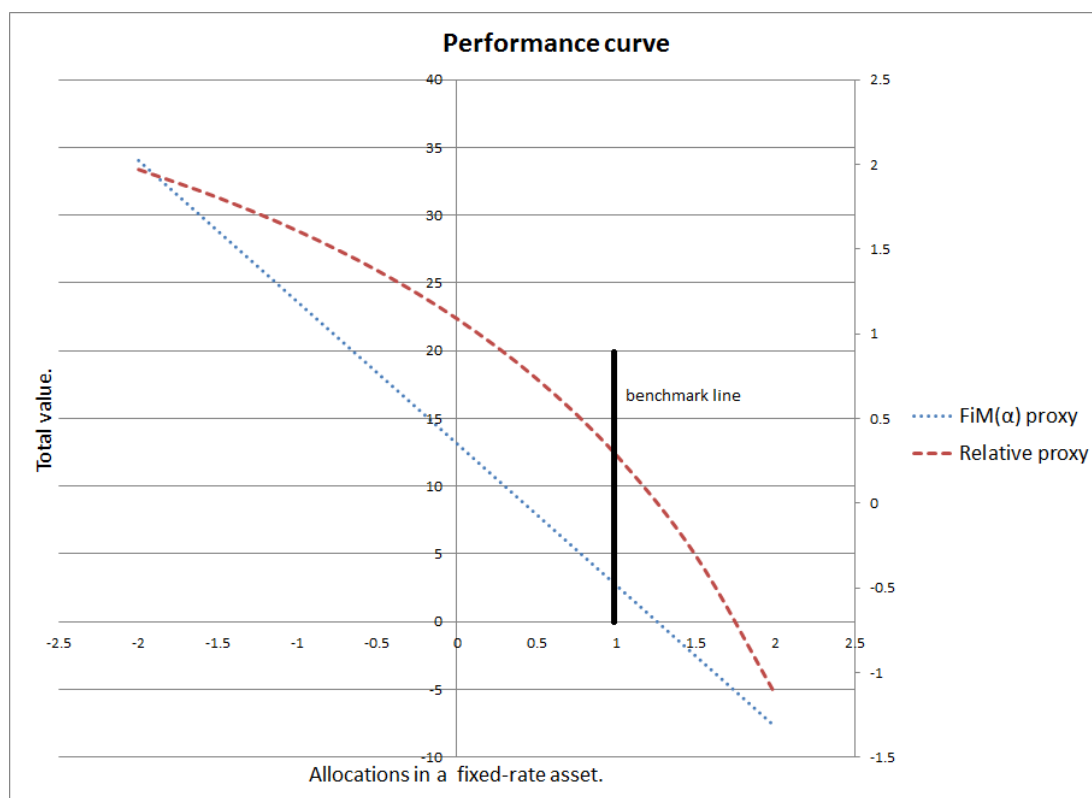


Figure 6.7.2: Performance of a proxy strategy of a risky asset of AFGRI Limited.

The relative ratio of a buy and hold strategy ranges from a factor of -1.5 to a factor of 2 as shown in figure 6.7.3 by a curve line. The proxy strategy attains an optimal total value of 3403.59 percent at the allocation of -200 percent in a fixed-rate asset. The proxy strategy underperforms at allocations above a complete allocation in a fixed-rate asset. The performance of a fixed-mix proxy strategy is shown in figure 6.7.2.

The fixed-mix proxy strategy with complete allocations in a risky security outperforms the risky return of a risky asset at a total value of 1314.31 percent by a relative factor of 1.089 to a buy and hold strategy. The buy and hold portfolio strategy with 50 percent allocations in a fixed-rate asset grows at a rate of 0.003 which gradually

decreases to 0.00012 and the fixed-mix proxy strategy grows at a rate below the growth rate of an index to 0.00011. The growth rate of a proxy strategy with -200 percent allocations in a fixed-rate asset is shown in the appendix in chart A.2.1.2. The performance of completely-mixed fixed-mix portfolio strategies of a risky asset of AFGRI Limited increases with an increasing allocation in a fixed-rate asset. The fixed-mix strategies yield negative total value over allocations from -200 percent to -150 percent in a fixed-rate asset as a result of a large spread of trades of a risky asset with an initial jump of asset prices which stabilises towards the end of the investment duration and closes with short positions in a risky asset. The annual return of a fixed-mix strategy with a complete allocation in a risky asset is 0.67 percent which underperforms a buy and hold strategy at an annual return of 16.98 percent. The annual return of fixed-mix strategy increases to 11.47 percent at an allocation of 200 percent in a fixed-rate asset where the annual return of a buy and hold strategy is 13.21 percent. The annual return of a fixed-mix proxy strategy is 6.77 percent at a complete allocation in a fixed-rate asset and increases up to an annual return of 24.08 percent with an increasing allocation in a risky asset.

The positive Sharpe ratios of a buy and hold strategy show the excess return over leveraged allocations in a fixed-rate asset up to a complete allocation. The discount excess return is realised for allocations above a complete allocation in a fixed-rate asset where the trend following strategy underperforms the benchmark return. The negative Sharpe ratios show fixed-mix strategies to underperform the benchmark return over leveraged and completely-mixed allocations and above the complete allocation the strategy underperforms with high market risk. The fixed-mix proxy strategy outperforms the benchmark return over leveraged allocations up to a complete allocation in a risky asset. The allocations above a complete allocation in a risky asset underperform the strategy over a higher expectancy of market risk.

6.7.3 Performance of a proxy strategy in a portfolio of a risky asset of Anglo Gold Ashanti Limited

The performance of the proxy strategy on a risky asset of Anglo Gold Ashanti Limited is shown in figure 6.7.3.

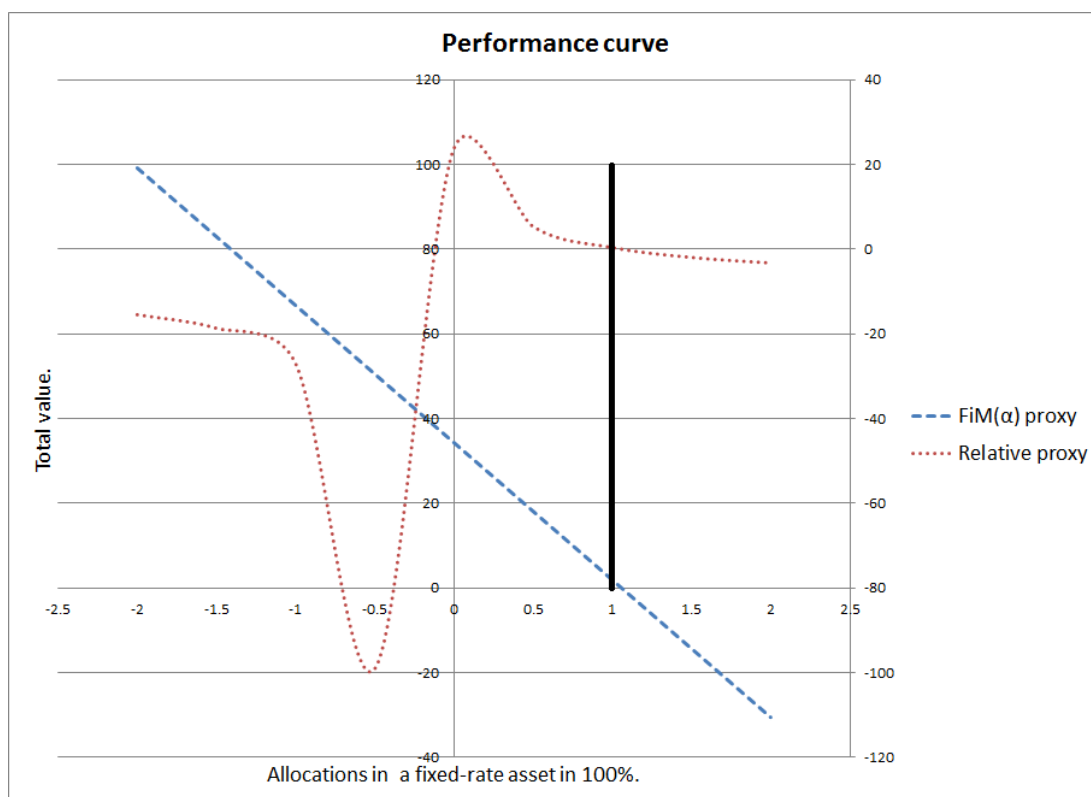


Figure 6.7.3: Performance of a proxy strategy of a risky asset of Anglo Gold Ashanti Limited.

The completely-mixed fixed-mix proxy strategies perform relatively optimal to a buy and hold strategy. The fixed-mix proxy strategy performs as allocations in a fixed-rate asset decrease. The buy and hold portfolio strategy underperforms over leveraged positions by incurring costs into a portfolio. The fixed-mix proxy strategy attains an optimal total value of 9915.49 percent at the allocation of -200 percent. The annual return of 3.30 percent is realised at a complete allocation in a risky asset corresponding to a total value return of 147 percent with a relative ratio of 23 to an outperforming proxy strategy. The annual return increases up to an annual return of 19.05 percent at the allocation of 200 percent in a fixed-rate asset with a relative ratio

below zero where proxy strategy underperforms in shorting the market risk. The leveraged fixed-mix strategies underperform by losing 97.85 percent of the initial allocation at the allocation of -200 percent in a fixed-rate asset to a leveraged allocation of -50 percent by losing 36.32 percent of the initial allocations. The completely-mixed allocations attain an optimal annual return of 7.19 percent at a complete allocation in a fixed-rate asset.

The fixed-mix proxy strategy attains an annual return of 5.38 percent at a complete allocation in a risky asset. The annual return increases up to 39.24 percent at the allocation of -200 percent in a fixed-rate asset. The negative Sharpe ratios show the buy and hold portfolio strategy to underperform the benchmark return of 539.23 percent over allocations from -200 percent to 200 percent in a fixed-rate asset. The Sharpe ratios show a higher expectancy of the market risk over allocations above complete allocation in a fixed-rate asset. The Sharpe ratios show the fixed-mix strategy to underperform the benchmark return over all allocations. The fixed-mix proxy strategy outperforms the benchmark return for leveraged allocations up to 50 percent allocations in a fixed-rate asset. The allocation in a fixed-rate asset at and above a complete allocation underperforms. The long run growth rate of a fixed-mix proxy strategy is 0.0002 below the rate of a buy and hold strategy for a portfolio with 50 percent allocation in a risky asset. The growth rates of a proxy strategy with -200 percent allocations in a fixed-rate asset is shown in the appendix in chart A.2.1.4 from a rate of 0.0111 which decreases gradually to a long run rate of 0.0003.

6.7.4 Performance of a proxy strategy in a portfolio of a risky asset of Anglo American Platinum Limited

The fixed-mix proxy strategy attains an optimal total value of 12894.03 percent at the allocation of -200 percent in a fixed-rate asset with a relative performance ratio of 64.02 to a buy and hold strategy. The fixed-mix proxy strategy with a complete allocation in a risky asset outperforms the benchmark return at a total value of 4477.79 percent. The fixed-mix proxy strategy with 50 percent allocations in a fixed-rate asset grows from a rate of 0.07 which decreases gradually to a rate of 0.008. The

index of the portfolio grows at a rate of 0.0006. The growth rates of a proxy strategy with -200 percent allocations in a fixed-rate asset are shown in the appendix in chart A.2.1.5 with a rate of 0.0253 which decreases gradually to 0.0013. The performance of a fixed-mix proxy strategy is shown in figure 6.7.4.

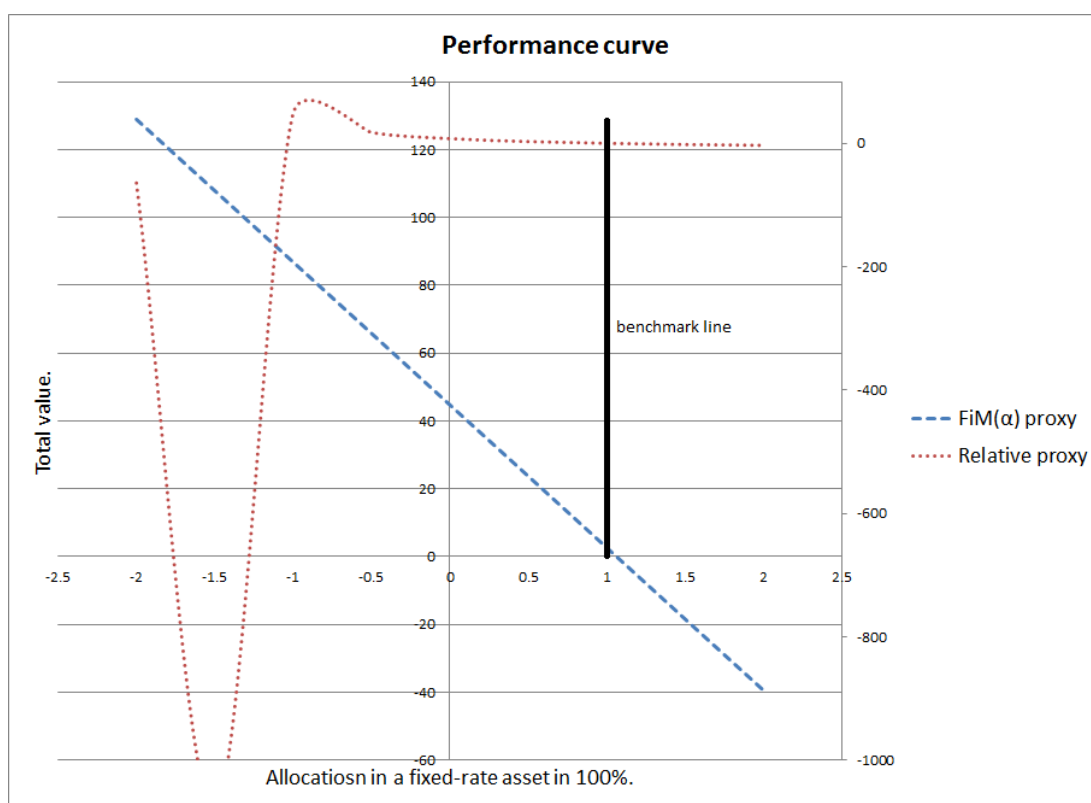


Figure 6.7.4: Performance of a proxy strategy of a risky asset of Anglo American Platinum Limited.

The buy and hold strategy of a portfolio of a risky asset of Anglo American Platinum Limited incurs costs in a portfolio over allocations from -200 percent to -150 percent in a fixed-rate asset. The annual return of 0.69 percent is realised at the allocation of -100 percent in a fixed-rate asset up to an annual return of 17.85 percent at the allocation of 200 percent. The fixed-mix portfolio underperforms the initial allocation in a portfolio by losing 96.25 percent at the allocation of -200 percent and 29.22 percent at the allocation of -100 percent in a fixed-rate asset. The annual return of a fixed-mix strategy increases from 3.81 percent at the allocation of -50 percent in a fixed-rate asset to an annual return of 8.78 percent at the allocation of 50 percent in a

fixed-rate asset. The annual return of a fixed-mix proxy strategy is 6.77 percent at a complete allocation up to an annual return of 33.17 percent at the allocation of -200 percent in a fixed-rate asset. The Sharpe ratios show the performance of a buy and hold strategy underperforming the benchmark return over leveraged allocation in a fixed-rate asset. The proxy strategy outperforms the benchmark return at a complete allocation in a fixed-rate asset.

6.7.5 Performance of a proxy strategy in a portfolio of a risky asset of Sasol Limited

The performance of a proxy strategy is shown in figure 6.7.5.

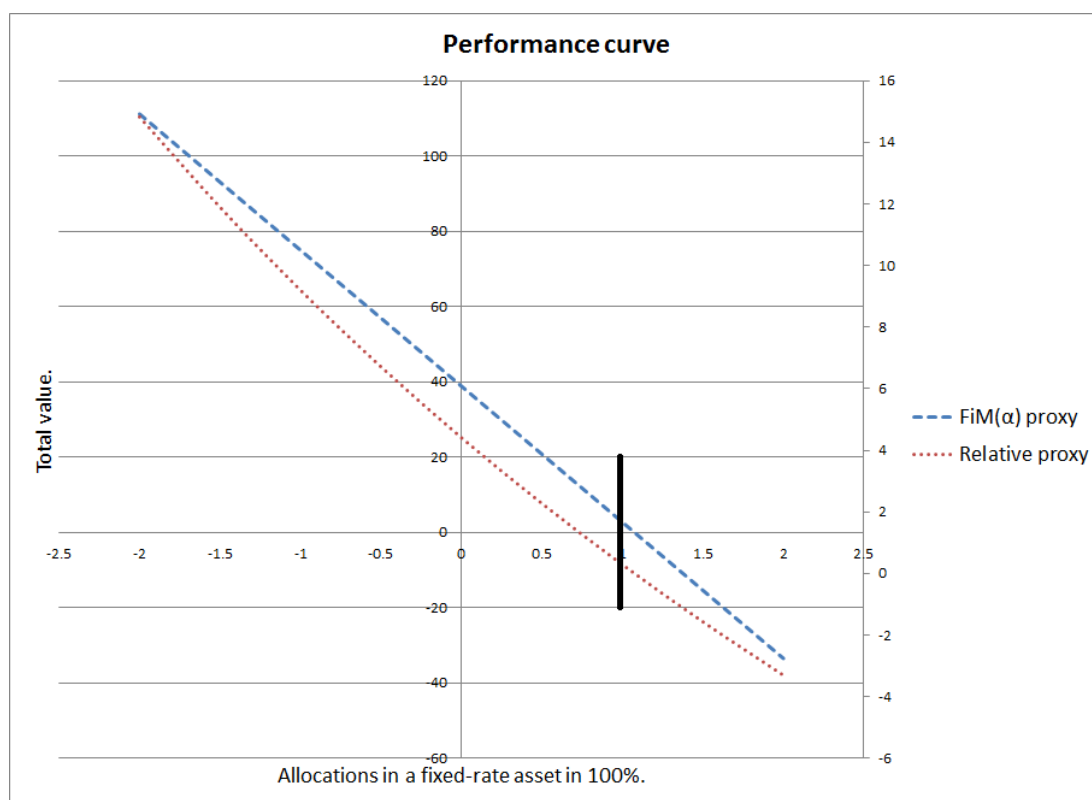


Figure 6.7.5: Performance of a proxy strategy of a risky asset of Sasol Limited.

The fixed-mix proxy strategy performs linearly relative to a buy and hold strategy over allocations in a fixed-rate asset. The fixed-mix proxy strategy attains an optimal

total value of 11115.78 percent at -200 percent allocations in a fixed-rate asset. The complete allocation in a risky asset attains a total value of 3885.05 percent. The fixed-mix proxy strategy above 100 percent allocations in a fixed-rate asset yields negative total value where a buy and hold strategy attains an optimal total value of 1012.67 percent at 200 percent allocations in a fixed-rate asset. The fixed-mix proxy strategy with 50 percent allocations in a fixed-rate asset grows initially at a rate of 0.0018 which decreases gradually to a rate of 0.0002 above the growth rate of the index of 0.0001. The growth rates of strategies with -200 percent allocations in a fixed-rate asset are shown in the appendix in chart A.2.1.3 with a rate decreasing gradually from 0.0096 to 0.0003.

The buy and hold strategy attains an annual return of 14.03 percent at the allocation of -200 percent in a fixed-rate asset. The annual return increases up to a return of 16.26 percent at the allocation of 200 percent in a fixed-rate asset. The fixed-mix strategy under performs the initial allocations in a portfolio over allocations from -200 percent to -150 percent in a fixed-rate asset. The annual return of 0.81 percent is attained at the allocation of -50 percent in a fixed-rate asset to an optimal return of 7.89 percent at a complete allocation in a fixed-rate asset. The annual return of a fixed-mix proxy strategy is 6.96 percent at a complete allocation in a fixed-rate asset up to an annual return of 33.03 percent at the allocation of -200 percent in a fixed-rate asset. The Sharpe ratios show the buy and hold strategy to underperform the benchmark return over leveraged and completely-mixed allocations in a fixed-rate asset and above the complete allocation the strategy outperforms the benchmark return. The fixed-mix strategy underperforms the benchmark return with a higher expectancy of market risk over a completely-mixed allocation. The fixed-mix proxy strategy outperforms the benchmark return over leveraged allocation and completely-mixed allocations. The proxy strategy underperforms with higher expectancy of market risk at 150 percent to 200 percent allocations in a fixed-rate asset.

6.7.6 Performance of a proxy strategy in a portfolio of a risky asset of Standard Bank Limited

The performance of a proxy strategy is shown in figure 6.7.6.

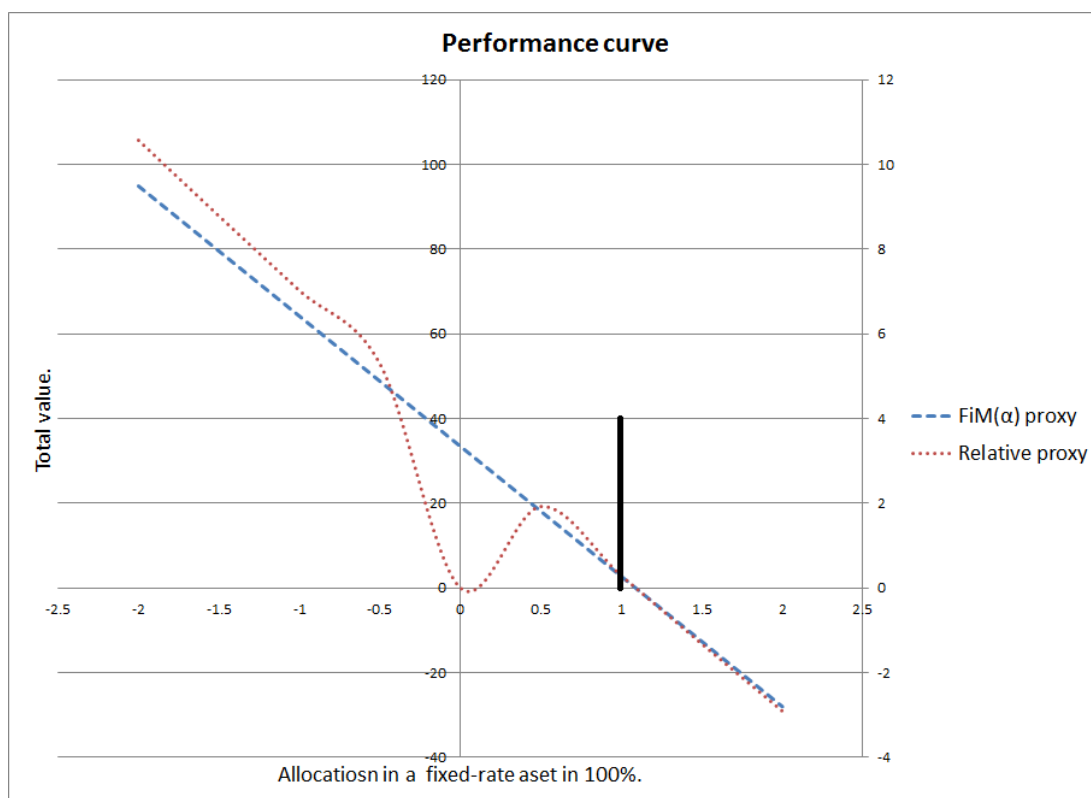


Figure 6.7.6: Performance of a proxy strategy of a risky asset of Standard Bank.

The proxy strategy yield negative total value above a complete allocation in a fixed-rate asset. The fixed-mix proxy strategy grows linearly up to an optimal total value of 9488.80 percent at the allocation of -200 percent in a fixed-rate asset. The fixed-mix proxy strategy attains a total value of 3342.72 percent at a complete allocation in a risky asset with a relative outperformance ratio of 3.59 to a buy and hold portfolio strategy. The fixed-mix proxy strategy underperforms with negative total values at allocations above 100 percent in a fixed-rate asset where a buy and hold strategy attains optimal total value of 963.35 percent at an allocation of 200 percent in a fixed-rate asset. The fixed-mix proxy strategy with 50 percent in a fixed-rate asset grows initially from a rate of 0.0055 which decreases gradually to a rate of 0.0008 above the growth of an index at a rate of 0.0006. The growth rates of a proxy strategy with -

200 percent allocations in a fixed-rate asset are shown in the appendix in chart A.2.1.6 with a rate from 0.0300 to a long run rate of 0.0012.

The buy and hold portfolio strategy of a risky asset of Standard Bank attains an annual return of 14.59 percent at the allocation of -200 percent in a fixed-rate asset. The annual return increases up to a return of 15.58 percent at the allocation of 200 percent in a fixed-rate asset. The fixed-mix strategy underperforms the initial allocations in the portfolio at -200 percent allocations in a fixed-rate asset. The fixed-mix strategy attains an optimal return of 9.56 percent at the allocation of 50 percent in a fixed-rate asset. The fixed-mix proxy strategy attains an annual return of 6.77 percent at a complete allocation in a fixed-rate asset up to an annual return of 31.07 percent at the allocation of -200 percent in a fixed-rate asset. The Sharpe ratios show the performance of a buy and hold strategy underperforms the benchmark strategy over leveraged and completely-mixed allocations. The strategy outperforms the benchmark at the allocation of 150 percent to 200 percent in a fixed-rate asset. The fixed-mix strategy underperforms the benchmark return. The fixed-mix proxy strategy outperforms the benchmark return over leveraged allocations up to 50 percent allocations in a fixed-rate asset. The proxy strategy underperforms the benchmark return at the allocation of 100 percent to 200 percent in a fixed-rate asset.

The empirical performance of portfolio systems constructed by a fixed-rate asset and risky assets show that the fixed-mix strategy yields non-negative growth below the benchmark return. The performance of a fixed-mix proxy strategy outperforms the benchmark return over completely-mixed allocations.

6.8 Summary of the interpretation

The quantitative performance of the computed portfolio strategies is found to yield returns which are above the performance of individual assets for appropriate allocations of assets. The trend-following strategies yield superior performance for fixed-mix portfolio strategies particularly for leveraged positions in a fixed-rate asset. The

optimal performance of fixed-mix strategies is mostly determined over completely-mixed portfolios in a fixed-rate asset on the range of -100 percent to 100 percent positions. The fast exponential growth in a fixed-mix portfolio is not realised since the stationary condition of asset prices is not attained in the considered trending risky assets. However, the fast exponential growth is of portfolios of a risky asset of AF-GRI Limited and Anglo Ashanti Limited is realised with high variability of the portfolio value which is dependents of the spread of trades. The stationary tradable securities release the fast exponential growth by rendering the stationary condition which is derived over trending risky asset prices.

6.9 Conclusion

The attained performance of active portfolio management of fixed-mix strategies is shown to outperform the buy and hold strategies over trending market conditions. The risk-adjusted returns which arise in the application of fixed-mix strategies are retained by using a portfolio insurance strategy which ensures positive contributions into a self-financing strategy. The realised portfolio growth from the volatility of asset prices arises from the market price of risk which is used by portfolio insurance strategies with a lookback straddle option to attain stationary tradable securities for fixed-mix portfolio strategies. The excess performance of fixed-mix portfolios is at discount to a performance of a buy and hold strategy the realized performance is below the performance of a fixed-rate asset. The discount results from trades which are dominated by selling transaction over trending market conditions. The determinant factor of the excess performance of active managed fixed-mix portfolios is the excess performance of underperforming market conditions and over performing conditions. The low concavity of the ratio process of the performances of portfolio strategies in a proxy strategy shows the superiority of a trend-following strategy over mean-reverting strategy as indicated by increased returns of a financial proxy over the leveraged risky position. The time-effective fixed-mix portfolios are managed to accumulate rebalancing gains from risk-adjusted returns. The following chapter revisits the research objectives and evaluate the performance of a fixed-mix portfolio strategy and a buy and hold portfolio strategy.

Chapter 7

CONCLUSION

7.1 Introduction

The performance of portfolios of a fixed-rate asset and a risky asset of a company listed in a major index the FTSE/JSE Top 40 index is analysed quantitatively. The optimal application of fixed-mix portfolios is evaluated over 3736 discrete trading periods. The fixed-mix strategy as an active portfolio management strategy discretely trades assets after a change in asset prices to rebalance fixed-proportions of assets in a portfolio thus increasing the value of the portfolio consistently over trading periods in the market risk.

The aim of this chapter is to conclude the quantitative analysis of the computations of fixed-mix portfolios comparable to a performance of an inactive investment holding strategy. The second aim is to remark the effectiveness of the financial proxy using a lookback straddle derivative option in acquiring the improved results of fixed-mix portfolios which are theoretical considered to yield fast exponential of wealth in stationary market conditions. The last aim of the research is to consider practical matters in the investment practise and to highlight the focused shortcomings of fixed-mix portfolios and as well as the recommendations of further research in this line of practise.

The enhanced performance of fixed-mix portfolios determined in generic market conditions by acquiring a stationary financial condition provides an empirical evidence of the optimality of fixed-mix portfolios. The financial proxy which optimizes on de-trended stochastic asset prices is a transaction which ensures a deterministic positive return over trading periods. The deterministic positive return is stationary throughout the trading periods and uniform with a trend. The determined financial proxy is a dual transaction which is encountered in trending market conditions and in reversal market conditions. The transaction is adaptable to the reversals and the vola-

tility of the trend which sufficiently retains risk-adjusted gains in generic market conditions. The potential of financial engineering suggested by Dempster, Evstigneev and Schenk-Hoppé (2010) in the growth of fixed-mix portfolios is contributed to by an innovative application of constant rebalanced portfolios with option-based portfolio insurance using a lookback straddle option.

The literature of active portfolio management with fixed-mix portfolios (Dempster *et al.*, 2003, 2007, 2008, 2010), (Evstigneev and Schenk-Hoppé, 2002), appraises the volatility of asset prices as a positive contributor in the portfolio growth. Under the framework of the mean-variance model of Markowitz the perspective of risk is contrary to a modern portfolio theory perspective. The financial assets with most volatility are preferable to financial assets of least volatility to provide performance variations at a minimum cost. This view indicates the favourable asset types in fixed-mix portfolios to be premium valued assets which have frequent fluctuations to trade the spread of the value of the asset. The portfolio insurance strategy using a lookback straddle option is used to effectively de-trending stochastic asset prices in order to retain strictly positive returns over discrete rebalancing.

The considered daily asset prices of a fixed-rate asset with a short rate determined as an effective rate of quoted prime rates in the performed computations showed less significant differences over up-trending and down-trending prime rates conditions. The computations of fixed-mix portfolios showed an increasing value over a long run with increased variability resulting from the spread of trades. The variability from the spread indicated the potential of actively and sufficiently rebalancing the portfolio to manage the cost and optimizing the rebalancing gains. The released performance of fixed-mix portfolios shows the consistency performance of fixed-mix portfolios which are optimal in market conditions with stationary reversals outperforming a buy and hold strategy. The growth rates of mean-reverting fixed-portfolios are increased over the growth rates of buy and hold portfolios with non-decreasing performances. The potential of fixed-mix strategies is realised over a long run with less performance gains over trending markets. The proxy strategy holds the condition of trend-stationarity which is favourable for trend-following strategies. The performance of a

proxy strategy exceeds the buy and hold strategy in trending market conditions. The low concavity of the relative performance ratios of a proxy strategy to a buy and hold strategy indicates the superiority of fixed-mix portfolios for trend-following strategies in a trending market with reversals. The superiority of the proxy strategy is indicated by relative performance ratios of the strategy relative to a buy and hold strategy which outperforms in trending markets. The dominating strategies of buy and hold portfolios exceed the performance of fixed-mix portfolios by large factors which indicate unfavourable market conditions for fixed-mix portfolios in the market risk of trending conditions.

The fixed-mix strategy outperforms in markets with reversals as found in the portfolios of a risky asset of Anglo Gold Ashanti Limited where the strategy outperforms the risky return of an asset. The optimal performance of a fixed-mix strategy for all considered risky portfolios range from an annual return of 7 percent to an annual return of 12 percent which indicates a benchmark performance. The fixed-mix portfolios are effective particularly for portfolio objectives which are conserving funds and using less leveraging strategies.

The proxy hypothesis which holds the condition of fixed-mix portfolios comes at an expense of an innovative strategy. The realised gains outperform the performances of the utilized assets which show the strategy to be sufficient in generating growth and hedging funds for self-financing portfolios. The self-financing portfolios are found to be effective in dealing with liabilities under the validity of the proxy hypothesis. The growth rates of fixed-mix portfolios under the proxy hypothesis are determined to strictly exceed the growth of individual assets and to be stable in the long run. The volatility of individual assets and appropriate risk managing tools are shown to be more effective in holding gains in a volatile market.

7.2 Computations of a financial proxy

The analysis focused on realizing the impact of volatility and to design a trading strategy that efficiently utilizes the presence of volatility. The computations of a fi-

nancial proxy showed an optimal performance of fixed-mix portfolios for de-trended stochastic asset prices in trending markets which dominates the performance of buy and hold portfolios. The tactic of using a combination of lookback options effectively hedge the market risk. The constructed approach of de-trending asset prices sufficiently holds the condition of stationarity in a self-financing fixed-mix portfolio. The constructed approach is effective for trend-following strategies which leveraged the gains of a risky asset.

A major problem of the unrealized potential of stochastic asset prices in generic markets conditions particularly in a trending market is resolved by active portfolio management approach which focuses on the market risk and the financial risk of the strategy. The unrealised gains of a fixed-mix strategy in a portfolio of a fixed-rate asset and a risky asset are determined over the spread of trades at discrete rebalancing. The spread of trade is the difference of discrete portfolios. The unrealised gains over discrete portfolios are quantified by a portfolio insurance strategy under the market price of risk of discrete portfolios. The theoretical hypothesis in Dempster *et al.* (2007) asserts fast exponential yield under the condition of stationary markets. The portfolio insurance strategy enhanced the performance by hedging the financial risk of the fixed-mix portfolio to construct a robust approach of tradable stationary asset processes which evolve uniform returns of an option to a rather fast exponential growth.

The approach implements fixed-mix strategies by effectively dealing with counter implications of the market price of risk in generic financial markets. The replication strategy of portfolio insurance constructs stationary tradable securities using a look-back straddle which ensures positive gains over multi-period transactions in a trending risky asset as well as in the market risk with large reversals.

The feasibility of the approach to determine a financial proxy which optimises the application of stationary tradable securities for fixed-mix portfolio strategies is the analytical results of Dempster *et al.* (2010). This result is, under a non-degeneracy condition and the stationarity of relative price ratios the fixed-mix portfolio in the

long run grows at a rate strictly exceeding the rate of a trending price index. The risks and returns of portfolios of a riskless asset and risky asset are optimum dependently. Non negative derivative returns are retained discretely. The retained returns are increased in volatile market conditions. The performed empirical analysis of a financial proxy of fixed-mix portfolio strategy under the findings of growth induced by volatility in Dempster *et al.* (2007) show the inefficient portfolios of increased volatility in the mean-variance model sufficient to yield risk-adjusted growth over trending stochastic asset prices.

The unrealised potential of stochastic processes under fixed-mix strategies arise in the randomness of the stochastic asset prices and the rebalancing method with is associated with the costs. The market price of risk measured by an excess return over a downside risk is retained by a lookback straddle option which is a financial derivative instrument suggested by Darius *et al.* (2012) as a trend-following strategy. The unrealized gains are subjected to transaction costs which are prone to frequent trading and excess gains over transactions. Moreover, the use of a portfolio insurance strategy compensates for the cost of frequent transactions on the excess returns of the market.

The quantification of market price of risk by derivative instruments to retain a yield that exceeds the cost over risk-adjusted returns and anticipated risk in a rebalancing period enhances the condition of stationarity. The excess growth or the spread performance of the trend-following strategies over a fixed-mix strategy shows the superiority of the trend-following strategy to enhance the performance of fixed-mix strategies over rebalancing periods. The optimally quantified market risk in the mean-variance framework gradually yields positive excess growth for any favourable arbitrary portfolio. The quantification of a derivative strategy induces the condition of stationarity by deriving stationary tradable securities. The tradable stationary security is required to model risk-adjusted return differentials in order to hedge the expected fast exponential growth of a fixed-mix portfolio. The time effective portfolios with financial engineered instruments are favourable to portfolio objectives to induce risk-adjusted growth for an appropriate financial transaction over trading periods.

The designed strategy which hedges the fast exponential growth in generic markets considers a risky security which retains a return of risky asset on the minimum price to a maximum price over the duration of a security. The retained gains are consistent in volatile market conditions.

7.3 Practical matters

The quantitative analysis of arbitrary performance of constant rebalanced self-financing portfolio investment strategies shows dependence on periodic rebalancing where in practice the rebalancing periods are monthly, quarterly or semi-annually. The realised performance of a proxy strategy in fixed-mix portfolios used a weekly rebalancing. High frequency rebalancing is shown to increase the value of a portfolio under fair consideration of limitations of fixed-mix portfolios. The fixed-rate asset on prime rates serves as an appropriate tool for benchmarking liabilities.

The performance of arbitrary fixed-mix proxy strategies shows the portfolio of a fixed-rate asset and a risky asset to be favourable on leveraged allocations in a fixed-rate asset. The dynamics of asset allocation in fixed-mix portfolios show a performance reduction for strategies which yield on the benchmark return by shorting a risky asset. The fixed-mix proxy strategy which yields on the risky return outperforms exceedingly. The fixed-mix decision rules as determined by Mulvey *et al.* (2007) are favorable for index funds and exchange traded funds of minimal transaction costs, tax exempt and tax deferred accounts. The research shows a growing potential of fixed-mix portfolio strategies particularly for the hedge funds community where the trading practice has less restrictions and high innovations. James (2012) noted that a determined-analytical approach and appropriate use of derivatives is effective when consolidated in the risk management framework under the objectives of the portfolio. In the investment practice, rewarding a risk-control consistent plan of action is proven successful. The active portfolio management with fixed-mix portfolios determines innovative tools under portfolio objectives which is shown to have

sophisticated solutions of analytic, numeric and Monte Carlo methods as noted in Markowitz (2013) and in Kung *et al.* (2013).

The outcome of the research restores the confidence of long term investors in volatile markets and emerging markets. Practical matters in placing funds for the investment community demand a forward scrutiny interest to challenge co-movements that come with diversification in the prospect of the future. It's is noted in (James *et al.*, 2012) that markets have become extra ordinarily connected under the categories of common shocks which are sudden changes in the interest rate and financial linkages such as investor behaviours and opinions with connections across asset classes. The fixed-mix portfolios retain the excess performance of over performance and underperformance in the market risk with the portfolio growth value emerging from less efficient portfolios of low cost.

7.4 The focused short comings of fixed-mix strategies

Theoretical fixed-mix portfolios are effectively on a market with stationary asset prices such that the non-degeneracy condition of the price ratios holds by the non-constancy in the portfolio, indicating a benchmarked variability of a stochastic. Moreover, the condition of stationarity which does not generally holds in practice, Dempster *et al.* (2007), strategic tactical implementations are recommended to attain a price ratio process of strictly positive returns to evolve revolutionary into a strategy. The costs of frequent rebalancing which are prone in the time deterministic hedging or the rebalancing time are not evaluated in particular. This is regarded to be a minor drawback in the potential of multi-periods active optimal transaction returns over the stochastic. The financial risk of sub optimal rebalancing gains in a fixed-mix portfolio strategy of asset prices is encountered at a cost of a maximal price differential derivative option, the lookback straddle, which is expensive to a plain vanilla straddle derivative option that attains a price level differential over a stochastic of the underlying asset prices.

7.5 Recommendations

The active portfolio management practise has been shown to increase the value of managing risky assets in the equity market. High frequency trading under fixed-mix strategies is recommended to analyse the consistency of rebalancing gains and to outperform high frequency trading benchmarks in the short term view. The limitations of the research are costs associated with the strategy and the sufficient empirical market risk which incorporates the determined financial risk.

Future research aims to analyse risk-adjusted returns in volatility trading and to analyse the time liability value of fixed-mix portfolios empirically in generic markets with volatile inefficient portfolios of low costs and to ensure the degrading of the financial growth over the stochastic condition. The research recommends a further analysis of stationary conditions in markets such as the currency market and to involve exchange traded funds and other assets classes to optimize the financial proxy. Furthermore, there is a need to consider convergence trades under non-degeneracy conditions and the potential of re-balancing financial engineered returns in multi-periods. Future research recommends examining the proficiency and the limitations of the financial proxy which is a dual transaction of lookback options.

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A.1 Appendix: The Models

1. The market model

The market model considers three tradable assets in the market namely the fixed-rate asset, the risky asset and the derivative instrument. The price of a fixed-rate asset at time t ,

$$P(t) = P(t + 1)e^{-r\delta t} \quad \delta t \text{ is a discrete time period.}$$

r is a fixed rate over a discrete period.

$$\Delta P(t) = \ln\left(\frac{P(t+1)}{P(t)}\right) = (r_{t+1} - r_t)\delta t \quad \text{is a discrete change.}$$

The log model of risky asset prices is

$$S(t) = \ln(s(t) / s(0)) \quad \text{is the initialised price of an asset.}$$

$$\Delta S(t) = S(t) - S(t - 1) \quad \text{is the rate of return.}$$

$$\delta = S(t)/t \quad \text{is the growth rate of asset prices.}$$

$$\mu = \frac{\Delta S(1) + \dots + \Delta S(t)}{t}$$

is a drift rate of an asset price.

$$\sigma = \sqrt{\text{var}(\Delta S(t))}$$

is the volatility.

The rate of the empirical parameter σ in a distribution of T observations is computed by $\sigma \sqrt{dt}$ where $dt = \frac{T}{255}$ (annually) and $dt = \frac{1}{255}$ (daily).

The definitions of the derivative instruments are given below,

1.1 (Darius *et al.*, 2002)

Straddle purchase is a combination of a short put and a short call option

$$S(t)_p = -(p(t) + c(t))$$

1.2 (Darius *et al.*, 2002)

Straddle Long is a combination of long put and long call options

$$S(t)_c = p(t) + c(t)$$

1.3 (Darius *et al.*, 2002)

Lookback put is a put option with a strike price determined as the maximum value of the stock price during the life on an option.

$$p(t)_l = K - S(t) \text{ where}$$

$$K = \max_{0 \leq t \leq T} S(t)$$

1.4 (Darius *et al.*, 2002)

Lookback call is a call option with a strike price determined as the minimum value during the life on an option.

$$c(t)_l = S(t) - K \text{ where}$$

$$K = \min_{0 \leq t \leq T} S(t)$$

1.5 (Darius *et al.*, 2002)

Lookback straddle is a combination of a lookback put and a lookback call

$$S(t)_l = p(t)_l + c(t)_l$$

2. The trading model

The trading model considers a financial market model of two assets which is described by the following vector,

$$S(t) = [S(t)_1, S(t)_2].$$

A trading portfolio specify the number of each asset held in the portfolio at each trading period by the following vector,

$$h(t) = [h(t)_1, h(t)_2] .$$

2.1 The buy and hold trading portfolio strategy with a number of assets specified by an asset allocation ratio α is described by the matrix equation

$$V(t) = h(0)S(t), \quad -2 \leq \alpha \leq 2,$$

$$h(0)_1 = \alpha V(0)/S(0)_1 \text{ and}$$

$$h(0)_2 = (1 - \alpha)V(0)/S(0)_2.$$

2.2 The fixed-mix trading portfolio strategy with a fixed proportion α is described by the matrix equation,

$$V(t) = h(t)S(t), \quad -2 \leq \alpha \leq 2,$$

$$h(t)_1 = \alpha V(t)/S(t)_1 \text{ and } h(t)_2 = (1 - \alpha)V(t)/S(t)_2.$$

2.3 The proxy strategy of a fixed-mix portfolio with a fixed-proportion α is described by the vector equation

$$V(t) = h(t)S(t), \quad -2 \leq \alpha \leq 2,$$

$$V(t) = V(t - 1) + \Delta V(t)$$

$$\Delta V = \alpha \Delta S(t)_1 + (1 - \alpha)S(t)_l,$$

$$h(t)_1 = \alpha V(t)/S(t)_1 \text{ and}$$

$$h(t)_2 = (1 - \alpha)V(t)/S(t)_2.$$

3. The Merton model of asset prices

(Dempster *et al.*, 2007)

The rate of returns of an asset price process is described by a Merton's stochastic differential equation,

$$dS(t) = S(t)(\mu dt + \sigma dW_t),$$

μ is a drift rate; σ is the volatility and W_t is a Wiener process such that the variance and the mean are t and zero respectively, implying an instantaneous time differential variance, $var[dW_t] = dt$.

4. The Capital asset pricing model

(Fama, French, 2004)

The Sharpe-Lintner capital asset pricing model defines the expected rate of return of an asset as the risk-free asset's rate of return and the risk premium which is the excess return on the market to a risk-free asset times the beta of the asset which is given by the equation.

$$E[r] = r_0 + \beta E[\mu - r_0]$$

$$\beta = \frac{Cov(r, \mu)}{\sigma^2(\mu)}$$

r_0 - rate of return of a risk-free asset.

r – rate of return of an asset.

μ - rate of return of a market portfolio.

β - risk premium coefficient.

5. The Markowitz's mean-variance model

(Farshid, Erkkö, 2012)

The Markowitz model

The model determines portfolio weights for the investment objectives which are to maximize the expected portfolio returns for a specified level of risk or to minimize the portfolio risk for a specified level of portfolio performance. The portfolio objectives are given by the following mean-variance optimization problem.

(Optimal return problem)

$$\begin{aligned} \max_{\alpha} & E[\alpha R] \\ \text{s.t. } & \sigma(\alpha R) < \sigma_{\text{TARGET}}; \Sigma \alpha \leq 1 \end{aligned}$$

(Minimum risk problem)

$$\begin{aligned} \min_{\alpha} & \sigma(\alpha R) \\ \text{s.t. } & E[\alpha R] = \mu_{\text{TARGET}}; \Sigma \alpha \leq 1 \end{aligned}$$

6. The Sharpe ratio

(John *et al.*, 2003)

The Sharpe ratio is a performance measure of a portfolio strategy to a benchmark performance. The ratio is defined as the excess return of the strategy to a benchmark return over a standard deviation of returns of the strategy. The ratio is defined below,

$$\text{Sharpe ratio} = \frac{(r - r_0)}{\sigma(r)}$$

r_0 - return of a benchmark.

r - return of a strategy.

$\sigma(r)$ - risk of a strategy.

7. The ARIMA model

(Brooks, 2002)

The Integrated Autoregressive moving average

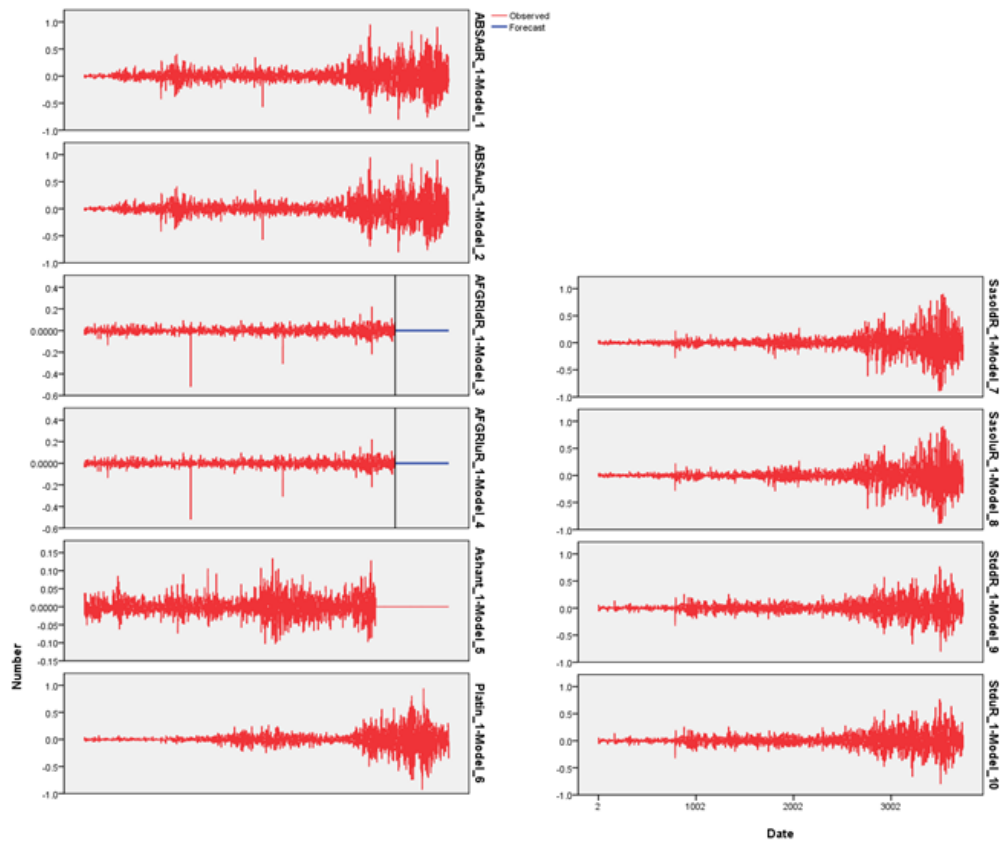
$$y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \theta_1 u_{t-1} + \theta_2 u_{t-2} + \cdots + \theta_q u_{t-q} + u_t$$

$$E[u_t] = 0; E[u_t^2] = \sigma^2; E[u_t u_s] = 0, t \neq s.$$

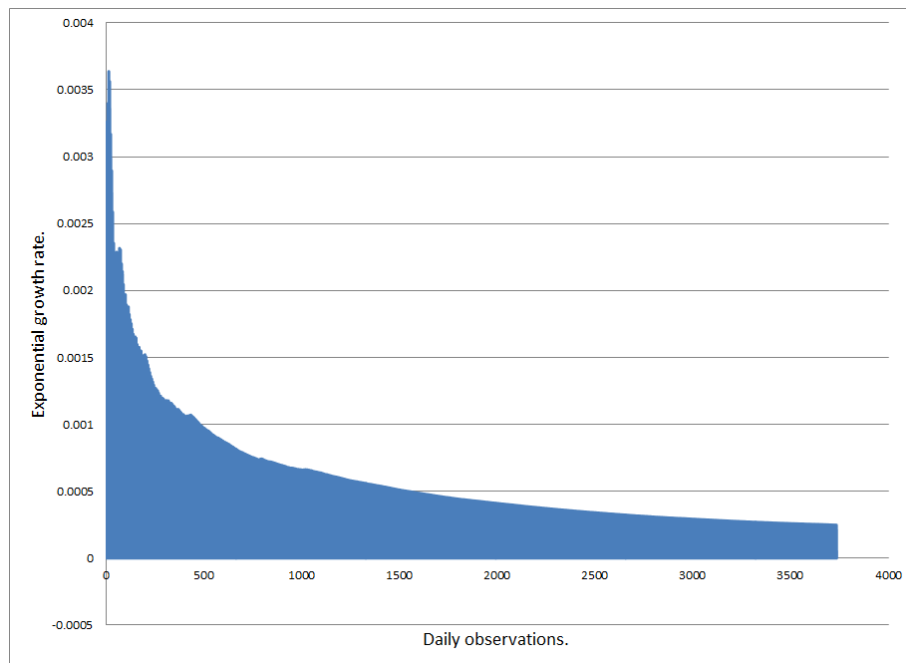
A.2 Appendix: Charts and Tables of performances

2.1 Charts

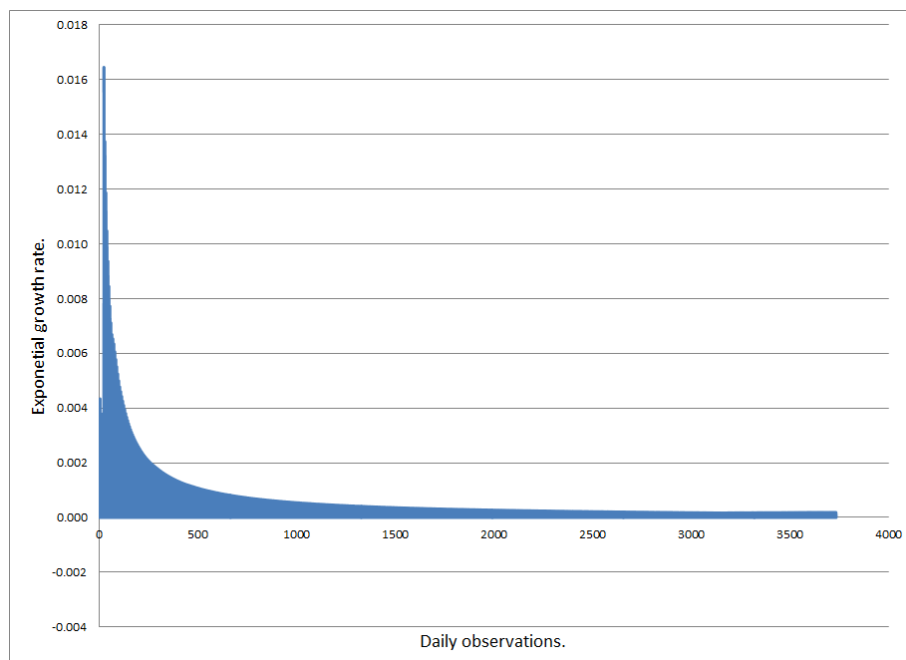
A.2.1.1 Chart: The ARIMA models of differenced relative price ratios.



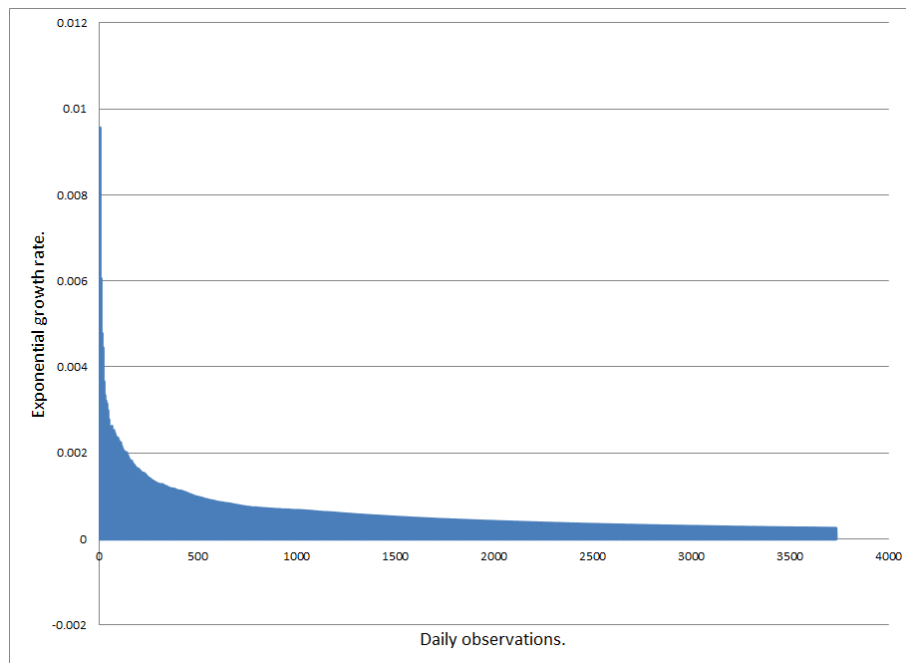
The ARIMA (0, 0, 0) models of differenced relative price ratios of ABSA Group Limited, AFGRI Limited, Anglo Gold Ashanti Limited, Anglo American Platinum, Sasol Limited and Standard Bank in down-trending prime rates and in up-trending prime rates.



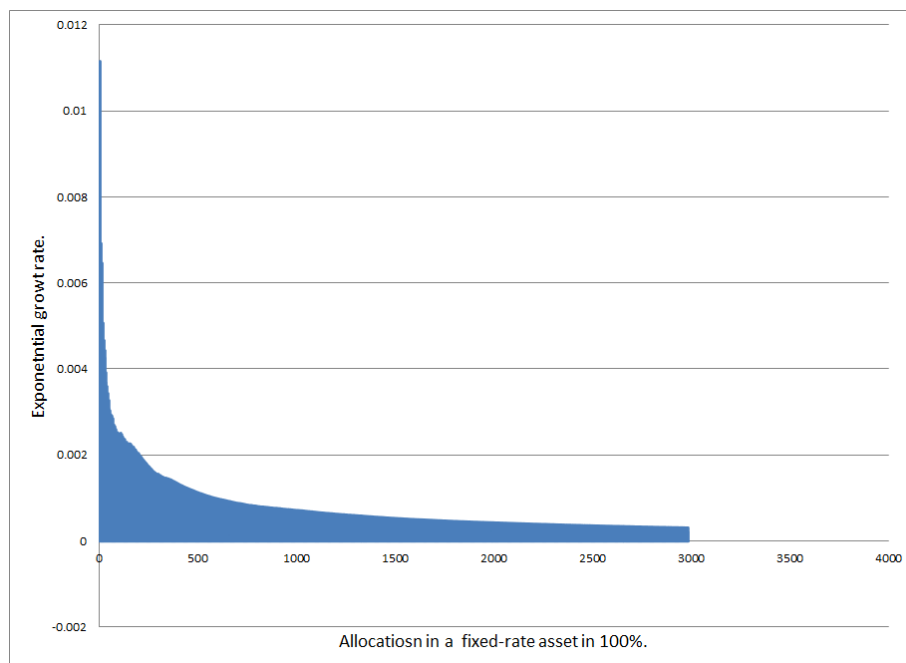
A.2.1.2 Chart: Growth rates of a proxy fixed-mix portfolio of a risky asset of ABSA Group Limited. The fixed-rate asset allocation is -200 percent.



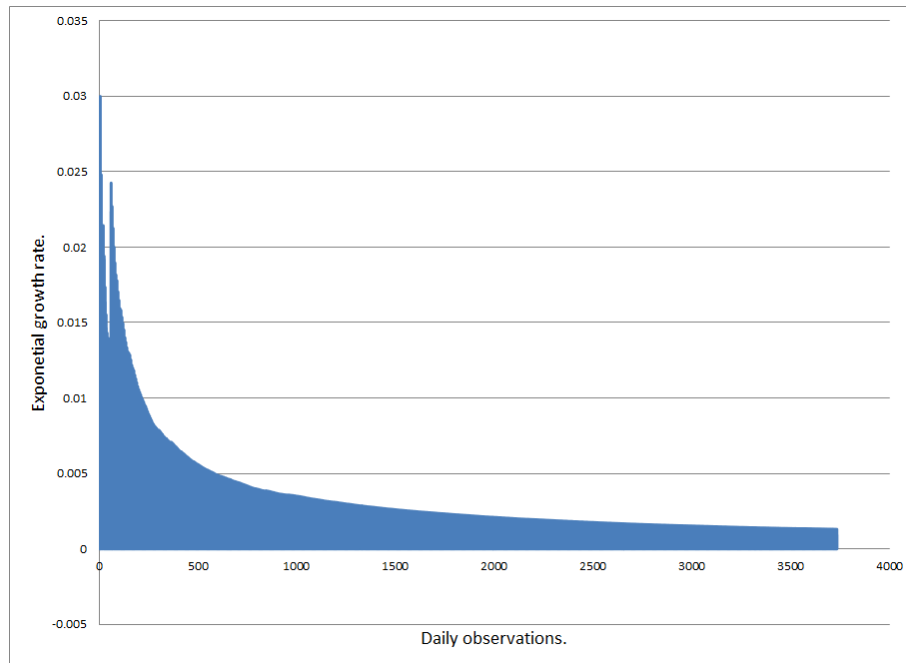
A.2.1.3 Chart: Growth rates of a proxy fixed-mix portfolio of a risky asset of AFGRI Limited. The fixed-rate asset allocation is -200 percent.



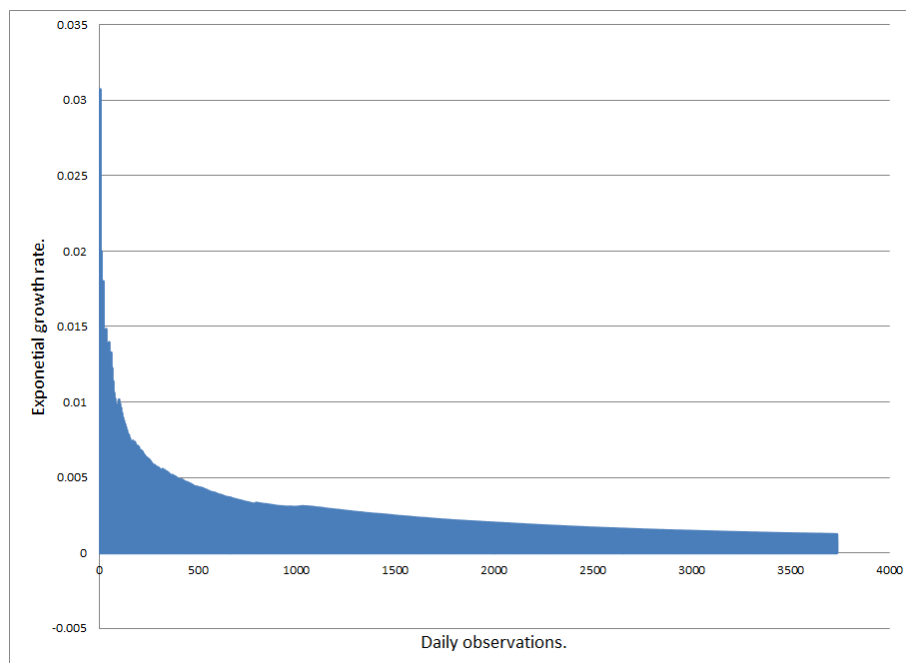
A.2.1.4 Chart: Growth rates of a proxy fixed-mix portfolio of a risky asset of Sasol Limited. The fixed-rate asset allocation is -200 percent.



A.2.1.5 Chart: Growth rates of a proxy fixed-mix portfolio of a risky asset of Anglo Gold Ashanti Limited. The fixed-rate asset allocation is -200 percent.



A.2.1.6 Chart: Growth rates of a proxy fixed-mix portfolio of a risky asset of Anglo American Platinum Limited. The fixed-rate asset allocation is -200 percent.



A.2.1.7 Chart: Growth rates of a proxy fixed-mix portfolio of a risky asset of Standard Bank Limited. The fixed-rate asset allocation is -200 percent.

2.2 Tables

Total returns and annual returns of portfolios of a fixed-mix proxy strategy, fixed-mix strategy and a buy and hold strategy, relative performance ratios to a buy and hold strategy and a performance measure to a benchmark asset over a duration of the investment. (n/a implies not available – not countable exponentially)

A.2.2.1 Table: Performance of portfolios of a risky asset of ABSA Group Limited.

				1.2693	proxy	FiM(α) proxy	FiM(α)	BoH(α)	FiM(α) proxy	FiM(α)	BoH(α)
α	FiM(α) proxy	FiM(α)	BoH(α)	Relative ratio	Relative ratio	annual return	annual return	annual return	Sharpe ratio	Sharpe ratio	Sharpe ratio
-2	101.7375	0.1217	17.1329	140.7790	5.9381	0.3155	-0.1438	0.1939	54.4412	-0.7788	0.0218
-1.5	85.2307	0.3928	15.8568	40.3681	5.3750	0.3034	-0.0638	0.1886	54.8637	-0.7160	0.0217
-1	68.7239	0.9768	14.5807	14.9275	4.7133	0.2887	-0.0016	0.1829	55.0303	-0.6387	0.0213
-0.5	52.2171	1.8850	13.3047	7.0581	3.9247	0.2700	0.0433	0.1767	54.5044	-0.4928	0.0200
0	35.7103	2.8371	12.0286	4.2398	2.9688	0.2440	0.0712	0.1698	51.8495	-0.3642	0.0173
0.5	19.2036	3.3398	10.7525	3.2195	1.7860	0.2017	0.0823	0.1621	39.6460	-0.3467	0.0116
1	2.6968	3.0792	9.4764	3.0776	0.2846	0.0677	0.0768	0.1535	-449.8290	-0.5113	0.0000
1.5	-13.8100	2.2228	8.2003	3.6892	-1.6841	n/a	0.0545	0.1436	n/a	-1.1496	-0.0193
2	-30.3168	1.2542	6.9242	5.5210	-4.3784	n/a	0.0155	0.1321	n/a	-2.4149	-0.0387

A.2.2.2 Table: Performance of portfolios of a risky asset of AFGRI Limited.

				1.2693	proxy	FiM(α) proxy	FiM(α)	BoH(α)	FiM(α) proxy	FiM(α)	BoH(α)
α	FiM(α) proxy	FiM(α)	BoH(α)	Relative ratio	Relative ratio	annual return	annual return	annual return	Sharpe ratio	Sharpe ratio	Sharpe ratio
-2	34.0359	-0.1770	17.1329	-96.8199	1.9866	0.2473	n/a	0.1992	3.0244	n/a	0.0088
-1.5	28.8127	-0.1122	15.8568	-141.2956	1.8171	0.2356	n/a	0.1938	2.9252	n/a	0.0093
-1	23.5895	0.0830	14.5807	175.6634	1.6179	0.2216	-0.1745	0.1879	2.7225	-0.1025	0.0100
-0.5	18.3663	0.4715	13.3047	28.2167	1.3804	0.2041	-0.0527	0.1815	2.3210	-0.1162	0.0108
0	13.1431	1.1025	12.0286	10.9102	1.0927	0.1806	0.0068	0.1744	1.4444	-0.1596	0.0119
0.5	7.9200	1.9875	10.7525	5.4100	0.7366	0.1451	0.0482	0.1665	-1.2032	-0.2644	0.0128
1	2.6968	3.0792	9.4764	3.0776	0.2846	0.0696	0.0789	0.1577	-454.9332	-0.4851	0.0000
1.5	-2.5264	4.2628	8.2003	1.9237	-0.3081	n/a	0.1017	0.1475	n/a	-0.1821	-0.0077
2	-7.7496	5.3696	6.9242	1.2895	-1.1192	n/a	0.1178	0.1357	n/a	-0.0813	-0.0104

A.2.2.3 Table: Performance of portfolios of a risky asset of Anglo Gold Ashanti Limited.

				0.2729	proxy	FiM(α) proxy	FiM(α)	BoH(α)	FiM(α) proxy	FiM(α)	BoH(α)
α	FiM(α) proxy	FiM(α)	BoH(α)	Relative ratio	Relative ratio	annual return	annual return	annual return	Sharpe ratio	Sharpe ratio	Sharpe ratio
-2	99.1549	0.0215	-6.3704	-295.7706	-15.5650	0.3924	-0.3276	null	1.8578	-3.5238	n/a
-1.5	82.9422	0.0840	-4.4099	-52.4697	-18.8080	0.3772	-0.2114	null	1.6925	-2.5768	n/a
-1	66.7295	0.2598	-2.4495	-9.4287	-27.2422	0.3586	-0.1151	null	1.3418	-1.6176	n/a
-0.5	50.5168	0.6368	-0.4890	-0.7679	-103.2965	0.3348	-0.0385	null	1.2828	-1.2248	n/a
0	34.3040	1.2381	1.4714	1.1884	23.3139	0.3018	0.0182	0.0330	9.8770	-7.8542	-6.9328
0.5	18.0913	1.9071	3.4318	1.7995	5.2716	0.2472	0.0551	0.1053	23.2007	-19.9213	-8.6610
1	1.8786	2.3223	5.3923	2.3219	0.3484	0.0538	0.0719	0.1438	-541.1866	-432.3442	0.0000
1.5	-14.3342	2.2280	7.3527	3.3001	-1.9495	n/a	0.0684	0.1703	n/a	-28.9660	10.1627
2	-30.5469	1.6761	9.3132	5.5565	-3.2800	n/a	0.0441	0.1905	n/a	-22.4597	10.5034

A.2.2.4 Table: Performance of portfolios of a risky asset of Anglo American Platinum Limited.

				0.5584	proxy	FiM(α) proxy	FiM(α)	BoH(α)	FiM(α) proxy	FiM(α)	BoH(α)
α	FiM(α) proxy	FiM(α)	BoH(α)	Relative ratio	Relative ratio	annual return	annual return	annual return	Sharpe ratio	Sharpe ratio	Sharpe ratio
-2	128.9403	0.0375	-3.0778	-82.0915	-41.8938	0.3317	-0.2241	n/a	2.2514	-2.4620	n/a
-1.5	107.8997	0.1977	-0.9854	-4.9842	-109.4956	0.3195	-0.1106	n/a	1.6329	-1.3860	n/a
-1	86.8591	0.7078	1.1069	1.5638	78.4678	0.3047	-0.0236	0.0069	1.6931	-0.7661	-0.1461
-0.5	65.8185	1.7465	3.1993	1.8318	20.5728	0.2858	0.0381	0.0794	8.0344	-0.4262	-0.0883
0	44.7779	3.0020	5.2917	1.7627	8.4620	0.2595	0.0750	0.1137	6.4665	-0.2752	-0.0565
0.5	23.7373	3.6212	7.3840	2.0391	3.2147	0.2162	0.0878	0.1365	9.0360	-0.2695	-0.0278
1	2.6968	3.0792	9.4764	3.0776	0.2846	0.0677	0.0768	0.1535	-663.1696	-0.5113	0.0000
1.5	-18.3438	1.8491	11.5688	6.2563	-1.5856	n/a	0.0420	0.1671	n/a	-1.4179	0.0214
2	-39.3844	0.7838	13.6611	17.4302	-2.8830	n/a	-0.0166	0.1785	n/a	-1.5905	0.0335

A.2.2.5 Table: Performance of portfolios of a risky asset of Sasol Limited.

				0.9269	proxy	FiM(α) proxy	FiM(α)	BoH(α)	FiM(α) proxy	FiM(α)	BoH(α)
α	FiM(α) proxy	FiM(α)	BoH(α)	Relative ratio	Relative ratio	annual return	annual return	annual return	Sharpe ratio	Sharpe ratio	Sharpe ratio
-2	111.1578	0.0292	7.3977	253.7371	15.0260	0.3303	-0.2479	0.1403	64.3784	n/a	n/a
-1.5	93.0810	0.1339	7.7442	57.8520	12.0195	0.3179	-0.1410	0.1435	59.7378	-1.7294	-0.0093
-1	75.0042	0.4503	8.0906	17.9655	9.2705	0.3027	-0.0559	0.1466	58.1568	-0.7659	-0.0082
-0.5	56.9273	1.1228	8.4371	7.5146	6.7473	0.2834	0.0081	0.1495	36.7112	-0.3866	-0.0069
0	38.8505	2.0909	8.7835	4.2008	4.4231	0.2566	0.0517	0.1523	59.4937	-0.2466	-0.0053
0.5	20.7736	2.9239	9.1299	3.1226	2.2753	0.2127	0.0752	0.1551	46.7438	-0.2192	-0.0031
1	2.6968	3.0792	9.4764	3.0776	0.2846	0.0696	0.0789	0.1577	-462.0734	-0.3051	0.0000
1.5	-15.3801	2.4444	9.8228	4.0185	-1.5657	n/a	0.0627	0.1602	n/a	-0.6330	0.0043
2	-33.4569	1.4612	10.1693	6.9597	-3.2900	n/a	0.0266	0.1626	n/a	-1.2590	0.0097

A.2.2.6 Table: Performance of portfolios of a risky asset of Standard bank Limited.

				0.9649	proxy	FiM(α) proxy	FiM(α)	BoH(α)	FiM(α) proxy	FiM(α)	BoH(α)
α	FiM(α) proxy	FiM(α)	BoH(α)	Relative ratio	Relative ratio	annual return	annual return	annual return	Sharpe ratio	Sharpe ratio	Sharpe ratio
-2	94.8880	0.5113	8.4778	16.5804	11.1926	0.3107	-0.0458	0.1459	2.0822	-19.7719	-0.0065
-1.5	79.5228	1.2404	8.6442	6.9689	9.1995	0.2987	0.0147	0.1472	1.9318	-17.3537	-0.0060
-1	64.1576	2.3641	8.8106	3.7269	7.2818	0.2840	0.0587	0.1485	1.7490	-16.9748	-0.0054
-0.5	48.7924	3.5622	8.9771	2.5201	5.4352	0.2653	0.0867	0.1498	1.5158	-18.8646	-0.0046
0	33.4272	4.2615	9.1435	2.1456	3.6558	0.2395	0.0989	0.1511	1.1938	-24.4898	-0.0034
0.5	18.0620	4.0579	9.3100	2.2943	1.9401	0.1975	0.0956	0.1523	0.6696	-38.0271	-0.0019
1	2.6968	3.0792	9.4764	3.0776	0.2846	0.0677	0.0768	0.1535	-0.9498	-62.6326	0.0000
1.5	-12.6684	1.8618	9.6428	5.1794	-1.3138	n/a	0.0424	0.1547	n/a	-67.6308	0.0021
2	-28.0336	0.8958	9.8093	10.9499	-2.8579	n/a	-0.0075	0.1558	n/a	n/a	0.0040

A.3 Appendix: Test program

FiMBoH2port

The program FiMBoH2port implements fixed - mix trading strategies and buy and hold trading strategies. The simulation is carried on a two asset portfolio. The program reads daily prices of a risky asset and daily short rates for a fixed-rate asset. The program output the updated fixed-mix portfolio and a current value of a buy and hold portfolio.

START PROGRAM:

T : Number of investment periods.
 $V[0] = 1$: Initial amount in a portfolio.
 $V[t]$: Total amount in a portfolio at time t .
 $p1[t]$: Initialised price of a fixed-rate asset at current time t , $p1[0] = 1$.
 $p2[t]$: Initialised price of a risky asset at current time t , $p2[0] = 1$.
 $h1[t]$: number of fixed-rate assets.
 $h2[t]$: number of risky assets.

$t=1$: Initial holdings
 $boh_1 = (r1 * V[t]) / p1[t];$
 $boh_2 = (r2 * V[t]) / p2[t];$
 $t = 1, 2 \dots T-1, T$
: Time rebalancing
 $V[t] = h1[t] * p1[t] + h2[t] * p2[t];$
 $h1[t] = (r1 * V[t]) / p1[t];$
 $h2[t] = (r2 * V[t]) / p2[t];$

$V_t = boh_1 * p1[t] + boh_2 * p2[t];$

END PROGRAM:

proxyFiMBoH2port

The program proxyFiMBoH2port implements fixed - mix trading strategies and buy and hold trading strategies. The simulation is carried on a two asset portfolio. The program reads daily prices of a risky asset and daily short rates for a fixed-rate asset. The derivative instrument reads the price of the underlying risky asset and returns a payoff over duration of a derivative. The program output the updated fixed-mix portfolio and a current value of a buy and hold portfolio.

START PROGRAM:

T : Number of investment periods.
V[0] = 1 : Initial amount in a portfolio.
V[t] : Total amount in a portfolio at time t.
p1[t] : Initialised price of a fixed-rate asset at current time t, p1[0] =1.
p2[t] : Initialised price of a risky asset at current time t, p2[0] =1.
h1[t] : number of fixed-rate assets.
h2[t] : number of risky assets.

t=0 : Initial holdings

boh_1 = (r1*V[0])/p1[0];

boh_2 = (r2*V[0])/p2[0];

t =5, 10... T-5, T

dp1 = p1[t]-p1[t-5] : change of a fixed-rate asset in 5 discrete periods

dp2 = max (p2[t]) - min (p2[t]) : payoff of an option over a duration of 5 discrete periods.

Deterministic time hedging

V[t] = V[t-1]+ r1*dp1 + r2*dp2

h1[t] = (r1*V[t])/p1[t];

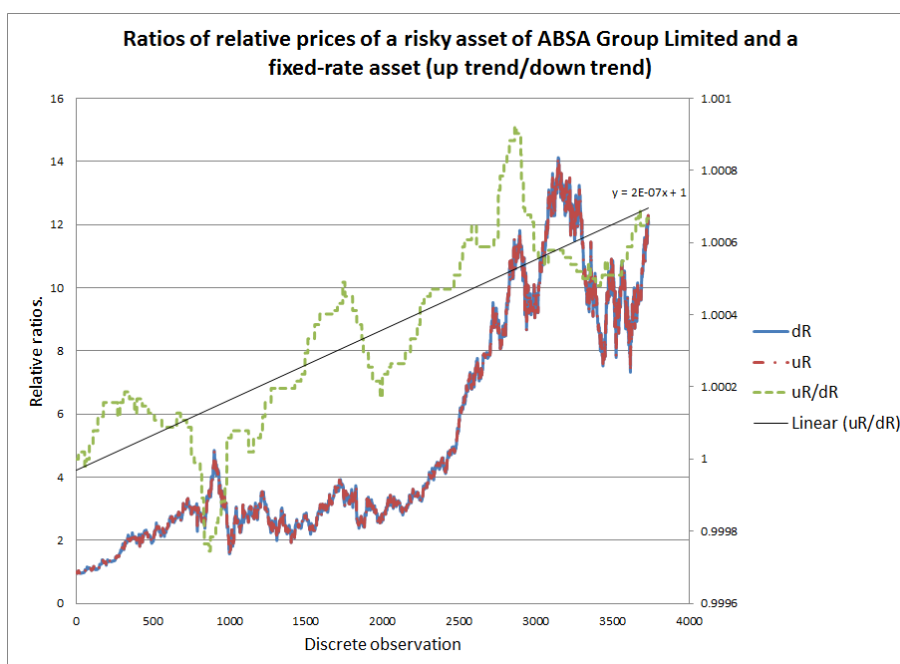
h2[t] = (r2*V[t])/p2[t];

Vt = boh_1*p1[t] + boh_2*p2[t];

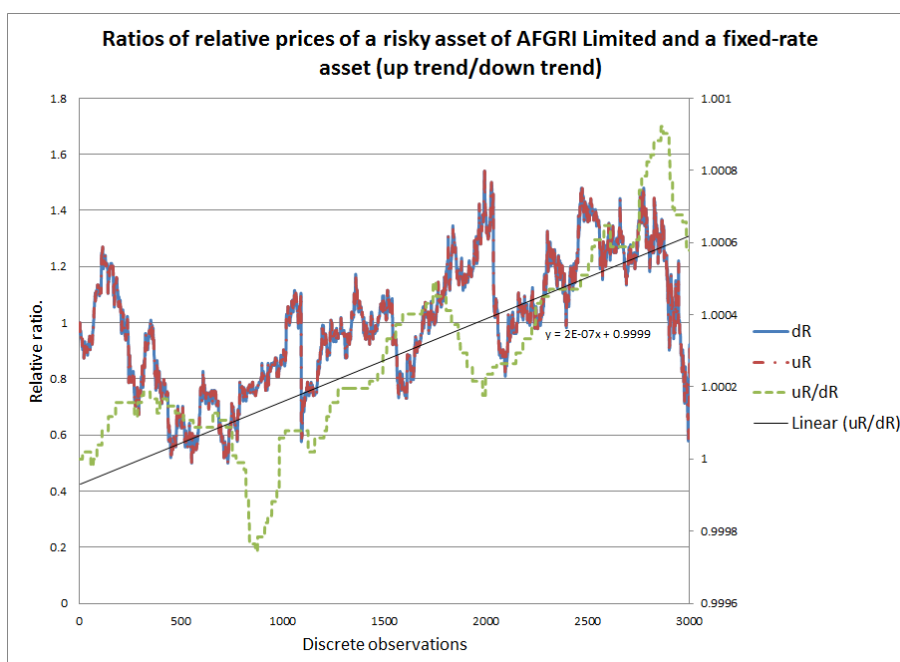
END PROGRAM:

A.4 Appendix: Chart and Tables of preliminaries

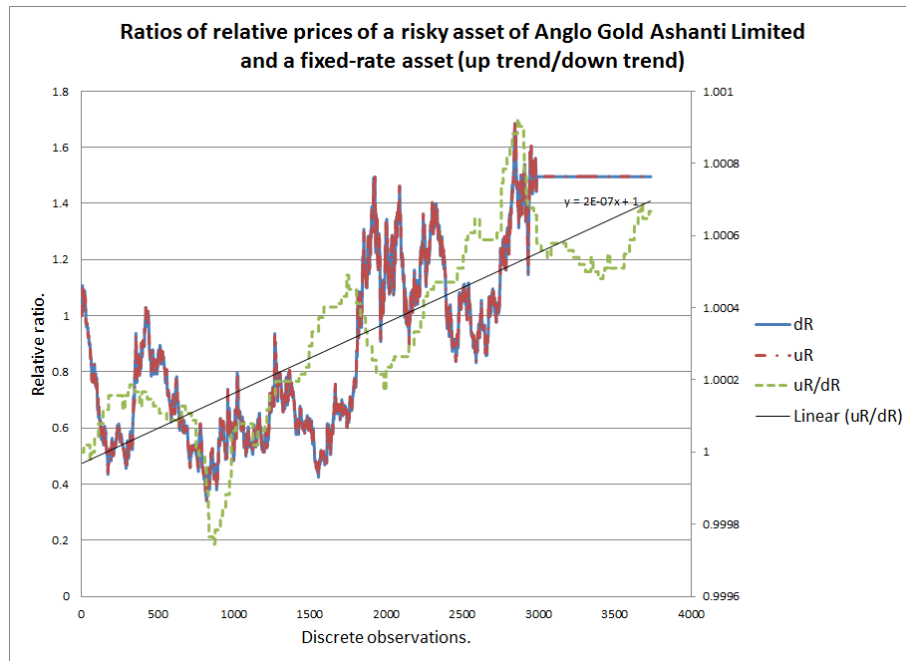
A.4.1 Charts



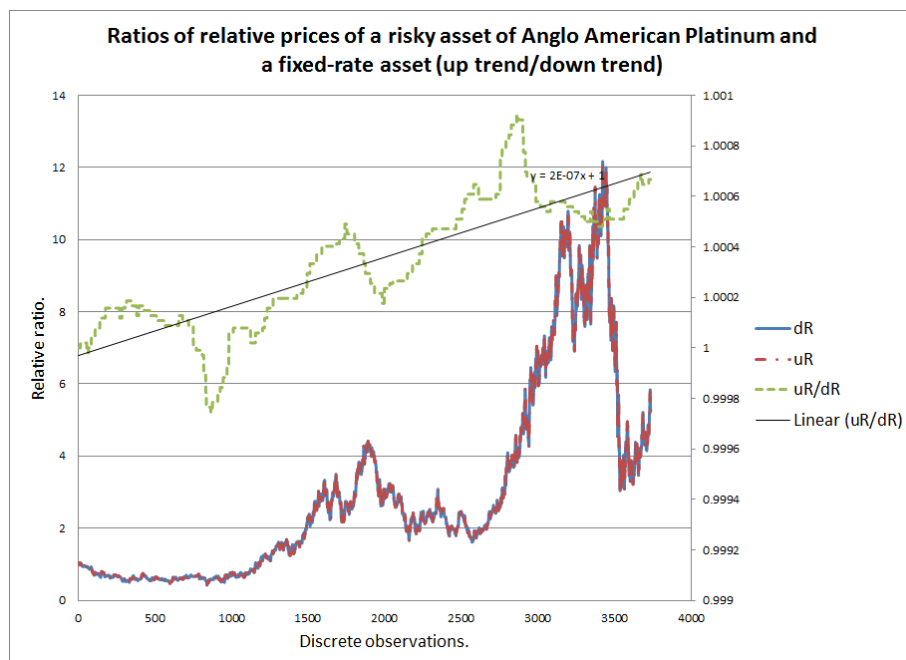
A.4.1.1 Chart: Relative price ratios of a portfolio of a risky asset of ABSA Group Limited.



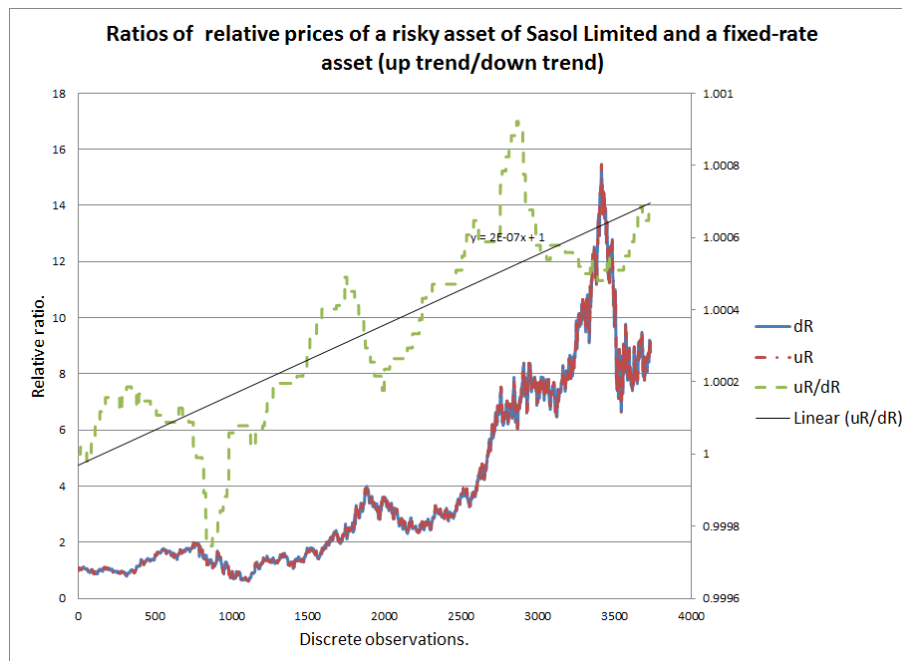
A.4.1.2 Chart: Relative price ratios of a portfolio of a risky asset of AFRI Limited.



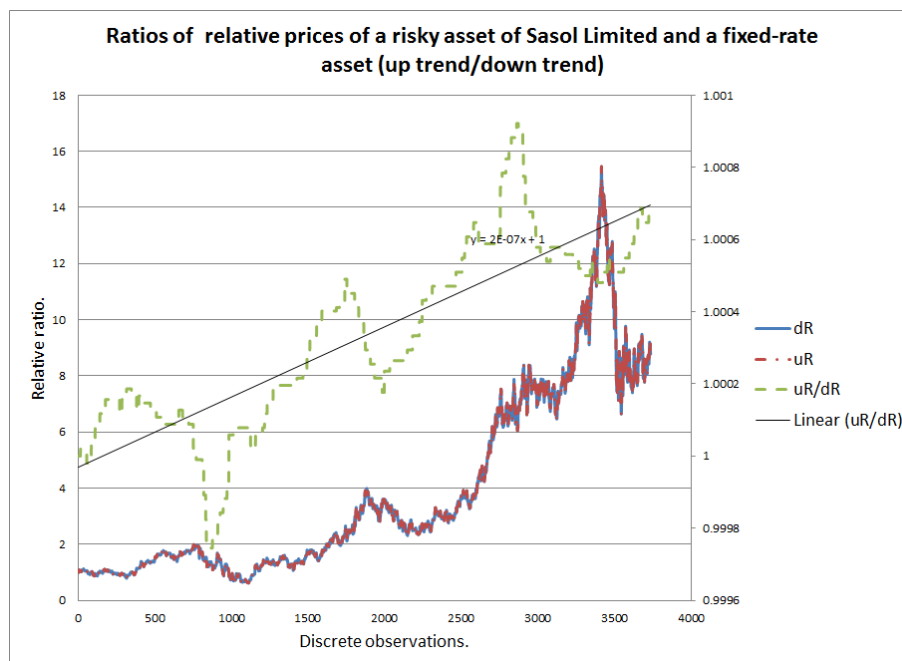
A.4.1.3 Chart: Relative price ratios of a portfolio of a risky asset of Anglo Ashanti Limited.



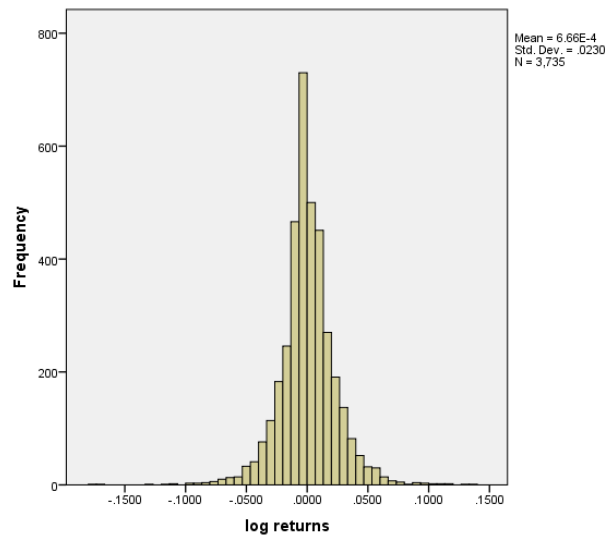
A.4.1.4 Chart: Relative price ratios of a portfolio of a risky asset of Anglo American Platinum Limited.



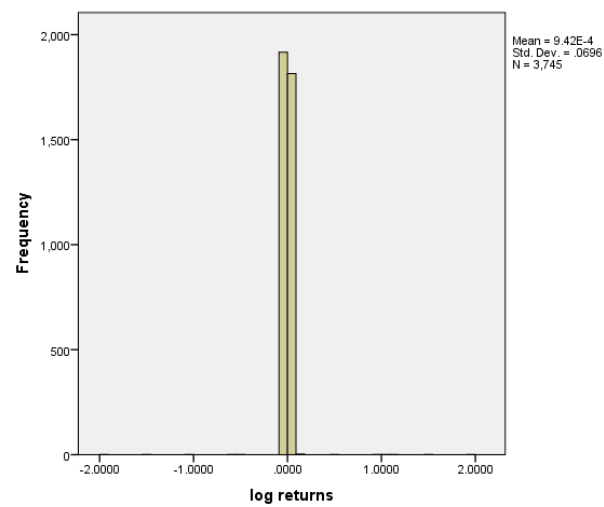
A.4.1.5 Chart: Relative price ratios of a portfolio of a risky asset of Sasol Limited.



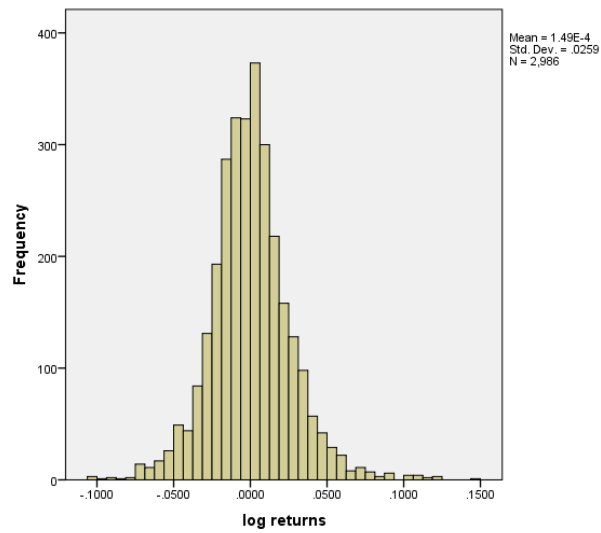
A.4.1.6 Chart: Relative price ratios of a portfolio of a risky asset of Standard Bank.



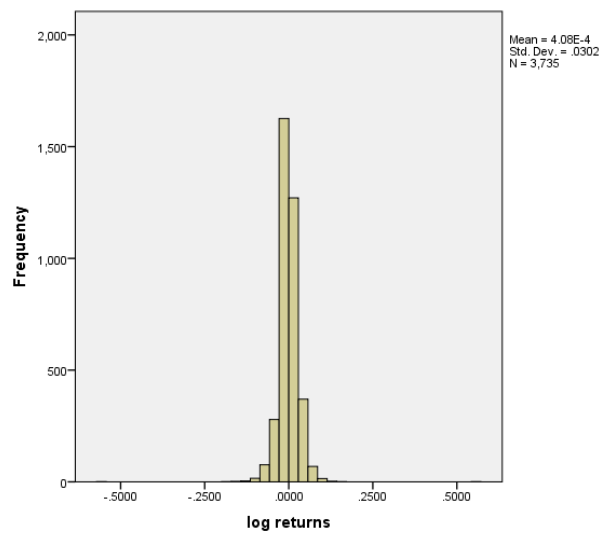
A.4.1.7 Chart: Log returns of daily prices of a risky asset of ABSA Group Limited.



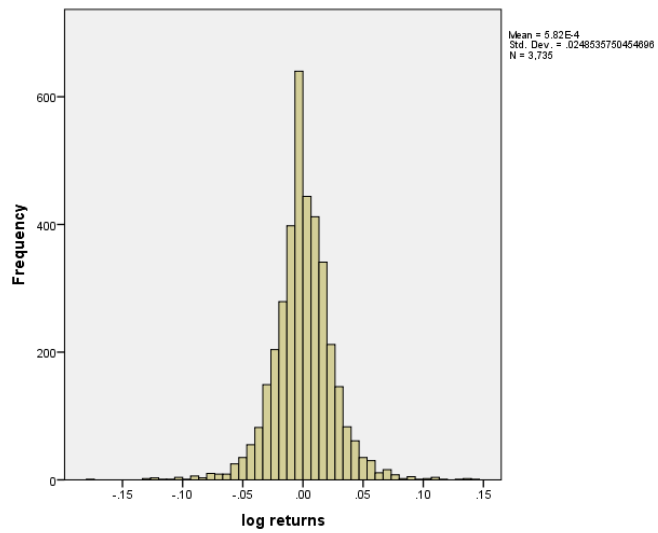
A.4.1.8 Chart: Log returns of daily prices of a risky asset of AFGRI Limited



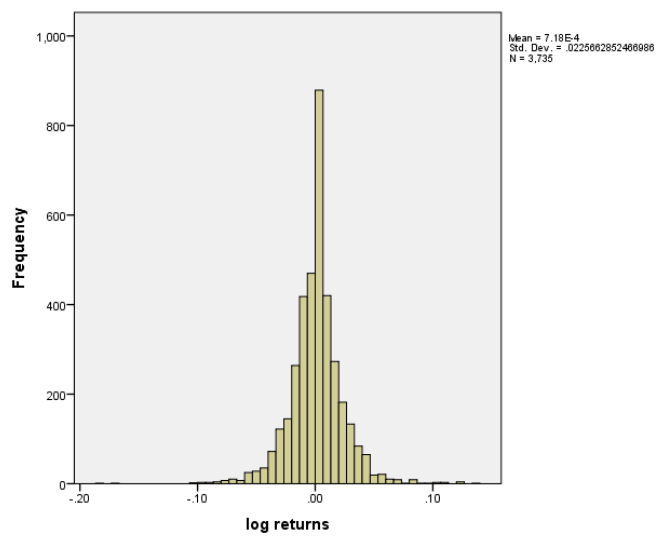
A.4.1.9 Chart: Log returns daily prices of a risky asset of Anglo Gold Ashanti Limited



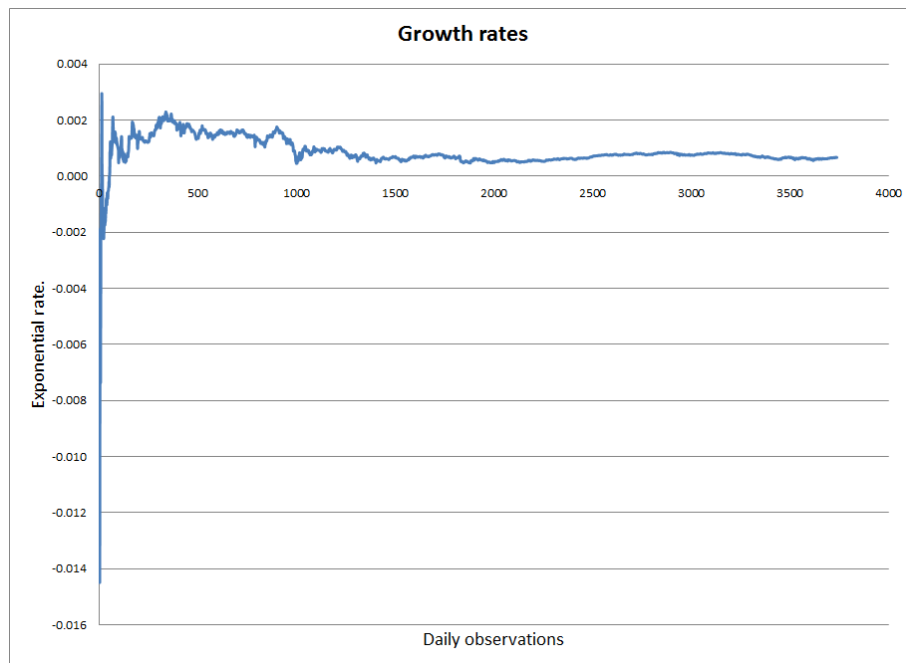
A.4.1.10 Chart: Log returns of daily prices of a risky asset of Anglo American Platinum Limited



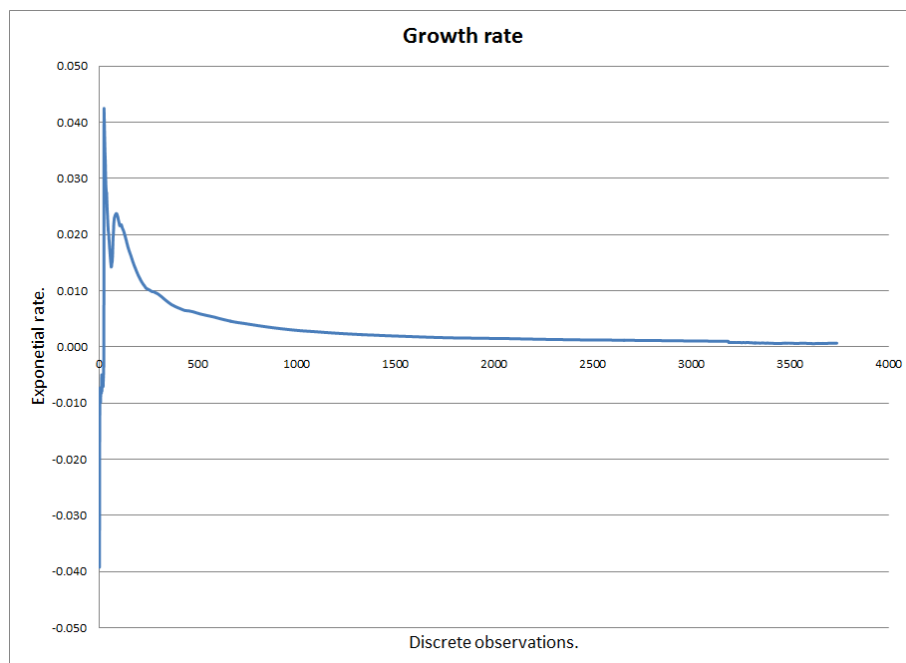
A.4.1.11 Chart: Log returns of daily prices of a risky asset of Sasol Limited



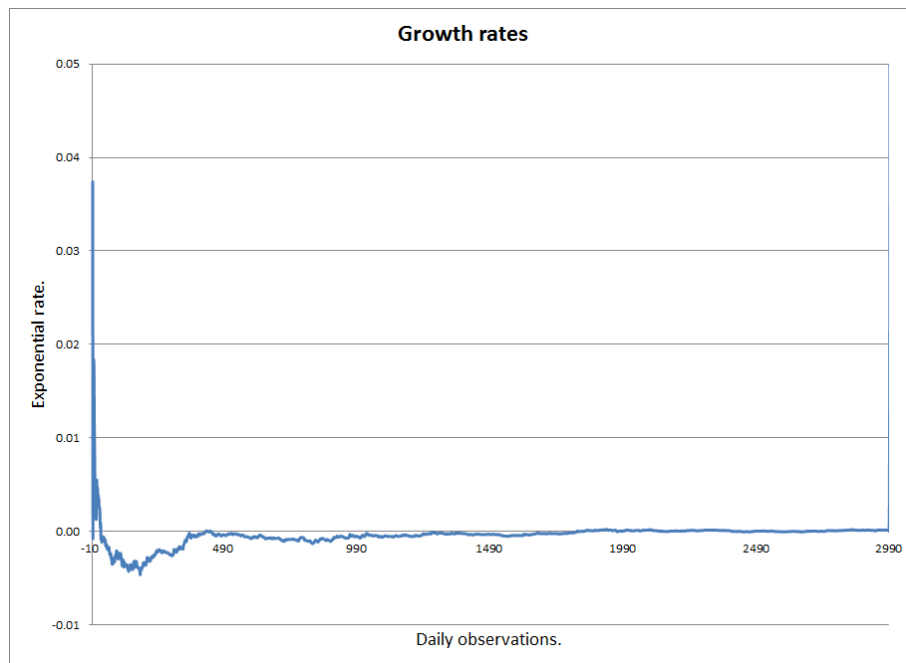
A.4.1.12 Chart: Log returns of daily prices of a risky asset of Standard Bank Limited



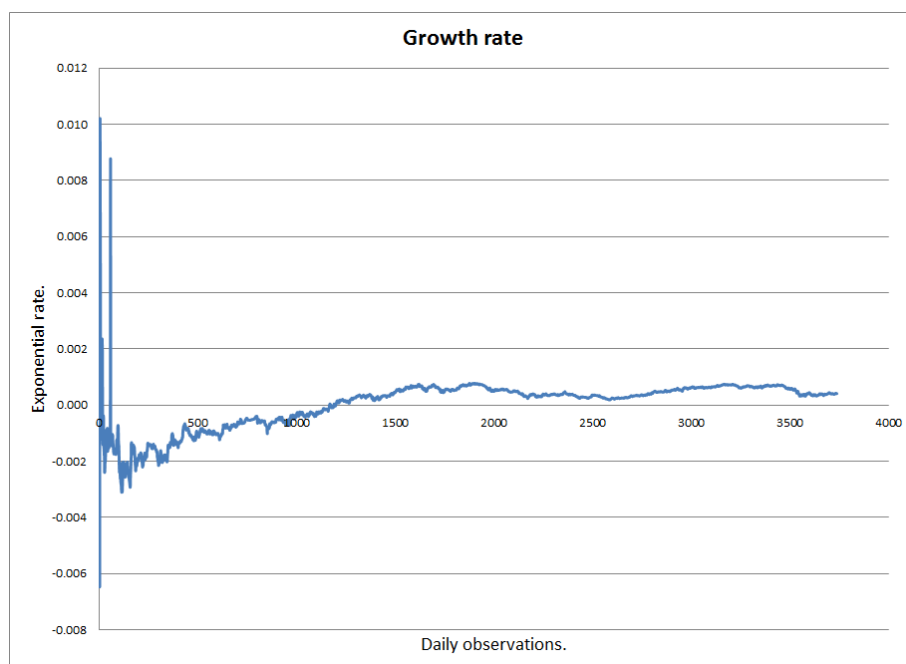
A.4.1.13 Chart: Growth rates of a risky asset of ABSA Group Limited.



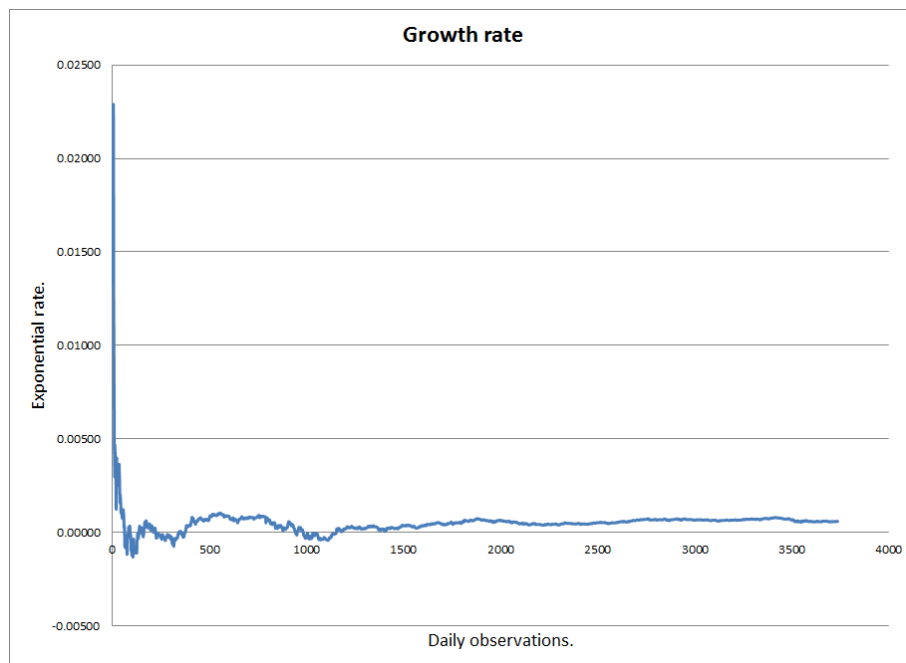
A.4.1.14 Chart: Growth rates of a risky asset of AFGRI Limited.



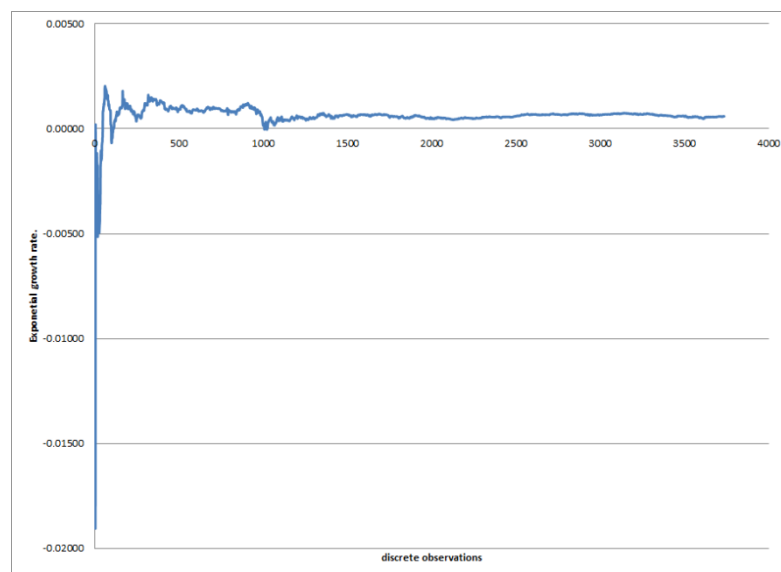
A.4.1.15 Chart: Growth rates of a risky asset of Anglo Gold Ashanti Limited



A.4.1.16 Chart: Growth rates of a risky asset of Anglo American Platinum Limited.



A.4.1.17 Chart: Growth rates of a risky asset of Sasol Limited.



A.4.1.18 Chart: Growth rates of a risky asset of Standard Bank Limited.

A.4.2. Tables

A.4.2.1 Table: Descriptive statistics of daily log prices of risky assets.

	N	Range	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
Log price of a risky asset of AFGRI Limited	3187	3.3178	-0.1335	3.1842	2.8310	0.4292	-3.3070	15.7070
Log price of a risky asset of ABSA Group Limited	3736	2.6967	-0.0488	2.6479	1.3976	0.6932	0.1690	-0.9670
Log price of a risky asset of Standard Bank	3736	2.4994	-0.1126	2.3868	1.1617	0.6757	0.1800	-1.1530
Log price of a risky asset of Sasol Limited	3736	3.2170	-0.4788	2.7382	1.0046	0.8241	0.2790	-1.1540
Log price of a risky asset of Anglo Gold American Platinum	3736	3.3642	-0.8655	2.4987	0.6855	0.9017	0.1160	-1.0680
Log price of a risky asset of Anglo Gold Ashanti Limited	2987	1.6015	-1.0800	0.5215	-0.2238	0.3553	0.0720	-1.1240

A.4.2.2 Table: Descriptive statistics of daily total returns of a fixed-rate asset and of risky assets.

	N	Range	Minimum	Maximum	Mean	Std. deviation	Skewness	Kurtosis
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic
Prime rate (normal rate- up trending)	3736	15.5	10	25.5	15.3518	3.4872	0.5073	-0.3098
Log price of a Fixed-rate asset (normal rate)	3736	2.2488	0.0000	2.2488	1.0115	0.6362	0.2701	-1.074
Total returns of a Fixed-rate asset log price (normal rate)	3735	0.0006	0.0004	0.0010	0.0006	0.0001	0.5073	-0.3102
Log price of a Fixed-rate asset (down trending)	3736	2.2485	0.0000	2.2485	1.2372	0.6361	-0.27	-1.0738
Total returns of a Fixed-rate asset log price (down trending)	3735	0.0006	0.0004	0.0010	0.0006	0.0001	0.5077	-0.3098
Total returns of a log price of a risky asset of ABSA Group Limited	3735	0.3167	-0.1796	0.1371	0.0007	0.0230	-0.116	4.982
Total returns of a log price of a risky asset of Standard Bank	3735	0.3242	-0.1863	0.1379	0.0007	0.0226	-0.035	5.585
Total returns of a log price of a risky asset of AFGRI Limited	3187	4.0000	-2.0000	2.0000	0.0009	0.0696	1.708	532.177
Total returns of a log price of a risky asset of Sasol Limited	3735	0.3165	-0.1735	0.1429	0.0006	0.0249	-0.095	4.053
Total returns of a log price of a risky asset of Anglo American Platinum	3735	1.1180	-0.5601	0.5579	0.0004	0.0302	-0.182	64.391
Total returns of a log price of a risky asset of Anglo Gold Ashanti Limited	2986	0.2547	-0.1053	0.1494	0.0001	0.0259	0.418	2.366

A.5 Appendix: Charts and tables of value processes

A.5.1 Charts

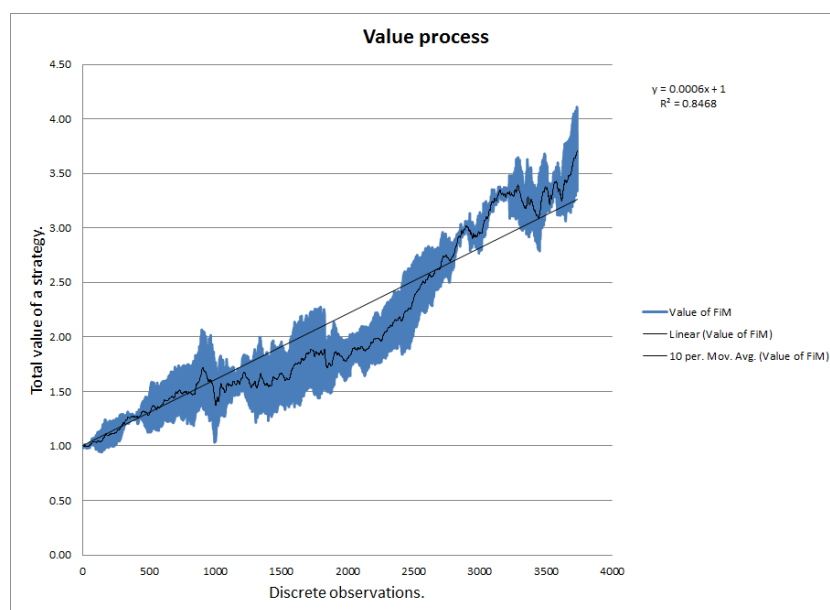


Chart 1

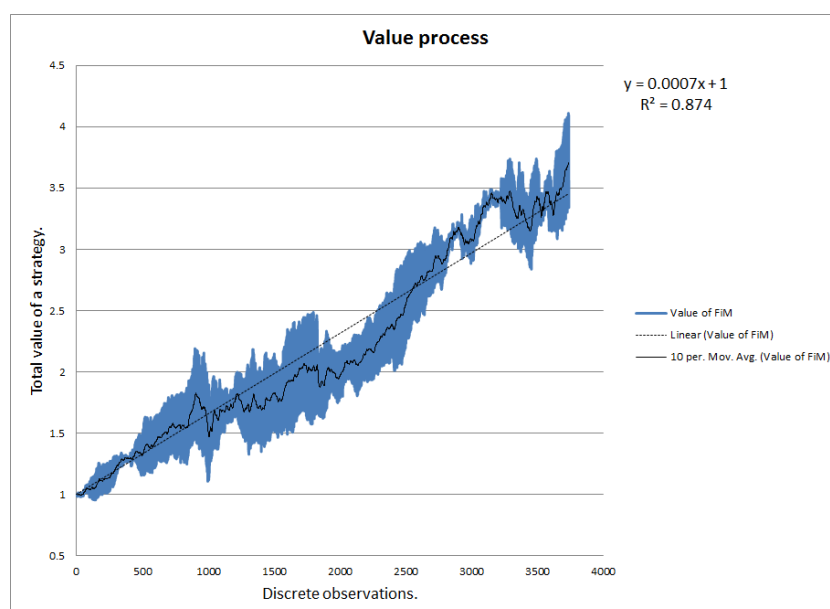


Chart 2

A.5.1.1 Value processes of a fixed-mix portfolio strategy of a risky asset of ABSA Group Limited. Chart 1: Fixed 50 percent in a fixed-rate asset in up-trending prime rates. Chart 2: Fixed 50 percent in a fixed-rate asset in down-trending prime rates.

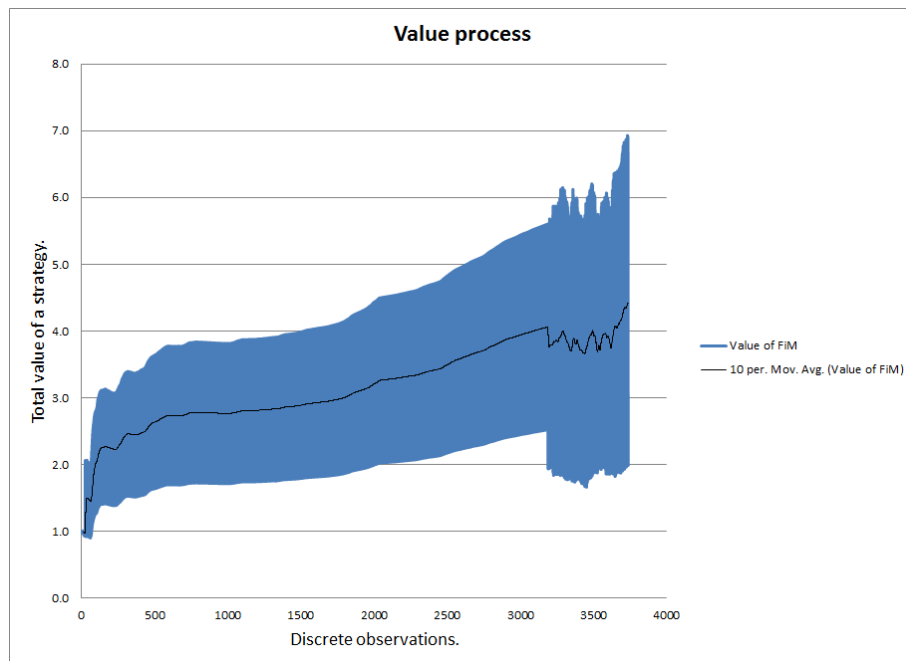


Chart 1

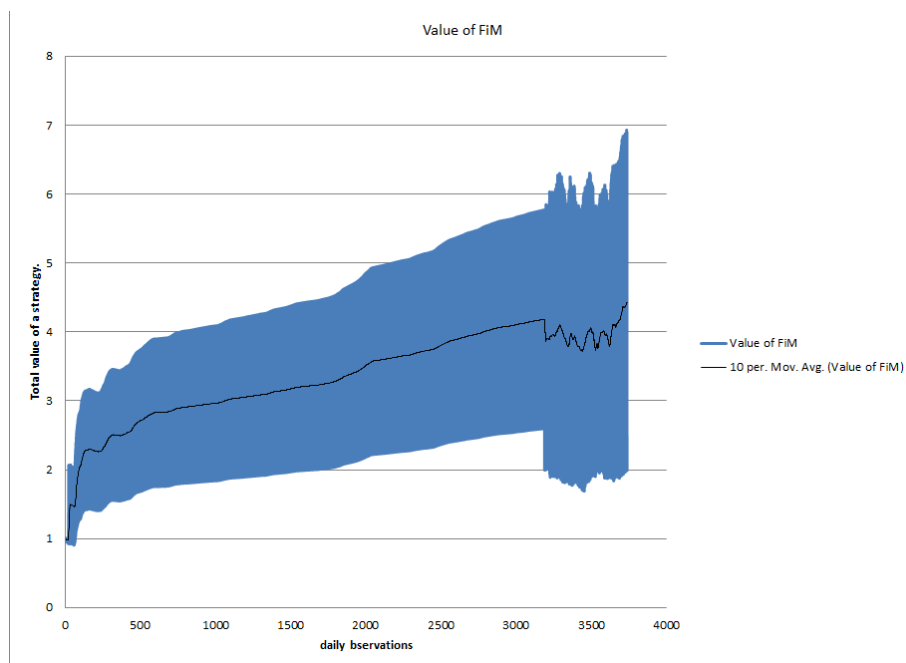


Chart 2

A.5.1.2 Chart: Value processes of a fixed-mix portfolio strategy in a portfolio of a risky asset of AFGRI Limited. Chart 1: Fixed 50 percent in a fixed-rate asset in up-trending prime rates. Chart 2: Fixed 50 percent in a fixed-rate asset in down-trending prime rates.

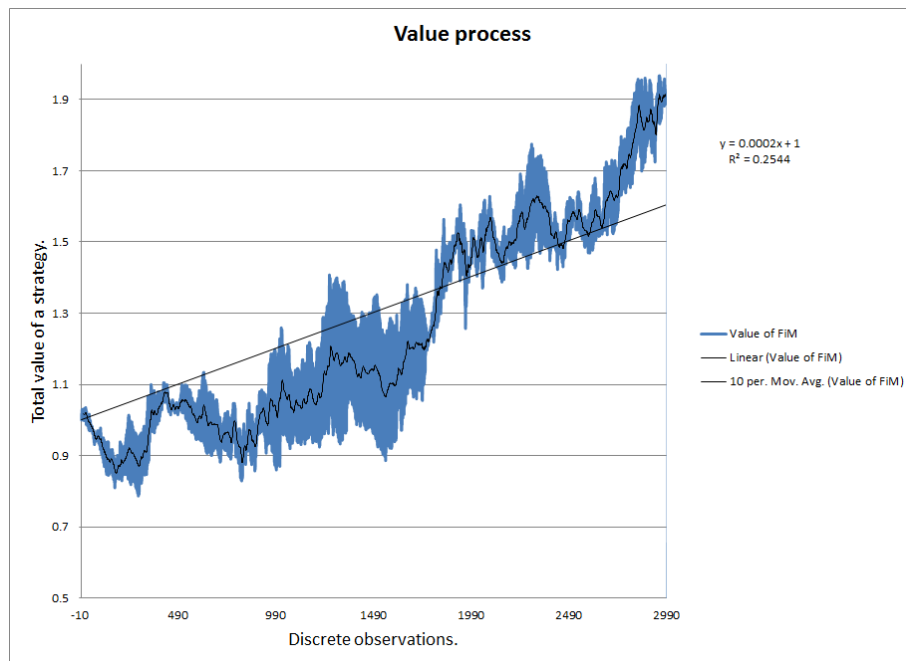


Chart 1

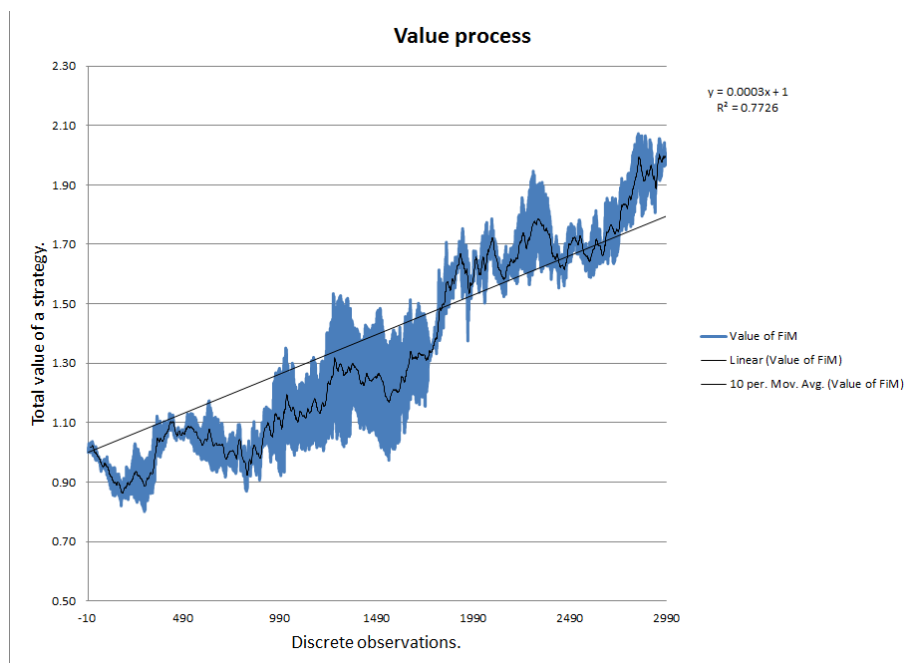


Chart 2

A.5.1.3 Value processes of a fixed-mix portfolio strategy of a risky asset of Anglo Gold Ashanti Limited. Chart1: Fixed 50 percent in a fixed-rate asset in up-trending prime rates. Chart2: Fixed 50 percent in a fixed-rate asset in down-trending prime rates.

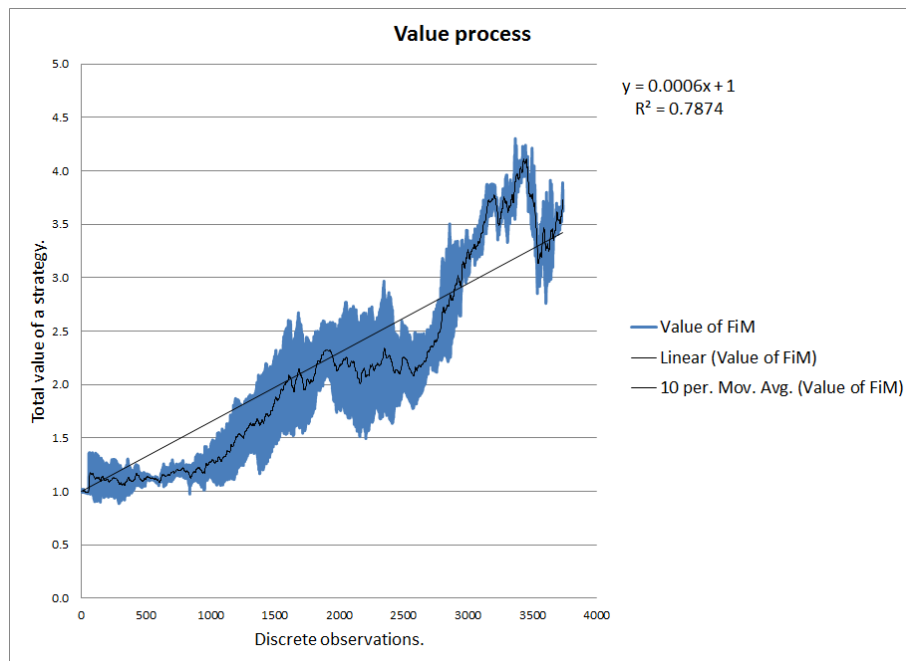


Chart 1

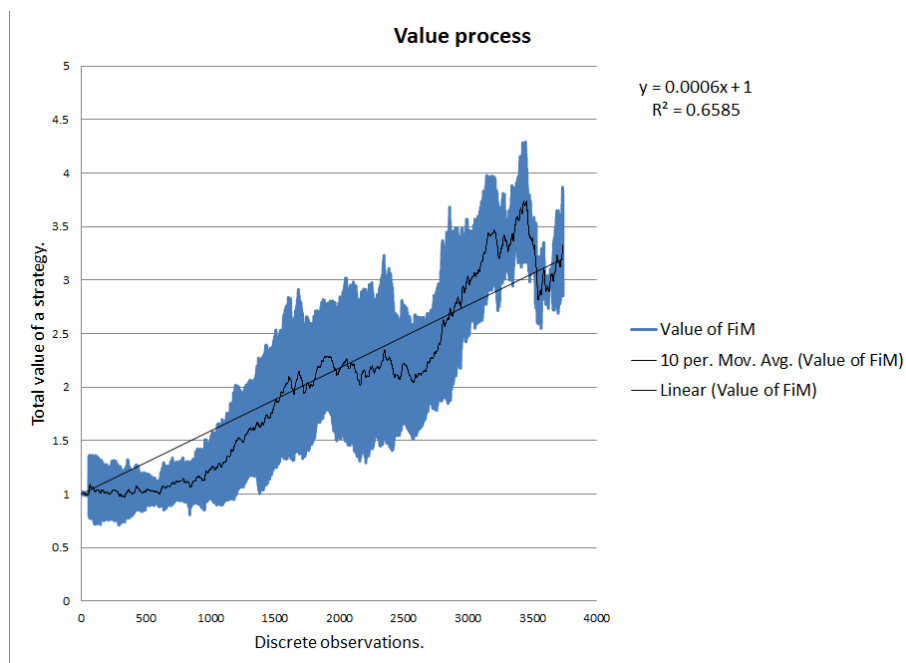


Chart 2

A.5.1.4 Chart: Value processes of a fixed-mix portfolio strategy of a risky asset of Anglo American Platinum Limited. Chart 1: Fixed 50 percent in a fixed-rate asset in up-trending prime rates. Chart2: Fixed 50 percent in a fixed-rate in down-trending prime rates.

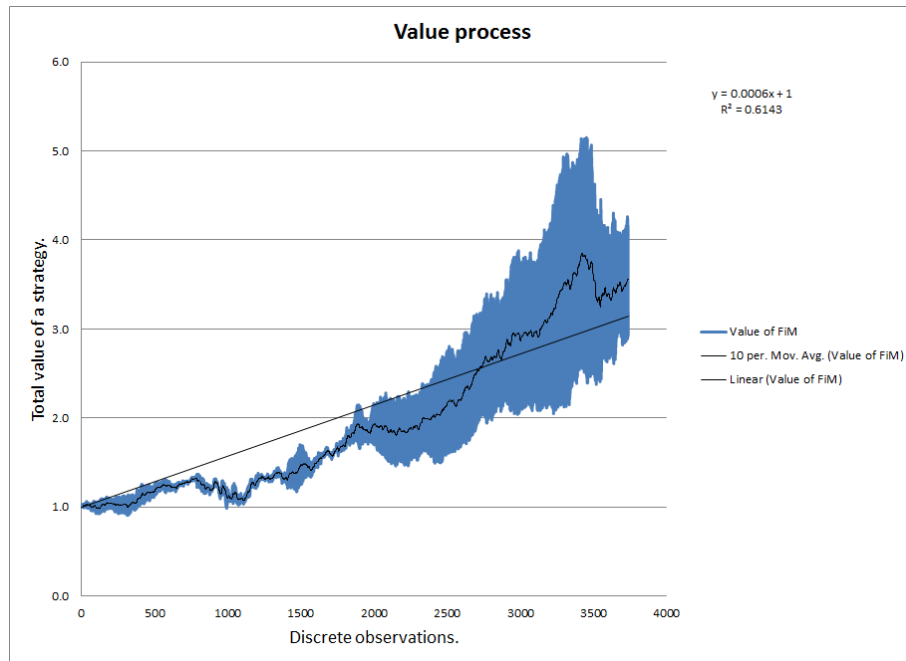


Chart 1

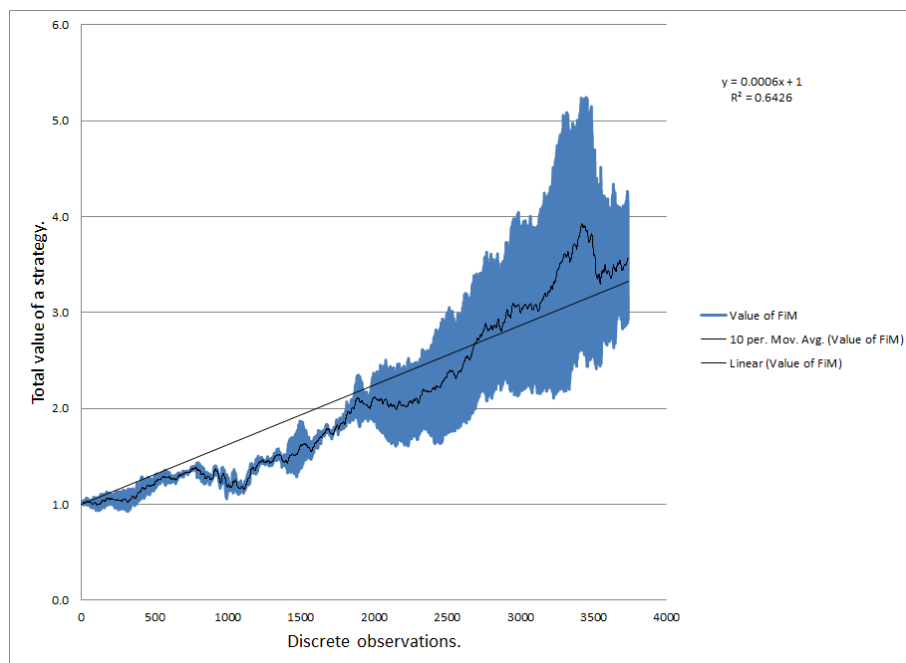


Chart 2

A.5.1.5 Chart: Value processes of a fixed-mix portfolio strategy of a risky asset of Sasol Limited. Chart 1: Fixed 50 percent in a fixed-rate asset in up-trending prime rates. Chart 2: Fixed 50 percent in a fixed-rate asset in down-trending prime rates.

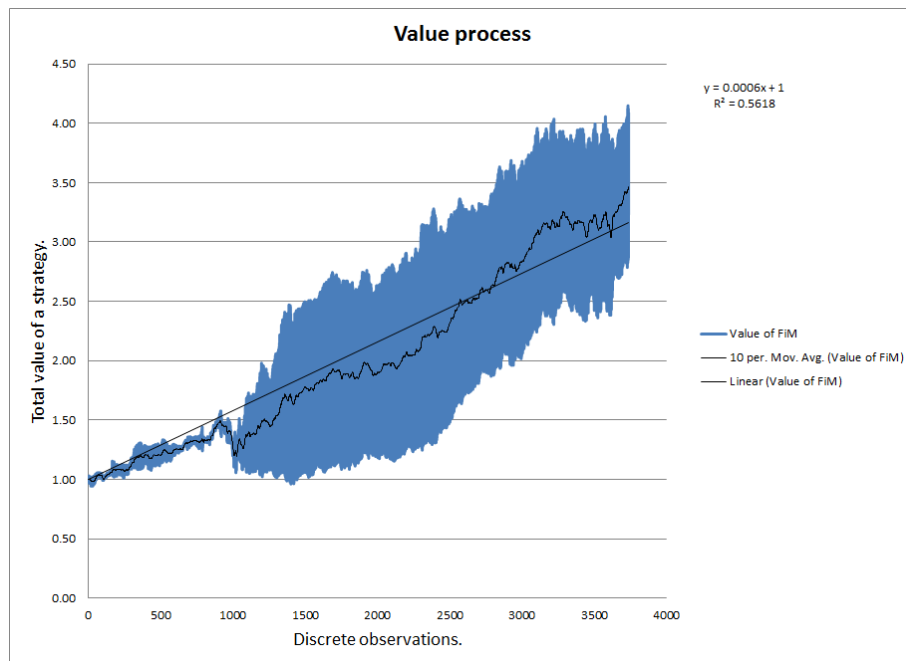


Chart 1

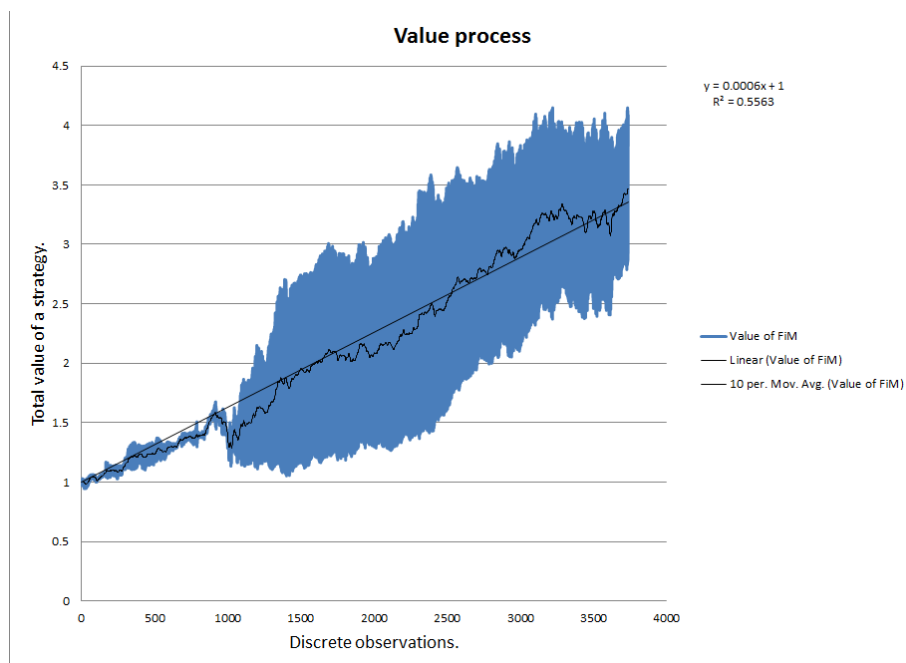


Chart 2

A.5.1.6 Chart: Value processes of a fixed-mix portfolio strategy of a risky asset of Standard Bank. Chart 1: Fixed 50 percent in a fixed-rate asset in up-trending prime rates. Chart 2: Fixed 50 percent in a fixed-rate asset in down-trending prime rates.

A.5.2 Tables

A.5.2.1 Table: Descriptive statistics of discrete total returns of portfolio strategies of a risky asset of ABSA Group Limited in up-trending prime rates.

ABSA Group Limited, up trending prime rate:	N	Range	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
BoH(α), FiM(α)	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic
Total returns of a Fixed-mix portfolio strategy FiM(-2)	3736	11.5638	0.1002	11.6640	1.0985	1.4609	3.0930	11.427
Total returns of a Buy and Hold portfolio strategy BoH(-2)	3736	29.4243	0.8319	30.2562	8.6564	7.0975	1.1900	0.297
Total returns of a Fixed-mix portfolio strategy FiM(-1.5)	3736	8.9030	0.2129	9.1160	1.3523	1.1615	2.5240	8.556
Total returns of a Buy and Hold portfolio strategy BoH(-1.5)	3736	25.3611	0.8620	26.2231	7.7767	6.1877	1.1390	0.142
Total returns of a Fixed-mix portfolio strategy FiM(-1)	3736	6.4112	0.3632	6.7744	1.6934	0.9295	1.2610	2.817
Total returns of a Buy and Hold portfolio strategy BoH(-1)	3736	21.2979	0.8922	22.1900	6.8969	5.2928	1.0690	-0.071
Total returns of a Fixed-mix portfolio strategy FiM(-0.5)	3736	4.2395	0.5508	4.7903	2.0201	0.8560	0.2610	-1.018
Total returns of a Buy and Hold portfolio strategy BoH(-0.5)	3736	17.2347	0.9223	18.1569	6.0172	4.4219	0.9760	-0.361
Total returns of a Fixed-mix portfolio strategy FiM(0)	3736	3.4938	0.7798	4.2736	2.1865	0.8651	0.3010	-1.12
Total returns of a Buy and Hold portfolio strategy BoH(0)	3736	13.1714	0.9524	14.1238	5.1374	3.5927	0.8660	-0.714
Total returns of a Fixed-mix portfolio strategy FiM(0.5)	3736	3.1644	0.9452	4.1096	2.0894	0.7859	0.4850	-1.012
Total returns of a Buy and Hold portfolio strategy BoH(0.5)	3736	9.8910	0.9798	10.8708	4.2577	2.8417	0.8060	-0.903
Total returns of a Fixed-mix portfolio strategy FiM(1)	3736	2.0792	1.0000	3.0792	1.7462	0.5744	0.6740	-0.674
Total returns of a Buy and Hold portfolio strategy BoH(1)	3736	8.4764	1.0000	9.4764	3.3779	2.2488	1.0610	0.046
Total returns of a Fixed-mix portfolio strategy FiM(1.5)	3736	1.5513	0.7316	2.2828	1.2841	0.3295	0.7690	0.208
Total returns of a Buy and Hold portfolio strategy BoH(1.5)	3736	9.3396	0.0042	9.3438	2.4988	1.9626	1.7890	2.672
Total returns of a Fixed-mix portfolio strategy FiM(2)	3736	1.2519	0.4037	1.6556	0.8506	0.2188	0.1610	-0.851
Total returns of a Fixed-mix portfolio strategy BoH(2)	3736	12.0257	-2.0086	10.0171	1.6368	2.1166	1.6380	2.678

A.5.2.2 Table: Descriptive statistics of discrete total returns of portfolio strategies of a risky asset of ABSA Group Limited in down-trending prime rates.

ABSA Group Limited, down trending prime rate:	N	Range	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
BoH(α), FiM(α)	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic
Total returns of a Fixed-mix portfolio strategy FiM(-2)	3736	9.1753	0.0685	9.2438	0.9044	1.2222	2.9130	9.584
Total returns of a Buy and Hold portfolio strategy BoH(-2)	3736	28.8434	-0.2483	28.5951	7.1199	6.9427	1.2090	0.384
Total returns of a Fixed-mix portfolio strategy FiM(-1.5)	3736	7.4954	0.1618	7.6573	1.1577	1.0073	2.4500	7.719
Total returns of a Buy and Hold portfolio strategy BoH(-1.5)	3736	24.5196	0.4577	24.9773	6.6222	6.0651	1.1710	0.225
Total returns of a Fixed-mix portfolio strategy FiM(-1)	3736	5.7149	0.3160	6.0309	1.5214	0.8407	1.2140	2.607
Total returns of a Buy and Hold portfolio strategy BoH(-1)	3736	20.4774	0.8820	21.3595	6.1287	5.2064	1.1100	-0.004
Total returns of a Fixed-mix portfolio strategy FiM(-0.5)	3736	4.0058	0.5139	4.5197	1.9139	0.8205	0.2750	-1.001
Total returns of a Buy and Hold portfolio strategy BoH(-0.5)	3736	16.8244	0.9172	17.7416	5.6330	4.3755	1.0140	-0.316
Total returns of a Fixed-mix portfolio strategy FiM(0)	3736	3.4938	0.7798	4.2736	2.1865	0.8651	0.3010	-1.12
Total returns of a Buy and Hold portfolio strategy BoH(0)	3736	13.1714	0.9524	14.1238	5.1374	3.5927	0.8660	-0.714
Total returns of a Fixed-mix portfolio strategy FiM(0.5)	3736	3.1543	0.9555	4.1098	2.2060	0.7998	0.3070	-1.148
Total returns of a Buy and Hold portfolio strategy BoH(0.5)	3736	9.8932	0.9808	10.8739	4.6418	2.8972	0.6650	-1.052
Total returns of a Fixed-mix portfolio strategy FiM(1)	3736	2.0788	1.0000	3.0788	1.9492	0.5898	0.1330	-1.114
Total returns of a Buy and Hold portfolio strategy BoH(1)	3736	8.4732	1.0000	9.4732	4.1462	2.3673	0.5190	-0.817
Total returns of a Fixed-mix portfolio strategy FiM(1.5)	3736	1.4580	0.8716	2.3295	1.5189	0.3590	-0.0620	-1.013
Total returns of a Buy and Hold portfolio strategy BoH(1.5)	3736	9.2464	0.5876	9.8340	3.6505	2.1304	0.6470	0.005
Total returns of a Fixed-mix portfolio strategy FiM(2)	3736	1.6737	0.5100	2.1837	1.0665	0.2859	0.6100	-0.463
Total returns of a Fixed-mix portfolio strategy FiM(2)	3736	11.5031	-0.8324	10.6707	3.1549	2.2796	0.6940	-0.19

A.5.2.3 Table: Descriptive statistics of discrete total returns of portfolio strategies of a risky asset of AFGRI Limited in up-trending prime rates.

AFGRI Limited, Up trending prime rate:	N	Range	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
BoH(α), FiM(α)	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic
Total returns of a Fixed-mix portfolio strategy FiM(-2)	3736	125.1895	-0.9416	124.2480	30.6078	33.4284	0.9850	-0.117
Total returns of a Buy and Hold portfolio strategy BoH(-2)	3736	62.5974	0.6098	63.2072	47.4734	16.6089	-1.3750	0.536
Total returns of a Fixed-mix portfolio strategy FiM(-1.5)	3710	69.6778	-0.3897	69.2882	20.4492	19.8092	0.6730	-0.841
Total returns of a Buy and Hold portfolio strategy BoH(-1.5)	3736	52.3434	0.6761	53.0195	40.1242	13.6577	-1.3510	0.517
Total returns of a Fixed-mix portfolio strategy FiM(-1)	3587	37.1319	0.1001	37.2320	13.8412	11.3671	0.3190	-1.452
Total returns of a Buy and Hold portfolio strategy BoH(-1)	3736	42.6486	0.7424	43.3910	32.7749	10.7185	-1.3150	0.503
Total returns of a Fixed-mix portfolio strategy FiM(-0.5)	3736	18.6564	0.3719	19.0283	8.5333	6.3994	0.1220	-1.802
Total returns of a Buy and Hold portfolio strategy BoH(-0.5)	3736	32.9574	0.8087	33.7661	25.4257	7.8048	-1.2520	0.525
Total returns of a Fixed-mix portfolio strategy FiM(0)	3736	10.1983	0.7988	10.9972	5.3184	3.3399	0.0730	-1.855
Total returns of a Buy and Hold portfolio strategy BoH(0)	3736	23.2741	0.8750	24.1491	18.0764	4.9618	-1.1270	0.78
Total returns of a Fixed-mix portfolio strategy FiM(0.5)	3736	6.0281	0.9002	6.9282	3.1647	1.4643	0.4960	-1.041
Total returns of a Buy and Hold portfolio strategy BoH(0.5)	3736	13.9217	0.9413	14.8630	10.7272	2.4496	-0.8790	2.535
Total returns of a Fixed-mix portfolio strategy FiM(1)	3736	2.0792	1.0000	3.0792	1.7462	0.5744	0.6740	-0.674
Total returns of a Buy and Hold portfolio strategy BoH(1)	3736	8.4764	1.0000	9.4764	3.3779	2.2488	1.0610	0.046
Total returns of a Fixed-mix portfolio strategy FiM(1.5)	3736	4.3742	0.0038	4.3780	0.8416	1.0873	1.4900	1.792
Total returns of a Buy and Hold portfolio strategy BoH(1.5)	3734	18.2755	-8.9317	9.3438	-3.9735	4.6683	1.6780	1.531
Total returns of a Fixed-mix portfolio strategy FiM(2)	3736	9.0099	-2.9139	6.0960	0.2981	1.7317	1.1300	1.509
Total returns of a Fixed-mix portfolio strategy FiM(2)	3734	29.2541	-19.2370	10.0171	-11.3266	7.4940	1.6550	1.312

A.5.2.4 Table: Descriptive statistics of discrete total returns of portfolio strategies of a risky asset of AFGRI Limited in down-trending prime rates.

AFGRI Limited, Down trending prime rate:	N	Range	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
BoH(a), FiM(a)	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic
Total returns of a Fixed-mix portfolio strategy FiM(-2)	3736	110.5764	-0.8379	109.7385	24.4135	27.7111	1.2510	.768
Total returns of a Buy and Hold portfolio strategy BoH(-2)	3736	62.2339	0.5972	62.8312	45.9369	16.1859	-1.3810	.565
Total returns of a Fixed-mix portfolio strategy FiM(-1.5)	3736	63.4627	-0.3565	63.1061	17.0913	16.9981	0.8450	-.315
Total returns of a Buy and Hold portfolio strategy BoH(-1.5)	3736	51.9530	0.6667	52.6196	38.9718	13.3314	-1.3590	.545
Total returns of a Fixed-mix portfolio strategy FiM(-1)	3736	34.9100	0.0644	34.9744	11.8368	10.2643	0.4350	-1.225
Total returns of a Buy and Hold portfolio strategy BoH(-1)	3736	41.6720	0.7361	42.4081	32.0067	10.4919	-1.3240	.528
Total returns of a Fixed-mix portfolio strategy FiM(-0.5)	3736	18.0753	0.3656	18.4409	8.0516	6.0429	0.1290	-1.776
Total returns of a Buy and Hold portfolio strategy BoH(-0.5)	3736	32.3830	0.8056	33.1886	25.0415	7.6840	-1.2610	.545
Total returns of a Fixed-mix portfolio strategy FiM(0)	3736	10.1983	0.7988	10.9972	5.3184	3.3399	0.0730	-1.855
Total returns of a Buy and Hold portfolio strategy BoH(0)	3736	23.2741	0.8750	24.1491	18.0764	4.9618	-1.1270	.780
Total returns of a Fixed-mix portfolio strategy FiM(0.5)	3736	6.0233	0.9045	6.9278	3.3499	1.5369	0.4010	-1.265
Total returns of a Buy and Hold portfolio strategy BoH(0.5)	3736	14.3130	0.9444	15.2574	11.1113	2.6089	-0.8400	2.166
Total returns of a Fixed-mix portfolio strategy FiM(1)	3736	2.0788	1.0000	3.0788	1.9492	0.5898	0.1330	-1.114
Total returns of a Buy and Hold portfolio strategy BoH(1)	3736	8.4732	1.0000	9.4732	4.1462	2.3673	0.5190	-.817
Total returns of a Fixed-mix portfolio strategy FiM(1.5)	3736	4.4645	0.0042	4.4687	0.9793	1.1991	1.1490	.553
Total returns of a Buy and Hold portfolio strategy BoH(1.5)	3736	18.4868	-8.6528	9.8340	-2.8190	4.5840	1.4380	1.136
Total returns of a Fixed-mix portfolio strategy FiM(2)	3736	9.5619	-3.0205	6.5414	0.3403	1.9803	0.8480	.429
Total returns of a Fixed-mix portfolio strategy FiM(2)	3736	29.5325	-18.8617	10.6707	-9.7841	7.2824	1.5300	1.175

A.5.2.5 Table: Descriptive statistics of discrete total returns of portfolio strategies of a risky asset of Anglo Gold Ashanti Limited in up-trending prime rates.

Anglo Ashanti Limited, Up trending prime rate:	N	Range	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
BoH(α), FiM(α)	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic
Total returns of a Fixed-mix portfolio strategy FiM(-2)	2987	1.1756	0.0086	1.1842	0.1444	0.1888	2.3940	6.831
Total returns of a Buy and Hold portfolio strategy BoH(-2)	2987	8.1962	-6.8771	1.3191	-2.2668	1.7023	-0.5680	-0.28
Total returns of a Fixed-mix portfolio strategy FiM(-1.5)	2987	1.1251	0.0301	1.1551	0.2254	0.1976	1.7470	3.301
Total returns of a Buy and Hold portfolio strategy BoH(-1.5)	2987	6.1374	-4.8712	1.2661	-1.4872	1.2421	-0.5050	-0.283
Total returns of a Fixed-mix portfolio strategy FiM(-1)	2987	1.0394	0.0863	1.1257	0.3690	0.1904	1.1250	1.075
Total returns of a Buy and Hold portfolio strategy BoH(-1)	2987	4.0786	-2.8654	1.2132	-0.7077	0.7896	-0.3530	-0.295
Total returns of a Fixed-mix portfolio strategy FiM(-0.5)	2987	0.8804	0.2154	1.0958	0.6008	0.1702	0.1490	-0.409
Total returns of a Buy and Hold portfolio strategy BoH(-0.5)	2987	2.0198	-0.8595	1.1603	0.0718	0.3740	0.2060	-0.324
Total returns of a Fixed-mix portfolio strategy FiM(0)	2987	0.9378	0.4684	1.4063	0.9178	0.2024	-0.0930	-0.737
Total returns of a Buy and Hold portfolio strategy BoH(0)	2987	1.3450	0.3396	1.6846	0.8514	0.3031	0.5240	-0.825
Total returns of a Fixed-mix portfolio strategy FiM(0.5)	2987	1.1800	0.7870	1.9669	1.2586	0.3019	0.4110	-1.031
Total returns of a Buy and Hold portfolio strategy BoH(0.5)	2987	2.7007	0.7589	3.4596	1.6309	0.6925	0.8050	-0.412
Total returns of a Fixed-mix portfolio strategy FiM(1)	2987	1.3223	1.0000	2.3223	1.5116	0.3552	0.4890	-0.816
Total returns of a Buy and Hold portfolio strategy BoH(1)	2987	4.3923	1.0000	5.3923	2.4104	1.1415	0.8060	-0.335
Total returns of a Fixed-mix portfolio strategy FiM(1.5)	2987	1.3289	0.9719	2.3009	1.5824	0.3061	0.2110	-0.808
Total returns of a Buy and Hold portfolio strategy BoH(1.5)	2987	6.4063	0.9487	7.3550	3.1900	1.6006	0.7890	-0.312
Total returns of a Fixed-mix portfolio strategy FiM(2)	2987	1.6662	0.8961	2.5622	1.4637	0.3042	0.6040	-0.028
Total returns of a Fixed-mix portfolio strategy FiM(2)	2987	8.4258	0.8957	9.3216	3.9695	2.0632	0.7760	-0.303

A.5.2.6 Table: Descriptive statistics of discrete total returns of portfolio strategies of a risky asset of Anglo Gold Ashanti Limited in down-trending prime rates.

Anglo Ashanti Limited, Down trending prime rate:	N	Range	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
BoH(α), FiM(α)	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic
Total returns of a Fixed-mix portfolio strategy FiM(-2)	2987	1.1756	0.0086	1.1842	0.1444	0.1888	2.3940	6.831
Total returns of a Buy and Hold portfolio strategy BoH(-2)	2987	8.1962	-6.8771	1.3191	-2.2668	1.7023	-0.5680	-0.28
Total returns of a Fixed-mix portfolio strategy FiM(-1.5)	2987	1.1251	0.0301	1.1551	0.2254	0.1976	1.7470	3.301
Total returns of a Buy and Hold portfolio strategy BoH(-1.5)	2987	6.1374	-4.8712	1.2661	-1.4872	1.2421	-0.5050	-0.283
Total returns of a Fixed-mix portfolio strategy FiM(-1)	2987	1.1743	0.0062	1.1806	0.1256	0.1831	2.6480	8.392
Total returns of a Buy and Hold portfolio strategy BoH(-1)	2987	10.2830	-8.9666	1.3164	-3.8889	2.5800	-0.1180	-0.961
Total returns of a Fixed-mix portfolio strategy FiM(-0.5)	2987	0.8978	0.1961	1.0939	0.5645	0.1666	0.2670	-0.193
Total returns of a Buy and Hold portfolio strategy BoH(-0.5)	2987	2.6375	-1.4778	1.1596	-0.3337	0.5521	0.2070	-0.443
Total returns of a Fixed-mix portfolio strategy FiM(0)	2987	0.9378	0.4684	1.4063	0.9178	0.2024	-0.0930	-0.737
Total returns of a Buy and Hold portfolio strategy BoH(0)	2987	1.3450	0.3396	1.6846	0.8514	0.3031	0.5240	-0.825
Total returns of a Fixed-mix portfolio strategy FiM(0.5)	2987	1.2715	0.8011	2.0726	1.3482	0.3391	0.2560	-1.29
Total returns of a Buy and Hold portfolio strategy BoH(0.5)	2987	3.1642	0.7862	3.9504	2.0364	0.9117	0.4140	-1.152
Total returns of a Fixed-mix portfolio strategy FiM(1)	2987	1.5229	1.0000	2.5229	1.7374	0.4517	0.0450	-1.244
Total returns of a Buy and Hold portfolio strategy BoH(1)	2987	5.3641	1.0000	6.3641	3.2215	1.5883	0.3340	-1.145
Total returns of a Fixed-mix portfolio strategy FiM(1.5)	2987	1.7812	0.9729	2.7541	1.9472	0.4574	-0.3070	-1.005
Total returns of a Buy and Hold portfolio strategy BoH(1.5)	2987	7.8633	0.9507	8.8140	4.4065	2.2729	0.2970	-1.128
Total returns of a Fixed-mix portfolio strategy FiM(2)	2987	2.7845	0.9447	3.7292	1.9167	0.4701	0.6200	0.58
Total returns of a Fixed-mix portfolio strategy FiM(2)	2987	10.3684	0.8984	11.2668	5.5916	2.9601	0.2770	-1.116

A.5.2.7 Table: Descriptive statistics of discrete total returns of portfolio strategies of a risky asset of Anglo American Platinum Limited in up-trending prime rates.

Anglo American PLatinum, Up trending prime rate:	N	Range	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
BoH(α), FiM(α)	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic
Total returns of a Fixed-mix portfolio strategy FiM(-2)	3736	4.0705	0.0061	4.0766	0.5435	0.5871	2.2540	5.995
Total returns of a Buy and Hold portfolio strategy BoH(-2)	3736	29.3079	-7.7606	21.5473	2.1068	5.2452	1.4720	1.971
Total returns of a Fixed-mix portfolio strategy FiM(-1.5)	3736	4.8124	0.0437	4.8562	0.8693	0.7294	1.8340	4.063
Total returns of a Buy and Hold portfolio strategy BoH(-1.5)	3736	24.2187	-5.0166	19.2021	2.3186	4.5234	1.6060	2.081
Total returns of Fixed-mix portfolio strategy FiM(-1)	3736	4.8903	0.2157	5.1060	1.3186	0.8847	1.1430	0.847
Total returns of a Buy and Hold portfolio strategy BoH(-1)	3736	19.1295	-2.2725	16.8570	2.5305	3.8385	1.7050	2.165
Total returns of a Fixed-mix portfolio strategy FiM(-0.5)	3736	4.4542	0.5150	4.9692	1.8041	1.0366	0.6780	-0.851
Total returns of a Buy and Hold portfolio strategy BoH(-0.5)	3736	14.6559	-0.1441	14.5118	2.7423	3.2140	1.6930	2.05
Total returns of a Fixed-mix portfolio strategy FiM(0)	3736	4.5709	0.7191	5.2900	2.1359	1.0913	0.5290	-0.983
Total returns of a Buy and Hold portfolio strategy BoH(0)	3736	11.7458	0.4208	12.1667	2.9542	2.6924	1.4570	1.332
Total returns of a Fixed-mix portfolio strategy FiM(0.5)	3736	3.4162	0.8854	4.3016	2.1245	0.9325	0.5120	-0.99
Total returns of a Buy and Hold portfolio strategy BoH(0.5)	3736	8.9789	0.8427	9.8215	3.1661	2.3433	1.0540	-0.047
Total returns of a Fixed-mix portfolio strategy FiM(1)	3736	2.0792	1.0000	3.0792	1.7462	0.5744	0.6740	-0.674
Total returns of a Buy and Hold portfolio strategy BoH(1)	3736	8.4764	1.0000	9.4764	3.3779	2.2488	1.0610	0.046
Total returns of a Fixed-mix portfolio strategy FiM(1.5)	3736	1.2748	0.6424	1.9172	1.1840	0.3011	0.1590	-0.823
Total returns of a Buy and Hold portfolio strategy BoH(1.5)	3736	11.1826	0.7103	11.8929	3.5898	2.4384	1.6810	2.727
Total returns of a Fixed-mix portfolio strategy FiM(2)	3736	1.6057	0.1538	1.7594	0.6681	0.4094	0.4470	-1.259
Total returns of a Fixed-mix portfolio strategy FiM(2)	3736	14.0755	0.3975	14.4730	3.8016	2.8563	2.1870	4.881

A.5.2.8 Table: Descriptive statistics of discrete total returns of portfolio strategies of a risky asset of Anglo American Platinum Limited in down-trending prime rates.

Anglo American Platinum, Down trending prime rates:	N	Range	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
BoH(a), FiM(a)	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic
Total returns of a Fixed-mix portfolio strategy FiM(-2)	3736	3.0309	-0.2652	2.7657	0.2785	0.5085	1.9070	5.995
Total returns of a Buy and Hold portfolio strategy BoH(-2)	3736	28.7837	-8.3937	20.3900	0.5702	5.1898	1.5800	1.971
Total returns of a Fixed-mix portfolio strategy FiM(-1.5)	3736	3.7552	-0.1337	3.6215	0.4794	0.6770	1.4190	4.063
Total returns of a Buy and Hold portfolio strategy BoH(-1.5)	3736	23.8256	-5.4914	18.3342	1.1662	4.4680	1.7240	2.081
Total returns of a Fixed-mix portfolio strategy FiM(-1)	3736	4.1512	0.0342	4.1854	0.8102	0.8690	1.0640	0.847
Total returns of a Buy and Hold portfolio strategy BoH(-1)	3736	18.8677	-2.5894	16.2783	1.7622	3.7893	1.8280	2.165
Total returns of a Fixed-mix portfolio strategy FiM(-0.5)	3736	4.1053	0.1738	4.2791	1.2703	1.0294	0.7870	-0.851
Total returns of a Buy and Hold portfolio strategy BoH(-0.5)	3736	14.5385	-0.3160	14.2225	2.3582	3.1813	1.7930	2.05
Total returns of a Fixed-mix portfolio strategy FiM(0)	3736	4.1906	0.4125	4.6031	1.7456	1.0807	0.5660	-0.983
Total returns of a Buy and Hold portfolio strategy BoH(0)	3736	11.7458	0.4208	12.1667	2.9542	2.6924	1.4570	1.332
Total returns of a Fixed-mix portfolio strategy FiM(0.5)	3736	3.5809	0.7097	4.2906	2.0227	0.9277	0.3700	-0.99
Total returns of a Buy and Hold portfolio strategy BoH(0.5)	3736	9.2367	0.8741	10.1108	3.5502	2.3965	0.8380	-0.047
Total returns of a Fixed-mix portfolio strategy FiM(1)	3736	2.0788	1.0000	3.0788	1.9492	0.5898	0.1330	-0.674
Total returns of a Buy and Hold portfolio strategy BoH(1)	3736	8.4732	1.0000	9.4732	4.1462	2.3673	0.5190	0.046
Total returns of a Fixed-mix portfolio strategy FiM(1.5)	3736	2.1052	0.6602	2.7655	1.5827	0.5131	0.3460	-0.823
Total returns of a Buy and Hold portfolio strategy BoH(1.5)	3736	11.2876	0.7380	12.0256	4.7422	2.6137	0.8060	2.727
Total returns of a Fixed-mix portfolio strategy FiM(2)	3736	2.8178	0.2371	3.0549	1.1237	0.8160	0.5440	-1.259
Total returns of a Fixed-mix portfolio strategy FiM(2)	3736	14.4427	0.4345	14.8772	5.3381	3.0700	1.1730	4.881

A.5.2.9 Table: Descriptive statistics of discrete total returns of portfolio strategies of a risky asset of Sasol Limited in up-trending prime rates.

Sasol Limited, Up trending prime rates:	N	Range	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
BoH(a), FiM(a)	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic
Total returns of a Fixed-mix portfolio strategy FiM(-2)	3736	3.9468	0.0183	3.9651	0.6211	0.6610	1.7070	3.602
Total returns of a Buy and Hold portfolio strategy BoH(-2)	3736	33.1416	-1.6158	31.5258	4.7657	5.8386	1.7170	2.923
Total returns of a Fixed-mix portfolio strategy FiM(-1.5)	3736	6.4895	0.0849	6.5745	0.9641	1.0675	2.4400	6.888
Total returns of a Buy and Hold portfolio strategy BoH(-1.5)	3736	28.5661	-1.0570	27.5091	4.5344	5.1650	1.6340	2.529
Total returns of Fixed-mix portfolio strategy FiM(-1)	3736	8.3801	0.2452	8.6254	1.3802	1.4806	2.5430	6.764
Total returns of a Buy and Hold portfolio strategy BoH(-1)	3736	23.9906	-0.4981	23.4924	4.3031	4.5049	1.5230	2.01
Total returns of a Fixed-mix portfolio strategy FiM(-0.5)	3736	8.5559	0.4772	9.0331	1.7587	1.6446	2.3680	5.407
Total returns of a Buy and Hold portfolio strategy BoH(-0.5)	3736	19.4150	0.0607	19.4757	4.0718	3.8654	1.3770	1.336
Total returns of a Fixed-mix portfolio strategy FiM(0)	3736	6.9514	0.7103	7.6617	1.9764	1.4395	2.0260	3.531
Total returns of a Buy and Hold portfolio strategy BoH(0)	3736	14.8395	0.6195	15.4590	3.8405	3.2584	1.2000	0.525
Total returns of a Fixed-mix portfolio strategy FiM(0.5)	3736	4.2438	0.9075	5.1513	1.9657	0.9889	1.3940	1.195
Total returns of a Buy and Hold portfolio strategy BoH(0.5)	3736	10.4836	0.9587	11.4423	3.6092	2.7060	1.0360	-0.187
Total returns of a Fixed-mix portfolio strategy FiM(1)	3736	2.0792	1.0000	3.0792	1.7462	0.5744	0.6740	-0.674
Total returns of a Buy and Hold portfolio strategy BoH(1)	3736	8.4764	1.0000	9.4764	3.3779	2.2488	1.0610	0.046
Total returns of a Fixed-mix portfolio strategy FiM(1.5)	3736	1.6855	0.9512	2.6367	1.4032	0.3985	1.4340	1.016
Total returns of a Buy and Hold portfolio strategy BoH(1.5)	3736	8.9968	0.9563	9.9531	3.1466	1.9548	1.5430	2.387
Total returns of a Fixed-mix portfolio strategy FiM(2)	3736	1.6468	0.3638	2.0106	1.0391	0.3677	0.1660	-0.673
Total returns of a Fixed-mix portfolio strategy FiM(2)	3736	11.2441	-0.6078	10.6363	2.9153	1.9013	2.0880	4.924

A.5.2.10 Table: Descriptive statistics of discrete total returns of portfolio strategies of a risky asset of Sasol Limited in down-trending prime rates.

Sasol Limited, Down trending prime rates:	N	Range	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
BoH(α), FiM(α)	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic
Total returns of a Fixed-mix portfolio strategy FiM(-2)	3736	3.5723	0.0173	3.5896	0.5248	0.5973	1.8630	4.273
Total returns of a Buy and Hold portfolio strategy BoH(-2)	3736	33.2151	-2.8603	30.3549	3.2291	5.8181	1.8050	3.364
Total returns of a Fixed-mix portfolio strategy FiM(-1.5)	3736	6.0352	0.0702	6.1053	0.8411	0.9837	2.6160	7.945
Total returns of a Buy and Hold portfolio strategy BoH(-1.5)	3736	28.6212	-1.9903	26.6309	3.3820	5.1372	1.7330	2.93
Total returns of a Fixed-mix portfolio strategy FiM(-1)	3736	8.0068	0.2059	8.2127	1.2546	1.4008	2.6770	7.564
Total returns of a Buy and Hold portfolio strategy BoH(-1)	3677	24.0273	-1.1204	22.9069	3.5924	4.4882	1.6040	2.27
Total returns of a Fixed-mix portfolio strategy FiM(-0.5)	3736	8.4183	0.4429	8.8612	1.6738	1.6007	2.4370	5.783
Total returns of a Buy and Hold portfolio strategy BoH(-0.5)	3706	19.4334	-0.2504	19.1830	3.7180	3.8444	1.4480	1.511
Total returns of a Fixed-mix portfolio strategy FiM(0)	3736	6.9514	0.7103	7.6617	1.9764	1.4395	2.0260	3.531
Total returns of a Buy and Hold portfolio strategy BoH(0)	3745	17.4590	-2.0000	15.4590	3.8312	3.2605	1.1980	0.529
Total returns of a Fixed-mix portfolio strategy FiM(0.5)	3736	4.3176	0.9247	5.2422	2.0728	1.0075	1.2790	0.92
Total returns of a Buy and Hold portfolio strategy BoH(0.5)	3745	13.7350	-2.0000	11.7350	3.9837	2.7540	0.8520	-0.455
Total returns of a Fixed-mix portfolio strategy FiM(1)	3736	2.0788	1.0000	3.0788	1.9492	0.5898	0.1330	-1.114
Total returns of a Buy and Hold portfolio strategy BoH(1)	3745	11.4732	-2.0000	9.4732	4.1362	2.3740	0.5090	-0.802
Total returns of a Fixed-mix portfolio strategy FiM(1.5)	3736	1.8044	0.9591	2.7635	1.6632	0.4493	0.7120	-0.529
Total returns of a Buy and Hold portfolio strategy BoH(1.5)	3745	12.0941	-2.0000	10.0941	4.2887	2.1877	0.3800	-0.261
Total returns of a Fixed-mix portfolio strategy FiM(2)	3736	2.0774	0.4004	2.4778	1.3080	0.4689	0.0850	-0.718
Total returns of a Fixed-mix portfolio strategy BoH(2)	3745	12.8673	-2.0000	10.8673	4.4411	2.2436	0.4170	-0.011

A.5.2.11 Table: Descriptive statistics of discrete total returns of portfolio strategies of a risky asset of Standard Bank in up-trending prime rates.

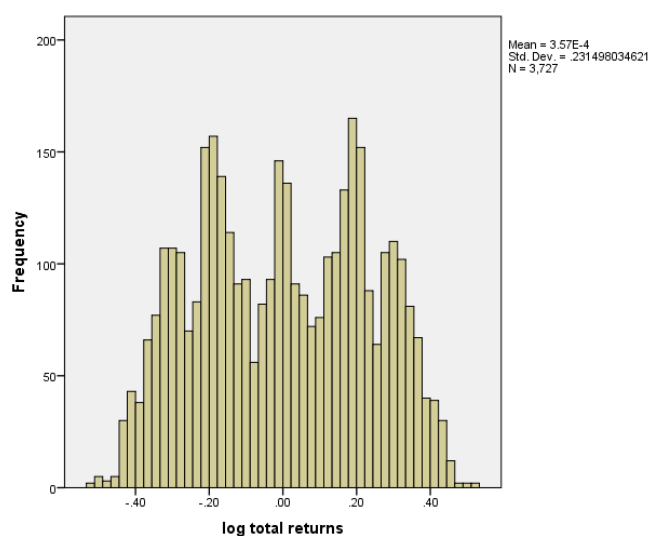
Standard Bank, Up trending prime rates:	N	Range	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
BoH(a), FiM(a)	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic
Total returns of a Fixed-mix portfolio strategy FiM(-2)	3736	8.9063	0.0173	8.9236	1.5982	1.8434	1.4820	1.77
Total returns of a Buy and Hold portfolio strategy BoH(-2)	3736	20.6253	-0.5928	20.0324	5.4309	4.4686	1.2400	0.509
Total returns of a Fixed-mix portfolio strategy FiM(-1.5)	3736	8.7345	0.0548	8.7893	1.9572	2.0869	1.1300	0.241
Total returns of a Buy and Hold portfolio strategy BoH(-1.5)	3736	17.8120	-0.1117	17.7003	5.0887	3.9865	1.1680	0.305
Total returns of a Fixed-mix portfolio strategy FiM(-1)	3736	7.7533	0.1473	7.9007	2.2137	2.1236	0.8660	-0.726
Total returns of a Buy and Hold portfolio strategy BoH(-1)	3736	15.1231	0.2450	15.3681	4.7466	3.5288	1.0630	0.01
Total returns of a Fixed-mix portfolio strategy FiM(-0.5)	3736	6.2130	0.3268	6.5398	2.3111	1.9052	0.7200	-1.155
Total returns of a Buy and Hold portfolio strategy BoH(-0.5)	3736	12.4342	0.6016	13.0359	4.4044	3.1063	0.9310	-0.38
Total returns of a Fixed-mix portfolio strategy FiM(0)	3736	4.8203	0.5928	5.4131	2.2386	1.4657	0.6650	-1.191
Total returns of a Buy and Hold portfolio strategy BoH(0)	3736	9.9852	0.8935	10.8787	4.0622	2.7353	0.8070	-0.778
Total returns of a Fixed-mix portfolio strategy FiM(0.5)	3736	3.2041	0.9440	4.1481	2.0312	0.9442	0.6290	-0.951
Total returns of a Buy and Hold portfolio strategy BoH(0.5)	3736	8.4343	0.9533	9.3875	3.7201	2.4395	0.7970	-0.843
Total returns of a Fixed-mix portfolio strategy FiM(1)	3736	2.0792	1.0000	3.0792	1.7462	0.5744	0.6740	-0.674
Total returns of a Buy and Hold portfolio strategy BoH(1)	3736	8.4764	1.0000	9.4764	3.3779	2.2488	1.0610	0.046
Total returns of a Fixed-mix portfolio strategy FiM(1.5)	3736	2.1832	0.7157	2.8988	1.4371	0.5339	0.6250	-0.865
Total returns of a Buy and Hold portfolio strategy BoH(1.5)	3736	9.3213	0.8699	10.1912	3.0357	2.1907	1.6160	2.077
Total returns of a Fixed-mix portfolio strategy FiM(2)	3736	2.1783	0.3246	2.5029	1.1411	0.6086	0.4440	-1.085
Total returns of a Buy and Hold portfolio strategy BoH(2)	3736	11.5873	0.1444	11.7317	2.6936	2.2754	2.1520	4.138

A.5.2.12 Table: Descriptive statistics of discrete total returns of portfolio strategies of a risky asset of Standard Bank in down-trending prime rates.

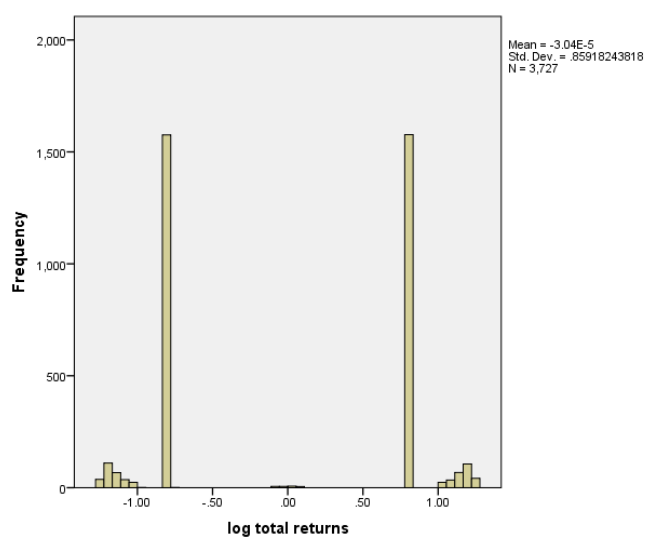
Standard Bank, Down trending prime rates:	N	Range	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
BoH(α), FiM(α)	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic
Total returns of a Fixed-mix portfolio strategy FiM(-2)	2629	6.1943	0.0132	6.2074	1.7221	1.2394	1.0610	0.842
Total returns of a Buy and Hold portfolio strategy BoH(-2)	3736	19.9270	-1.5642	18.3629	3.8944	4.2747	1.3500	0.911
Total returns of a Fixed-mix portfolio strategy FiM(-1.5)	3191	6.6284	0.0431	6.6715	1.8716	1.6008	0.7990	-0.411
Total returns of a Buy and Hold portfolio strategy BoH(-1.5)	3736	17.3816	-0.9336	16.4481	3.9364	3.8335	1.2870	0.63
Total returns of a Fixed-mix portfolio strategy FiM(-1)	3736	6.4530	0.1223	6.5753	1.9517	1.8067	0.7800	-0.891
Total returns of a Buy and Hold portfolio strategy BoH(-1)	3736	14.8362	-0.3029	14.5333	3.9783	3.4212	1.1800	0.237
Total returns of a Fixed-mix portfolio strategy FiM(-0.5)	3736	5.6877	0.2981	5.9858	2.1766	1.7711	0.7000	-1.17
Total returns of a Buy and Hold portfolio strategy BoH(-0.5)	3736	12.2908	0.3277	12.6185	4.0203	3.0496	1.0180	-0.263
Total returns of a Fixed-mix portfolio strategy FiM(0)	3736	4.8203	0.5928	5.4131	2.2386	1.4657	0.6650	-1.191
Total returns of a Buy and Hold portfolio strategy BoH(0)	3736	9.9852	0.8935	10.8787	4.0622	2.7353	0.8070	-0.778
Total returns of a Fixed-mix portfolio strategy FiM(0.5)	3736	3.2042	0.9454	4.1496	2.1466	0.9836	0.5480	-1.133
Total returns of a Buy and Hold portfolio strategy BoH(0.5)	3736	8.4318	0.9585	9.3903	4.1042	2.5000	0.6050	-1.059
Total returns of a Fixed-mix portfolio strategy FiM(1)	3736	2.0788	1.0000	3.0788	1.9492	0.5898	0.1330	-1.114
Total returns of a Buy and Hold portfolio strategy BoH(1)	3736	8.4732	1.0000	9.4732	4.1462	2.3673	0.5190	-0.817
Total returns of a Fixed-mix portfolio strategy FiM(1.5)	3736	2.0679	0.9394	3.0072	1.7032	0.6353	0.5550	-1.274
Total returns of a Buy and Hold portfolio strategy BoH(1.5)	3736	9.6906	1.0000	10.6906	4.1881	2.3548	0.6120	-0.119
Total returns of a Fixed-mix portfolio strategy FiM(2)	3736	3.1601	0.4689	3.6290	1.4478	0.8613	0.8500	-0.542
Total returns of a Buy and Hold portfolio strategy BoH(2)	3736	11.5195	0.8781	12.3976	4.2301	2.4641	0.8030	0.531

A.6. Appendix: Frequency distribution of log total returns

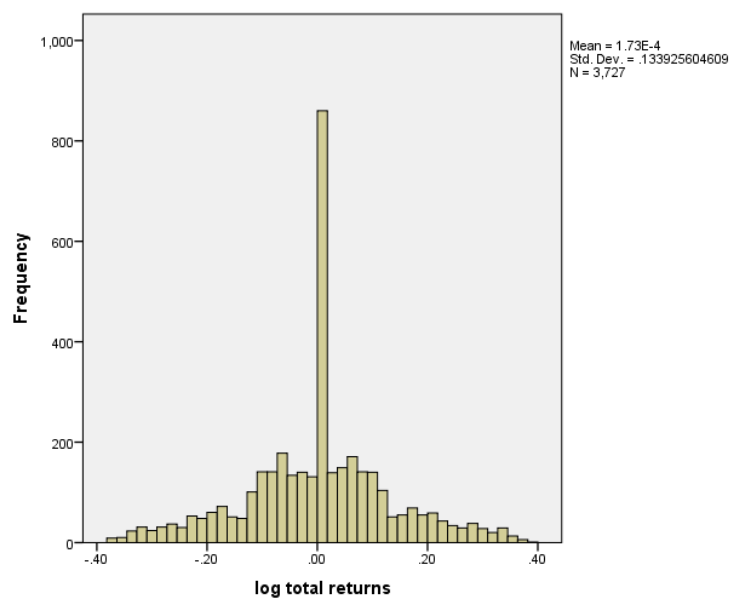
A.6.1 Charts



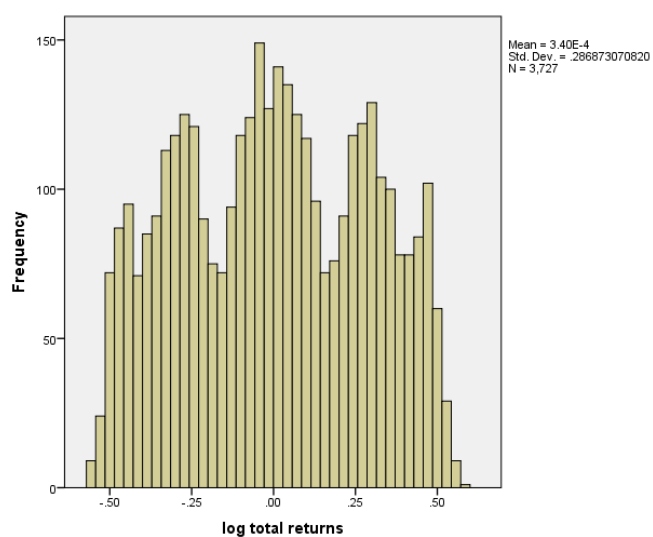
A.6.1.1 Frequency distribution of discrete total returns of a fixed-mix portfolio strategy of a risky asset of ABSA Group Limited. Fixed 50 percent in a fixed-rate asset.



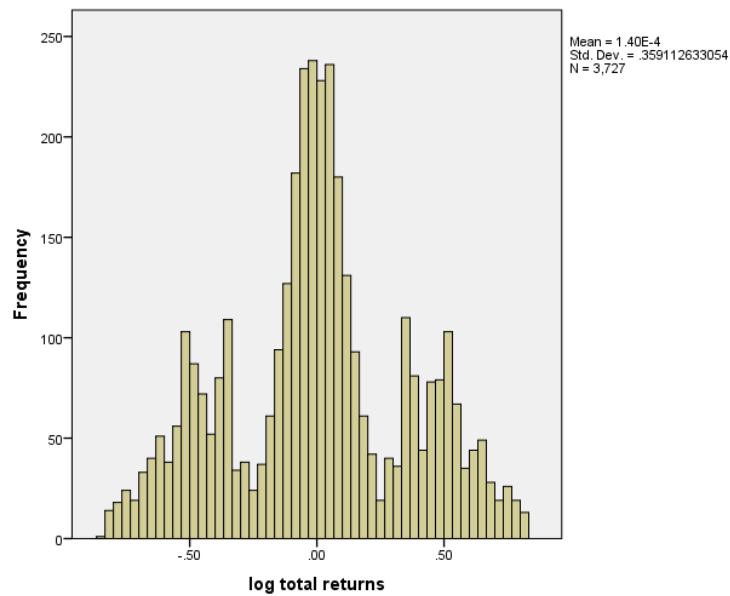
A.6.1.2 Frequency distribution of discrete total returns of a fixed-mix portfolio strategy of a risky asset of AFGRI Limited. Fixed 50 percent in a fixed-rate asset.



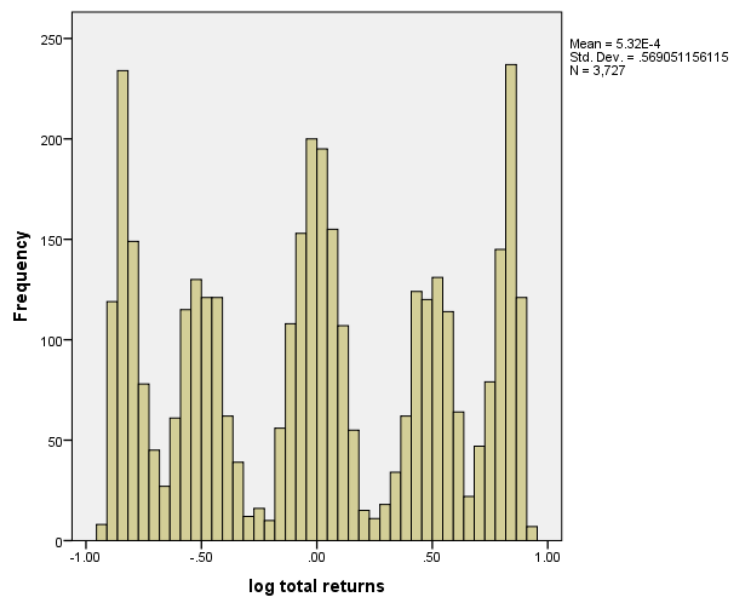
A.6.1.3 Frequency distribution of discrete total returns of a fixed-mix portfolio strategy of a risky asset of Anglo Gold Ashanti Limited. Fixed 50 percent in a fixed-rate asset.



A.6.1.4 Frequency distribution of discrete total returns of a fixed-mix portfolio strategy of a risky asset of Anglo American Platinum Limited. Fixed 50 percent in a fixed-rate asset.



A.6.1.5 Frequency distribution of discrete total returns of a fixed-mix portfolio strategy of a risky asset of Sasol Limited. Fixed 50 percent in a fixed-rate asset.



A.6.1.6 Frequency distribution of discrete total returns of a fixed-mix portfolio strategy of a risky asset of Standard Bank Limited. Fixed 50 percent in a fixed-rate asset.

A.7 Price formulas of lookback options

The formula of a price of a lookback call option, put option and straddle option at time t , maturing at time T , is given below following from (Darius *et al.*, 2002).

T – Time to maturity.

t – Current time.

r – Continuously compounded risk-free rate.

S_t – Current stock price at time t .

σ^2 – Variance of the stock price returns, assumed constant.

$N(x)$ – Cumulative normal distribution function.

A_t – minimum stock price over each contract duration of $\tau = \tau_{T,1}, \tau_{T,2}, \dots, \tau_{T,n}$, at time t .

B_t – maximum stock price over each contract duration of $\tau = \tau_{T,1}, \tau_{T,2}, \dots, \tau_{T,n}$, at time t .

$x \in \{a_i, b_i\}$ for $i = 1, 2, 3$, is the input value in the cumulative normal distribution function.

$\Gamma_\tau = \sum_{i=1}^n \tau_{T,i}$, is the term of the investment.

A.7.1 The formula of a price of a lookback call option.

$$c_t^T = S_t(N(a_1) - \frac{\sigma^2}{2r}N(-a_1)) - A_t(e^{-r(T-t)}N(a_2) - \frac{\sigma^2}{2r}\left(\frac{A_t}{S_t}\right)^{\frac{r-\frac{\sigma^2}{2}}{2\sigma^2}}N(-a_3))$$

(A.7.1.1)

$$a_i = \log \frac{S_t}{A_t} + \frac{I_i\left(r + I_i \frac{\sigma^2}{2}\right)(T-t)}{\sigma(T-t)^{1/2}}, \text{ where } I_i = \begin{cases} -1, & i = 3 \\ 1, & i = 1 \end{cases}. \quad (\text{A.7.1.2})$$

$$a_2 = a_1 + \sigma(T-t)^{1/2}.$$

$$A_t = \min_{0 \leq t \leq T} S_t. \quad (\text{A.7.1.3})$$

The probability factors of a payoff structure for the moneyness condition of the call option are considered independently for the initialised option price. The probability factor of a realised stock to call in the money or out of the money is,

$$P(G(x = a), t) = N(a_1) - \frac{\sigma^2}{2r} N(-a_1). \quad (\text{A.7.1.4})$$

The probability factor of a realised strike to call in the money is,

$$P(Z(x = a), t) = e^{-r(T-t)} N(a_2) - \frac{\sigma^2}{2r} \left(\frac{A_t}{S_t} \right)^{\frac{r - \frac{\sigma^2}{2}}{2\sigma^2}} N(-a_3). \quad (\text{A.7.1.5})$$

The price of an option at time t , maturing at time T , is

$$c_t^T = S_t P(G(a), t) - A_t P(Z(a), t). \quad (\text{A.7.1.6})$$

The initialised prices of a lookback call option at time t with the independent price probability factors, $P(G(a), t)$ and $P(Z(a), t)$, are determined by

$$2c_t^T = S_t P(G(a), t) - A_t P(Z(a), t) = S_t. \quad (\text{A.7.1.7})$$

A.7.2 The formula of a price of a lookback put option.

$$p_t^T = B_t \left(e^{-r(T-t)} N(b_1) - \frac{\sigma^2}{2r} \left(\frac{B_t}{S_t} \right)^{\frac{2r}{\sigma^2} - 1} N(-b_3) \right) - S_t \left(N(b_2) - \frac{\sigma^2}{2r} N(-b_2) \right)$$

(A.7.2.1)

$$b_i = \log \frac{S_t}{B_t} - \frac{I_i \left(r - \frac{\sigma^2}{2} \right) (T-t)}{\sigma(T-t)^{1/2}}, \text{ where } I_i = \begin{cases} -1, & i = 3 \\ 1, & i = 1 \end{cases}. \quad (\text{A.7.2.2})$$

$$b_2 = b_1 - \sigma(T-t)^{1/2}.$$

$$B_t = \max_{0 \leq t \leq T} S_t. \quad (\text{A.7.2.3})$$

The independent probability factors of a payoff structure for the moneyness condition of the put option. The probability factor of a realised stock to call in the money or out of the money is,

$$P(G(x = b), t) = N(b_2) - \frac{\sigma^2}{2r} N(-b_2). \quad (\text{A.7.2.4})$$

The probability factor of a realised strike to put in the money is,

$$P(Z(x = b), t) = e^{-r(T-t)} N(b_1) - \frac{\sigma^2}{2r} \left(\frac{B_t}{S_t} \right)^{\frac{2r}{\sigma^2} - 1} N(-b_3). \quad (\text{A.7.2.5})$$

The prices of a lookback put option at time t , maturing at time T , are determined by

$$p_t^T = B_t P(Z(b), t) - S_t P(G(b), t). \quad (\text{A.7.2.6})$$

The initialised prices of a call option at time t , with the independent price probability factors, $P(Z(b), t)$ and $P(G(b), t)$, are

$$2p_t^T = B_t P(Z(b), t) - S_t P(G(b), t) = S_t. \quad (\text{A.7.2.7})$$

A.7.3 The price of a lookback straddle derivative option

From the above settings (A.7.1 and A.7.2) the price of the derivative option is equal to the price of a stock at the time of writing the contract, (from equation A.7.1.1 and A.7.2.7)

$$s_t^T = p_t^T + c_t^T = \frac{S_t}{2} + \frac{S_t}{2} = S_t. \quad (\text{A.7.3.1})$$